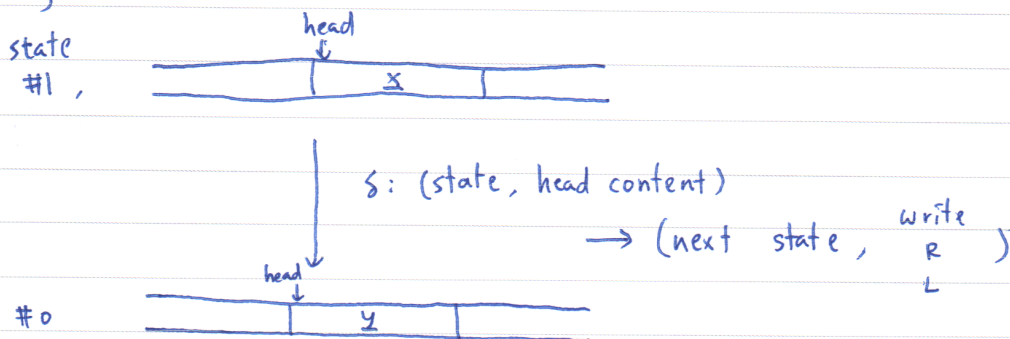


* computing model :

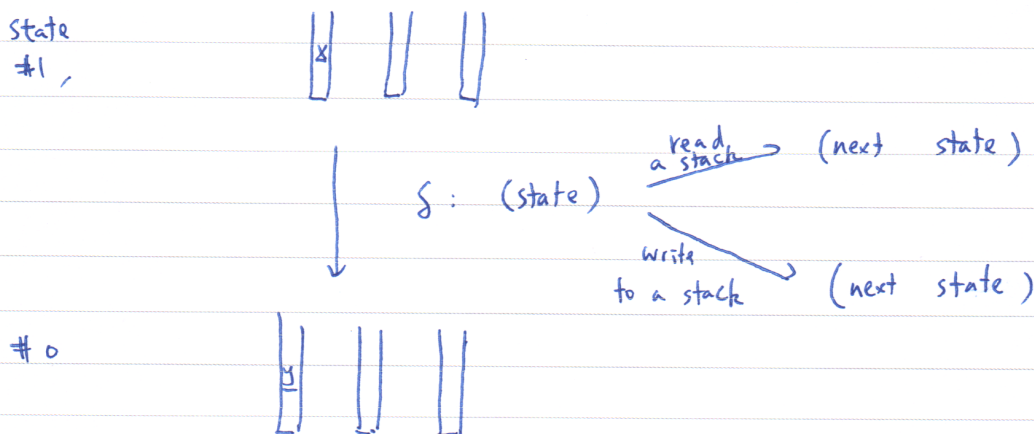
(ideally) mechanical process ("algorithm") to generate

$$\begin{array}{ccc} \text{output } y & = & f(x) \\ & & \downarrow \text{input} \\ & & \text{computable function} \end{array}$$

Turing machine



Stack machine



* universal computing model U

run program as input to "simulate"

$$U(\underbrace{P_f}_\text{program of } f, x) = f(x)$$

* K_U is (almost) "model-independent"

$$K_{U_1}(\underline{x}) \leq \underbrace{K_{U_2}(\underline{x})}_{c_2} + \underbrace{c_1}_{c_1}$$

P_2 : program of U_2 on U_1

P_s : shortest program that generates $\underline{x}$ on U_2

$$U_1(P_2 P_s) = U_2(P_s) = \underline{x}$$

$$\text{so } K_{U_1}(\underline{x}) \leq \underbrace{|P_2|}_{c_2} + \underbrace{|P_s|}_{K_{U_2}}$$

$$K_{U_2}(\underline{x}) \leq K_{U_1}(\underline{x}) + c_1$$

$\Downarrow$

$$K_{U_2}(\underline{x}) - c \leq K_{U_1}(\underline{x}) \leq K_{U_2}(\underline{x}) + c \quad \text{w/ } c = \max(c_1, c_2)$$

* K not too big

$$K(\underline{x}) \leq |\underline{x}| + \underbrace{c}_{\text{trivial program}}$$

* K will be big for some $|\underline{x}|$

$$\exists |\underline{x}| \text{ s.t. } K(\underline{x}) \geq |\underline{x}| \quad (\text{cannot always compress})$$

* K can be very short

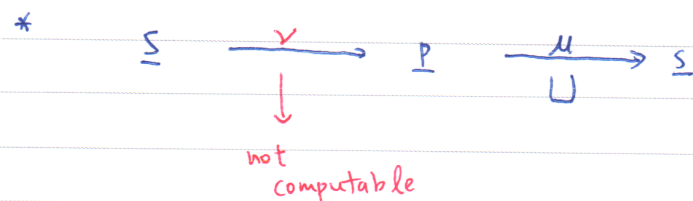
$$K(0^N) \leq \underbrace{\log N}_{\text{input + variable}} + \underbrace{c}_{\text{for loop}}$$

* very few strings can be compressed to short program

$$\# \text{ programs with } < N-M \text{ bits : } \frac{2^{N-M} - 1}{2^N}$$

$$\# \text{ strings of length } N : 2^N$$

fraction of $\underline{x} \in \{0,1\}^N$
 $\psi K(\underline{x}) \leq N-10$
 < 0.001



if Σ computable

$\Rightarrow K(\Sigma)$ computable

$\Rightarrow$ exists M for computing $K(\Sigma)$

$\Rightarrow$ function $F(L) \{$

for $\Sigma \in \{0,1\}^*$ from short to long $\{$

run M on Σ to get $K(\Sigma)$

if $\boxed{K(\Sigma) > L}$

return Σ

$\{ \}$

length:

$$\log L + |\text{code}| + |M|$$

returned string: $K(\Sigma) > L$

$$\text{program} \leq \log L + |\text{code}| + |M|$$

) contradiction