# Convolutional Neural Network

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April 24, 2020

# Motivation and Definition

# Fully-Connected Neural Network on Images

issue: too many weights for model/computation complexity

# Neurons that Act Locally

$$\equiv \text{fix } w_{ij,\hat{i}\hat{j}}^{(1)} = 0 \text{ when } |i - \hat{i}| > \Delta \text{ or } |j - \hat{j}| > \Delta$$

# Translation-Invariant Neurons that Act Locally

$$s_{\hat{i}\hat{j}} = b_{\hat{i}\hat{j}} + \sum_{|i-\hat{i}|\le\Delta} \sum_{|j-\hat{j}|\le\Delta} w_{ij,\hat{i}\hat{j}} x_{ij}$$

need: $w_{ij,\hat{i}\hat{j}} = w_{(i+3)(j+4),(\hat{i}+3)(\hat{j}+4)} = v_{(i-\hat{i})(j-\hat{j})}$

# Image Filter and Kernel

$$s_{\hat{i}\hat{j}} = b_{\hat{i}\hat{j}} + \sum_{-\Delta \le p \le \Delta} \sum_{-\Delta \le q \le \Delta} v_{pq} x_{(\hat{i}+p)(\hat{j}+q)}$$

scores $s_{\hat{i}\hat{j}}$ computed by constrained neurons $\equiv$ apply linear image filter with kernel $v$ on image

# Cross-Correlation and Convolution

$$
\begin{aligned}
s_{\hat{i}} &= b_{\hat{i}} + \sum_{-\Delta \le p \le \Delta} v_p x_{\hat{i+p}} \\
&= b_{\hat{i}} + \sum_{-\Delta \le p \le \Delta} u_{-p} x_{\hat{i+p}} \\
&= b_{\hat{i}} + \sum_{-\Delta \le p \le \Delta} u_p x_{\hat{i-p}}
\end{aligned}
$$

convolution neural network: NNet that contains some convolution nodes/layers

# Technical Details

# Image Size Preservation after Convolution Layer

$$\Delta = 1 : 3 * 3 \text{ kernel}$$

$$128 * 128 \text{ image} \Longrightarrow 126 * 126 \text{ filtered image}$$

common patch: zero-padding

# Sub-Sampling with Stride

stride: top-left sub-sampling