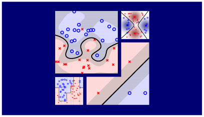
# Machine Learning Techniques
# (機器學習技法)



## Lecture 8: Adaptive Boosting

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# Roadmap

1. Embedding Numerous Features: Kernel Models
2. Combining Predictive Features: Aggregation Models

## Lecture 7: Blending and Bagging

**blending** known diverse hypotheses **uniformly**, **linearly**, or even **non-linearly**; obtaining diverse hypotheses from **bootstrapped data**

## Lecture 8: Adaptive Boosting

- Motivation of Boosting
- Diversity by Re-weighting
- Adaptive Boosting Algorithm
- Adaptive Boosting in Action

3. Distilling Implicit Features: Extraction Models

# Apple Recognition Problem

- is this a picture of an apple?
- say, want to teach a class of **6 year olds**
- gather photos under CC-BY-2.0 license on Flicker
  (**thanks to the authors below!**)

(APAL stands for Apple and Pear Australia Ltd)

Dan Foy
`https://flic.kr/p/jNQ55`

APAL
`https://flic.kr/p/jzP1VB`

adrianbartel
`https://flic.kr/p/bdy2hZ`

ANdrzej cH.
`https://flic.kr/p/51DKA8`

Stuart Webster
`https://flic.kr/p/9C3Ybd`

nachans
`https://flic.kr/p/9XD7Ag`

APAL
`https://flic.kr/p/jzRe4u`

Jo Jakeman
`https://flic.kr/p/7jwtGp`

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`https://flic.kr/p/jzPYNr`

APAL
`https://flic.kr/p/jzScif`

# Apple Recognition Problem

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- gather photos under CC-BY-2.0 license on Flicker (**thanks to the authors below!**)

Mr. Roboto.

https://flic.kr/p/i5BN85

Richard North

https://flic.kr/p/bHhPkB

Richard North

https://flic.kr/p/d8tGou

Emilian Robert Vicol

https://flic.kr/p/bpmGXW

Nathaniel McQueen

https://flic.kr/p/pZv1Mf

Crystal

https://flic.kr/p/kaPYp

jfh686

https://flic.kr/p/6vjRFH

skyseeker

https://flic.kr/p/2MynV

Janet Hudson

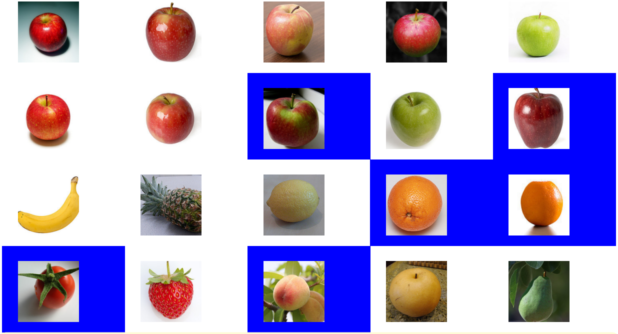
https://flic.kr/p/7QDBbm

Rennett Stowe

https://flic.kr/p/agmnrk

# Our Fruit Class Begins

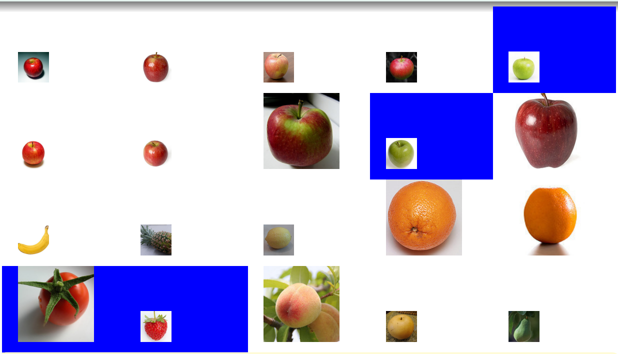
- Teacher: Please look at the pictures of apples and non-apples below. Based on those pictures, how would you describe an apple? Michael?
- Michael: I think apples are **circular**.



(Class): Apples are **circular**.

# Our Fruit Class Continues

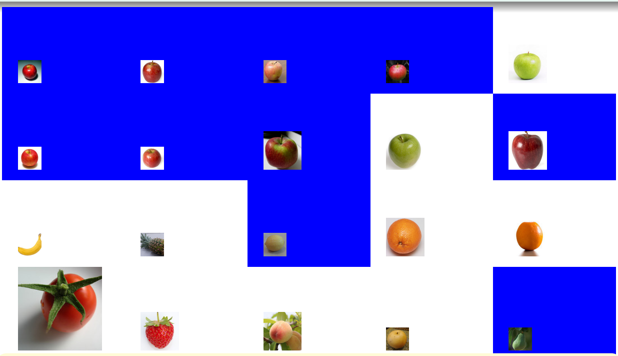
- Teacher: Being circular is a good feature for the apples. However, if you only say circular, you could make several mistakes. What else can we say for an apple? Tina?
- Tina: It looks like apples are **red**.



(Class): Apples are somewhat **circular** and somewhat **red**.

# Our Fruit Class Continues More

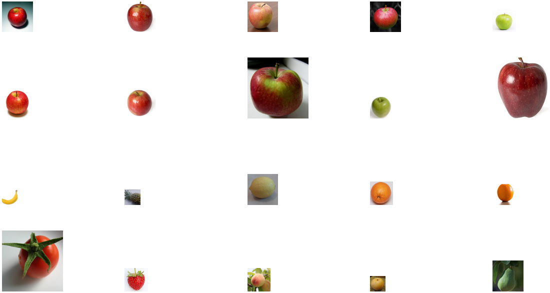
- Teacher: Yes. Many apples are red. However, you could still make mistakes based on circular and red. Do you have any other suggestions, Joey?
- Joey: Apples could also be **green**.



(Class): Apples are somewhat **circular** and somewhat **red** and possibly **green**.

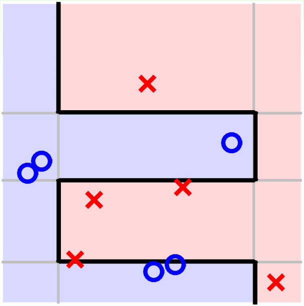
# Our Fruit Class Ends

- Teacher: Yes. It seems that apples might be circular, red, green. But you may confuse them with tomatoes or peaches, right? Any more suggestions, Jessica?
- Jessica: Apples have **stems** at the top.



(Class): Apples are somewhat **circular**, somewhat **red**, possibly **green**, and may have **stems** at the top.

# Motivation



- students: simple hypotheses $g_t$ (like vertical/horizontal lines)
- (Class): sophisticated hypothesis $G$ (like black curve)
- Teacher: a tactic learning algorithm that **directs the students to focus on key examples**

next: the '**math**' of such an algorithm

# Fun Time

Which of the following can help recognize an apple?

1. apples are often circular
2. apples are often red or green
3. apples often have stems at the top
4. all of the above

# Fun Time

Which of the following can help recognize an apple?

1. apples are often circular
2. apples are often red or green
3. apples often have stems at the top
4. all of the above

**Reference Answer: (4)**

Congratulations! **You have passed first grade. :-)**

# Bootstrapping as Re-weighting Process

$$\mathcal{D} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), (\mathbf{x}_3, y_3), (\mathbf{x}_4, y_4)\}$$

$$\overset{\text{bootstrap}}{\Longrightarrow} \quad \tilde{\mathcal{D}}_t = \{(\mathbf{x}_1, y_1), (\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), (\mathbf{x}_4, y_4)\}$$

## weighted $E_{\text{in}}$ on $\mathcal{D}$

$$E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{4} \sum_{n=1}^{4} u_n^{(t)} \cdot [\![ y_n \neq h(\mathbf{x}_n) ]\!]$$

$(\mathbf{x}_1, y_1)$, $u_1 = 2$
$(\mathbf{x}_2, y_2)$, $u_2 = 1$
$(\mathbf{x}_3, y_3)$, $u_3 = 0$
$(\mathbf{x}_4, y_4)$, $u_4 = 1$

## $E_{\text{in}}$ on $\tilde{\mathcal{D}}_t$

$$E_{\text{in}}^{0/1}(h) = \frac{1}{4} \sum_{(\mathbf{x}, y) \in \tilde{\mathcal{D}}_t} [\![ y \neq h(\mathbf{x}) ]\!]$$

$(\mathbf{x}_1, y_1)$, $(\mathbf{x}_1, y_1)$
$(\mathbf{x}_2, y_2)$

$(\mathbf{x}_4, y_4)$

each diverse $g_t$ in bagging:
by minimizing bootstrap-weighted error

# Weighted Base Algorithm

minimize (regularized)

$$E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{N}\sum_{n=1}^{N} u_n \cdot \text{err}(y_n, h(\mathbf{x}_n))$$

**SVM**

$E_{\text{in}}^{\mathbf{u}} \propto C \sum_{n=1}^{N} u_n \widehat{\text{err}}_{\text{SVM}}$ by dual QP

$\Leftrightarrow$ adjusted upper bound

$0 \leq \alpha_n \leq C u_n$

**logistic regression**

$E_{\text{in}}^{\mathbf{u}} \propto \sum_{n=1}^{N} u_n \text{err}_{\text{CE}}$ by SGD

$\Leftrightarrow$ sample $(\mathbf{x}_n, y_n)$ with probability proportional to $u_n$

**example-weighted** learning:
extension of **class-weighted** learning in Lecture 8 of ML Foundations

# Re-weighting for More Diverse Hypothesis

'improving' bagging for binary classification:
how to re-weight for **more diverse hypotheses**?

$$g_t \leftarrow \operatorname*{argmin}_{h\in\mathcal{H}} \left( \sum_{n=1}^{N} u_n^{(t)} [\![ y_n \neq h(\mathbf{x}_n) ]\!] \right)$$

$$g_{t+1} \leftarrow \operatorname*{argmin}_{h\in\mathcal{H}} \left( \sum_{n=1}^{N} u_n^{(t+1)} [\![ y_n \neq h(\mathbf{x}_n) ]\!] \right)$$

if $g_t$ '**not good**' for $\mathbf{u}^{(t+1)}$ $\Longrightarrow$ $g_t$-like hypotheses not returned as $g_{t+1}$
$\Longrightarrow$ $g_{t+1}$ diverse from $g_t$

idea: **construct $\mathbf{u}^{(t+1)}$** to make $g_t$ **random-like**

$$\frac{\sum_{n=1}^{N} u_n^{(t+1)} [\![ y_n \neq g_t(\mathbf{x}_n) ]\!]}{\sum_{n=1}^{N} u_n^{(t+1)}} = \frac{1}{2}$$

# 'Optimal' Re-weighting

want: $\frac{\sum_{n=1}^{N} u_n^{(t+1)} [\![ y_n \neq g_t(\mathbf{x}_n) ]\!]}{\sum_{n=1}^{N} u_n^{(t+1)}} = \frac{\blacksquare_{t+1}}{\blacksquare_{t+1} + \bullet_{t+1}} = \frac{1}{2}$, where

$$\blacksquare_{t+1} = \sum_{n=1}^{N} u_n^{(t+1)} [\![ y_n \neq g_t(\mathbf{x}_n) ]\!], \bullet_{t+1} = \sum_{n=1}^{N} u_n^{(t+1)} [\![ y_n = g_t(\mathbf{x}_n) ]\!]$$

- need: $\underbrace{(\text{total } u_n^{(t+1)} \text{ of incorrect})}_{\blacksquare_{t+1}} = \underbrace{(\text{total } u_n^{(t+1)} \text{ of correct})}_{\bullet_{t+1}}$
- one possibility by **re-scaling (multiplying) weights**, if

| | |
|---|---|
| (total $u_n^{(t)}$ of incorrect) = 1126 ; (weighted incorrect rate) = $\frac{1126}{7337}$ | (total $u_n^{(t)}$ of correct) = 6211 ; (weighted correct rate) = $\frac{6211}{7337}$ |
| incorrect: $u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot 6211$ | correct: $u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot 1126$ |

'optimal' re-weighting under weighted incorrect rate $\epsilon_t$:

multiply incorrect $\propto (1 - \epsilon_t)$; multiply correct $\propto \epsilon_t$

# Fun Time

For four examples with $u_n^{(1)} = \frac{1}{4}$ for all examples. If $g_1$ predicts the first example wrongly but all the other three examples correctly. After the 'optimal' re-weighting, what is $u_1^{(2)}/u_2^{(2)}$?

1. 4
2. 3
3. $1/3$
4. $1/4$

# Fun Time

For four examples with $u_n^{(1)} = \frac{1}{4}$ for all examples. If $g_1$ predicts the first example wrongly but all the other three examples correctly. After the 'optimal' re-weighting, what is $u_1^{(2)}/u_2^{(2)}$?

1. 4
2. 3
3. $1/3$
4. $1/4$

## Reference Answer: 2

By 'optimal' re-weighting, $u_1$ is scaled proportional to $\frac{3}{4}$ and every other $u_n$ is scaled proportional to $\frac{1}{4}$. So example 1 is now three times more important than any other example.

# Scaling Factor

**'optimal' re-weighting**: let $\epsilon_t = \frac{\sum_{n=1}^{N} u_n^{(t)} [\![ y_n \neq g_t(\mathbf{x}_n) ]\!]}{\sum_{n=1}^{N} u_n^{(t)}}$,

multiply incorrect $\propto (1 - \epsilon_t)$; multiply correct $\propto \epsilon_t$

define scaling factor $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$

$$\begin{aligned} \text{incorrect} &\leftarrow \text{incorrect} \cdot \blacklozenge_t \\ \text{correct} &\leftarrow \text{correct} \, / \, \blacklozenge_t \end{aligned}$$

- **equivalent** to optimal re-weighting
- $\blacklozenge_t \geq 1$ iff $\epsilon_t \leq \frac{1}{2}$
  —physical meaning: **scale up incorrect**; **scale down correct**
  —like what Teacher does

**scaling-up incorrect** examples
leads to **diverse hypotheses**

# A Preliminary Algorithm

$\mathbf{u}^{(1)} = ?$

for $t = 1, 2, \ldots, T$

1. obtain $g_t$ by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$, where $\mathcal{A}$ tries to minimize $\mathbf{u}^{(t)}$-weighted 0/1 error
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$, where $\epsilon_t$ = weighted error (incorrect) rate of $g_t$

return $G(\mathbf{x}) = ?$

- want $g_1$ 'best' for $E_{\text{in}}$: $u_n^{(1)} = \frac{1}{N}$
- $G(\mathbf{x})$:
  - uniform? but $g_2$ very bad for $E_{\text{in}}$ (**why? :-)**)
  - linear, non-linear? **as you wish**

next: a special algorithm to aggregate **linearly on the fly** with theoretical guarantee

# Linear Aggregation on the Fly

$\mathbf{u}^{(1)} = [\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}]$

for $t = 1, 2, \ldots, T$

1. obtain $g_t$ by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$, where ...
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$, where ...
3. compute $\alpha_t = \ln(\blacklozenge_t)$

return $G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t g_t(\mathbf{x})\right)$

- wish: large $\alpha_t$ for 'good' $g_t \Longleftarrow \alpha_t = \text{monotonic}(\blacklozenge_t)$
- will take $\alpha_t = \ln(\blacklozenge_t)$
  - $\epsilon_t = \frac{1}{2} \Longrightarrow \blacklozenge_t = 1 \Longrightarrow \alpha_t = 0$ (bad $g_t$ zero weight)
  - $\epsilon_t = 0 \Longrightarrow \blacklozenge_t = \infty \Longrightarrow \alpha_t = \infty$ (super $g_t$ superior weight)

Adaptive Boosting = weak base learning algorithm $\mathcal{A}$ (Student)
+ optimal re-weighting factor $\blacklozenge_t$ (Teacher)
+ 'magic' linear aggregation $\alpha_t$ (Class)

# Adaptive Boosting (AdaBoost) Algorithm

$\mathbf{u}^{(1)} = [\frac{1}{N}, \frac{1}{N}, \cdots, \frac{1}{N}]$

for $t = 1, 2, \ldots, T$

1. obtain $g_t$ by $\mathcal{A}(\mathcal{D}, \mathbf{u}^{(t)})$,
   where $\mathcal{A}$ tries to minimize $\mathbf{u}^{(t)}$-weighted 0/1 error
2. update $\mathbf{u}^{(t)}$ to $\mathbf{u}^{(t+1)}$ by

   $[\![y_n \neq g_t(\mathbf{x}_n)]\!]$ (incorrect examples): $u_n^{(t+1)} \leftarrow u_n^{(t)} \cdot \blacklozenge_t$

   $[\![y_n = g_t(\mathbf{x}_n)]\!]$ (correct examples): $u_n^{(t+1)} \leftarrow u_n^{(t)} / \blacklozenge_t$

   where $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$ and $\epsilon_t = \frac{\sum_{n=1}^{N} u_n^{(t)} [\![y_n \neq g_t(\mathbf{x}_n)]\!]}{\sum_{n=1}^{N} u_n^{(t)}}$
3. compute $\alpha_t = \ln(\blacklozenge_t)$

return $G(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t g_t(\mathbf{x})\right)$

AdaBoost: provable **boosting property**

# Theoretical Guarantee of AdaBoost

- From VC bound

$$E_{\text{out}}(G) \le E_{\text{in}}(G) + O\left(\sqrt{\underbrace{O(d_{\text{VC}}(\mathcal{H}) \cdot T \log T)}_{d_{\text{VC}} \text{ of all possible } G} \cdot \frac{\log N}{N}}\right)$$

- **first term can be small**:
  $E_{\text{in}}(G) = 0$ after $T = O(\log N)$ iterations if $\epsilon_t \le \epsilon < \frac{1}{2}$ always
- **second term can be small**:
  overall $d_{\text{VC}}$ grows "slowly" with $T$

boosting view of AdaBoost:

if $\mathcal{A}$ is weak but always **slightly better than random** ($\epsilon_t \le \epsilon < \frac{1}{2}$),
then (AdaBoost+$\mathcal{A}$) can be strong ($E_{\text{in}} = 0$ and $E_{\text{out}}$ small)

# Fun Time

According to $\alpha_t = \ln(\blacklozenge_t)$, and $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$, when would $\alpha_t > 0$?

1. $\epsilon_t < \frac{1}{2}$
2. $\epsilon_t > \frac{1}{2}$
3. $\epsilon_t \neq 1$
4. $\epsilon_t \neq 0$

# Fun Time

According to $\alpha_t = \ln(\blacklozenge_t)$, and $\blacklozenge_t = \sqrt{\frac{1-\epsilon_t}{\epsilon_t}}$, when would $\alpha_t > 0$?

1. $\epsilon_t < \frac{1}{2}$
2. $\epsilon_t > \frac{1}{2}$
3. $\epsilon_t \neq 1$
4. $\epsilon_t \neq 0$

## Reference Answer: ①

The math part should be easy for you, and it is interesting to think about the physical meaning: $\alpha_t > 0$ ($g_t$ is useful for $G$) if and only if the weighted error rate of $g_t$ is better than random!

# Decision Stump

want: a '**weak**' base learning algorithm $\mathcal{A}$
that minimizes $E_{\text{in}}^{\mathbf{u}}(h) = \frac{1}{N}\sum_{n=1}^{N} u_n \cdot [\![ y_n \neq h(\mathbf{x}_n) ]\!]$ **a little bit**

## a popular choice: decision stump

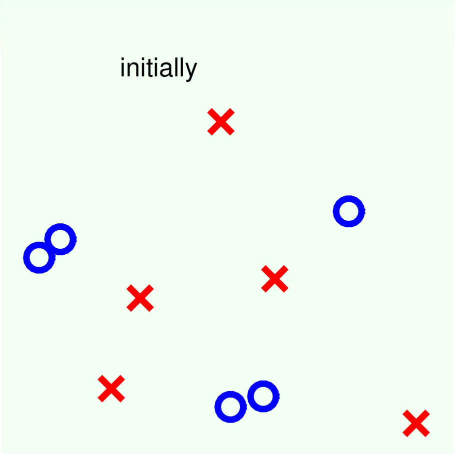
- **in ML Foundations Homework 2, remember? :-)**

$$h_{s,i,\theta}(\mathbf{x}) = s \cdot \text{sign}(x_i - \theta)$$

- positive and negative rays on some feature: three parameters (feature $i$, threshold $\theta$, direction $s$)
- physical meaning: vertical/horizontal lines in 2D
- efficient to optimize: $O(d \cdot N \log N)$ time

**decision stump** model:
allows efficient minimization of $E_{\text{in}}^{\mathbf{u}}$
but perhaps **too weak to work by itself**

# A Simple Data Set



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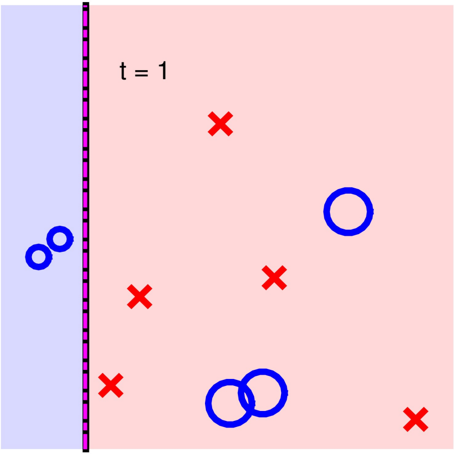

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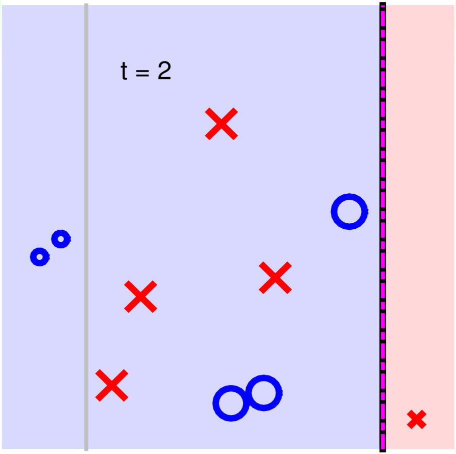

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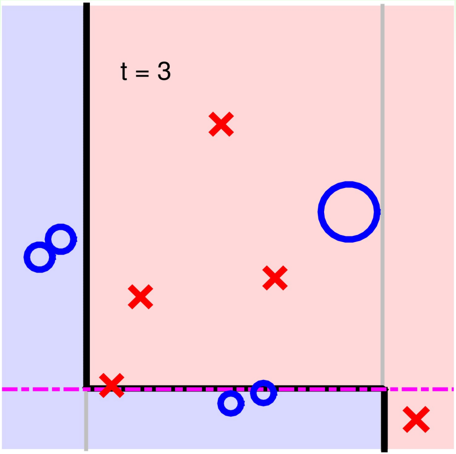

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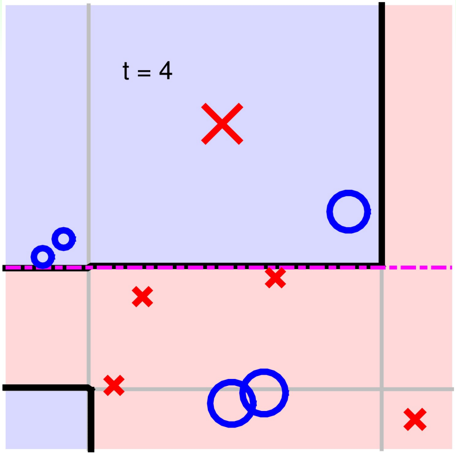

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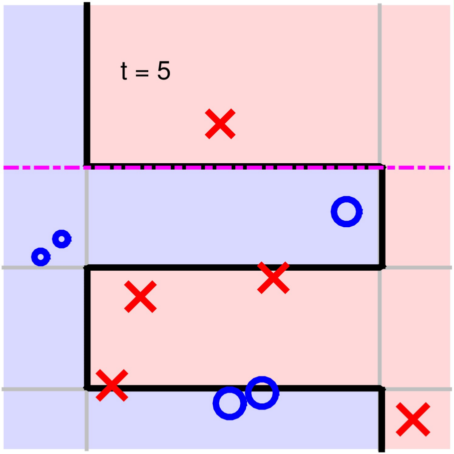
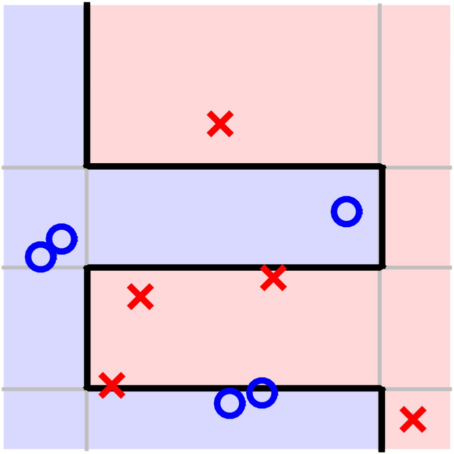
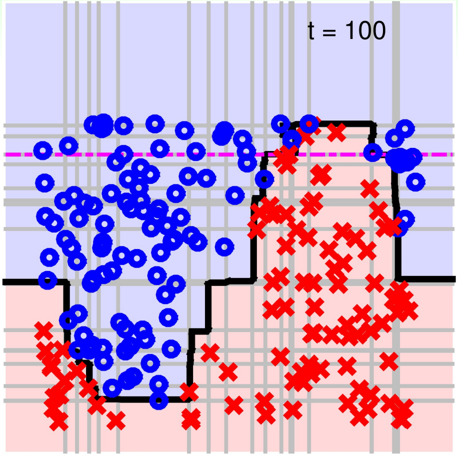


# A Simple Data Set



**'Teacher'-like algorithm works!**

# A Complicated Data Set




AdaBoost-Stump: **non-linear yet efficient**

# AdaBoost-Stump in Application

original picture by F.U.S.I.A. assistant and derivative work by Sylenius via Wikimedia Commons

## The World's First 'Real-Time' Face Detection Program

- AdaBoost-Stump as core model: linear aggregation of key patches selected out of 162,336 possibilities in 24x24 images
—**feature selection** achieved through AdaBoost-Stump
- modified linear aggregation $G$ to rule out non-face earlier
—**efficiency** achieved through modified linear aggregation

AdaBoost-Stump:
efficient feature selection and aggregation

# Fun Time

For a data set of size 9876 that contains $\mathbf{x}_n \in \mathbb{R}^{5566}$, after running AdaBoost-Stump for 1126 iterations, what is the number of distinct features within $\mathbf{x}$ that are effectively used by $G$?

1. $0 \leq \text{number} \leq 1126$
2. $1126 < \text{number} \leq 5566$
3. $5566 < \text{number} \leq 9876$
4. $9876 < \text{number}$

# Fun Time

For a data set of size 9876 that contains $\mathbf{x}_n \in \mathbb{R}^{5566}$, after running AdaBoost-Stump for 1126 iterations, what is the number of distinct features within $\mathbf{x}$ that are effectively used by $G$?

1. $0 \le \text{number} \le 1126$
2. $1126 < \text{number} \le 5566$
3. $5566 < \text{number} \le 9876$
4. $9876 < \text{number}$

## Reference Answer: ①

Each decision stump takes only one feature. So 1126 decision stumps need at most 1126 distinct features.

# Summary

1. Embedding Numerous Features: Kernel Models
2. Combining Predictive Features: Aggregation Models

## Lecture 8: Adaptive Boosting

- Motivation of Boosting
  aggregate weak hypotheses for strength
- Diversity by Re-weighting
  scale up incorrect, scale down correct
- Adaptive Boosting Algorithm
  two heads are better than one, theoretically
- Adaptive Boosting in Action
  AdaBoost-Stump useful and efficient

- **next: learning conditional aggregation instead of linear one**

3. Distilling Implicit Features: Extraction Models