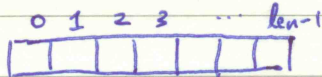


\* data structure

construct  $\leftarrow \dots \rightarrow$  destruct

get  $\Rightarrow$  find

put  
remove ) dynamic maintenance



\* C/c++ (build-in) array 有編號的櫃子.

① construct

Fixed Len Array

destruct

C: `int arr[] = (int*) malloc(sizeof(int) * len);` `free(arr);`

C++: `int arr[] = new arr[len];` `delete[] arr;`

initialized to 0

uninitialized

may need other initialization steps to EMPTY

int

```
① findAtIndex(int idx) {
    return arr[idx];
}
```

void

```
② putAtIndex(int idx, int& value) { // throw away original if non-empty
    arr[idx] = value;
}
```

fast "random" access efficiently (indexed)

by translating `arr[idx]` to

`* (arr + idx)`

start steps, each of length size of (int)

③ put (value) or insert (value)

for  $i \leftarrow 0$  to  $len-1$

if findAt Index ( $i$ ) = EMPTY
putAt Index ( $i$ , value)
return

// error (full) if here.

④ remove At Index ( $idx$ )

$arr[idx] = \text{EMPTY}$

\* Consecutive Array

a Fixed Len Array such that

all non-EMPTY elements are in

$arr[0], arr[1], \dots, arr[\text{end}-1]$

# elements

① construct :

from nothing : Fixed Len Array. construct()

from Fixed Len Array :

"compress" EMPTY spaces

① findAt Index : trivial

② putAt Index ( $idx$ , value)

if ( $idx < \text{end}$ )

else

// something needs to be done

③ insert (value) :

insert at "end" and  $\text{end} \leftarrow \text{end} + 1$ , perhaps

④ remove At Index ( $idx$ ) :

roughly, swap with  $arr[\text{end}]$  and then remove, perhaps

Consecutive Array "easier" insert by keeping "end"  
slightly harder putAtIndex and removeAtIndex

\* array for storing top game entries

```
class Record {
public:
    string name;
    int score;
};
```

need:

- ① often: list top 20 players in order
- ① often: update list by "inserting" new record & removing old one (lower)
- ② seldom: remove some records

\* if using Fixed Len Array of size 20

- ① find First, find Second, ..., find 19th, find 20th?
- same for using Consecutive Array

Ordered Array:

a Consecutive Array such that

$$\text{arr}[0] \geq \text{arr}[1] \geq \text{arr}[2] \geq \dots \geq \text{arr}[\text{end}-1]$$

• score

① construct

from nothing: ConsecutiveArray.construct()

from ConsecutiveArray: selectionSort()



(new) list Orderly ()

for  $i \leftarrow 0$  to  $len-1$

print  $arr[i]$

① findAt Index : trivial

② putAt Index : VERY restricted

③ insert (value) :

先"找到"

從頭 for  $i \leftarrow 0$  to  $end-1$

if  $arr[i] < value$

插隊

從尾 for  $i \leftarrow end-1$  to  $0$

if  $arr[i] > value$

插隊

④ remove At Index ( $idx$ ) :

"補位"

for  $i \leftarrow idx+1$  to  $end-1$

$arr[i-1] \leftarrow arr[i]$

$end \leftarrow end-1$

找到 + 插隊

can be combined here

(see textbook)

\* about "找到" find Index Of (value)

find Record of (key)

score here

naive: Sequential Search

$len-1$

non-Consecutive

for  $i \leftarrow 0$  to  $end-1$  Consecutive

if  $arr[i].score = key$

return  $arr[i]$

return FAIL

Ordered 偷吃步

else if  $arr[i].score < key$

return FAIL

\* 偷吃步 ① for Ordered

if  $\underset{\text{score}}{\text{arr}[i]} < \text{key}$  後面都不用看了

偷吃步 ② for Ordered

if  $\text{arr}[i] > \text{key}$  前面都不用看了

$[\text{left}, \text{right}) \leftarrow [0, \text{end})$

while  $\text{left} < \text{right}$  // not 空空如也

pick  $i$

if  $\text{arr}[i] = \text{key}$

return  $\text{arr}[i]$  // 找到

else if  $\text{arr}[i] < \text{key}$

find in ~~前面~~ // ①

$[\text{left}, i)$ : still Consecutive & Ordered

|||  
right  $\leftarrow i$

else if  $\text{arr}[i] > \text{key}$

find in ~~後面~~ // ②

$[i+1, \text{end})$ : still Consecutive & Ordered

|||  
left  $\leftarrow i+1$

return FAIL

\* "pick  $i$ ": ambiguous (violate definiteness)

- pick  $i$  within  $[\text{left}, \text{right})$  better, but how?
- pick  $i$  <sup>uniformly</sup> randomly within  $[\text{left}, \text{right})$ ?
- pick  $i \leftarrow \text{left}$  ?

if lucky, 找到 or 偷一大步

else 沒偷到.

## \* Binary Search

pick  $i \leftarrow \left\lfloor \frac{\text{left} + \text{right}}{2} \right\rfloor$  (usually named mid)

claim: after executing "pick  $i$ "  $\lceil \log_2 \text{end} \rceil + 1$  times  
 $\text{right} - \text{left} < 1$  (空空如也)

lemma: every <sup>non-found</sup> updating after "pick  $i$ " decreases  
 $\text{right} - \text{left}$  by at least half

case ①:  $\text{right} - \text{left}$  becomes  

$$\left\lfloor \frac{\text{left} + \text{right}}{2} \right\rfloor - \text{left} \leq \frac{\text{left} + \text{right}}{2} - \text{left}$$

$$= \frac{\text{right} - \text{left}}{2}$$

case ②:  $\text{right} - \text{left}$  becomes  

$$\text{right} - \left\lfloor \frac{\text{left} + \text{right}}{2} \right\rfloor - 1$$

$$\leq \text{right} - \left( \frac{\text{left} + \text{right}}{2} - 1 \right) - 1$$

$$= \frac{\text{right} - \text{left}}{2}$$

so after  $\lceil \log_2 \text{end} \rceil + 1$  times of "pick  $i$ "

$$\text{right} - \text{left} \leq \frac{\text{end} - 0}{2^{\lceil \log_2 \text{end} \rceil + 1}}$$

$$\leq \frac{\text{end}}{2^{\lceil \log_2 \text{end} \rceil + 1}}$$

$$\leq \frac{1}{2}$$

$$< 1$$

$\Rightarrow$  Binary search terminates after at most  $\lceil \log_2 \text{end} \rceil + 1$  iterations