More on Arrays and Structures

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What We Have Done

- Why Correctness Proof?
- Sequential and Binary Search
- Space Complexity
- Time Complexity
- Asymptotic Notations
- Practical Complexity
- Arrays
- Dense Array versus Sparse Array
- Concrete versus Abstract Data Type
- Reading Assignment:
  Some Examples of Space/Time Complexity, Dynamic 1-D Array

Arrays: from Implementation to Abstraction

C Implementation View

(One-dimensional) array is a block of consecutive memory that

- holds a list of N elements
- allows users to retrieve the k-th element
- allows users to store to the k-th location

An Abstract View

Abstract (one-dimensional) array

- holds a list of N elements
- allows users to retrieve the k-th element
- allows users to store to the k-th location

different implementations:
different space/time complexity

Dense Array versus Sparse Array

one abstract array, two possible implementations

```
int dense[10] = {1, 3, 0, 0, 0, 0, 0, 0, 0, 2};
int sparse[3][2] = {{0, 1}, {1, 3}, {9, 2}};
```

- dense array: store everything (consecutively), needs 10 positions
  - space: $O(N)$ for a length-N array
  - retrieving: $O(1)$
  - storing: $O(1)$
  - creating: $O(1)$
- sparse array: store only non-zero (index, element) pairs, needs 3 pairs
  - space: $O(E)$ for E elements, better than $O(N)$ if E small
  - retrieving: $O(\log E)$ if index ordered (HOW?)
  - storing: ???
  - creating: ???

note: often use **array** to mean dense array only

Concrete Data Type (Sec. 1.4)

array consists of ...

- objects: a set of (*index*, *element*) pairs (== a list of elements)
- actions: retrieve, store, create which sets/gets the objects

- concrete data type: the **actual outcome** of the type
  - object representation + action implementation
  - for actual coding, per-platform optimization, etc.

(dense 1-D) array in C

- object representation: a block of consecutive memory, with a chunk representing each *element* element for each *index*
- action implementation: $[\cdot]$ for retrieving and storing, `malloc` for creating, etc.

Abstract Data Type (Sec. 1.4)

array consists of ...

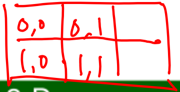
- objects: a set of (*index*, *element*) pairs (== a list of elements)
- actions: retrieve, store, create which sets/gets the objects

- abstract data type: the **pseudo essence (contract)** of the type
  - object specification + action specification
  - for illustration, high-level analysis, etc.

abstract 1-D array

- object specification: (*index*, *element*) pairs with *index* $\in \{0, \cdots, N-1\}$
- action specification:
  retrieve(*index*) returns the *element* associated with *index*;
  store(*index*, *element*) sets *element* to be associated with *index*;
  create(N) creates the objects, etc.
  (sometimes with time/space constraints)

will usually look at abstract data type first before going concrete



abstract rectangular 2-D array

- object specification: (*index*, *element*) pairs with $index \in \{(0,0),(0,1),\cdots,(N-1,M-1)\}$
- action specification: retrieve(*index*); store(*index*, *element*); create(N, M), etc.

2-D array by 1-D array in C

- object representation: a block of consecutive memory of size $N * M$, with a chunk representing each *element* for each *index*
- action implementation:

2-D Array (Subsec. 2.2.2): by 1-D Array

```
#define N (100)
#define M (200)
int* twodim = (int*)malloc(sizeof(int)*N*M);

int get(int* arr, int n, int m)
  { return arr[n*M + m]; }
```

2-D Array: by 1-D Array with Constant Folding

abstract rectangular 2-D array

- object specification: (*index*, *element*) pairs with $index \in \{(0,0),(0,1),\cdots,(N-1,M-1)\}$
- action specification: retrieve(*index*); store(*index*, *element*); create(N, M), etc.

2-D array by 1-D array with constant folding in C

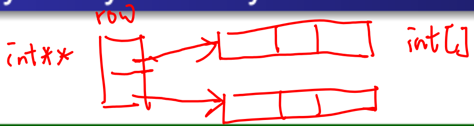
- object representation: a block of consecutive memory of size $N * M$, with a chunk representing each *element* for each *index*
- action implementation:

2-D Array: by 1-D Array with Constant Folding

```
#define N (100)
#define M (200)
int twodim[N][M];

int get(int arr[][M], int n, int m)
  { return arr[n][m];}
```

2-D Array: by Array of Arrays



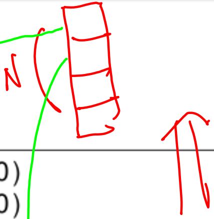
abstract rectangular 2-D array

- object specification: (*index*, *element*) pairs with $index \in \{(0,0),(0,1),\cdots,(N-1,M-1)\}$
- action specification:
  retrieve(*index*); store(*index*, *element*); create(N, M), etc.

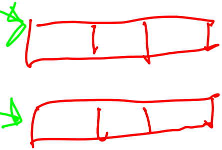
2-D array by array of arrays in C

- object representation: N blocks of consecutive memory of size M
- action implementation:

2-D Array: by Array of Arrays




```
#define N (100)
#define M (200)
int** twodim = (int**)malloc(sizeof(int*)*N);
for(int n=0;n<N;n++)
  twodim[n] = (int*)malloc(sizeof(int)*M);
int get(int** arr, int n, int m)
  { return arr[n][m];}
```



Comparison of Three Implementations

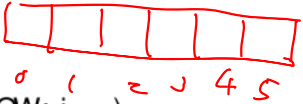
```
int* twodim = (int*)malloc(sizeof(int)*N*M);
int twodim[N][M];
int** twodim = (int**)malloc(sizeof(int*)*N);
```

	1	2	3
space	$N * M$ integers	$N * M$ int.	$N * M$ int. + N pointers
type	`int*`	`int*[M]`	`int**`
create	constant	constant	$O(N)$
retrieve	arithmetic+dereference	arith.+deref.	deref.+deref.

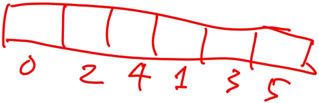
method 2 for static allocating; method 1 or 3 for dynamic allocating (your choice)

A Tale between Two Programs

```
int rowsum(){
  int i, j;
  int res = 0;
  for(i=0;i<MAXROW; i++)
    for(j=0;j<MAXCOL; j++)
      res += array[i][j];
}
```




```
int colsum(){
  int i, j;
  int res = 0;
  for(j=0;j<MAXCOL; j++)
    for(i=0;i<MAXROW; i++)
      res += array[i][j];
}
```



Reading Assignment

be sure to go ask the TAs or me if you are still confused

Polynomials (Sec. 2.4)

```c
typedef struct{
  int degree;
  double* coef;
} densepoly;
typedef struct{
  int nTerms;
  int* expo; /* expo[i] ordered */
  double* coef;
} sparsepoly;
```

- `densepoly` versus `sparsepoly`: like dense array versus sparse
- a simple polynomial adding algorithm
  - allocate the resulting poly
  - fill in the values
- trivial for `densepoly` (HW1), slightly harder for `sparsepoly`

Adding Sparse Polynomials

```c
typedef struct{
   int nTerms;
   int* expo; /* expo[i] ordered */
   double* coef;
} sparsepoly;
```

add $x^{100} + 2x^3 + 3$ and $4x^4 + 5x^3 + 6x + 7$

- allocate the resulting poly
- fill in the values

Reading Assignment

be sure to go ask the TAs or me if you are still confused

Sparse Matrix (Sec. 2.5)

$$\begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix}$$

Specialty

a rectangular 2-D array that contains many common elements (0) that we may not want to repeatedly store

Matrix Abstract Data Type

matrix consists of ...

- objects: a set of (*row*, *column*, *value*) triples where *value* is numerical
- actions: create, transpose, add, multiply, (retrieve, store)

- dense implementation: as 2D dense arrays
- array of array implementation:
  - "(dense 1D) of (sparse 1D)"
  - "(sparse 1D) of (sparse 1D)"
- ordered triples implementation: our next topic

Ordered Triples Implementation

$$\begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix}$$

Ordered(-by-row-then-by-col) Triples

6	6	8
row	*col*	*value*
0	0	15
0	3	22

- space complexity ?
- time complexity for retrieve?

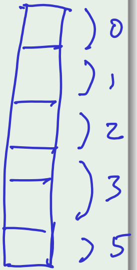
simple exercise: compare to unordered triple implementation

Transposing A Sparse Matrix

$$\begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 3 & 0 \\ 22 & 0 & -6 \\ 0 & 0 & 0 \\ -15 & 0 & 0 \end{bmatrix}$$

Ordered(-by-row-then-by-col) Triples

6	3	6
row	*col*	*value*
0	0	15
0	3	22
0	5	-15
1	1	11
1	2	3
2	3	-6



Transpose by Scanning Each *col* (Prog. 2.8)

set up a sparse matrix *res* of attributes (*col*, *row*, *elements*)
for $j \leftarrow 0$ to $col - 1$ **do**
  for $e \leftarrow 0$ to *elements* **do**
    if $e.col == j$
      append ($e.col$, $e.row$, $e.value$) to *res*
    end if
  end for
end for

- space complexity: $\Theta(elements)$ for *res*, constant for others
- time complexity: $\Theta(col * elements)$ (so $O(col * elements)$)

Netflix competition:
$row = 17700$, $col = 480189$, $elements = 100480507$
col * elements $\approx 5 \cdot 10^{13}$ ($>$ 5 hr on 2.8 GHz CPU)

Transpose by Scanning Once (Prog. 2.9) I

- to save time on transposing, we want to scan only once

set up a sparse matrix *res* of attributes (*col*, *row*, *elements*)
for ~~$j \leftarrow 0$ to $col - 1$~~ **do**
    for $e \leftarrow 0$ to *elements* **do**
        if ~~$e.col == j$~~
            append (*e.col*, *e.row*, *e.value*) to the (*e.col*)-th pile
        end if
    end for
end for

Transpose by Scanning Once (Prog. 2.9) II

- where's the j-th pile? pre-compute pile size and starting locations

set up two arrays *pilesize* and *pilestart*, each of length *col*
for $e \leftarrow 0$ to *elements* **do**
  if $e.col == j$
    $pilesize[j] \leftarrow pilesize[j] + 1$
  end if
end for
for $j \leftarrow 0$ to $col - 1$ **do**
  $pilestart[j] \leftarrow pilestart[j-1] + pilesize[j]$
end for

Transpose by Scanning Once (Prog. 2.9) III

- space complexity: $\Theta(elements)$ for *res*, $\Theta(col)$ for the helping arrays
- time complexity: $\Theta(col)$ for pre-computing, $\Theta(elements)$ for scanning

Netflix competition:
$row = 17700$, $col = 480189$, $elements = 100480507$
col * 2 + elements 10^8 ($0.04 * constant$ sec. on 2.8 GHz CPU)

Recap: Trade-off

Concrete Implementation of the SparseMatrix Data Structure

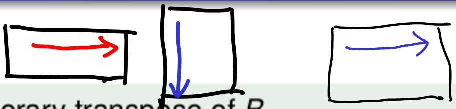
- unordered triples: simpler transpose $O(elements)$, time-consuming retrieval $O(elements)$
- ordered triples: harder transpose $O(col + elements)$, efficient retrieval $O(logrow * logcol)$

The Transpose Algorithm

- scanning each column: smaller space $O(elements)$, time-consuming algorithm $O(col * elements)$
- scanning once: bigger space $O(col + elements)$, efficient algorithm $O(col + elements)$

Good Programmer (a.k.a. you):
understand the trade-off clearly and make wise choices!

Matrix Multiplication (Subsec. 2.5.4)



do a temporary transpose of B
for $i \leftarrow 0$ to $rowA - 1$ **do**
  for $j \leftarrow 0$ to $colB - 1$ **do**
    compute $C[i, j]$ by multiplying $A[i, :]$ and $B[:, j]^T$
  end for
end for

- multiplying $A[i, :]$ and $B[:, j]^T$: similar to (sparse) polynomial adding —keep it in your toolbox
- time complexity:
  - each multiplying takes $O(\#A_i + \#B_j)$
  - total (careful counting): $O(colB * \#A + rowA * \#B)$

~~Structures and Unions~~ (Sec. 2.6)

Multi Dim   Matrix

Reading Assignment

be sure to go ask the TAs or me if you are still confused

Reading Assignment

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Strings: Pattern Matching (Subsec. 2.7.3)

find the position that *pat* (first) shows up in *string*

for $i \leftarrow 0$ to $len(string) - 1$ **do**
  if *pat* matches $strings[i, i + len(pat) - 1]$
    matching found
  end if
end for
matching not found

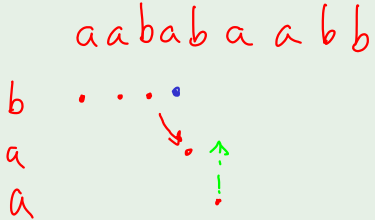
string aababaabb

pat baa

- the IF takes $O(m)$ for $m = len(pat)$
  —can use heuristic on comparing *begin* and *end* first, but still $O(m)$ in the worst case
- so total $O(n * m)$ for $n = len(string)$

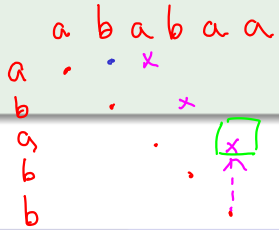
Slow Pattern Matching

$i, j \leftarrow 0$
while $i < len(string)$ and $j < len(pat)$ **do**
  if $pat[j] == string[i]$
    $i \leftarrow i + 1, j \leftarrow j + 1$ (continue matching)
  else
    $i \leftarrow i - j + 1$
    $j \leftarrow 0$
    (fail and totally go back)
  end if
end while
check matching status



Fast Pattern Matching

$i, j \leftarrow 0$
while $i < len(string)$ and $j < len(pat)$ **do**
  if $pat[j] == string[i]$
    $i \leftarrow i + 1, j \leftarrow j + 1$ (continue matching)
  else
    $i \leftarrow i$
    decrease j such that $pat[0, j-1]$ matches $string[i-j, i-1]$
    (fail but continue partially)
  end if
end while
check matching status



Fast Pattern Matching

$i, j \leftarrow 0$
while $i < len(string)$ and $j < len(pat)$ **do**
  if $pat[j] == string[i]$
    $i \leftarrow i + 1, j \leftarrow j + 1$ (continue matching)
  else
    $i \leftarrow i$
    decrease j to t such that $pat[0, t-1]$ matches $string[i-t, i-1]$
    (fail but continue partially)
  end if
end while
check matching status

- number of $i \leftarrow i + 1$: $O(len(string))$
- number of decrease j: $O($ number of $j \leftarrow j + 1$ $)$ because j is always > 0
- total: $O(len(string))$ IF the decrease step is $O(1)$

How To Make the Decrease Step $O(1)$

- main tool: pre-compute and store (like fast transpose)
- how to decrease j to t such that $pat[0, t-1]$ matches $string[i-t, i-1]$
  - originally, $pat[0, j-1]$ matches $string[i-j, i-1]$
  - want, $pat[0, t-1]$ matches $string[i-t, i-1]$
  - so, $pat[0, t-1]$ matches $pat[j-t, j-1]$

  —want: largest t such that $pat[0, t-1]$ matches $pat[j-t, j-1]$
- a trivial algorithm of $O(len(pat)^2)$ (how?)
- faster algorithm: see textbook