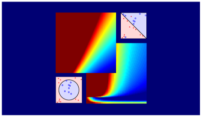
Machine Learning Foundations
(機器學習基石)



Lecture 2: Learning to Answer Yes/No, Extended

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Handling $\text{sign}(\cdot) = 0$

Perceptron Learning Algorithm

start from some $\mathbf{w}_0$ (say, $\mathbf{0}$), and 'correct' its mistakes on $\mathcal{D}$

When $\mathbf{w}_0 = \mathbf{0}$, technically $\text{sign}(\mathbf{w}_0^T \mathbf{x}_{n(0)}) = 0$, shall we update?

- convention -1: $\text{sign}(0) = -1$ (update if $y_{n(0)} = +1$)
- convention +1: $\text{sign}(0) = +1$ (update if $y_{n(0)} = +1$)
- convention 0: $\text{sign}(0) = 0$ (always update)
- convention r: $\text{sign}(0) =$ random flip (50% chance of update)

—usually does not matter much, **as long as $\mathbf{w}_1$ often becomes non-zero**

$\mathbf{w}_t^T \mathbf{x}_{n(t)} = 0$ **rarely happens in practice**

Updating w_0

Perceptron Learning Algorithm

For $t = 0, 1, \ldots$

1. find a mistake of $\mathbf{w}_t$ called $\left(\mathbf{x}_{n(t)}, y_{n(t)}\right)$: $\text{sign}\left(\mathbf{w}_t^T \mathbf{x}_{n(t)}\right) \neq y_{n(t)}$
2. (try to) correct the mistake by

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + y_{n(t)} \mathbf{x}_{n(t)}, \text{i.e.,}$$

$$\begin{bmatrix} w_{t+1,0} \\ w_{t+1,1} \\ \ldots \\ w_{t+1,d} \end{bmatrix} = \begin{bmatrix} w_{t,0} \\ w_{t,1} \\ \ldots \\ w_{t,d} \end{bmatrix} + y_{n(t)} \begin{bmatrix} x_0 (= \text{what?}) \\ x_{n(t),1} \\ \ldots \\ x_{n(t),d} \end{bmatrix}$$

. . . until no more mistakes
return last $\mathbf{w}$ (called $\mathbf{w}_{\text{PLA}}$) as g

each update changes $w_{t,0}$ by $y_{n(t)}$

PLA Mistake Bound

inner product grows fast

$$\mathbf{w}_f^T \mathbf{w}_{t+1} \geq \mathbf{w}_f^T \mathbf{w}_t + \underbrace{\min_n y_n \mathbf{w}_f^T \mathbf{x}_n}_{\rho}$$

length^2 grows slowly

$$\|\mathbf{w}_{t+1}\|^2 \leq \|\mathbf{w}_t\|^2 + \underbrace{\max_n \|\mathbf{x}_n\|^2}_{R^2}$$

Magic Chain!

$$\begin{aligned}
\mathbf{w}_f^T \mathbf{w}_1 &\geq \mathbf{w}_f^T \mathbf{w}_0 + \rho \\
\mathbf{w}_f^T \mathbf{w}_2 &\geq \mathbf{w}_f^T \mathbf{w}_1 + \rho \\
\mathbf{w}_f^T \mathbf{w}_3 &\geq \mathbf{w}_f^T \mathbf{w}_2 + \rho \\
&\dots \\
\mathbf{w}_f^T \mathbf{w}_T &\geq \mathbf{w}_f^T \mathbf{w}_{T-1} + \rho
\end{aligned}$$

Magic Chain!

$$\begin{aligned}
\|\mathbf{w}_1\|^2 &\leq \|\mathbf{w}_0\|^2 + R^2 \\
\|\mathbf{w}_2\|^2 &\leq \|\mathbf{w}_1\|^2 + R^2 \\
\|\mathbf{w}_3\|^2 &\leq \|\mathbf{w}_2\|^2 + R^2 \\
&\dots \\
\|\mathbf{w}_T\|^2 &\leq \|\mathbf{w}_{T-1}\|^2 + R^2
\end{aligned}$$

start from $\mathbf{w}_0 = \mathbf{0}$, after T mistake corrections,

$$1 \geq \frac{\mathbf{w}_f^T}{\|\mathbf{w}_f\|} \frac{\mathbf{w}_T}{\|\mathbf{w}_T\|} \geq \frac{T\rho}{1\sqrt{T}R} \Longrightarrow T \leq \left(\frac{R}{\rho}\right)^2$$