From Ordinal Ranking to Binary Classification

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Which Age-Group?

2

infant (1)

child (2)

teen (3)

adult (4)

rank: a finite ordered set of labels $\mathcal{Y} = \{1, 2, \cdots, K\}$

Properties of Ordinal Ranking (1/2)

ranks represent **order** information

infant (1)

$<$

child (2)

$<$

teen (3)

$<$

adult (4)

general classification cannot properly use order information

How Much Did You Like These Movies?

http://www.netflix.com

rank: natural representation of human preferences

Properties of Ordinal Ranking (2/2)

ranks do **not** carry numerical information

- ★★★★★ not 2.5 times “better” than ★★☆☆☆
- actual metric may be hidden

infant
(ages 1–3)

child
(ages 4–12)

teen
(ages 13–19)

adult
(ages 20–)

general regression deteriorates without correct numerical information

Ordinal Ranking

Setup

input space $\mathcal{X}$; rank space $\mathcal{Y}$ (a finite ordered set)

- `age-group`: $\mathcal{X}$ = encoding(human pictures), $\mathcal{Y} = \{1, \cdots, 4\}$
- `netflix`: $\mathcal{X}$ = encoding(movies), $\mathcal{Y} = \{1, \cdots, 5\}$

Given

N examples (input x_n, rank y_n) $\in \mathcal{X} \times \mathcal{Y}$

Goal

a ranker (decision function) $r(x)$ that closely predicts the ranks y associated with some **unseen** inputs x

How to say closely predict?

Formalizing (Non-)Closeness: Cost

- ranks carry no numerical information: how to say “close”?
- artificially quantify the **cost** of being wrong

  > e.g. loss of customer loyalty when the system says ★★★★★ but you feel ★★☆☆☆

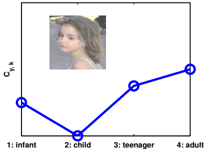
- cost vector $\mathbf{c}$ of example $(x, y, \mathbf{c})$:
  $\mathbf{c}[k]$ = cost when predicting (x, y) as rank k
  e.g. for (`Sweet Home Alabama` , ★★☆☆☆), a proper cost is $\mathbf{c} = (1, 0, 2, 10, 15)$

closely predict: small cost during testing

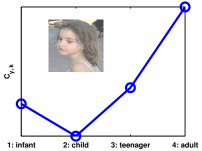
Ordinal Cost Vectors

For an ordinal example $(x, y, \mathbf{c})$, the cost vector $\mathbf{c}$ should

- be consistent with rank y: $\mathbf{c}[y] = \min_k \mathbf{c}[k] \, (= 0)$
- respect order information: V-shaped (**ordinal**) or even convex (**strongly ordinal**)



V-shaped: pay more when predicting further away



convex: pay **increasingly** more when further away

$\mathbf{c}[k] = [\![y \neq k]\!]$	$\mathbf{c}[k] = \lvert y - k \rvert$	$\mathbf{c}[k] = (y - k)^2$
classification: ordinal	absolute: strongly ordinal	squared: strongly ordinal
$(1, 0, 1, 1, 1)$	$(1, 0, 1, 2, 3)$	$(1, 0, 1, 4, 9)$

Our Contributions

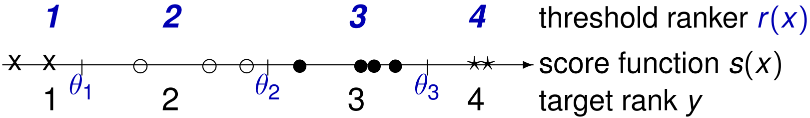
a theoretical and algorithmic foundation of ordinal ranking, which reduces ordinal ranking to binary classifaction, and ...

- provides a methodology for designing new ordinal ranking algorithms with **any** ordinal cost **effortlessly**
- takes many existing ordinal ranking algorithms as **special cases**
- introduces **new theoretical guarantee** on the generalization performance of ordinal rankers
- leads to **superior experimental results**

If I have seen further it is by standing on the shoulders of Giants—I. Newton

Threshold Ranker

- if getting an ideal score $s(x)$ of a movie x, how to construct the discrete $r(x)$ from an analog $s(x)$?



quantize $s(x)$ by **ordered** (non-uniform) thresholds θ_k

- commonly used in previous work:
  - threshold perceptrons (PRank, Crammer and Singer, 2002)
  - threshold hyperplanes (SVOR, Chu and Keerthi, 2005)
  - threshold ensembles (ORBoost, Lin and Li, 2006)

threshold ranker: $r(x) = \min\{k\colon s(x) < \theta_k\}$

Key Idea: Associated Binary Queries

getting the rank using a threshold ranker

1. is $s(x) > \theta_1$? Yes
2. is $s(x) > \theta_2$? No
3. is $s(x) > \theta_3$? No
4. is $s(x) > \theta_4$? No

generally, how do we query the rank of a movie x?

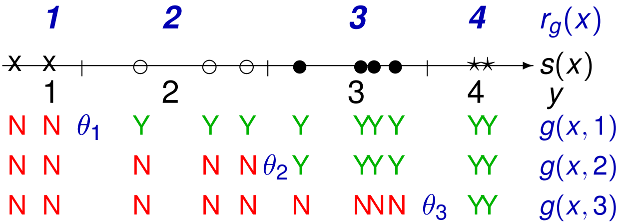
1. is movie x better than rank 1? Yes
2. is movie x better than rank 2? No
3. is movie x better than rank 3? No
4. is movie x better than rank 4? No

associated binary queries:

is movie x better than rank k?

More on Associated Binary Queries

say, the machine uses $g(x, k)$ to answer the query
"*is movie x better than rank k?*"
e.g. for threshold ranker: $g(x, k) = \text{sign}(s(x) - \theta_k)$



associated binary examples:

$$\left(\underbrace{(x, k)}_{k\text{-th associated binary query}}, \underbrace{(z)_k}_{\text{desired answer}} \right)$$

Computing Ranks from Associated Binary Queries

when $g(x, k)$ answers "*is movie x better than rank k?*"

Consider $\big(g(x,1), g(x,2), \cdots, g(x,K-1)\big)$,

- consistent predictions: (Y, Y, N, N, N, N, N)
- extracting the rank from consistent predictions:
  - minimum index searching: $r_g(x) = \min\{k\colon g(x,k) = \text{N}\}$
  - counting: $r_g(x) = 1 + \sum_k [\![g(x,k) = \text{Y}]\!]$
- two approaches equivalent for consistent predictions
- mistaken/inconsistent predictions? e.g. (Y, N, Y, Y, N, N, Y)
  —counting: simpler to analyze and robust to mistake

are all associated examples of the same importance?

Importance of Associated Binary Examples

- given movie x with rank $y = 2$, and $\mathbf{c} = (y - k)^2$

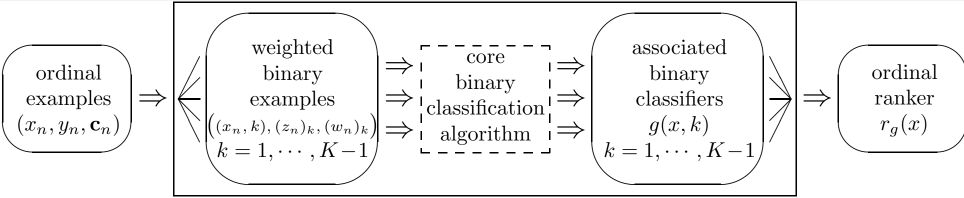
	g_1	g_2	g_3	g_4
is x better than rank 1?	N	Y	Y	Y
is x better than rank 2?	N	N	Y	Y
is x better than rank 3?	N	N	N	Y
is x better than rank 4?	N	N	N	N
$r_g(x)$	1	2	3	4
$\mathbf{c}\left[r_g(x)\right]$	1	0	1	4

- 3 more for answering query 3 wrong; only 1 more for answering query 1 wrong
- $(w)_k \equiv \Big|\mathbf{c}[k+1] - \mathbf{c}[k]\Big|$: the importance of $\big((x, k), (z)_k\big)$

per-example cost bound (Li and Lin, 2007):
for **consistent predictions** or **strongly ordinal costs**

$$\mathbf{c}\left[r_g(x)\right] \leq \sum_{k=1}^{K-1} (w)_k \left[\!\left[(z)_k \neq g(x, k)\right]\!\right]$$

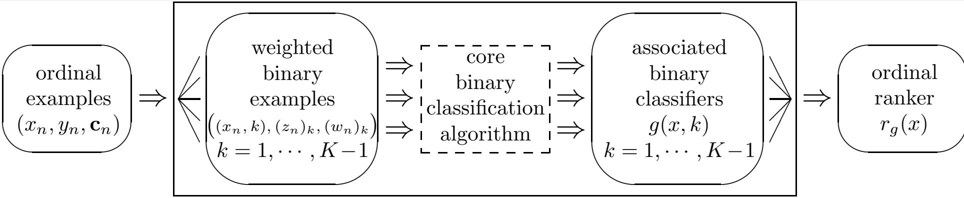
The Reduction Framework (1/2)



1. transform ordinal examples $(x_n, y_n, \mathbf{c}_n)$ to weighted binary examples $\big((x_n, k), (z_n)_k, (w_n)_k\big)$
2. use your favorite algorithm on the weighted binary examples and get $K-1$ binary classifiers (i.e., one big joint binary classifier) $g(x, k)$
3. for each new input x, predict its rank using $r_g(x) = 1 + \sum_k [\![g(x, k) = \text{Y}]\!]$

the reduction framework:
systematic & easy to implement

The Reduction Framework (2/2)



- **performance guarantee**:
  accurate binary predictions $\Longrightarrow$ correct ranks
- **wide applicability**:
  works with any ordinal **c** & any binary classification algorithm
- **simplicity**:
  mild computation overheads with $O(NK)$ binary examples
- **state-of-the-art**:
  allows new improvements in binary classification to be immediately inherited by ordinal ranking

Theoretical Guarantees of Reduction (1/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**

error transformation theorem (Li and Lin, 2007)

For **consistent predictions** or **strongly ordinal costs**,
if g makes test error Δ in the induced binary problem,
then r_g pays test cost at most Δ in ordinal ranking.

- a one-step extension of the per-example cost bound
- conditions: general and minor
- performance guarantee in the absolute sense

what if no "absolutely good" binary classifier?

Theoretical Guarantees of Reduction (2/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**
2. relatively good binary classifier
   $\Longrightarrow$ relatively good ranker? **YES!**

regret transformation theorem (Lin, 2008)

For **consistent predictions** or **strongly ordinal costs**,
if g is ϵ-close to the optimal binary classifier g_*,
then r_g is ϵ-close to the optimal ranker r_*.

"reduction to binary" sufficient for algorithm design,
but necessary?

Theoretical Guarantees of Reduction (3/3)

1. absolutely good binary classifier
   $\Longrightarrow$ absolutely good ranker? **YES!**
2. relatively good binary classifier
   $\Longrightarrow$ relatively good ranker? **YES!**
3. algorithm producing relatively good binary classifier
   $\Longleftrightarrow$ algorithm producing relatively good ranker? **YES!**

equivalence theorem (Lin, 2008)

For a general family of **ordinal costs**,
a good ordinal ranking algorithm exists
if & only if a good binary classification algorithm exists
for the corresponding learning model.

ordinal ranking is **equivalent to** binary classification

Unifying Existing Algorithms

ordinal ranking = reduction + cost + binary classification

ordinal ranking	cost	binary classification algorithm
PRank (Crammer and Singer, 2002)	absolute	modified perceptron rule
kernel ranking (Rajaram et al., 2003)	classification	modified hard-margin SVM
SVOR-EXP SVOR-IMC (Chu and Keerthi, 2005)	classification absolute	modified soft-margin SVM modified soft-margin SVM
ORBoost-LR ORBoost-All (Lin and Li, 2006)	classification absolute	modified AdaBoost modified AdaBoost

- development and implementation time could have been saved
- algorithmic structure revealed (SVOR, ORBoost)

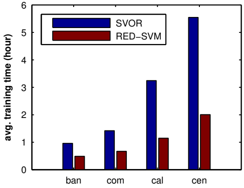
variants of existing algorithms can be designed quickly by tweaking reduction

Designing New Algorithms Effortlessly

ordinal ranking = reduction + cost + binary classification

ordinal ranking	cost	binary classification algorithm
RED-SVM	absolute	standard soft-margin SVM
RED-C4.5	absolute	standard C4.5 decision tree
(Li and Lin, 2007)		

SVOR (modified SVM) v.s. RED-SVM (standard SVM):



advantages of core binary classification algorithm inherited in the new ordinal ranking one

Proving New Generalization Theorems

Ordinal Ranking (Li and Lin, 2007)

For RED-SVM/SVOR, with pr. $> 1 - \delta$,

$$\text{expected test cost of } r \leq \underbrace{\frac{\beta}{N}\sum_{n=1}^{N}\sum_{k=1}^{K-1}[\![\bar{\rho}(r(x_n), y_n, k) \leq \Phi]\!]}_{\substack{\text{ambiguous training}\\ \text{predictions w.r.t.}\\ \text{criteria } \Phi}} + \underbrace{O\left(\text{poly}\left(K, \frac{\log N}{\sqrt{N}}, \frac{1}{\Phi}, \sqrt{\log \frac{1}{\delta}}\right)\right)}_{\substack{\text{deviation that decreases}\\ \text{with stronger criteria or}\\ \text{more examples}}}$$

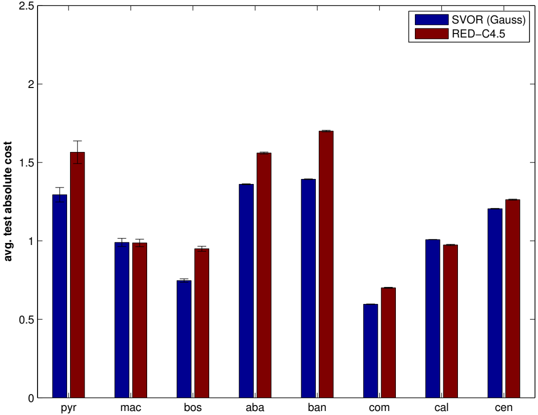
Bi. Cl. (Bartlett and Shawe-Taylor, 1998)

For SVM, with pr. $> 1 - \delta$,

$$\text{expected test err. of } g \leq \underbrace{\frac{1}{N}\sum_{n=1}^{N}[\![\bar{\rho}(g(x_n), y_n) \leq \Phi]\!]}_{\substack{\text{ambiguous training}\\ \text{predictions w.r.t.}\\ \text{criteria } \Phi}} + \underbrace{O\left(\text{poly}\left(\frac{\log N}{\sqrt{N}}, \frac{1}{\Phi}, \sqrt{\log \frac{1}{\delta}}\right)\right)}_{\substack{\text{deviation that decreases}\\ \text{with stronger criteria or}\\ \text{more examples}}}$$

new ordinal ranking theorem
= reduction + any cost + bin. thm. + math derivation

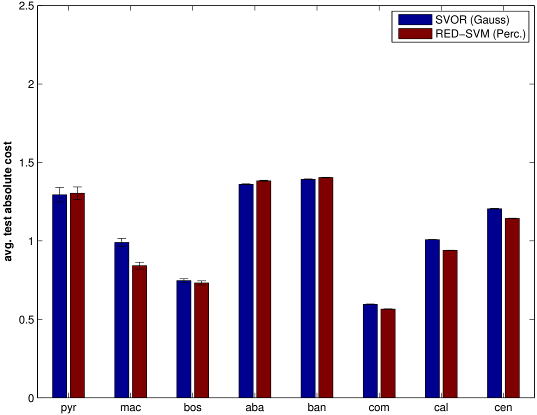
Reduction-C4.5 v.s. SVOR



- C4.5: a (too) simple binary classifier —decision trees
- SVOR: state-of-the-art ordinal ranking algorithm

even simple Reduction-C4.5 sometimes beats SVOR

Reduction-SVM v.s. SVOR



- SVM: one of the most powerful binary classification algorithm
- SVOR: state-of-the-art ordinal ranking algorithm extended from modified SVM

Reduction-SVM without modification often better than SVOR and faster

Conclusion

- reduction framework: simple but useful
  - **establish** equivalence to binary classification
  - **unify** existing algorithms
  - **simplify** design of new algorithms
  - **facilitate** derivation of new theoretical guarantees
- **superior** experimental results:
  better performance and faster training time

reduction keeps ordinal ranking
up-to-date with binary classification