# Cost-sensitive Multiclass Classification Using One-versus-one Comparisons

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# Which Digit Did You Write?

?

one (1) two (2) three (3) four (4)

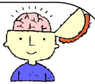
- a **classification** problem
  —grouping “pictures” into different “categories”

**How can machines learn to classify?**

# Supervised Machine Learning

Parent

(picture, category) pairs

Kid's brain  → good decision function

possibilities

Truth $f(x)$ + noise $e(x)$

examples (picture $x_n$, category $y_n$)

learning algorithm  → good decision function $g(x) \approx f(x)$

learning model $\{g_\alpha(x)\}$

challenge:

see only $\{(x_n, y_n)\}$ without knowing $f(x)$ or $e(x)$

$\stackrel{?}{\Longrightarrow}$ **generalize** to unseen $(x, y)$ w.r.t. $f(x)$

# Mis-prediction Costs $(g(x) \approx f(x)?)$

?

- ZIP code recognition:
  1: wrong; 2: right; 3: wrong; 4: wrong
- check value recognition:
  1: one-dollar mistake; 2: no mistake;
  3: one-dollar mistake; 4: **two**-dollar mistake
- evaluation by formation similarity:
  1: not very similar; 2: very similar;
  3: somewhat similar; 4: a **silly** prediction

**different applications evaluate mis-predictions differently**

# ZIP Code Recognition

?

1: wrong; 2: right; 3: wrong; 4: right

- **regular** classification problem: only right or wrong
- wrong cost: 1; right cost: 0
- prediction error of $g$ on some $(x, y)$:

$$\text{classification cost} = [\![ y \neq g(x) ]\!]$$

regular classification: **well-studied**, many good algorithms

# Check Value Recognition

?

1: one-dollar mistake; 2: no mistake;
3: one-dollar mistake; 4: **two**-dollar mistake

- **cost-sensitive** classification problem:
  different costs for different mis-predictions
- e.g. prediction error of $g$ on some $(x, y)$:

$$\text{absolute cost} = |y - g(x)|$$

cost-sensitive classification: **new**, need more research

# Which Age-Group?

?

infant (1)

child (2)

teen (3)

adult (4)

- small mistake—classify a child as a teen;
  big mistake—classify an infant as an adult
- prediction error of $g$ on some $(x, y)$:

$$\mathcal{C}(y, g(x)), \text{ where } \mathcal{C} = \begin{pmatrix} 0 & 1 & 4 & 5 \\ 1 & 0 & 1 & 3 \\ 3 & 1 & 0 & 2 \\ 5 & 4 & 1 & 0 \end{pmatrix}$$

$\mathcal{C}$: **cost matrix**

# Cost Matrix $\mathcal{C}$

**regular classification**

$\mathcal{C}$ = classification cost $\mathcal{C}_c$:

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

**cost-sensitive classification**

$\mathcal{C}$ = anything other than $\mathcal{C}_c$:

$$\begin{pmatrix} 0 & 1 & 4 & 5 \\ 1 & 0 & 1 & 3 \\ 3 & 1 & 0 & 2 \\ 5 & 4 & 1 & 0 \end{pmatrix}$$

regular classification:

**special case** of cost-sensitive classification

# Cost-Sensitive Binary Classification (1/2)

medical profile $x$

?

| medical profile $x_1$ | medical profile $x_2$ |
|---|---|
| H1N1 (1) | NOH1N1 (2) |

- predicting H1N1 as NOH1N1:
  serious consequences to public health
- predicting NOH1N1 as H1N1:
  not good, but less serious
- cost-sensitive $\mathcal{C}$: $\begin{pmatrix} 0 & 1000 \\ 1 & 0 \end{pmatrix}$
- regular $\mathcal{C}_c$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

**how to change the entry from 1 to 1000?**

# Cost-Sensitive Binary Classification (2/2)

**copy each case labeled H1N1** 1000 **times**

**original problem**

evaluate w/ $\begin{pmatrix} 0 & 1000 \\ 1 & 0 \end{pmatrix}$

($x_1$,H1N1)
($x_2$,NOH1N1)
($x_3$,NOH1N1)
($x_4$,NOH1N1)
($x_5$,H1N1)

**equivalent problem**

evaluate w/ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

($x_1$,H1N1), $\cdots$, ($x_1$,H1N1)
($x_2$,NOH1N1)
($x_3$,NOH1N1)
($x_4$,NOH1N1)
($x_5$,H1N1), $\cdots$, ($x_5$,H1N1)

mathematically:

$$\begin{pmatrix} 0 & 1000 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1000 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

# Our Contribution

| | binary | multiclass |
|---|---|---|
| regular | well-studied | well-studied |
| cost-sensitive | known (Zadrozny, 2003) | ongoing (our work, among others) |

*a theoretical and algorithmic study of cost-sensitive classification, which ...*

- introduces a methodology for extending regular classification algorithms to cost-sensitive ones with **any cost**
- provides **strong theoretical support** for the methodology
- leads to some promising algorithms with **superior experimental results**

will describe the methodology and a concrete algorithm

# Key Idea: Cost Transformation

$$\underbrace{\begin{pmatrix} 0 & 1000 \\ 1 & 0 \end{pmatrix}}_{\mathcal{C}} = \underbrace{\begin{pmatrix} 1000 & 0 \\ 0 & 1 \end{pmatrix}}_{\#\text{ of copies}} \cdot \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\mathcal{C}_c}$$

$$\underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 3 & 2 & 3 & 4 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\mathcal{C}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{mixture weights } Q} \cdot \underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\mathcal{C}_c}$$

- **split** the cost-sensitive example:

  $(x, 2)$

  $\Longrightarrow$ a mixture of regular examples $\{(x, 1), (x, 2), (x, 2), (x, 3)\}$

  or a weighted mixture $\{(x, 1, 1), (x, 2, 2), (x, 3, 1)\}$

**why split?**

# Cost Equivalence by Splitting

$$\underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 3 & 2 & 3 & 4 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\mathcal{C}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{mixture weights } Q} \cdot \underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\mathcal{C}_c}$$

- $(x, 2)$
  $\Longrightarrow$ a weighted mixture $\{(x, 1, 1), (x, 2, 2), (x, 3, 1)\}$
- **cost equivalence**: for any classifier $g$,

$$\mathcal{C}(y, g(x)) = \sum_{\ell=1}^{K} Q(y, \ell) \, [\![\ell \neq g(x)]\!]$$

| | | |
|---|---|---|
| | $\min_g$ expected LHS | (original cost-sensitive problem) |
| $=$ | $\min_g$ expected RHS | (a regular problem when $Q(y, \ell) \geq 0$) |

# Cost Transformation Methodology: Preliminary

1. split each training example $(x_n, y_n)$ to a weighted mixture $\left\{(x_n, \ell, Q(y_n, \ell))\right\}_{\ell=1}^{K}$
2. apply regular classification algorithm on the weighted mixtures $\bigcup_{n=1}^{N} \left\{(x_n, \ell, Q(y_n, \ell))\right\}_{\ell=1}^{K}$

- by cost equivalence,

$$\begin{aligned} & \text{good } g \text{ for new regular classification problem} \\ = \; & \text{good } g \text{ for original cost-sensitive classification problem} \end{aligned}$$

- regular classification: needs $Q(y_n, \ell) \geq 0$

**but what if $Q(y_n, \ell)$ negative?**

# Similar Cost Vectors

$$\underbrace{\begin{pmatrix} 1 & 0 & 1 & 2 \\ 3 & 2 & 3 & 4 \end{pmatrix}}_{\text{costs}} = \underbrace{\begin{pmatrix} 1/3 & 4/3 & 1/3 & -2/3 \\ 1 & 2 & 1 & 0 \end{pmatrix}}_{\text{mixture weights } Q(y,\ell)} \cdot \underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\text{classification costs}}$$

- negative $Q(y, \ell)$: cannot split
- but $\hat{\mathbf{c}} = (1, 0, 1, 2)$ is **similar** to $\mathbf{c} = (3, 2, 3, 4)$: for any classifier $g$,

$$\hat{\mathbf{c}}[g(x)] + \text{constant} = \mathbf{c}[g(x)] = \sum_{\ell=1}^{K} Q(y, \ell) \, [\![\ell \neq g(x)]\!]$$

- constant can be dropped during minimization

$$\begin{array}{lll} & \min_g \text{ expected } \hat{\mathcal{C}}(y, g(x)) & \text{(original cost-sensitive problem)} \\ = & \min_g \text{ expected RHS} & \text{(regular problem w/ } Q \geq 0\text{)} \end{array}$$

# Cost Transformation Methodology: Revised

1. shift each row of original cost $\hat{\mathcal{C}}$ to a similar and "splittable" $\mathcal{C}(y, :)$
2. split $(x_n, y_n)$ to a weighted mixture $\left\{(x_n, \ell, Q(y_n, \ell))\right\}_{\ell=1}^{K}$ with $\mathcal{C}$
3. apply regular classification algorithm on the weighted mixtures $\bigcup_{n=1}^{N} \left\{(x_n, \ell, Q(y_n, \ell))\right\}_{\ell=1}^{K}$

- **splittable**: $Q(y_n, \ell) \geq 0$
- by cost equivalence after shifting:

$$\begin{aligned} & \text{good } g \text{ for new regular classification problem} \\ = \; & \text{good } g \text{ for original cost-sensitive classification problem} \end{aligned}$$

**but infinitely many similar and splittable $\mathcal{C}$!**

# Uncertainty in Mixture

- a single example $\{(x, 2)\}$
  —**certain** that the desired label is 2
- a mixture $\{(x, 1, 1), (x, 2, 2), (x, 3, 1)\}$ sharing the same $x$
  —**uncertainty** in the desired label (25%: 1, 50%: 2, 25%: 3)
- over-shifting adds unnecessary mixture uncertainty:

$$\underbrace{\begin{pmatrix} 3 & 2 & 3 & 4 \\ 33 & 32 & 33 & 34 \end{pmatrix}}_{\text{costs}} = \underbrace{\begin{pmatrix} 1 & 2 & 1 & 0 \\ 11 & 12 & 11 & 10 \end{pmatrix}}_{\text{mixture weights}} \cdot \underbrace{\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}}_{\mathcal{C}_c}$$

should choose a similar and splittable **c**
with **minimum mixture uncertainty**

# Cost Transformation Methodology: Final

1. shift original cost $\hat{\mathcal{C}}$ to a similar and splittable $\mathcal{C}$ with minimum "mixture uncertainty"
2. split $(x_n, y_n)$ to a weighted mixture $\left\{(x_n, \ell, Q(y_n, \ell)\right\}_{\ell=1}^{K}$ with $\mathcal{C}$
3. apply regular classification algorithm on the weighted mixtures $\bigcup_{n=1}^{N} \left\{(x_n, \ell, Q(y_n, \ell))\right\}_{\ell=1}^{K}$

- mixture uncertainty: entropy of each normalized $Q(y, :)$
- a simple and unique optimal shifting exists for every $\hat{\mathcal{C}}$

$$\begin{aligned} & \text{good } g \text{ for new regular classification problem} \\ = \; & \text{good } g \text{ for original cost-sensitive classification problem} \end{aligned}$$

# From OVO to CSOVO

**One-Versus-One: A Popular Classification Meta-Method**

1. for a pair $(i, j)$, take all examples $(x_n, y_n)$ that $y_n = i$ or $j$
2. train a binary classifier $g^{(i,j)}$ using those examples
3. repeat the previous two steps for all different $(i, j)$
4. predict using the votes from $g^{(i,j)}$

**cost-sensitive one-versus-one:**
**cost transformation + one-versus-one**

# Cost-Sensitive One-Versus-One (CSOVO)

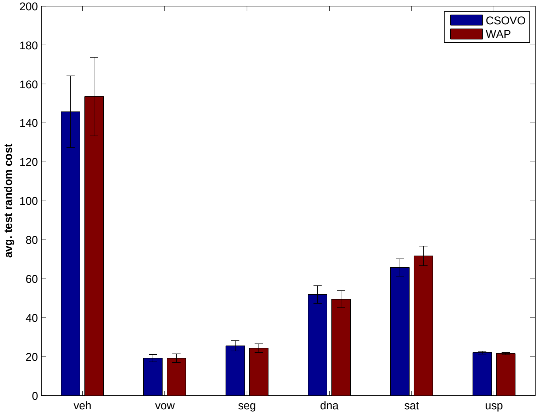
1. for a pair $(i, j)$, transform all examples $(x_n, y_n)$ to $\left(x_n, \underset{k \in \{i,j\}}{\operatorname{argmin}} \mathcal{C}(y_n, k)\right)$ with weight $\left|\mathcal{C}(y_n, i) - \mathcal{C}(y_n, j)\right|$
2. train a binary classifier $g^{(i,j)}$ using those examples
3. repeat the previous two steps for all different $(i, j)$
4. predict using the votes from $g^{(i,j)}$

- comes with **good theoretical guarantee**:

$$\text{test cost of final classifier} \leq 2 \sum_{i<j} \text{test cost of } g^{(i,j)}$$

**simple**, **efficient**, and takes original OVO as **special case**

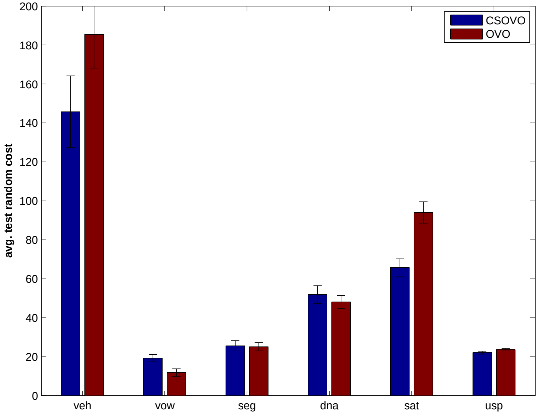
# CSOVO v.s. WAP




- a general cost-sensitive setup with "random" cost
- WAP (Abe et al., 2004): related to CSOVO, but more complicated and slower
- couple both meta-methods with SVM

**CSOVO simpler, faster, with similar performance —a preferable choice**

# CSOVO v.s. OVO




- OVO: popular regular classification meta-method, NOT cost-sensitive
- couple both meta-methods with SVM

**CSOVO often better suited for cost-sensitive classification**

# Conclusion

- **cost transformation** methodology:
  makes **any** (robust) regular classification algorithm cost-sensitive
- theoretical guarantee: **cost equivalence**
- algorithmic use: a **novel and simple** algorithm CSOVO
- experimental performance of CSOVO: **superior**

many more cost-sensitive algorithms can be designed similarly