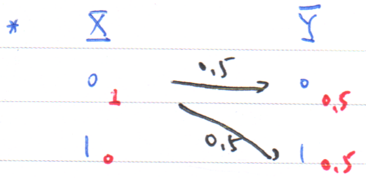
Subject : Intro. to Info. Theory

No. : 1

Date : 10 / 29 / 2019

*



$H(Y)$ contains mostly "transition uncertainty"

regardless of the original info in X

(i.e. typical in X^N)

avg. transition uncertainty w.r.t. x

$$\underbrace{\sum_x p(x) \overbrace{\sum_y p(y|x) \log \frac{1}{p(y|x)}}}_{H(Y|X)}$$

(note : use $p(y|x)$ instead of $q(y|x)$ for simplicity)

* will show that "best" VQ related to coding of Y "–" $Y|X$

* consider $p(x,y) = p(x)\, p(y|x) = p(y)\, p(x|y)$

$$H(X,Y) = \sum_{x,y} p(x,y) \log \frac{1}{p(x,y)}$$ (joint ensemble entropy)

$$H(X) = \sum_x p(x) \log \frac{1}{p(x)} = \sum_{x,y} p(x,y) \log \frac{1}{p(x)}$$

$$H(Y) = \sum_y p(y) \log \frac{1}{p(y)} = \sum_{x,y} p(x,y) \log \frac{1}{p(y)}$$

$$H(Y|X) = \sum_x p(x) \sum_y p(y|x) \log \frac{1}{p(y|x)} = \sum_{x,y} p(x,y) \log \frac{1}{p(y|x)}$$

$$H(X|Y) = \quad \cdots \quad = \sum_{x,y} p(x,y) \log \frac{1}{p(x|y)}$$

$$= \sum_{x,y} p(x,y) \log \frac{p(y)}{p(x,y)}$$

$$= H(X,Y) - H(Y)$$

$$H(Y|X) = H(X,Y) - H(X)$$

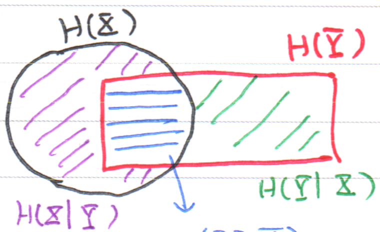
* want : Y "$-$" $Y \mid X$

$$H(Y) - H(Y \mid X)$$

$$= \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(y)\,p(x)}$$

$$= I(X;Y)$$

$\nwarrow$ mutual information



$$I(X;Y)$$
$$\parallel$$
$$I(Y;X)$$

* $$0 \leq \underbrace{I(Y;X)}_{H(Y)-H(Y|X)} \leq H(Y)$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$H(Y|X) \leq H(Y) \qquad H(Y \mid X) \geq 0$$

"=" iff $p(y|x) = 0$ or 1 always

$\langle$pf$\rangle$ $\displaystyle \sum p(x,y) \log \frac{p(x)p(y)}{p(x,y)}$

$$\leq \frac{1}{\ln 2} \sum p(x,y) \left(\frac{p(x)\,p(y)}{p(x,y)} - 1 \right)$$

"=" iff $p(x)\,p(y) = p(x,y)$

statistical independence

$$= \frac{1}{\ln 2} \left(\sum \sum p(x)\,p(y) - \sum p(x,y) \right)$$

$$= 0$$

* some "physical" meanings of $I(X;Y)$

typical in X^N : $p(x) \approx 2^{-N H(X)}$

typical in Y^N : $p(y) \approx 2^{-N H(Y)}$

typical in $(X,Y)^N$: $p(x,y) \approx 2^{-N H(X,Y)}$

$\to$ "jointly" typical : $\dfrac{p(x)\,p(y)}{p(x,y)} \approx 2^{-N\, I(X;Y)}$

Subject : Intro. to Info. Theory

No. : 3

Date : 10/29/2019

* typical in $\mathbb{X}^N$: # $\approx 2^{NH(\mathbb{X})}$
  typical in $\mathbb{Y}^N$: # $\approx 2^{NH(\mathbb{Y})}$
  typical in $(\mathbb{X}, \mathbb{Y})^N$: # $\approx 2^{NH(\mathbb{X}, \mathbb{Y})}$

chance of getting "matching" $(\underline{x}, \underline{y})$

by typical $\underline{x} \sim p(\underline{x})$, typical $\underline{y} \sim p(\underline{y})$

$$\Downarrow$$

$$\frac{2^{NH(\mathbb{X})}\; 2^{NH(\mathbb{Y})}}{2^{NH(\mathbb{X},\mathbb{Y})}} = 2^{-N}$$

* recall

per-symbol quantization distortion w.r.t. scheme q

$$\bar{d}_q = \sum_x p(x) \sum_y q(y|x)\, d(x,y)$$

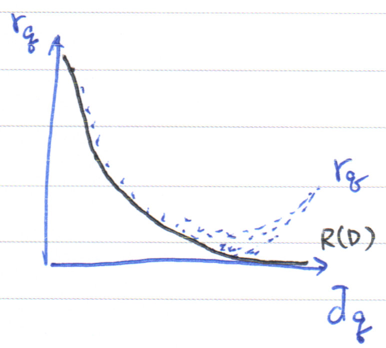
q results in ensemble $\mathbb{Y}$

$$I(\mathbb{X};\mathbb{Y}) = \sum_x \sum_y p(x)\, q(y|x) \log \frac{p(x)\; q(y|x)}{p(x)\; \underset{\sum_x p(x)\, q(y|x)}{\underset{\parallel}{p(y)}}}$$

* Q : (min) # bits to achieve distortion constraint D (given $\mathbb{X}$)

$\Downarrow$ $\boxed{\text{will show}}$

$$R(D) \equiv \left[\min_q \underbrace{I_q(\mathbb{X};\mathbb{Y})}_{\substack{\text{will call "rate"} \\ r_q}} \quad \text{s.t.} \quad \bar{d}_q \le D \right]$$



R(D)

- monotonic
- convex (see notes)