Subject : Intro. to Info. Theory

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* random programs $\underline{p}$ $\xrightarrow[U]{\text{universal model}}$ random computable strings $\underline{s}$

"monkey-typing" distribution "simple-string" distribution

* monkey-typing distribution

$$\boxed{\begin{array}{lll} |\underline{p}|=1 : & Pr(\underline{p}) = \frac{1}{2} & \# \underline{p} = 2 \\ |\underline{p}|=2 : & Pr(\underline{p}) = \frac{1}{4} & \# \underline{p} = 4 \\ \vdots & & \vdots \end{array}}$$

$\sum_{\underline{p}} Pr(\underline{p})$ not converge

* fix : ① all "valid" (halting) $\underline{p}$?

halting not decidable

& no guarantee about $\sum_{\underline{p}} Pr(\underline{p})$

② decrease $Pr(\underline{p})$? e.g. $Pr(\underline{p}) = 2^{-2|\underline{p}|}$

possible, but not seen in literature

* "popular fix" : consider prefix-free programs

$\boxed{\{\text{valid programs}\} \text{ form prefix-free set of codes}}$

$\sum_{\text{valid } \underline{p}} 2^{-|\underline{p}|} \leq 1$ immediately from Kraft's

* want : universal (Turing Machine) w/ prefix-free programs

e.g. original program $\underline{p} = p_1 p_2 \cdots p_n$ $\Rightarrow$ prefix-free program $0p_0 0p_1 0p_2 \cdots 0p_n 1$ (also-called self-delimiting)

prefix-free universal TM "crashes" for invalid $\underline{p}'$

another angle

line 1

line 2

return

~~line 4~~ not accepted for prefix-free program

* $\boxed{\text{simple-string}}$ probability
universal

$$P(\underline{s}) = \sum_{\underset{\text{prefix-free program}}{\underline{p}} : \underset{\text{prefix-free universal TM}}{U}(\underline{p}) \underset{\text{halts w/o crashing}}{=} \underline{s}} \underbrace{2^{-|\underline{p}|}}_{\text{monkey typing}}$$

* $\sum_{\underline{s}} P(\underline{s}) \leq 1$, "unnormalized" (semi-) probability

* "normalization constant"

$$\Omega = \sum_{\underline{p} : U(\underline{p}) \text{ halts}} 2^{-|\underline{p}|}$$

prefix-free program

Chaitin's magical number

cannot be (approximately) computed

approximate Ω to N-bits
+ run all N-bits programs until enough halts) other $\underline{p}$'s won't halt

$\Downarrow$

decide all N-bits $\underline{p}$ on halting

* universal probability for prediction

given $\underline{S}_{t-1} = S_1 S_2 \cdots S_{t-1}$?

$\uparrow$

S_t

$$P_*(\underline{s}) = \sum_{*} P(\underline{s}*) = \sum_{U(\underline{p}) = \underline{s}*} 2^{-|\underline{p}|}$$