Subject : Intro. to Info. Theory

No. : 1
Date : 9 / 17 / 2019

* which digit are you thinking?

5	3	2
6	4	4
7	7	6
8	8	8

$\{1, 2, 3, 4, 5, 6, 7, 8\} \longrightarrow$?

*

$$\text{alternatives} \xrightarrow[\text{information}]{\text{specify}} \text{outcome}$$

(uncertain) (received) (certain)

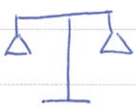
* 4 criteria
    - alternatives — a set $\mathcal{X} = \{a_1, a_2, \cdots, a_K\}$
    - abstract
    - bidirectional — to be specified
    - additive — additively

* deterministic information (necessary information for **0-error**)

$$H_0(\mathcal{X}) = \log_2 K \qquad \text{(bits to specify)}$$

$$K = |\mathcal{X}|$$

* ball-balance game



$\{L, B, R\}$ 2 times $\Rightarrow \log_2 9$ bits (recei

10 balls, 1 heavier $\Rightarrow \log_2 10$ bits (needed)

(received) < (needed) : impossible

(received) $\geq$ (needed) : possible $\boxed{\text{only if}}$ physical

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No. : 2
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* formal reason for impossibility

pigeon hole principle

if $\underbrace{|X|}_{\downarrow} > \underbrace{|Y|}_{\downarrow}$, there is no one-to-one function from X to $|Y|$

pigeons cages

(alternatives) (names)

one-to-one ν : $\forall\ a_1 \neq a_2$, $\nu(a_1) \neq \nu(a_2)$

$\Downarrow$

$\exists\ \mu$ s.t. $\mu(\nu(a_k)) = a_k$

* e.g. impossible to use $< \log(n!)$ comparisons to sort n different elements

* e.g.

impossible to use $< \log n$ comparisons to search for $a \in X$

possible to use $\log n$ only if a_k's sorted

* additivity of H_0

$$\underbrace{H_0(A \times B)}_{\log(K_A \cdot K_B)} = \underbrace{H_0(A)}_{\log K_A} + \underbrace{H_0(B)}_{\log K_B}$$

* Can we do better than H_0?

No (as pigeon hole principle says) if zero-error

Yes (?) if $\boxed{\text{some error}}$

* 10 balls, 1 heavier } if $\text{Prob}(\text{ball } i \text{ heavy}) = \frac{i}{55}$

2 uses of scale

ignore 1st ball, identify $\{a_2, \cdots, a_{10}\}$ w/ 2 uses

↓

error prob. $= \frac{1}{55} < 2\%$

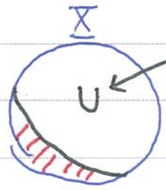
* probabilistic information w/ error $\leq \delta$

$\underbrace{\mathbb{X}, \quad p}_{\text{called an ensemble}}$ on $\mathbb{X}$ as probability $\begin{cases} p(a_k) \geq 0 \\ \sum_k p(a_k) = 1 \end{cases}$

$\Pr(U)$ for $\underbrace{U}_{\text{event}} \subseteq \mathbb{X} = \sum_{a_k \in U} p(a_k)$

$$H_\delta(\mathbb{X}) = \log_2 \min\{|U| \mid U \subseteq \mathbb{X} \text{ and } \Pr(U) \geq 1-\delta\}$$

e.g. $H_{\frac{1}{55}}(\mathbb{X}) \underset{\leq}{=} \log_2 9$



as small as possible

$\leq \delta$ in probability

e.g. many "possible" images, few realistic ones

* $\underset{\text{set}}{\underset{\downarrow}{H_0(\mathbb{X})}} = \underset{\text{ensemble}}{\underset{\downarrow}{H_0(\mathbb{X})}}$ if $\boxed{\forall k \;\; p(a_k) > 0}$

every element needed in U

* if $\delta < 1$

$\Rightarrow$ requiring $\Pr(U) > 0$

$\Rightarrow$ requiring $|U| \geq 1$

$\Rightarrow$ $H_\delta(\mathbb{X}) \geq 0$