

# Convolutional Neural Network

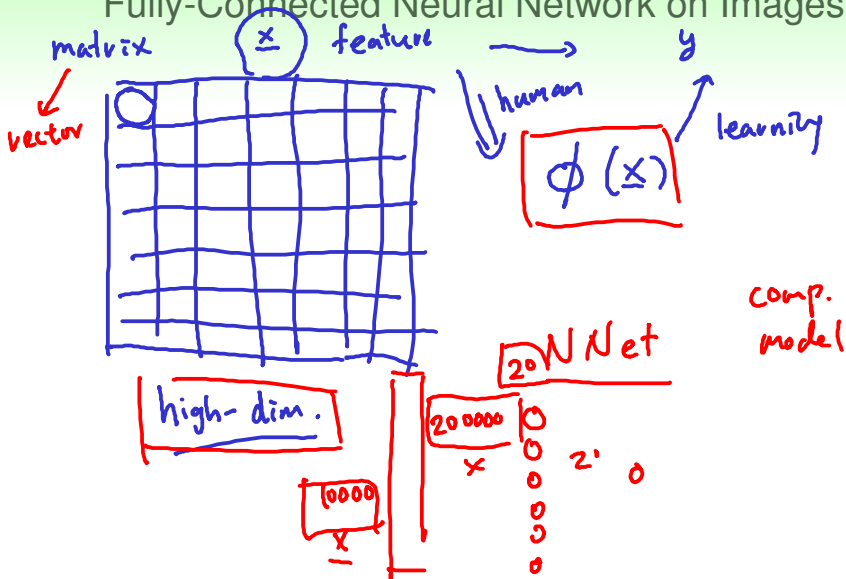
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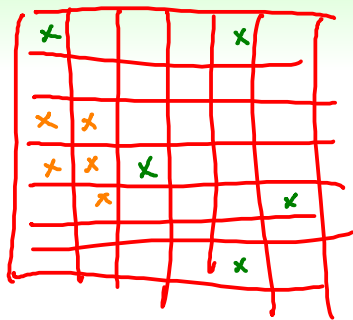
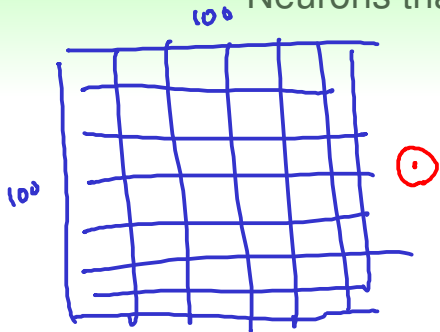
## Motivation and Definition

# Fully-Connected Neural Network on Images



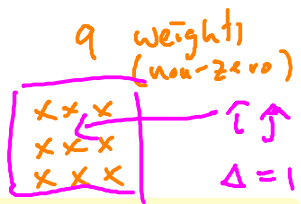
issue: too many weights for model/computation complexity

# Neurons that Act Locally



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集中

NNet + constraint

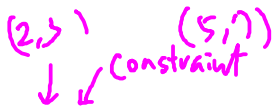
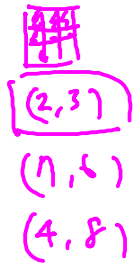


$$\equiv \text{fix } w_{ij, \hat{i}\hat{j}}^{(1)} = 0 \text{ when } |i - \hat{i}| > \Delta \text{ or } |j - \hat{j}| > \Delta$$

# Translation-Invariant Neurons that Act Locally

$$s_{\hat{i}\hat{j}} = b_{\hat{i}\hat{j}} + \sum_{|i-\hat{i}| \leq \Delta} \sum_{|j-\hat{j}| \leq \Delta} w_{ij, \hat{i}\hat{j}} x_{ij}$$

image size



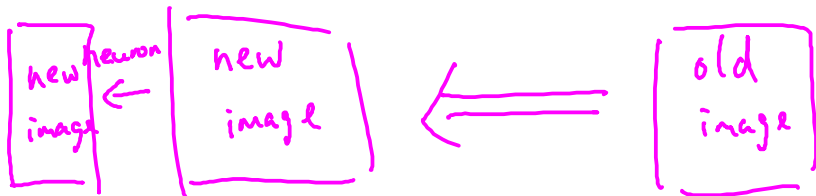
need:  $w_{ij, \hat{i}\hat{j}} = w_{(i+3)(j+4), (\hat{i}+3)(\hat{j}+4)} = v_{(i-\hat{i})(j-\hat{j})}$

## Image Filter and Kernel

 $x^{(i)}$ 

$$s_{ij} = b_{ij} + \sum_{-\Delta \leq p \leq \Delta} \sum_{-\Delta \leq q \leq \Delta} v_{pq} x_{(i+p)(j+q)}$$

$$s_{ij}$$

$$x_{ij}$$


scores  $s_{ij}$  computed by constrained neurons  $\equiv$  apply linear image filter with kernel  $v$  on image

# Cross-Correlation and Convolution

$$\begin{aligned}
 \underline{s_i} &= b_i + \sum_{-\Delta \leq p \leq \Delta} v_p x_{i+p} \\
 &= b_i + \sum_{-\Delta \leq p \leq \Delta} u_{-p} x_{i+p} \\
 &= b_i + \sum_{-\Delta \leq p \leq \Delta} u_p x_{i-p}
 \end{aligned}$$

Handwritten annotations:
 

- Red arrows point to  $v_p x_{i+p}$  and  $u_{-p} x_{i+p}$  in the first two equations, with the label "cross correlation".
- A red arrow points to the third equation, with the label "CONV.".



$$\begin{aligned}
 & \left[ \begin{array}{c} u \\ -\Delta \quad \Delta \end{array} \right] \\
 & \left[ \text{rev}(x) \right]
 \end{aligned}$$

convolution neural network: NNet that contains **some** convolution nodes/layers

## Technical Details



# Image Size Preservation after Convolution Layer

$\Delta = 1$ :  $3 \times 3$  kernel



~~128 \* 128 image~~  $\Rightarrow$  126 \* 126 filtered image



constant (126)  
 zero

noise

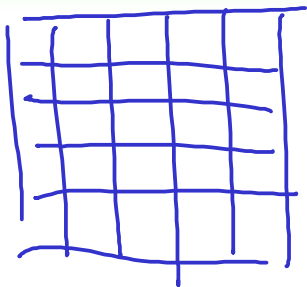
copy boundary

from another image  
 pattern

common patch: zero-padding

## Sub-Sampling with Stride

$$\underline{128 \times 128} \rightarrow \underline{64 \times 64}$$



```
for ( ; ; f++ )
    i+=2
```

stride: top-left sub-sampling