Data Structures and Algorithms (資料結構與演算法)

Lecture 1: Algorithm

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Roadmap

1. the one where it all began

   Lecture 1: Algorithm
   - definition of algorithm
   - pseudo code of algorithm
   - criteria of algorithm
   - correctness proof of algorithm

2. the data structures awaken
3. fantastic trees and where to find them
4. the search revolutions
5. sorting: the final frontier

definition of algorithm

Name Origin of Algorithm

Muhammad ibn Mūsā al-Ḵwārizmī on a Soviet Union stamp

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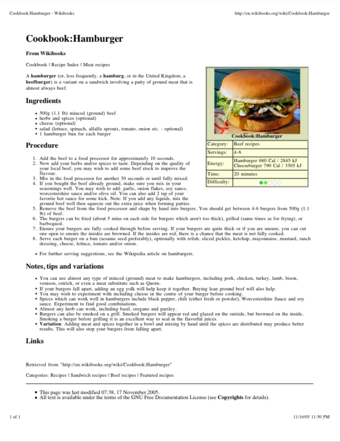
algorithm

- named after al-Ḵwārizmī (780–850), Persian mathematician and father of algebra
- algebra: rules to calculate with symbols
- algorithm: instructions to compute with variables

algorithm: recipe-like instructions for computing

Recipe for Cooking Dish



Cookbook:Hamburger - Wikibooks http://en.wikibooks.org/wiki/Cookbook:Hamburger

Cookbook:Hamburger

From Wikibooks

Cookbook | Recipe Index | Meat recipes

A **hamburger** (or, less frequently, a **hamburg**, or in the United Kingdom, a **beefburger**) is a variant on a sandwich involving a patty of ground meat that is almost always beef.

Cookbook:Hamburger

Category:	Beef recipes
Servings:	4-6
Energy:	Hamburger 680 Cal / 2845 kJ Cheeseburger 790 Cal / 3305 kJ
Time:	20 minutes
Difficulty:	●●○○○

Ingredients

- 500g (1.1 lb) minced (ground) beef
- herbs and spices (optional)
- cheese (optional)
- salad (lettuce, spinach, alfalfa sprouts, tomato, onion etc. - optional)
- 1 hamburger bun for each burger

Procedure

1. Add the beef to a food processor for approximately 10 seconds.
2. Now add your herbs and/or spices to taste. Depending on the quality of your local beef, you may wish to add some beef stock to improve the flavour.
3. Mix in the food processor for another 30 seconds or until fully mixed.
4. If you bought the beef already ground, make sure you mix in your seasonings well. You may wish to add: garlic, onion flakes, soy sauce, worcestershire sauce and/or olive oil. You can also add 2 tsp of your favorite hot sauce for some kick. Note: If you add any liquids, mix the ground beef well then squeeze out the extra juice when forming patties.
5. Remove the beef from the food processor and shape by hand into burgers. You should get between 4-6 burgers from 500g (1.1 lb) of beef.
6. The burgers can be fried (about 5 mins on each side for burgers which aren't too thick), grilled (same times as for frying), or barbequed.
7. Ensure your burgers are fully cooked through before serving. If your burgers are quite thick or if you are unsure, you can cut one open to ensure the insides are browned. If the insides are red, there is a chance that the meat is not fully cooked.
8. Serve each burger on a bun (sesame seed preferably), optionally with relish, sliced pickles, ketchup, mayonnaise, mustard, ranch dressing, cheese, lettuce, tomato and/or onion.

- For further serving suggestions, see the Wikipedia article on hamburgers.

Notes, tips and variations

- You can use almost any type of minced (ground) meat to make hamburgers, including pork, chicken, turkey, lamb, bison, venison, ostrich, or even a meat substitute such as Quorn.
- If your burgers fall apart, adding an egg yolk will help keep it together. Buying lean ground beef will also help.
- You may wish to experiment with including cheese in the centre of your burger before cooking.
- Spices which can work well in hamburgers include black pepper, chili (either fresh or powder), Worcestershire Sauce and soy sauce. Experiment to find good combinations.
- Almost any herb can work, including basil, oregano and parsley.
- Burgers can also be smoked on a grill. Smoked burgers will appear red and glazed on the outside, but browned on the inside. Smoking a burger before grilling it is an excellent way to seal in the flavorful juices.
- **Variation**: Adding meat and spices together in a bowl and mixing by hand until the spices are distributed may produce better results. This will also stop your burgers from falling apart.

Links

Retrieved from "http://en.wikibooks.org/wiki/Cookbook:Hamburger"

Categories: Recipes | Sandwich recipes | Beef recipes | Featured recipes

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1 of 1 11/16/05 11:50 PM

a recipe for hamburger on Wikibooks

figure by Gentgeen,

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recipe

Wikipedia: a set of instructions that describes how to prepare or make something, especially a dish of prepared food

recipe: instructions to complete a (cooking) task

Sheet Music for Playing Instrument

first page of manuscript of Bach's lute suite in G minor

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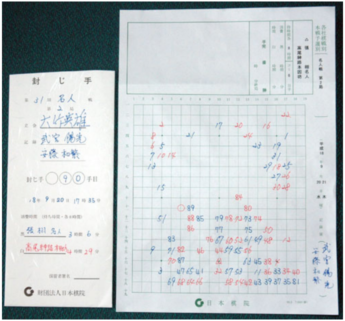
sheet music

Wikipedia: handwritten or printed form of musical notation ... to indicate the pitches, rhythms or chords of a song

sheet music: instructions to play instrument (well)

Kifu for Playing Go



a Japanese kifu

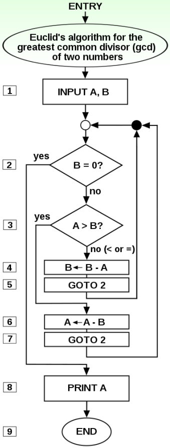
figure by Velobici,

kifu

go game record of steps that describe how the game had been played

kifu: instructions to mimic/learn to play go (professionally)

Algorithm for Computing



flowchart of Euclid's algorithm for calculating the greatest common divisor (g.c.d.) of two numbers

figure by Somepics,

algorithm

Wikipedia: algorithm is a finite sequence of well-defined, computer-implementable instructions, typically to solve a class of problems or to perform a computation

algorithm $\sim$ computing recipe:
(computable) instructions to solve a computing task
efficiently/correctly

Fun Time

Which of the following in the kitchen is the best metaphor for an algorithm?

1. recipe
2. chef
3. garbage
4. meat

Fun Time

Which of the following in the kitchen is the best metaphor for an algorithm?

1. recipe
2. chef
3. garbage
4. meat

Reference Answer: ①

algorithm $\sim$ computing recipe:
(computable) instructions to solve a computing task efficiently/correctly

pseudo code of algorithm

Pseudo Code for Get-Min-Index

C Version

```
/* return index to min. element
   in arr[0] ... arr[len-1] */
int getMinIndex
    (int arr[], int len){
  int i;
  int m=0;
  for(i=0;i<len;i++){
    if (arr[m] > arr[i]){
      m = i;
    }
  }
  return m;
}
```

Pseudo Code Version

```
Get-Min-Index(A)
m = 1
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

pseudo code: spoken language of programming

Bad Pseudo Code: Too Detailed

Unnecessarily Detailed

```
Get-Min-Index(A)
m = 1
for i = 2 to A.length
    // update if i-th element smaller
    Am = A[m]
    Ai = A[i]
    if Am > Ai
        m = i
    else
        m = m
return m
```

Concise

```
Get-Min-Index(A)
m = 1
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

goal of pseudo code: communicate efficiently

Bad Pseudo Code: Too Mysterious

Unnecessarily Mysterious

```
Get-Min-Index(A)
  x = 1
  for xx = 2 to A.length

      if A[x] > A[xx]
          xx = x
  return xx
```

Clear

```
Get-Min-Index(A)
  m = 1 // store current min. index
  for i = 2 to A.length
      // update if i-th element smaller
      if A[m] > A[i]
          m = i
  return m
```

goal of pseudo code: communicate correctly

Bad Pseudo Code: Too Abstract

Unnecessarily Abstract

```
Get-Min-Index(A)
    m = 1 // store current min. index
    run a loop through A
    that updates m in every iteration
    return m
```

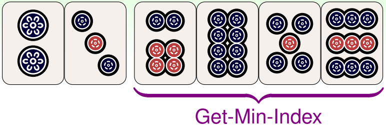
Concrete

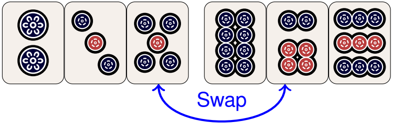
```
Get-Min-Index(A)
    m = 1 // store current min. index
    for i = 2 to A.length
        // update if i-th element smaller
        if A[m] > A[i]
            m = i
    return m
```

goal of pseudo code: communicate effectively

From Get-Min-Index to Selection-Sort

```
Get-Min-Index(A, ℓ, r)
m = ℓ // store current min. index
for i = ℓ + 1 to r
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```





Good Pseudo Code

- modularize, just like coding
- depends on speaker/listener
- usually no formal definition

```
Selection-Sort(A)
for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length)
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

follow any textbook if you really need a definition

Quick Demo of Selection Sort

```
Selection-Sort(A)
for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length))
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

```
Get-Min-Index(A, ℓ, r)
m = ℓ // store current min. index
for i = ℓ + 1 to r
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

instructions	$A[1]$	$A[2]$	$A[3]$	$A[4]$	$A[5]$	$A[6]$
$m \xleftarrow{2}$ Get-Min-Index$(A, 1, 6)$ Swap$(A[1], A[2])$						
$m \xleftarrow{2}$ Get-Min-Index$(A, 2, 6)$ ~~Swap$(A[2], A[2])$~~						
$m \xleftarrow{5}$ Get-Min-Index$(A, 3, 6)$ Swap$(A[3], A[5])$						
$m \xleftarrow{5}$ Get-Min-Index$(A, 4, 6)$ Swap$(A[4], A[5])$						
$m \xleftarrow{5}$ Get-Min-Index$(A, 5, 6)$ ~~Swap$(A[5], A[5])$~~						
$m \xleftarrow{6}$ Get-Min-Index$(A, 6, 6)$ ~~Swap$(A[6], A[6])$~~						

suggestion: draw, don't just watch

Fun Time

Which of the following can be used to describe good pseudo code?

1. clear
2. concise
3. concrete
4. all of the above

Fun Time

Which of the following can be used to describe good pseudo code?

1. clear
2. concise
3. concrete
4. all of the above

Reference Answer: 4

Have fun communicating with other programmers using good pseudo code! :-)

criteria of algorithm

Criteria of Recipe

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Cocktail Recipe: Screwdriver (from Wikipedia)

inputs: 5 cl vodka, 10 cl orange juice

1. mix inputs in a highball glass with ice
2. garnish with orange slice and serve

output: a glass of delicious cocktail

- input: ingredients
- definiteness: clear instructions
- effectiveness: feasible instructions
- finiteness: completable instructions
- output: delicious drink

algorithm $\sim$ recipe: same five criteria for algorithm

(Knuth, The Art of Computer Programming)

Input of Algorithm

... quantities which are given to it initially before the algorithm begins. These inputs are taken from specified sets of objects. (Knuth, TAOCP)

Get-Min-Index(A)

```
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

one algorithm, many uses (on different legal inputs)

Definiteness of Algorithm

Each step of an algorithm must be precisely defined; the actions to be carried out must be rigorously & unambiguously specified. (Knuth, TAOCP)

Clear

Get-Min-Index(A)

```
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

Ambiguous

Get-Zero-Index(A)

```
for i = 1 to A.length
    if A[m] is almost zero
        return m
// what to return here?
```

definiteness: clarity of algorithm

Effectiveness of Algorithm

. . . all of the operations to be performed in the algorithm must be sufficiently basic that they can in principle be done exactly and in a finite length of time by a man using paper and pencil. (Knuth, TAOCP)

Effective

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

Ineffective

```
Get-Min-Index-Robot(A)
m = 1 // store current min. index
form a ball Bm of weight A[1]
for i = 2 to A.length
    // update if i-th element smaller
    form a ball Bi of weight A[i]
    if Bi is lighter than Bm on a balance
        m = i
        replace Bm with Bi
return m
```

forming a ball (& other actions) are arguably ineffective on typical computers

Finiteness of Algorithm

An algorithm must always terminate after a finite number of steps . . .
a very finite number, a reasonable number. (Knuth, TAOCP)

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

finiteness (& efficiency): often need analysis for sophisticated algorithms (to be taught later)

Output of Algorithm

. . . quantities which have a specified relation to the inputs (Knuth, TAOCP)

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

output (correctness): need proving
with respect to requirements

Fun Time

What best describes the input/output relationship of the selection sort algorithm below?

1. input: an ascending array; output: the same array sorted in descending order
2. input: an arbitrary array; output: the same array sorted in descending order
3. input: an arbitrary array; output: the same array sorted in ascending order
4. none of the other choices

```
Selection-Sort(A)

for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length))
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

Fun Time

What best describes the input/output relationship of the selection sort algorithm below?

1. input: an ascending array; output: the same array sorted in descending order
2. input: an arbitrary array; output: the same array sorted in descending order
3. input: an arbitrary array; output: the same array sorted in ascending order
4. none of the other choices

```
Selection-Sort(A)
for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length))
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

Reference Answer: 3

The selection sort algorithm re-arranges an arbitrary array into ascending order.

correctness proof of algorithm

Claim

figure by Nick Youngson, licensed CC BY-SA 3.0 via Picpedia.Org

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

Correctness of Get-Min-Index

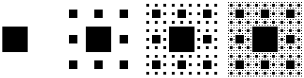
Upon exiting Get-Min-Index(A),

$$A[m] = \min_{1 \leq j \leq n} A[j]$$

with $n = A.length$

claim: math. statement that declares correctness

Invariant



invariants when constructing fractals
figures by Johannes Rössel,

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Correctness of Get-Min-Index

Upon exiting Get-Min-Index(A),

$$A[m] = \min_{1 \le j \le n} A[j]$$

with $n = A.length$

$\Uparrow$

Invariant within Get-Min-Index

Upon finishing the loop with $i = k$, denote m by m_k,

$$A[m_k] \le A[j] \text{ for } j = 1, 2, \ldots, k$$

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

(loop) invariant: property that algorithm maintains

Proof of Loop Invariant

Mathematical Induction

Base

when $i = 2$, invariant true because
. . .

- assume invariant true for $i = t - 1$
- when $i = t$,
  - if $A[m_{t-1}] > A[t] \Rightarrow m_t = t$

$$A[m_t] \begin{array}{ll} = A[t] & \leq A[t] \\ < A[m_{t-1}] & \leq A[j] \text{ for } j < t \end{array}$$

  - if $A[m_{t-1}] \leq A[t] \Rightarrow m_t = m_{t-1}$

$$A[m_t] \begin{array}{ll} = A[m_{t-1}] & \leq A[t] \\ = A[m_{t-1}] & \leq A[j] \text{ for } j < t \end{array}$$

—by mathematical induction,
invariant true for $i = 2, 3, \ldots, k$

$\Rightarrow$

```
Get-Min-Index(A)
m = 1 // store current min. index
for i = 2 to A.length
    // update if i-th element smaller
    if A[m] > A[i]
        m = i
return m
```

Correctness of Get-Min-Index

$\Uparrow$

Invariant within Get-Min-Index

Upon finishing the loop with $i = k$, denote m by m_k,

$$A[m_k] \leq A[j] \text{ for } j = 1, 2, \ldots, k$$

proof of (loop) invariants $\Rightarrow$ correctness claim of algorithm

Fun Time

Which of the following is a loop invariant to selection sort?

```
Selection-Sort(A)
for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length))
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

1. Upon finishing the loop with $i = k$, $A[1] \geq A[2] \geq \ldots \geq A[k]$.
2. Upon finishing the loop with $i = k$, $A[1] \leq A[2] \leq \ldots \leq A[k]$.
3. Upon finishing the loop with $i = k$, $A[k+1] \geq \ldots \geq A[A.length]$.
4. Upon finishing the loop with $i = k$, $A[k+1] \leq \ldots \leq A[A.length]$.

Fun Time

Which of the following is a loop invariant to selection sort?

```
Selection-Sort(A)
for i = 1 to A.length
    m = Get-Min-Index(A, i, A.length))
    if i ≠ m
        Swap(A[i], A[m])
return A // which has been sorted in place
```

1. Upon finishing the loop with $i = k$, $A[1] \geq A[2] \geq \ldots \geq A[k]$.
2. Upon finishing the loop with $i = k$, $A[1] \leq A[2] \leq \ldots \leq A[k]$.
3. Upon finishing the loop with $i = k$, $A[k+1] \geq \ldots \geq A[A.length]$.
4. Upon finishing the loop with $i = k$, $A[k+1] \leq \ldots \leq A[A.length]$.

Reference Answer: 2

The selection sort algorithm essentially picks the smallest element, the 2nd-smallest, and so on, and place them orderly. You can prove the loop invariant by mathematical induction.

Summary

Lecture 1: Algorithm

- definition of algorithm
  instructions to complete a task by computer
- pseudo code of algorithm
  communicate efficiently/correctly/effectively
- criteria of algorithm
  input, definite, effective, finite, output
- correctness proof of algorithm
  from (loop) invariants to claims

- next: data structures and their connections to algorithms