* see note for $R(D)$ of binary $\bar{X}, \bar{Y}$
  (and play in HW2 for symmetric $\bar{X}, \bar{Y}$)

* rate-distortion theorem ($\rightarrow$)

  if $R_I(D) = \min_{q:\, \bar{d}_q \le D} I_q(\bar{X}; \bar{Y}) = R$, for given ϵ

  there exists VQ scheme w/ $\underbrace{M \le 2^{(R+\epsilon)N}}_{\text{codewords}}$ and $\bar{d} \le D + \epsilon$

  $\Downarrow$ VQ rate $(R+\epsilon)$ $\qquad$ $\Downarrow$ VQ distortion $(D+\epsilon)$

  for N large enough

* VQ scheme :

  every $x \in \bar{X}^N \xrightarrow{\nu}$ one $\underline{y} \in \bar{Y}^N$ called $\underline{c}$ (codeword)

  $p(x)$ $\quad x_3 \quad \underline{c}_1$

  $d(x, \underline{y}) = \frac{1}{N} \sum d(x_n, y_n)$ $\qquad$ M codewords

  $\underline{c}_M$

  $\bar{d} = \sum_{\underline{x}} p(x)\, d(\underline{x}, \nu(\underline{x}))$

  will "construct" $\{\underline{c}_m\}$ probabilistically from "the" q
  that achieve $I_q = R$ and
  $\bar{d}_q \le D$

* idea 1 :

  - get typical $\underline{x}_i \in \bar{X}^N$ by $y_n \sim q(y \mid (\underline{x}_i)_n)$
    and assign typical $\underline{y} \in \bar{Y}^N \mid \underline{x}_i$ as $\nu(\underline{x}_i)$
    $\Downarrow$ $\quad \underline{y} = \underline{c}_i$
  - otherwise assign $\underline{c}_0$ and suffer d_* with small $p(\underline{x})$
  - $\bar{d}_q \le D \Rightarrow d(\text{typical } \underline{x}, \text{typical } \underline{y}) \lesssim D$ (small VQ distortion)
  - $M \approx NH(\bar{X})$ typical $\underline{x}_i \in \bar{X}^N$: (large VQ rate $\gtrsim H(\bar{X}) \ge R$)

* typicality revisited :

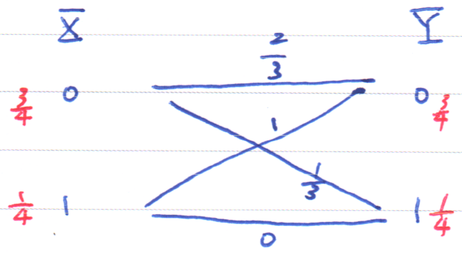
T_ϵ : typical set (of X^N) at width ϵ

$$2^{-(H+\epsilon)N} \leq P(\underline{x}) \leq 2^{-(H-\epsilon)N}$$

typical set of $(XY)^N$

$$2^{-(H(X,Y)+\epsilon)N} \leq P(\underline{x},\underline{y}) \leq 2^{-(H(X,Y)-\epsilon)N}$$

- cannot guarantee $\underline{x}$ to be typical



X	Y	
0	0	$\frac{1}{2}$
0	1	$\frac{1}{4}$
1	0	$\frac{1}{4}$
1	1	0

$(x_1, y_1), (x_2, y_2), \cdots, (x_N, y_N)$

w/ half $(0,0)$, half $(0,1)$, no $(1,0)$, no $(1,1)$

is in $T_\epsilon(X,Y)$ but $\underline{x} = (0,0,0,\cdots,0)$

not typical

- cannot guanrantee $d(\underline{x},\underline{y}) \approx \bar{d}_g$

* need stronger notion of typicality T:

$$\left| \frac{1}{N} \log \frac{1}{P(\underline{x},\underline{y})} - H(X,Y) \right| \leq \epsilon/6$$

and $\left| \frac{1}{N} \log \frac{1}{P(\underline{x})} - H(X) \right| \leq \epsilon/6$ jointly typical $\Rightarrow \frac{1}{N} \log \frac{P(\underline{x},\underline{y})}{P(\underline{x})P(\underline{y})} - I(X;Y)$

$\leq \epsilon/2$

and $\left| \frac{1}{N} \log \frac{1}{P(\underline{y})} - H(Y) \right| \leq \epsilon/6$

and $\left| d(\underline{x},\underline{y}) - \bar{d}_g \right| \leq \epsilon/2$ distortion-typical

$\downarrow$

still $\geq 1-\delta$ prob. when N large enough by letting each condition fail w/ prob. $< \frac{\delta}{4}$