Modulation

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Signal Contellation

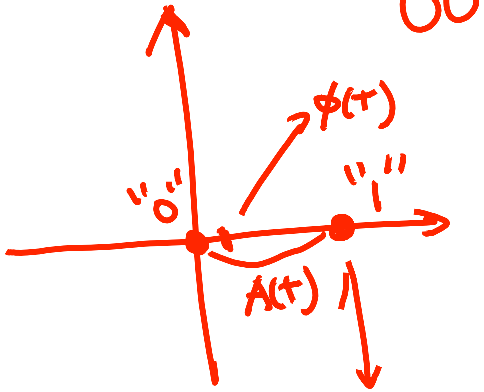
$$\bar{S}(t) = \cos(w_c t) = \cos(2\pi f_c t)$$

$$S(t) = A(t)\cos(2\pi f_c t + \phi(t))$$

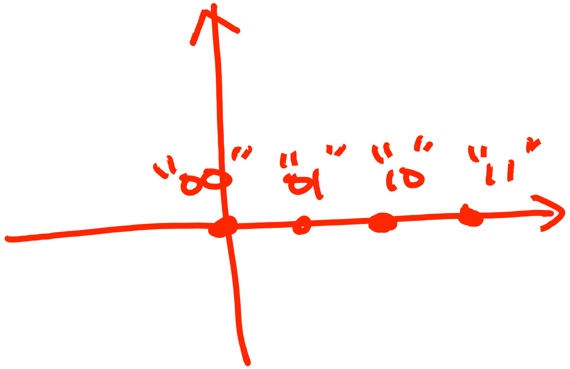
$\underline{A(t)}$: amp.

$\underline{\phi(t)}$: phase

OOK

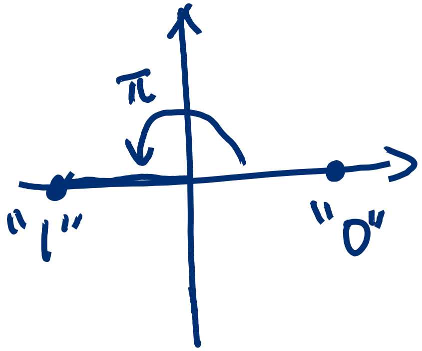


4ASK



bit pattern	極座標		wave
"0"	$A(t)=0$	$\phi(t)=0$	$0\cdot\cos(2\pi f_c t)$
"1"	$A(t)=1$	$\phi(t)=0$	$1\cdot\cos(2\pi f_c t)$
"00"	$A(t)=0$	$\phi(t)=0$	$0\cdot\cos(2\pi f_c t)$
"01"	" $= \frac{1}{3}$	$\phi(t)=0$	$\frac{1}{3}\cdot\cos(2\pi f_c t)$
"10"	" $= \frac{2}{3}$	"	$\frac{2}{3}\cdot$ "
"11"	" $= 1$	"	$1\cdot$ "

BPSK "B" ⇒ binary

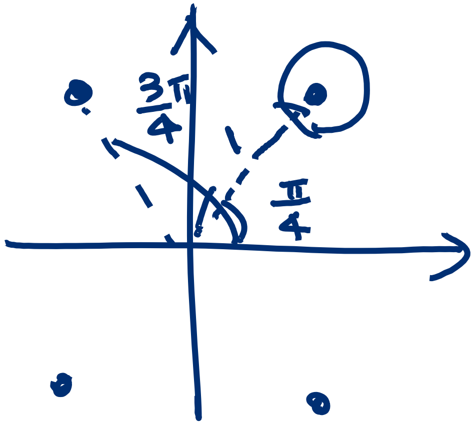


"0" $A(t)=1$ $\phi(t)=0$ $1\cdot\cos(2\pi f_c t)$

"1" $A(t)=1$ $\phi(t)=\pi$ $1\cdot\cos(2\pi f_c t + \pi)$

$= -\cos(2\pi f_c t)$

QPSK "Q" ⇒ 4
4PSK



"00" $A(t)=1$ $\phi(t)=\frac{\pi}{4}$ $\cos(2\pi f_c t+\frac{\pi}{4})$

"01" ⋮ $\phi(t)=\frac{3\pi}{4}$ $\cos(2\pi f_c t+\frac{3\pi}{4})$

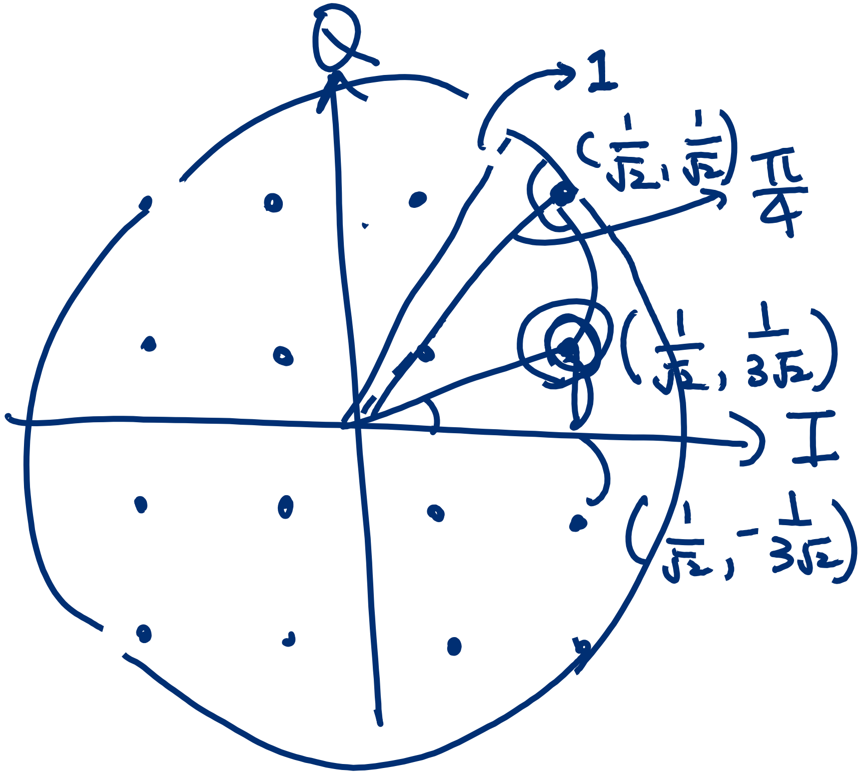
"10" ⋮ $\phi(t)=\frac{5\pi}{4}$ ⋮

"11" ⋮ $\phi(t)=\frac{7\pi}{4}$ $\cos(2\pi f_c t+\frac{7\pi}{4})$

$A=1$ $\phi=\frac{\pi}{4}$

$(1,\frac{\pi}{4}) \rightarrow (\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

16QAM Quadrature Amplitude Modulation



$$S(t) = \boxed{A}\cos(\omega_c t + \boxed{\phi})$$

$$= \underline{A\cos(\phi)}\cos(\omega_c t)$$

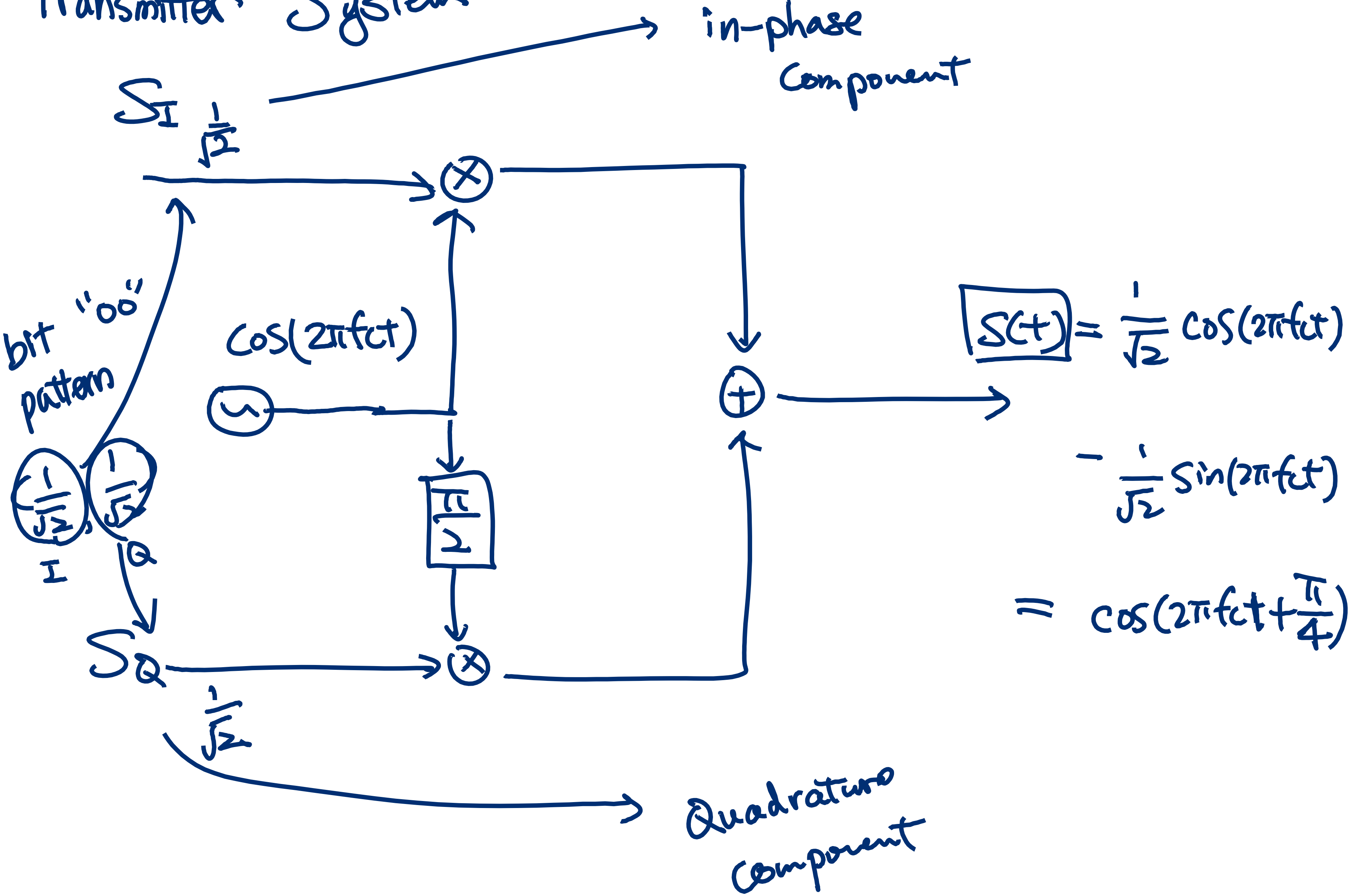
$$- \underline{A\sin(\phi)}\sin(\omega_c t)$$

$$= \underline{S_I}\cos(\omega_c t) - \underline{S_Q}\sin(\omega_c t)$$

$$(A, \phi) \Rightarrow (S_I, S_Q)$$

in-phase Quadrature

Transmitter System



$$\boxed{S(t)} = \frac{1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t) = \cos(2\pi f_c t + \frac{\pi}{4})$$

Error Margin

$$s(t) = \boxed{A(t)} \cos(2\pi f_c t + \boxed{\phi(t)})$$

noise

$$r(t) = \frac{\boxed{A(t)}}{\boxed{\rho}} \cos(2\pi f_c t + \boxed{\phi(t)} + \boxed{\theta}) + \boxed{n(t)}$$

attenuation [illegible]

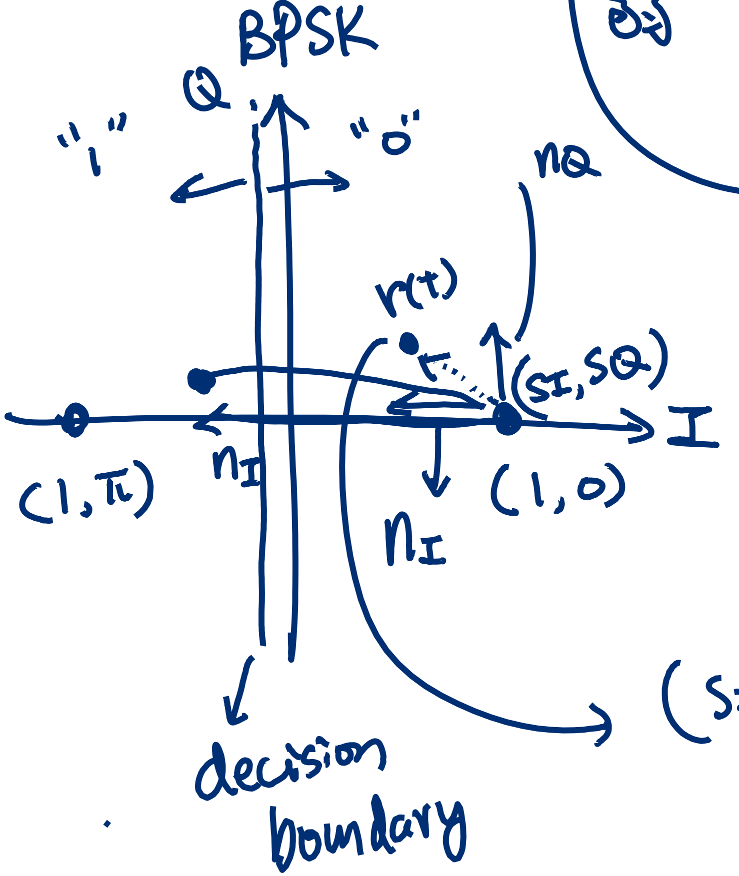
channel

$(t - \Delta t)$

channel → delay

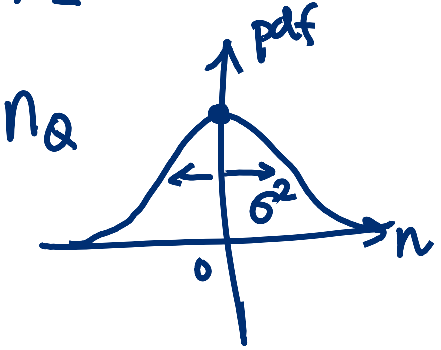
$$= S_I \cos(2\pi f_c t) + n_I$$

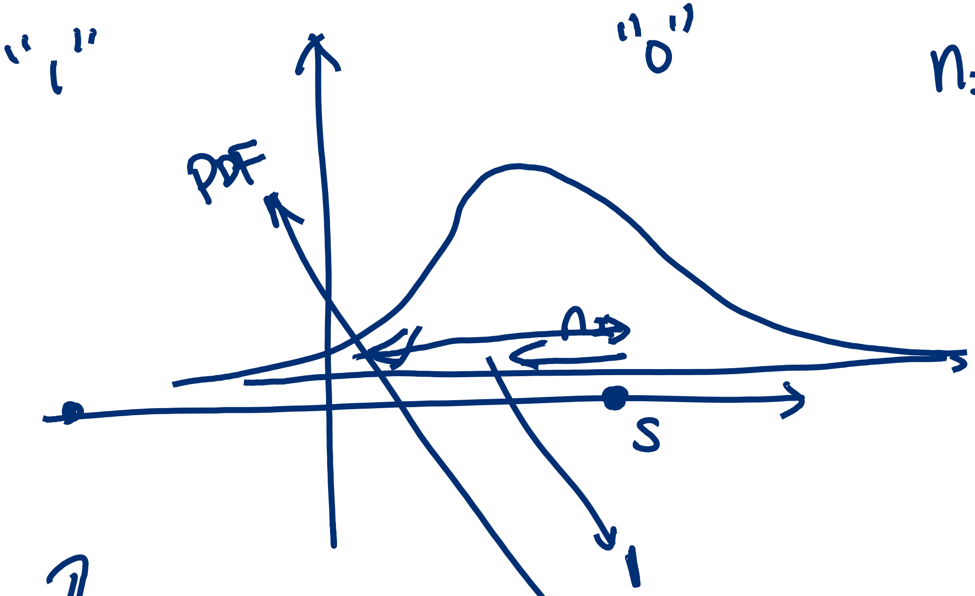
$$- S_Q \sin(2\pi f_c t) + n_Q$$



$$(S_I + n_I, S_Q + n_Q)$$

$$N(0, \sigma^2)$$



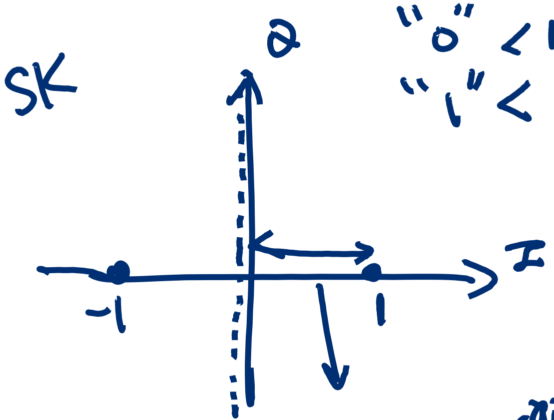


$$n_I \sim N(\underbrace{0}_{\text{mean}}, \underbrace{\sigma^2}_{\text{variance}})$$

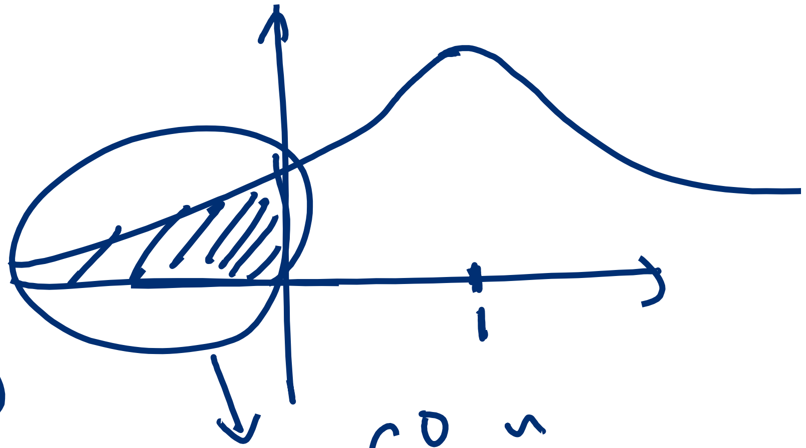
BPSK

"0" $\langle 1, 0 \rangle \equiv (1, 0)$

"1" $\langle 1, \pi \rangle \equiv (-1, 0)$

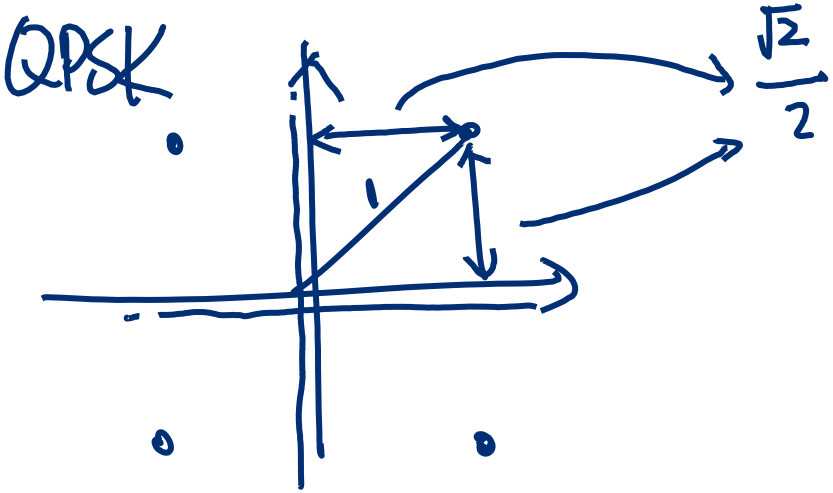


error margin=1

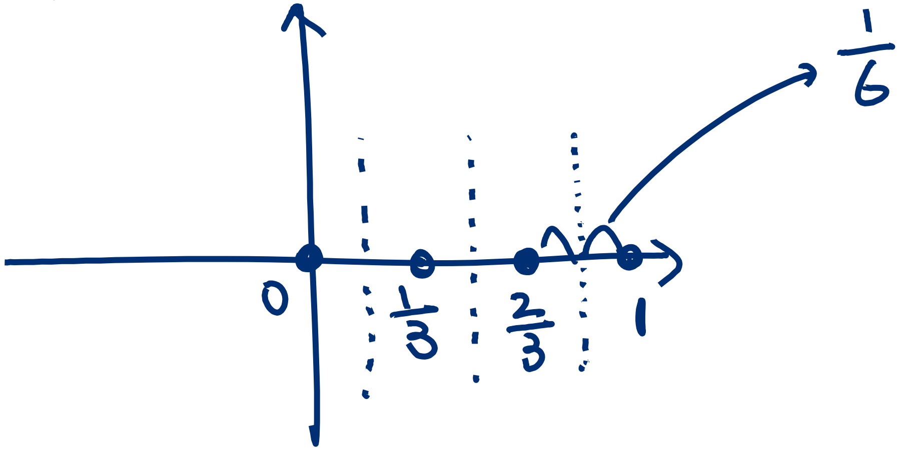


$$\underset{\text{Prob. of symbol error}}{P_e} = \int_{-\infty}^{0} \hat{f}(x)\,dx = \int_{-\infty}^{0} f(x-1)\,dx$$

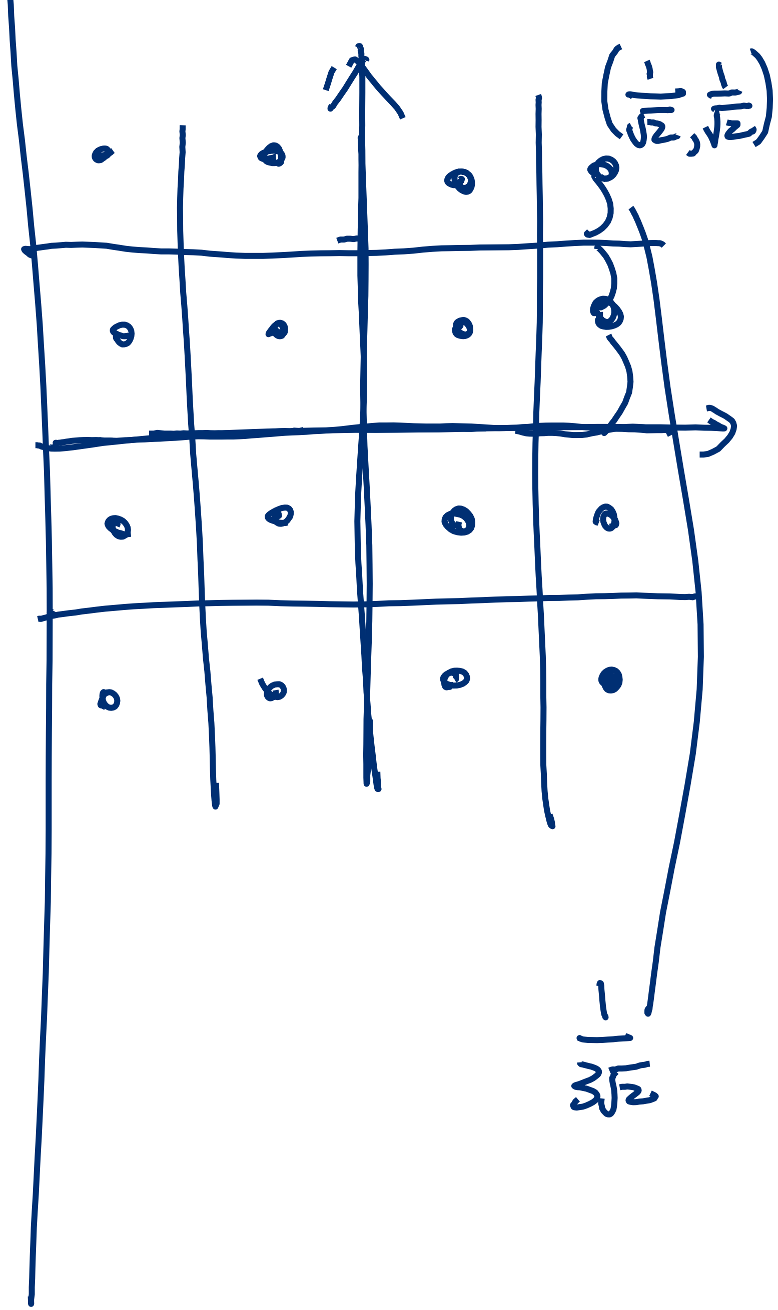
QPSK



4ASK

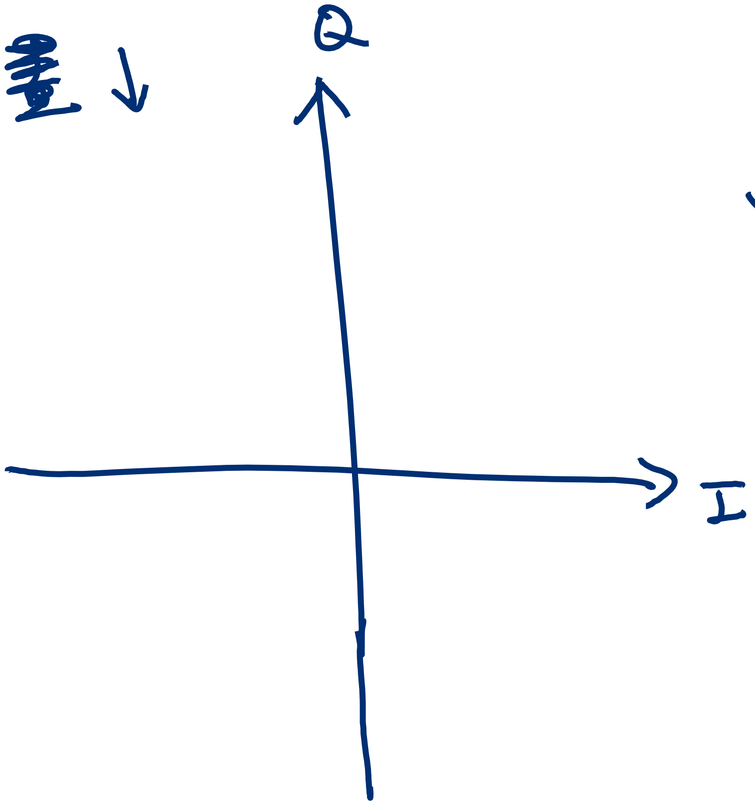


16QAM



Quiz #2

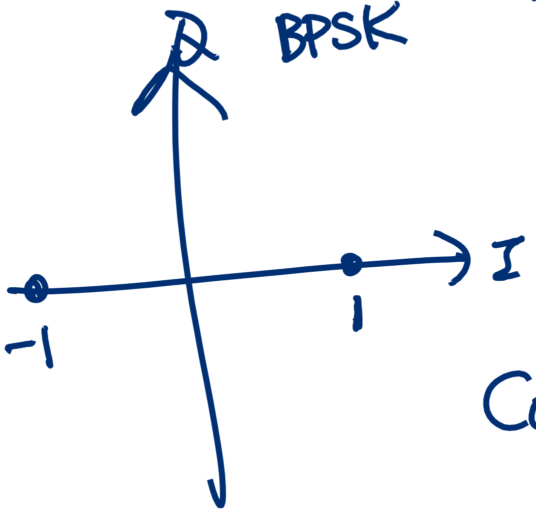
(1) 畫 ↓



8PSK

(2) error margin = ?

(3) 不出錯的 SNR = ?



$$r(t) = \frac{A(t)}{\rho} \cos\left(2\pi f_c t + \phi(t)\right)$$

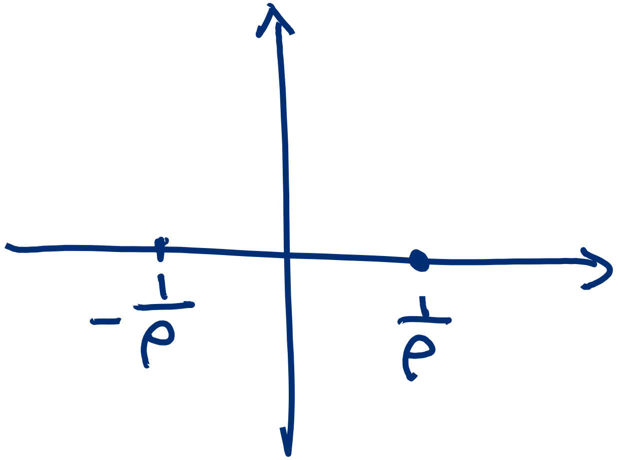
$$\Rightarrow \left(\frac{S_I}{\rho} , \frac{S_Q}{\rho} \right)$$

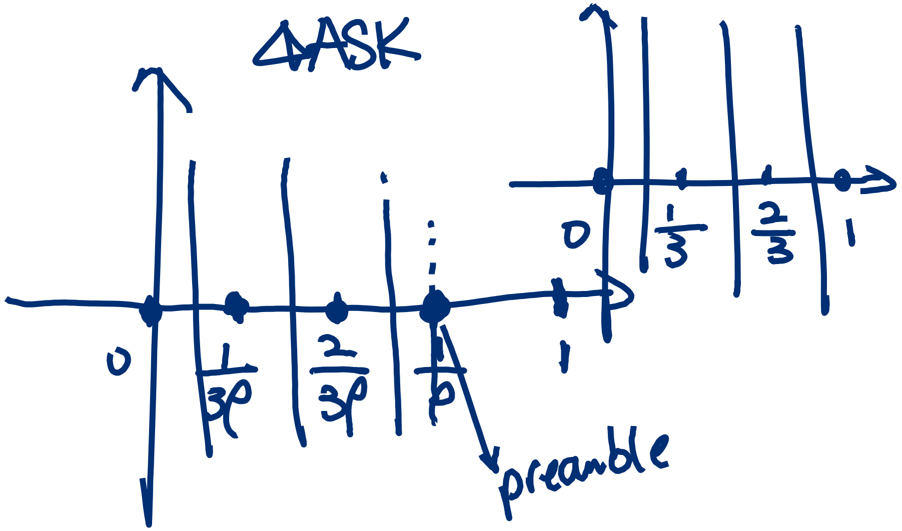
Complex number ↑

$$\boxed{S} = \frac{S_I}{\rho} + \frac{S_Q}{\rho} j$$

In-phase
→ real

Quadrature
→ imaginary





$$r(t)=\frac{\overset{a}{\uparrow}\,\textcircled{1}}{\rho}\cos(2\pi f_c t)$$

preamble: $\underline{\underline{\cos(2\pi f_c t)}}$ 已知

供 receiver 去除 channel 的影響. (1) ρ (attenuat)

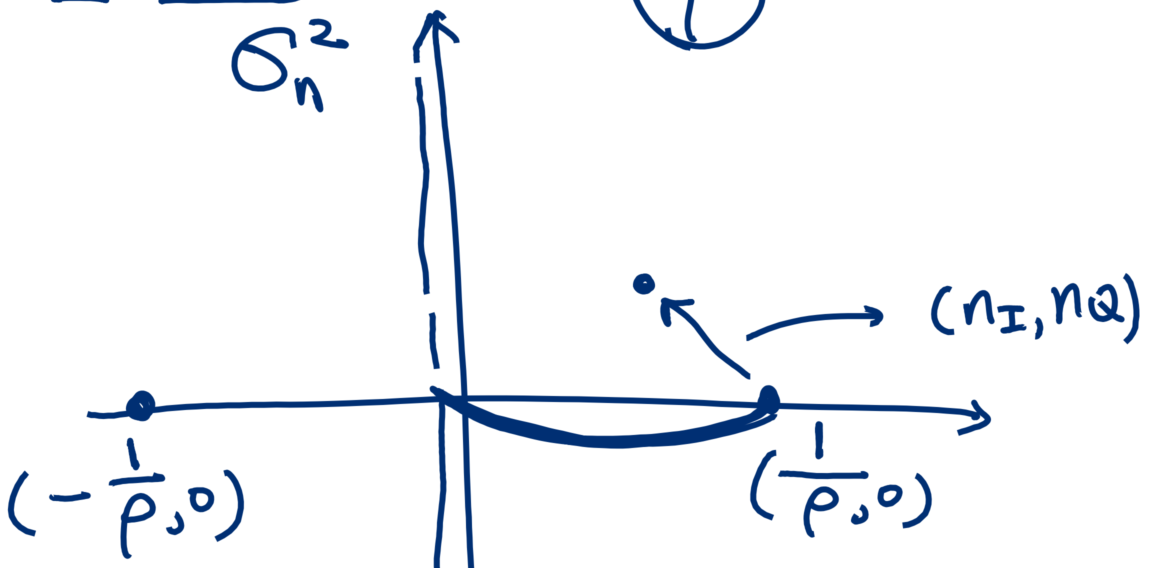
(2) ϕ (delay)

SNR: ⓢignal to ⓝoise ⓡatio

⇓

$$\gamma=\frac{\left(\frac{1}{\rho}\right)^2}{\sigma_n^2}$$

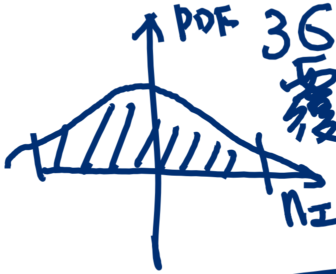
$$\bigcirc\!\!\!\!\!\frac{1}{\rho}$$



ρ ?

$n_I >$ error margin sym. err.

$n_I <$ error margin correct sym.



3σ 覆蓋大部分可能出現的情形

	noise amp. (std. deviation) $= \frac{\text{err. margin}}{3}$ σ	Signal amp $\boxed{E[S^2]}$	S/N $\frac{E[S^2]}{\sigma^2}$ ↓		per bit SNR
OOK	0.167	0.71 $\sqrt{\frac{0^2+1^2}{2}}$	12.6 (dB)	1	12.6
4ASK	0.053	0.62	21.4 (dB)	2 3dB	18.3
BPSK	0.333	1	9.5 (dB)	1	9.5
QPSK	0.237	1	12.5 (dB)	2 3dB	9.5
16QAM	0.077	0.74	25.8 (dB)	4 6 dB	13.7 19.8

dB → 大寫

decibel

"ratio"

比例

$\frac{子}{母}$

$$10\cdot\log_{10}\left(\frac{子}{母}\right)=\boxed{}\ (dB)$$

dB·ⓜ
dB W
dB V

$$10\cdot\log_{10}\left(100_{(倍)}\right)=20\ dB$$

$\underline{\underline{23dB}}$

20+3 dB

7dB ≒ 5
3dB ≒ 2
6dB ≒ 4

$$10\cdot\log_{10}\left(1000_{(倍)}\right)=30\ dB$$

$$10\cdot\log_{10}\left(2(倍)\right)=3\ dB$$

4倍

3+3=6dB

$$27dB=\underset{\substack{\downarrow\\100}}{20}+\underset{\substack{\downarrow\\5}}{7}$$

$$100\times5=500(倍)$$

dBm

1 mW

$$10\times\log_{10}\left(\frac{\text{ㄅ}}{1mW}\right)$$

ㄅ(大小) 是1 mW 的几倍

ㄅ=20mW

$$10\log_{10}\left(\frac{20\,mW}{1\,mW}\right) = 13\ dBm$$

dBW

$$10\times\log_{10}\left(\frac{500mW}{1\,mW}\right) = 27\ dBm$$

$$10\times\log_{10}\left(\frac{\textcircled{ㄅ}}{1W}\right)$$

WLAN AP TX power 50mW ~ 200mW

17dBm ~ 23dBm

$$-20dBm = 10\times\log_{10}\left(\frac{0.01}{1mW}\right)$$

SS

0.01 mW

$$= \frac{1}{100}\ mW$$

ρ: attenuation $= 10^5 \sim 10^{10}$

Claude Shannon Theorem

linear SNR

✗ dB

$$R \leq B \log_2 \left(1 + \frac{S}{N} \right)$$

→ SNR

R ↓ 傳多少資料

bit/s

B ↓ 頻寬 $\frac{1}{T_s}$

T_s: symbol duration

16MHz

$\boxed{\text{WiFi}}$

802.11 abgnac

SNR: 8.5dB

⇑

$$16 \times 10^6 \times \log_2 (1 + 7)$$

$$= 4.8 \times 10^7 = 48 \text{ Mbps}$$