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* How many bits to represent an ensemble $\mathbb{X}$?

$\approx H(\mathbb{X})$ by Huffman Coding

* " an ensemble of binary strings

$\mathbb{X} = \{ \underline{s}_1, \underline{s}_2, \cdots, \underline{s}_K \}$

where each $\underline{s}_i \in \{0,1\}^*$

?

symbol table + Huffman tree + Huffman decoder + encoded bits

$\underbrace{\sum |\underline{s}_k|}$ $O(K)$ const $\approx N \cdot H(\mathbb{X})$

can we further compress individual symbol strings ?

* compression of binary strings (loseless)

$$\underset{\in \{0,1\}^*}{\underline{s}} \xrightarrow{\nu} \underset{\in \{0,1\}^*}{\nu(\underline{s})} \xrightarrow{\mu} \mu(\nu(\underline{s})) = \underline{s}$$

* cannot "always compress" $\boxed{\text{for loseless}}$

e.g. $\mathbb{X} = \{0,1\}^N$ (all length-N binary strings)

if "always compress"

$$\mathbb{Y} = \nu(\mathbb{X}) \subseteq \bigcup_{n=0}^{N-1} \{0,1\}^n$$

$$|\mathbb{Y}| \leq 2^N - 1$$

$$\underbrace{H_0(\mathbb{X})}_{\overset{\shortparallel}{N}} > \underbrace{H_0(\mathbb{Y})}_{\overset{\wedge\shortparallel}{\log(2^N-1)}}$$

$\boxed{\text{no 1-1 mapping}}$ (no loseless)

(pigeon hole principle)

* easy binary strings : "0 0 0 0 ⋯ 0"

"0 1 0 1 0 1 ⋯ 0 1"

hard binary strings(?): "0 1 1 0 1 0 1 0 0 0 1 0 1 0 ⋯ " (is i prime ?)

" random string "

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* Kolmogorov (-Chaitin) Complexity

with respect to a universal computing model U

$$K_U(\underline{s}) = \min \{ |\underline{p}| : U(\underline{p}) = \underline{s} \}$$