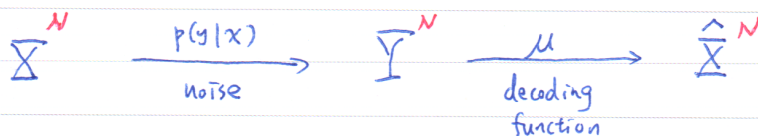


- * How many bits to represent $\bar{X}$ fixed-length? $H_0(\bar{X})$
(at least) $\bar{X}^N$ fixed-length with $\leq \delta$ error?
 $\approx N H(\bar{X})$
- $\bar{X}$ var-length? $\approx H(\bar{X})$ on average.
 $\bar{X}^N$ fixed-length with $\leq D$ distortion?
 $\approx N \cdot R_I(D)$
 $\min_{d_f \leq D} I(\bar{X}; \bar{Y}_f)$

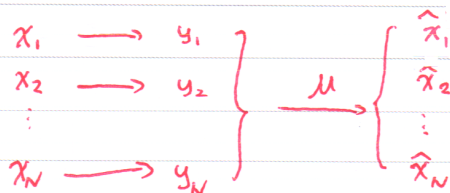
- * How many bits to transmit $\bar{X}^N$
(at most) through $p(y|x)$ "noise"
fixed-length with small error?
 $\leq N \cdot C$
 $\max_{\bar{Y}} I(\bar{X}; \bar{Y})$

* model:



error: $[\hat{\bar{X}} = \mu(\bar{Y}) \neq \bar{X}]$

N uses of channel



* contrast:

compression	quantization	transmission
$[\hat{\bar{X}} = \mu(\nu(\bar{X})) \neq \bar{X}]$	$d(\bar{x}, \nu(\bar{x}))$ $\bar{y}$	$[\bar{X} = \mu(\bar{Y}) \neq \bar{X}]$
total err.	avg. distortion	worst error
$\mathbb{E} [\dots]$	$\mathbb{E} d(\bar{x}, \nu(\bar{x}))$	$\max_{\bar{X}} P_E^{(\bar{X})} = \mathbb{E} [\dots]$

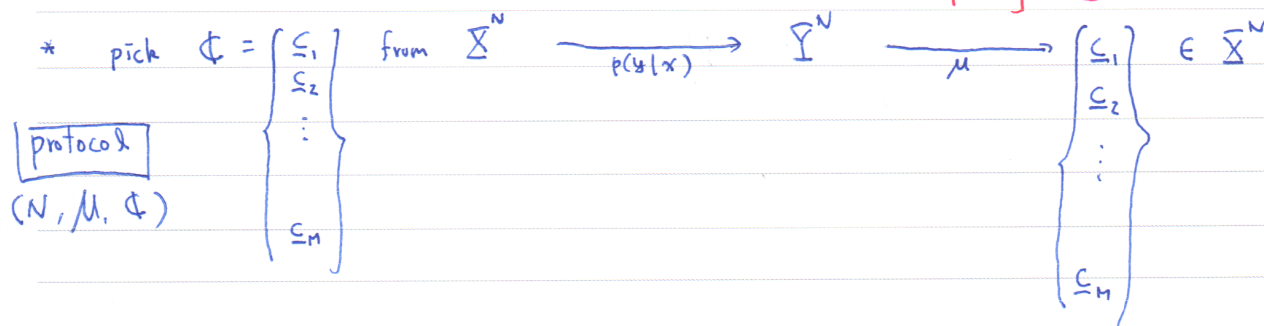
* "safe" transmission

transmit $\{x_1, x_2, \dots, x_M\}$ only such that $P_E^{(x_m)} \leq \epsilon$
||
 $P_E^{(m)}$ for all m

Q: what's maximum M ?

$$\text{rate } R = \frac{\log M}{N}$$

A: $\approx$ maximum mutual information $\max_{p(x)} I(\bar{X}_N; \bar{Y}_N)$



formal statement: given $0 < R < C$ and $\epsilon > 0$

there exists protocol with $\frac{\log M}{N} \geq R$ and $P_E^{(m)} \leq \epsilon$ for all m
rate

* conceptual proof:

① assume $q(x)$ achieves C , construct joint distribution $p(x, y)$
(rate-distortion = $q(y|x)$ " $R(D)$ ")

② construct $\mathcal{C}$ by sampling from $p(x)$ M times
(" $p(y)$ ")

③ let $\mu(y) = c$ iff only c is "typical" with y (jointly)
($\nu(x) = c$ iff $d(x, c)$ smallest)

existence
arg. $\neg$

④ bound atypical cases $\begin{cases} \text{no typical } c : \text{small prob. if } M \text{ large} \\ \text{many typical } c : \text{small prob. if } M \text{ not too large} \end{cases}$
($\begin{cases} \text{atypical } x : \text{small prob.} \\ \text{typical } x \text{ w/o typical } y : \text{small prob.} \end{cases}$)

⑤ handle max error by throw away some cases