DSA Homework #2 Reference Solution

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2.1

Algorithm 1 My algorithm

(1)

```
function BinSearch(recordarrayarr, integerlen, integervalue)
    left ← 0, right ← len
    ans ← −1
    while left < right do
        mid ← ⌊(left+right)/2⌋
        if arr[mid] = value then
            ans ← mid, left ← mid + 1
        else if arr[mid] > value then
            right ← mid
        else if arr[mid] < value then
            left ← mid + 1
    if ans = −1 then return NOTFOUND
    return arr[ans]
```

(2) They will appear in the same relative order, because that the new element will always be sorted to the end of the elements with the same value due to the while condition $A[j] > cur$ in the code.

2.2

(1) For $n \in \mathbb{N}$, $f(n) = \lceil g(n) \rceil \leq g(n) + 1 \leq g(n) + g(n)/\epsilon = (1 + 1/\epsilon)g(n)$ since $g(n) \geq \epsilon$.

(2) There exist $n_1, n_2 \in \mathbb{N}$ and $c_1, c_2 > 0$ such that for $i = 1, 2$ and $n \geq n_i$, $f_i(n) \leq c_i g_i(n)$. Then, for $n \geq \max\{n_1, n_2\}$, $f_1(n)f_2(n) \leq c_1 c_2 g_1(n) g_2(n)$.

(3) Let $n_0 = 1$ and $c = c'$. Then, $f(n) \leq cg(n)$ for all $n \geq n_0$.

(4) There exist $n_0 \in \mathbb{N}$ and $c > 0$ such that $f(n) \leq cg(n)$ for $n \geq n_0$. Let $c' = \max_{n<=n_0}\{f(n)/g(n), c\}$. Then, if $n < n_0$, we have $f(n) = f(n)/g(n) \cdot g(n) \leq c'g(n)$, and if $n \geq n_0$, $f(n) \leq cg(n) \leq c'g(n)$.

(5) The Horner's method requires n times of addition and n times of multiplication, so the total number of arithmetic operations this method executes is $2n = O(n)$.

(6) If $f(n) = n \bmod 2$ and $g(n) = 1 - (n \bmod 2)$, then neither $f(n) = O(g(n))$ or $g(n) = O(f(n))$ is true.

(7) Let $f(n) = 1/n$ and suppose that $f(n) = O(f(n)^2)$. Then, there exists $c > 0$ such that $f(n) \leq cf(n)^2$ for all $n \in \mathbb{N}$, i.e., $n \leq c$ for all n, a contradiction.