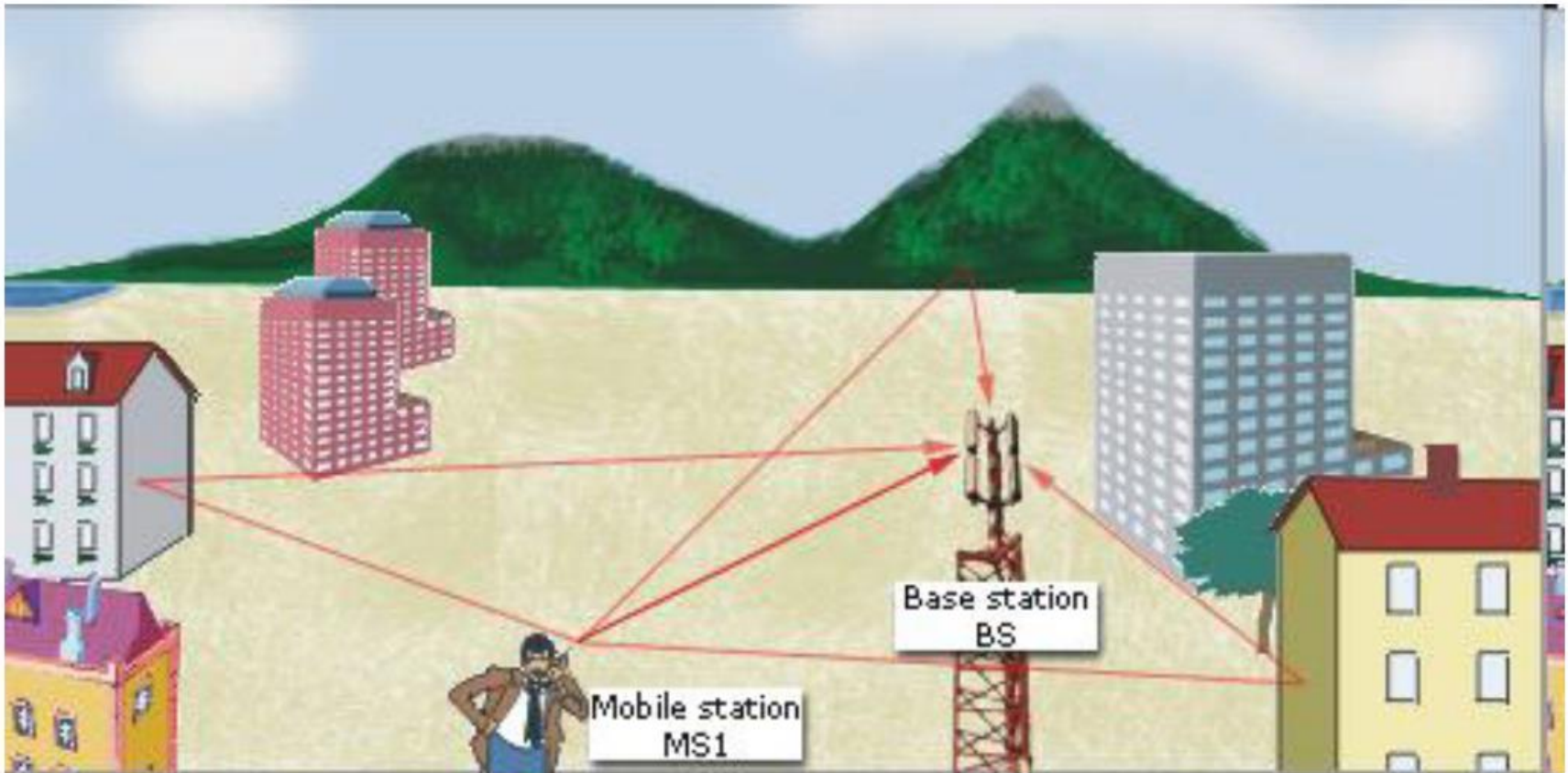


# Small-Scale Fading

PROF. MICHAEL TSAI

2016/04/15

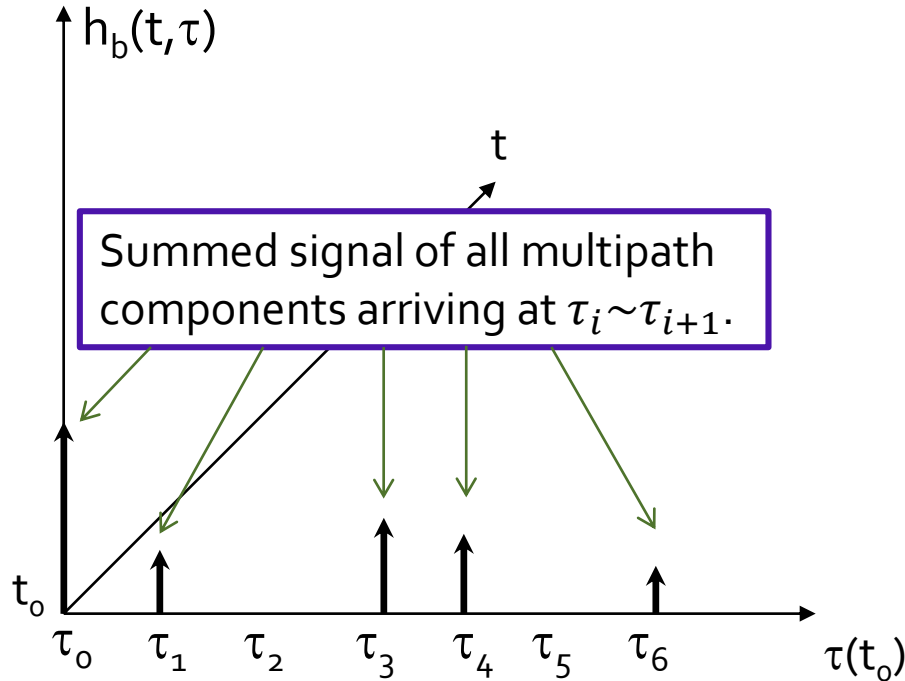
# Multipath Propagation



RX just sums up all Multi Path Component (MPC).

# Multipath Channel Impulse Response

An example of the time-varying discrete-time impulse response for a multipath radio channel



**Excess delay:** the delay with respect to the first arriving signal ( $\tau$ )

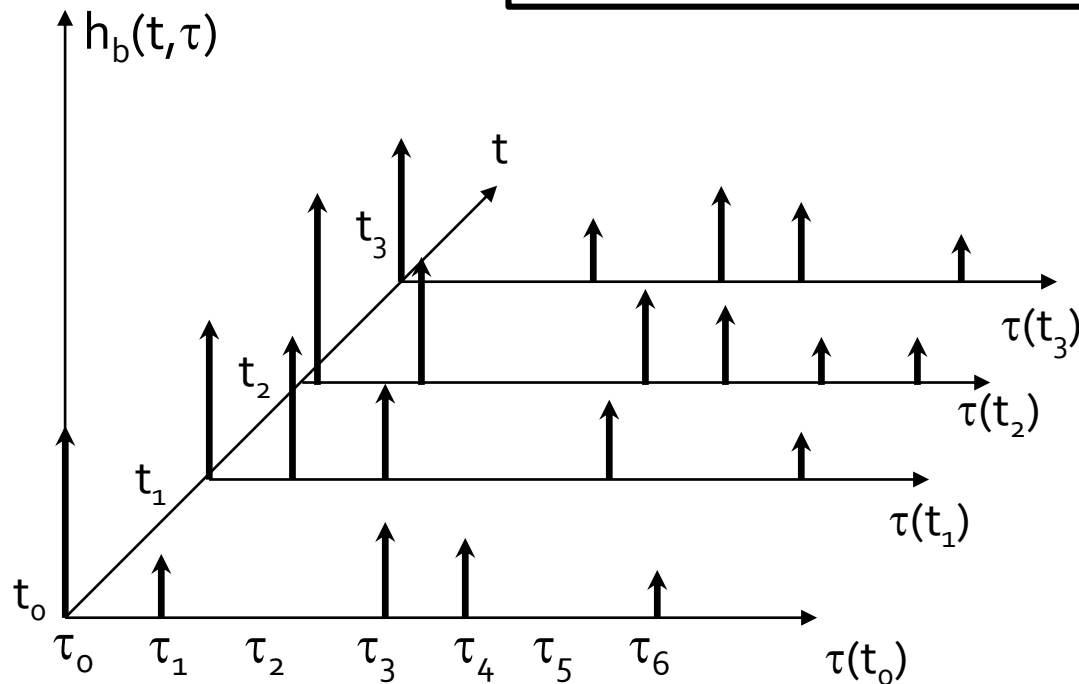
**Maximum excess delay:** the delay of latest arriving signal

The channel impulse response when  $t = t_0$  (what you receive at the receiver when you send an impulse at time  $t_0$ )

$\tau_0 = 0$ , and represents the time the first signal arrives at the receiver.

# Time-Variant Multipath Channel Impulse Response

Because the transmitter, the receiver, or the reflectors are moving, the impulse response is time-variant.



# Multipath Channel Impulse Response

- The channels impulse response is given by:

Summation over all MPC

Additional phase change due to reflections

$$h_b(t, \tau) = \sum_{i=0}^{N-1} a_i(t, \tau) \exp[-j\{2\pi f_c \tau(t_i) + \phi_i(t, \tau)\}] \delta(t - \tau_i(t))$$

Amplitude change (mainly path loss)

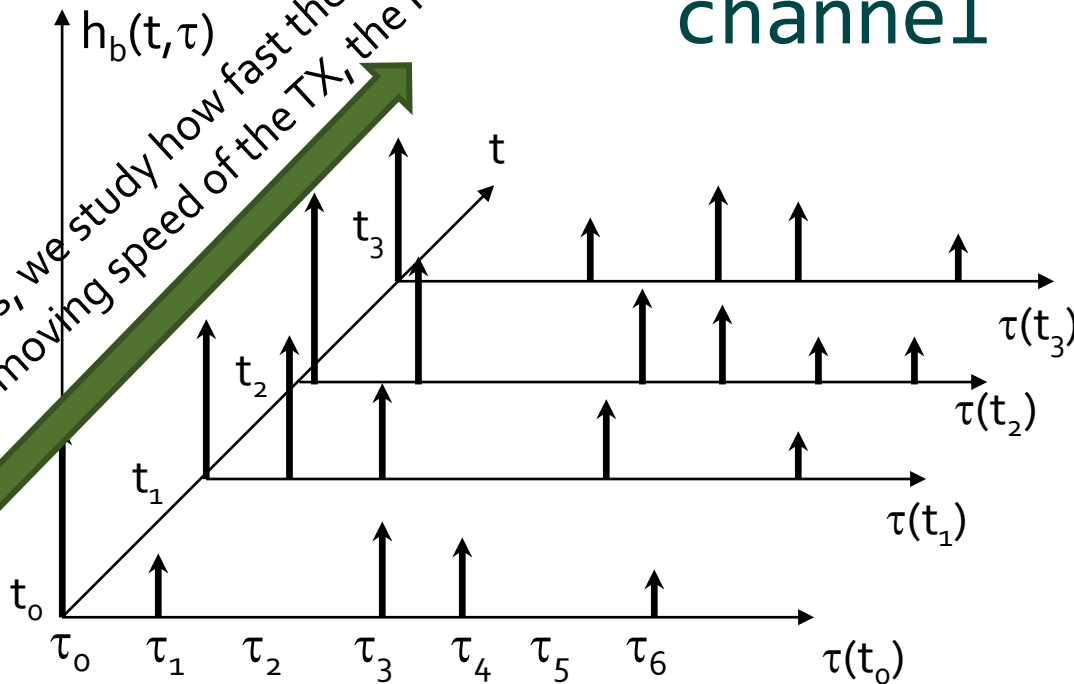
Phase change due to different arriving time

- **If assumed time-invariant (over a small-scale time or distance):**

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp[-j\theta_i] \delta(\tau - \tau_i)$$

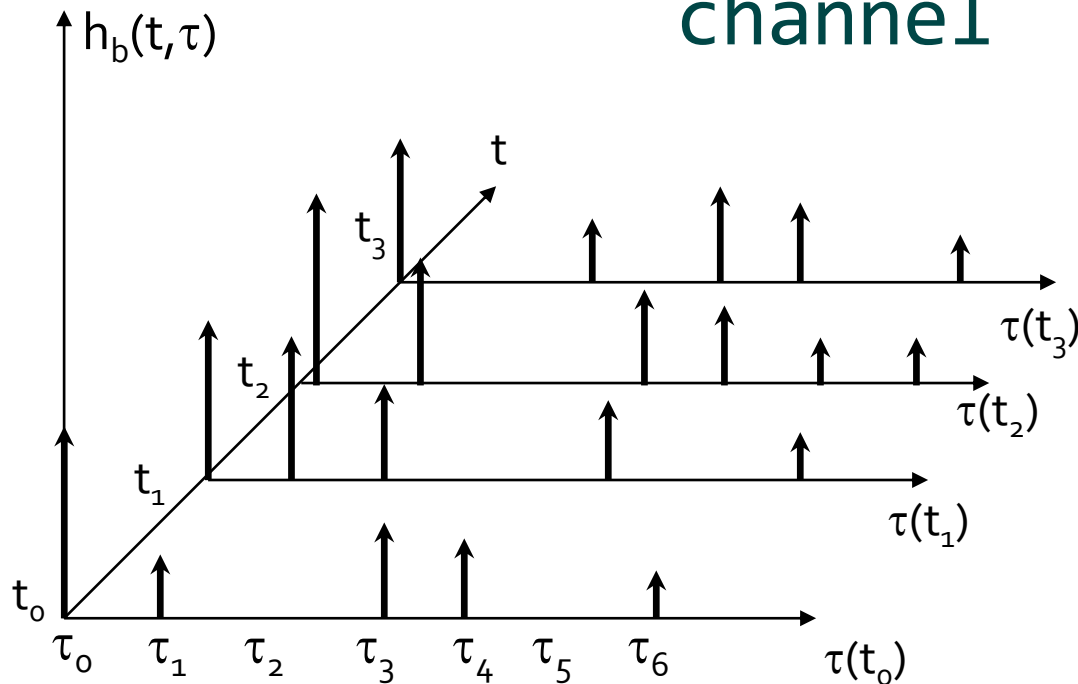
Following this axis, we study how fast the channel changes over time.  
(related to the moving speed of the TX, the RX, and the reflectors)

## Two main aspects of the wireless channel



Following this axis, we study how “spread-out” the impulse response are.  
(related to the physical layout of the TX, the RX, and the reflectors at a single time point)

# Two main aspects of the wireless channel



Following this axis, we study how “spread-out” the impulse response are. (related to the physical layout of the TX, the RX, and the reflectors at a single time point)

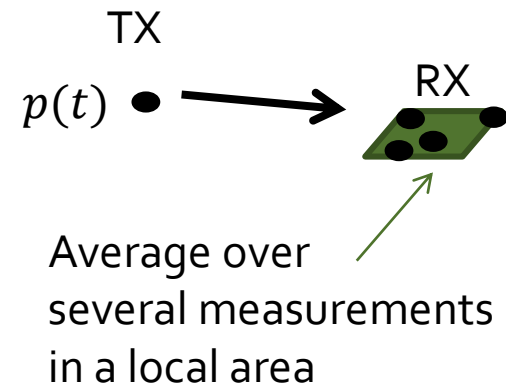
# Power delay profile

- To predict  $h_b(\tau)$  a probing pulse  $p(t)$  is sent s.t.

$$p(t) \approx \delta(t - \tau)$$

- Therefore, for small-scale channel modeling, **POWER DELAY PROFILE** is found by computing the spatial average of  $|h_b(t; \tau)|^2$  over a local area.

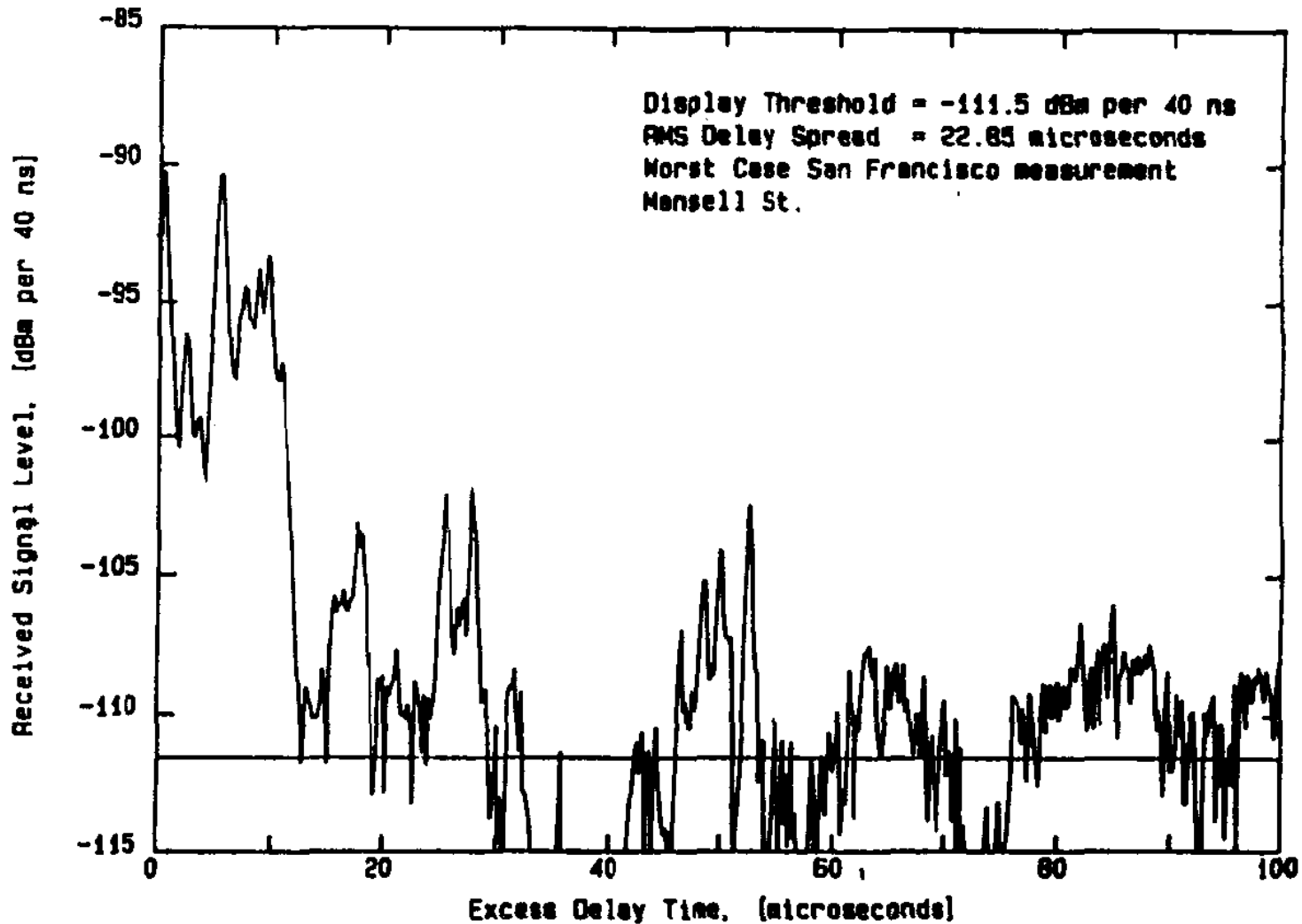
$$P(t; \tau) \approx k|h_b(t; \tau)|^2$$





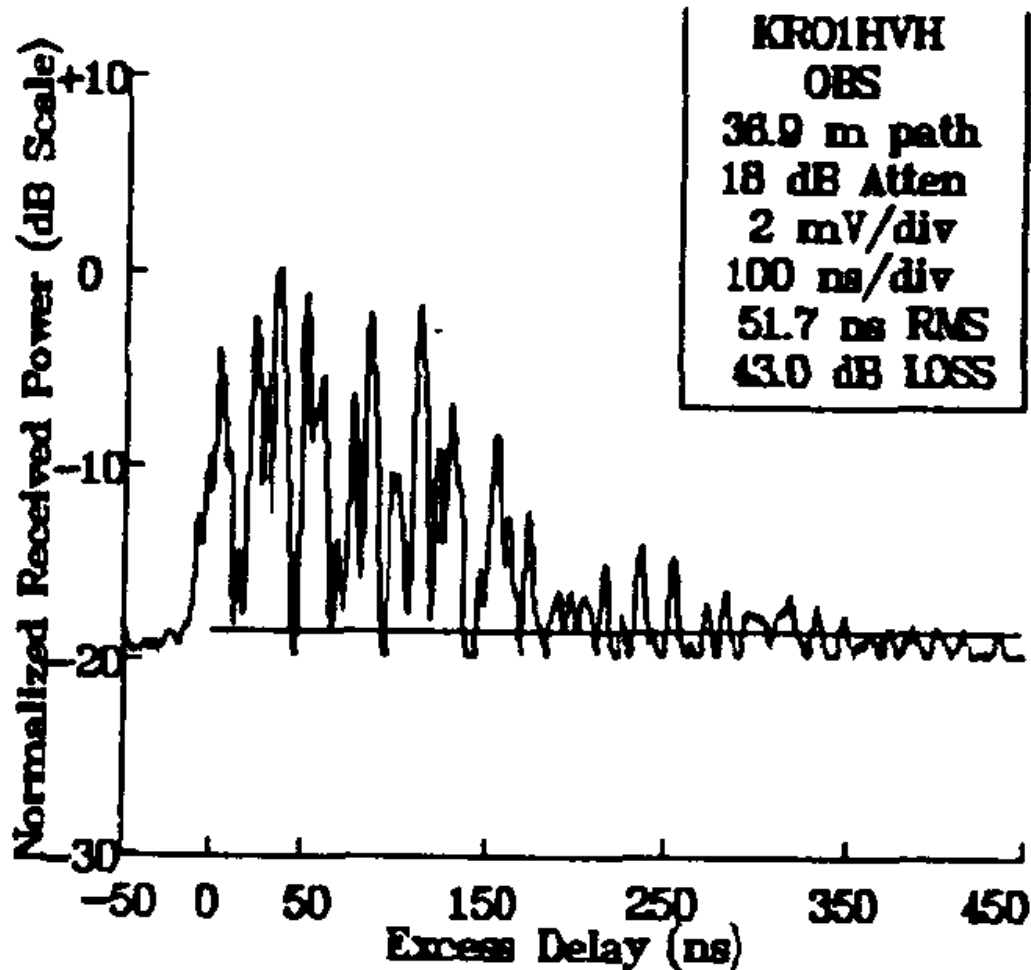
# Example: power delay profile

From a 900 MHz cellular system in San Francisco



# Example: power delay profile

Inside a grocery store at 4 GHz



# Time dispersion parameters

- Power delay profile is a good representation of the average “geometry” of the transmitter, the receiver, and the reflectors.
- To quantify “how spread-out” the arriving signals are, we use time dispersion parameters:

Already talked about this

- Maximum excess delay: the excess delay of the latest arriving MPC
- Mean excess delay: the “mean” excess delay of all arriving MPC
- RMS delay spread: the “standard deviation” of the excess delay of all arriving MPC

# Time dispersion parameters

- **Mean Excess Delay**

First moment of the power delay profile

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k}{\sum_k P(\tau_k)}$$

- **RMS Delay Spread**

Square root of the second central moment of the power delay profile

$$\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2}$$

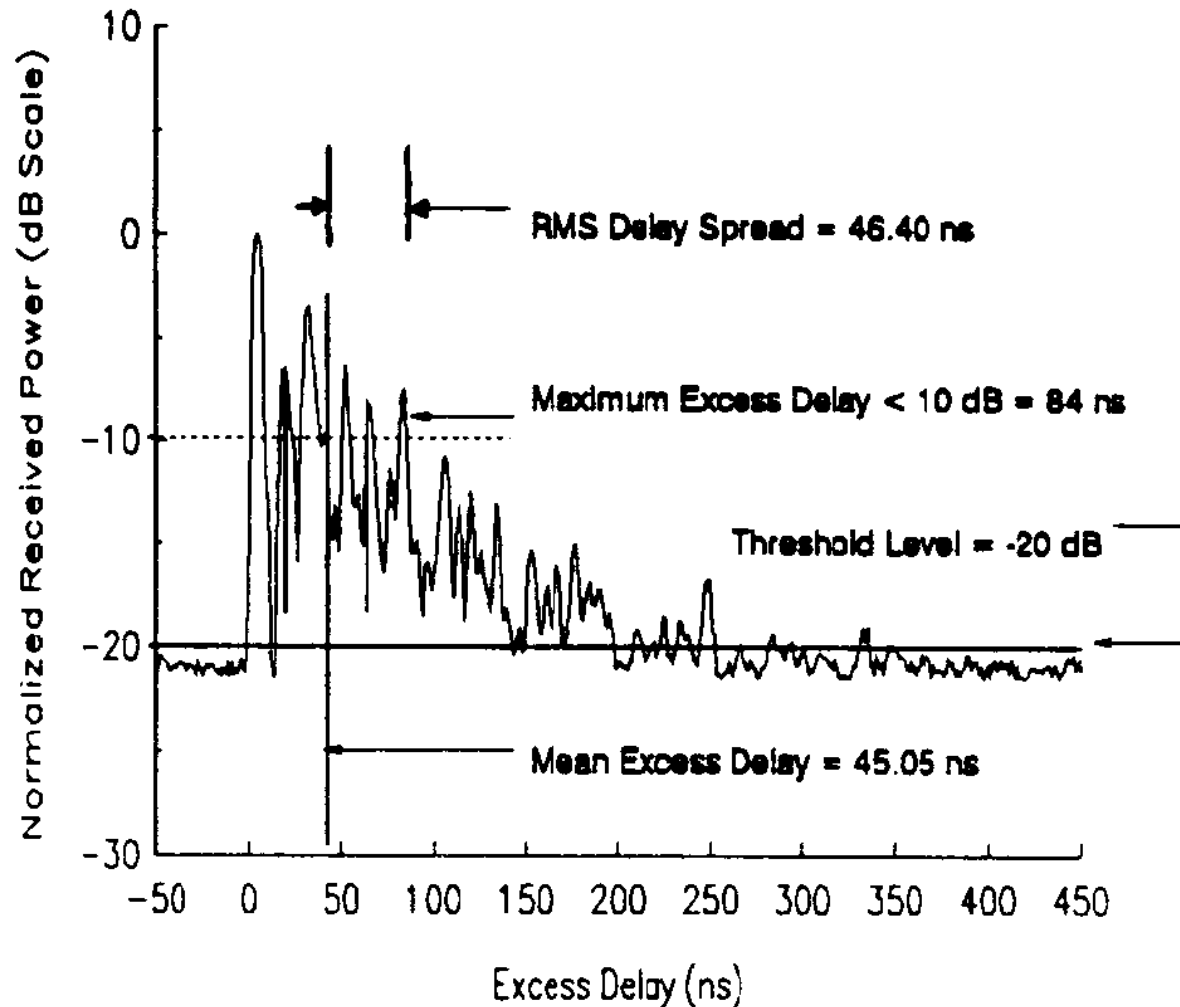
$$\bar{\tau}^2 = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)}$$

Second moment of the power delay profile

# Time dispersion parameters

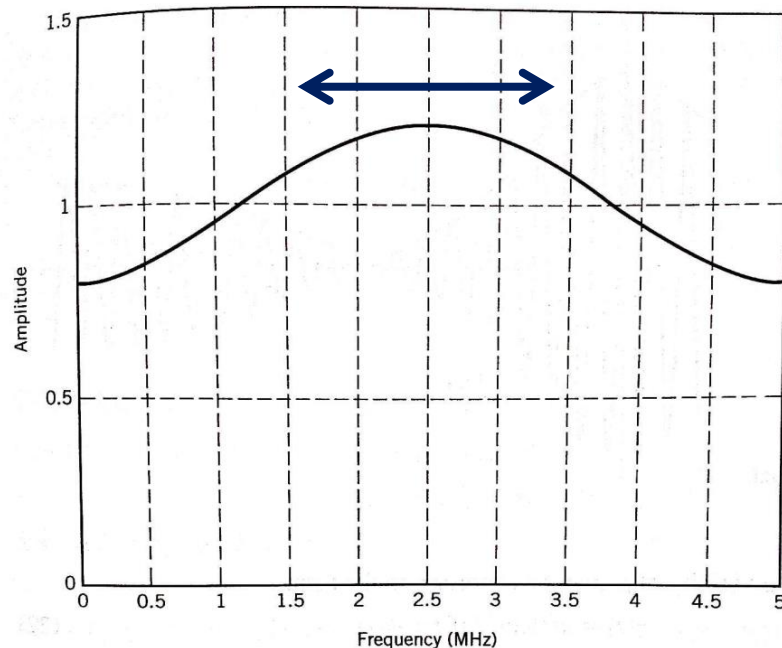
- **Maximum Excess Delay:**
  - Original version: the excess delay of the **latest** arriving MPC
  - In practice: the latest arriving could be smaller than the noise
  - No way to be aware of the “latest”
- **Maximum Excess Delay (practical version):**
  - The time delay during which multipath energy falls to X dB below the maximum.
- **This X dB threshold could affect the values of the time-dispersion parameters**
  - Used to differentiate the noise and the MPC
  - Too low: noise is considered to be the MPC
  - Too high: Some MPC is not detected

# Example: Time dispersion parameters



# Coherence Bandwidth

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered “flat”  
→ a channel passes all spectral components with approximately equal gain and linear phase.



# Coherence Bandwidth

- **Bandwidth over which Frequency Correlation function is above 0.9**

$$B_c \approx \frac{1}{50\sigma_\tau}$$

- **Bandwidth over which Frequency Correlation function is above 0.5**

$$B_c \approx \frac{1}{5\sigma_\tau}$$

Those two are approximations derived from empirical results.

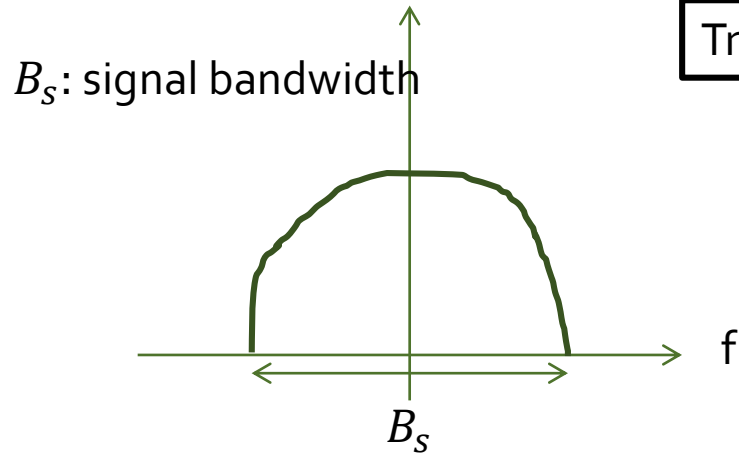


# Typical RMS delay spread values

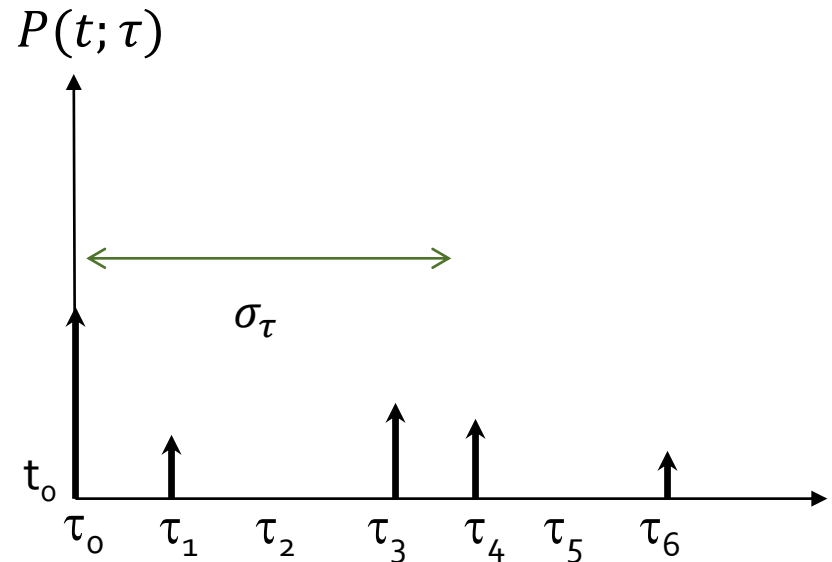
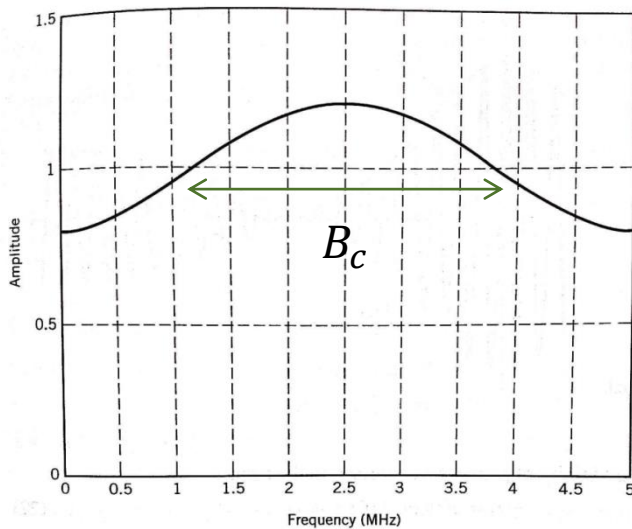
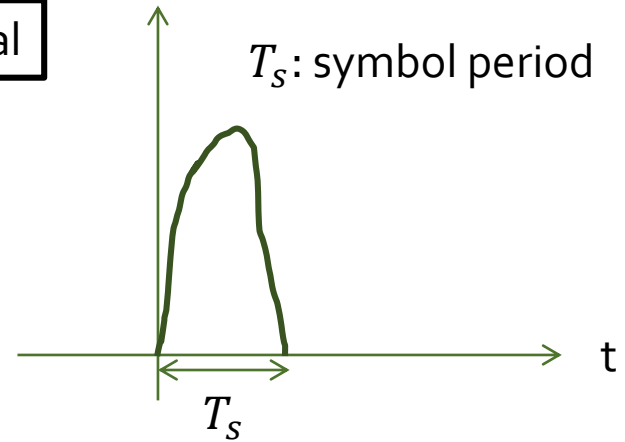
Environment	Frequency (MHz)	RMS Delay Spread ( $\sigma_\tau$ )	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 $\mu$ s	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

# Signal Bandwidth & Coherence Bandwidth

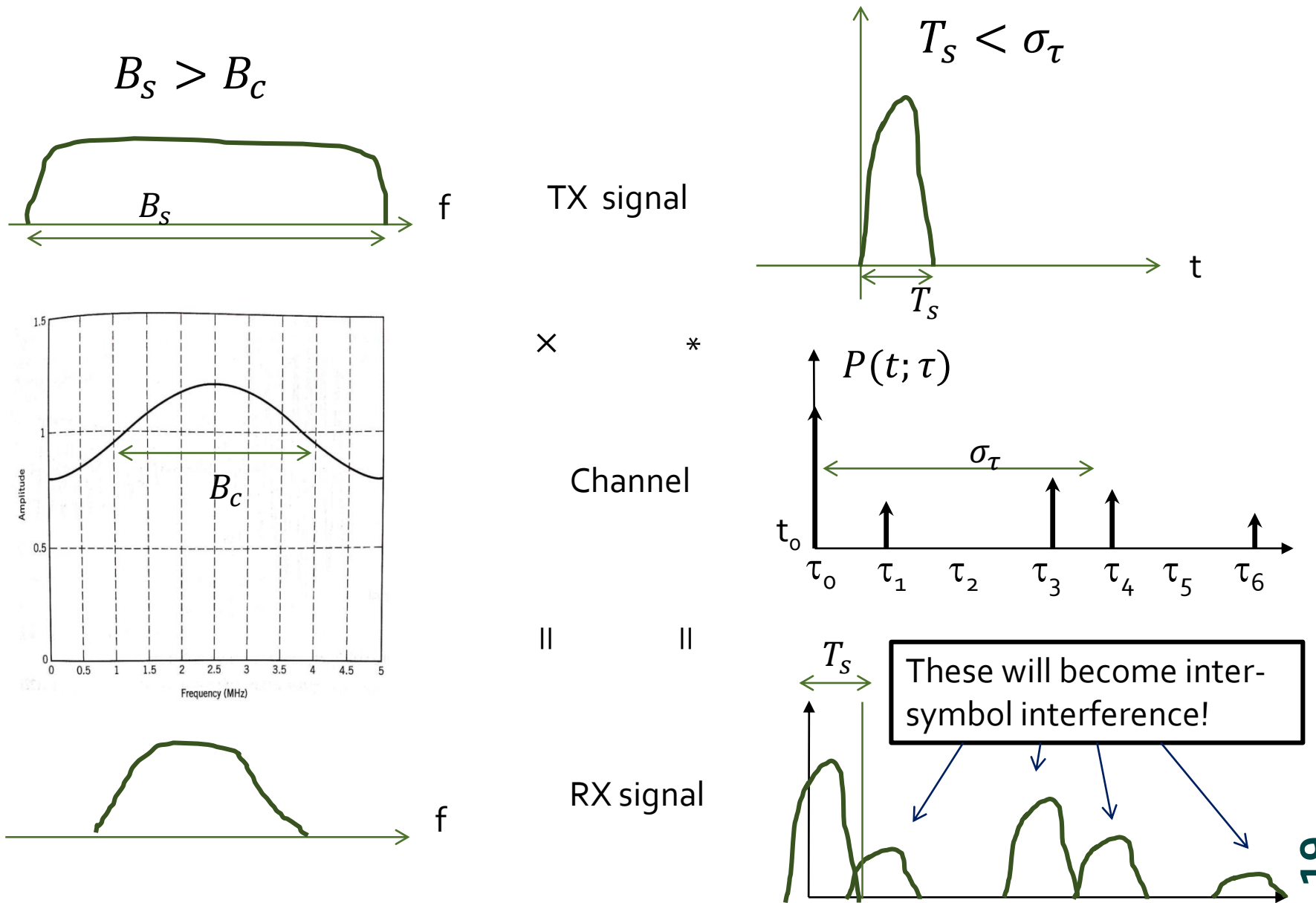
Transmitted Signal



$$T_s \approx \frac{1}{B_s}$$

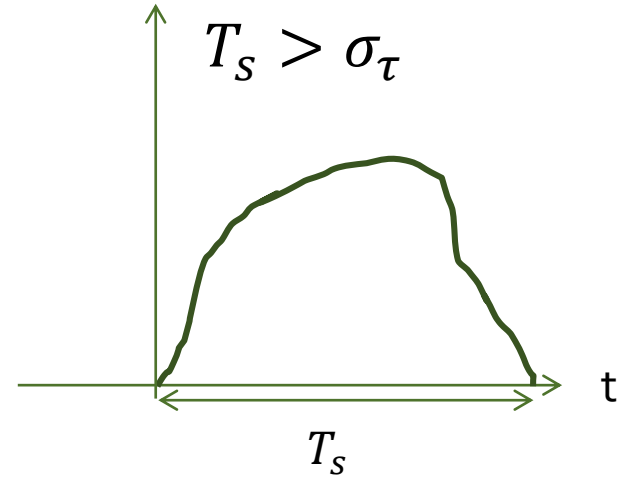
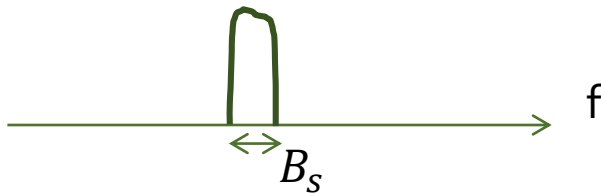


# Frequency-selective fading channel

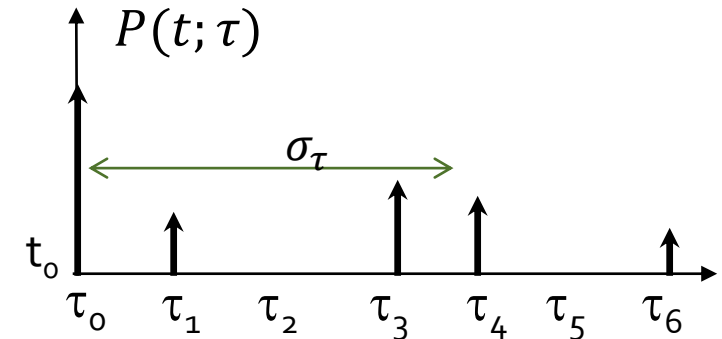


# Flat fading channel

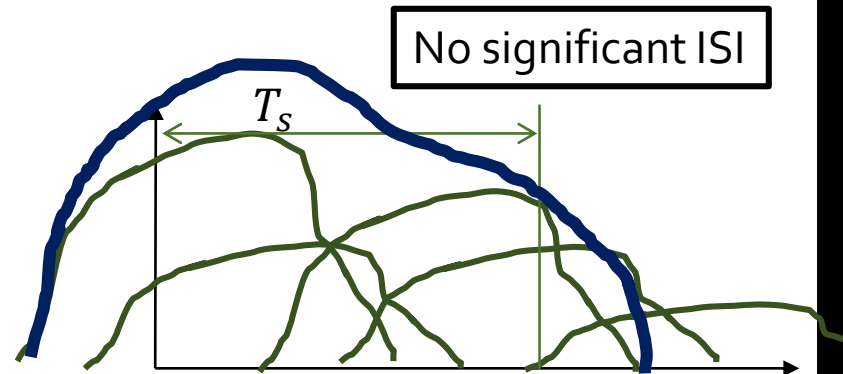
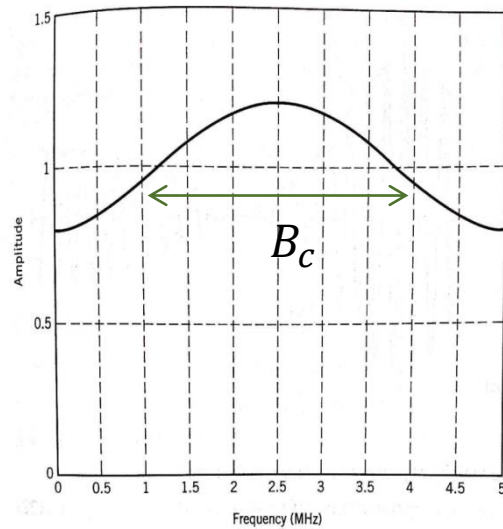
$$B_s < B_c$$



$\times$   $*$



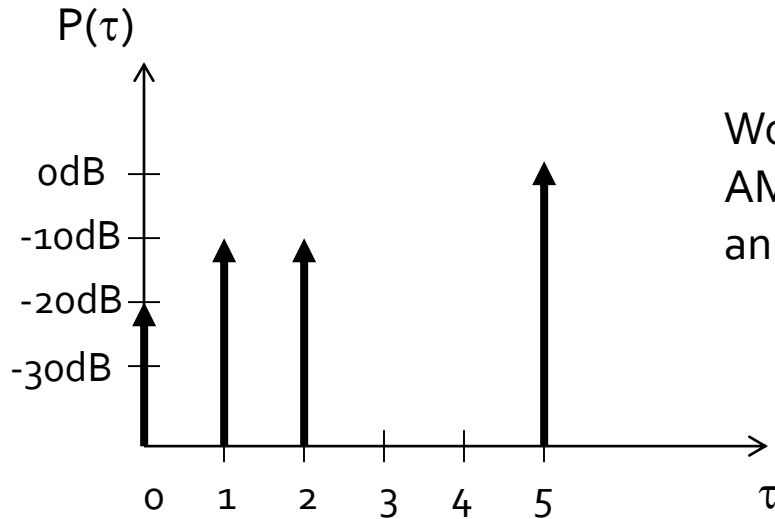
|| ||



# Equalizer 101

- **An equalizer is usually used in a frequency-selective fading channel**
  - When the coherence bandwidth is low, but we need to use high data rate (high signal bandwidth)
- **Channel is unknown and time-variant**
  - Step 1: TX sends a known signal to the receiver
  - Step 2: the RX uses the TX signal and RX signal to estimate the channel
  - Step 3: TX sends the real data (unknown to the receiver)
  - Step 4: the RX uses the estimated channel to process the RX signal
  - Step 5: once the channel becomes significantly different from the estimated one, return to step 1.

# Example



Would this channel be suitable for AMPS or GSM without the use of an equalizer?

$$\text{Mean Excess Delay} = \bar{\tau} = \frac{\sum_k P(\tau_k) \tau_k}{\sum_k P(\tau_k)} = \frac{5(1) + 2(0.1) + 1(0.1) + 0(0.01)}{1 + 0.1 + 0.1 + 0.01} = 4.38 \mu\text{s}$$

$$\bar{\tau}^2 = \frac{\sum_k P(\tau_k) \tau_k^2}{\sum_k P(\tau_k)} = \frac{(1)5^2 + (0.1)2^2 + (0.1)1^2 + (0.01)0^2}{1 + 0.1 + 0.1 + 0.01} = 21.07 \mu\text{s}^2$$

# Example

- Therefore:

$$\text{RMS Delay Spread} = \sigma_{\tau} = \sqrt{\overline{\tau^2} - (\overline{\tau})^2} = \sqrt{21.07 - (4.38)^2} = 1.37 \mu\text{s}$$

$$\text{Coherence Bandwidth} = B_C = \frac{1}{5\sigma_{\tau}} = \frac{1}{5(1.37 \mu\text{s})} = 146 \text{ KHz}$$

- Since  $B_C > 30 \text{ KHz}$ , AMPS would work without an equalizer.
- GSM requires  $200 \text{ KHz BW} > B_C \rightarrow$  An equalizer would be needed.