Multi-Input Multi-Output Systems (MIMO)

- Channel Model for MIMO
- MIMO Decoding
- MIMO Gains
- Multi-User MIMO Systems
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MIMO

• Each node has multiple antennas
  ▶ Capable of transmitting (receiving) multiple streams concurrently
  ▶ Exploit antenna diversity to increase the capacity
Channel Model (2x2)

\[ y_1 = h_{11}x_1 + h_{21}x_2 + n_1 \]
\[ y_2 = h_{12}x_1 + h_{22}x_2 + n_2 \]
\[ y = Hx + n \]

- Can be extended to \( N \times M \) systems
Antenna Space

M-antenna node receives in M-dimensional space

\[
\begin{pmatrix}
  y_1 \\
  y_2
\end{pmatrix} =
\begin{pmatrix}
  h_{11} \\
  h_{12}
\end{pmatrix} x_1 +
\begin{pmatrix}
  h_{21} \\
  h_{22}
\end{pmatrix} x_2 +
\begin{pmatrix}
  n_1 \\
  n_2
\end{pmatrix}
\]

\[
\tilde{y} = \tilde{\mathbf{h}}_1 x_1 + \tilde{\mathbf{h}}_2 x_2 + \tilde{n}
\]

\[
\tilde{h}_2 = (h_{21}, h_{22})
\]

\[
\tilde{y} = (y_1, y_2)
\]

\[
\tilde{h}_1 = (h_{11}, h_{12})
\]
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Zero-Forcing Decoding (algebra)

Given $x_1$, solve $x_2$ by successive interference cancellation (SIC)

To guarantee the full rank of $H$, antenna spacing at the transmitter and receiver must exceed half of the wavelength
Zero-Forcing Decoding (antenna space)

- To decode $x_1$, decode vector $y$ on the direction orthogonal to $x_2$
- To improve the SNR, use SIC decoding, which re-encodes the first detected signal, subtracts it from $y$, and decodes the second signal
Channel Estimation

• Estimate N x M matrix $H$

$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$
$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

Two equations, but four unknowns

Antenna 1 at Tx
Access code 1
Stream 1

Antenna 2 at Tx
Access code 2
Stream 2

Estimate $h_{11}, h_{12}$
Estimate $h_{21}, h_{22}$
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MIMO Gains

• **Multiplex Gain**
  - Exploit antennas to deliver multiple streams concurrently

• **Diversity Gain**
  - Exploit antenna diversity to increase the SNR of a single stream
  - Receive diversity and transmit diversity
Degree of Freedom

• For N x M MIMO channel

  ▶ Degree of Freedom (DoF): \( \min \{N,M\} \)
  ▶ Maximum diversity: \( NM \)
Multiplexing-Diversity Tradeoff

• Tradeoff between diversity gain and multiplex gain

• Say we have a $N \times N$ system
  ▶ Degree of freedom: $N$
  ▶ The transmitter can transmit $k$ streams concurrently, where $k \leq N$
  ▶ The optimal value of $k$ is determined by the tradeoff between the diversity gain and multiplex gain
Receive Diversity

• 1 x 2 example

\[ y_1 = h_1 x + n_1 \]
\[ y_2 = h_2 x + n_2 \]

- Decode the SNR of \((y_1 + y_2)\)
- Uncorrelated white Gaussian noise with zero mean
- Packet can be delivered through at least one of the many diver paths
Receive Diversity

- 1 x 2 example

\[
SNR = \frac{P(2X)}{P(n_1 + n_2)}
\]

where \( P \) refers to the power

\[
= \frac{E[(2X)^2]}{E[(n_1 + n_2)^2]} = \frac{E[(2X)^2]}{E[n_1^2 + n_2^2]}
\]

\[
= \frac{4E[X^2]}{2\sigma^2}, \text{ where } \sigma \text{ is the variance of AWGN}
\]

\[
= 2 * SNR_{\text{single antenna}}
\]

- Increase SNR by 3dB
- Especially beneficial for the low SNR link
Receive Diversity
Maximal Ratio Combining (MRC)

Multiply each $y$ with the conjugate of the channel

\[
\begin{align*}
    y_1 &= h_1 x + n_1 \\
    y_2 &= h_2 x + n_2
\end{align*}
\]

\[
\begin{align*}
    h_1^* y_1 &= \|h_1\|^2 x + h_1^* n_1 \\
    h_2^* y_2 &= \|h_2\|^2 x + h_2^* n_2
\end{align*}
\]

\[
\begin{align*}
    x &= \frac{h_1^* y_1 + h_2^* y_2}{\|h_1\|^2 + \|h_2\|^2}
\end{align*}
\]

\[
\begin{align*}
    SNR_{MRC} &= \frac{E[(\|h_1\|^2 + \|h_2\|^2)X]^2]}{E[(h_1^* n_1 + h_2^* n_2)^2]} \\
    &= \frac{(\|h_1\|^2 + \|h_2\|^2)^2 E(X^2)}{(\|h_1\|^2 + \|h_2\|^2)\sigma^2} \\
    &= \frac{(\|h_1\|^2 + \|h_2\|^2)E(X^2)}{\sigma^2}
\end{align*}
\]

\[
\begin{align*}
    SNR_{single} &= \frac{E[(\|h_1\|^2 X)^2]}{E[(h_1^* n_1)^2]} \\
    &= \frac{\|h_1\|^4 E(X^2)}{(\|h_1\|^2)\sigma^2} \\
    &= \frac{\|h_1\|^2 E(X^2)}{\sigma^2}
\end{align*}
\]

\[
\begin{align*}
    \text{gain} &= \frac{\|h_1\|^2 + \|h_2\|^2}{\|h_1\|^2}
\end{align*}
\]
Transmit Diversity

- Deliver a symbol twice in two consecutive time slots
- Repetitive code

\[
\begin{bmatrix}
  x_1 & 0 \\
  0 & x_1
\end{bmatrix}
\]

- Diversity: 2
- Data rate: 1/2 symbols/s/Hz
Transmit Diversity

- Alamouti code (space-time block code)

\[ x = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \]

- Diversity: 2
- Data rate: 1 symbols/s/Hz
Transmit Diversity

- Alamouti code (space-time block code)

\[
x = \begin{bmatrix}
  x_1 & -x_2^* \\
  x_2 & x_1^*
\end{bmatrix}
\]

\[
y(t) = h_1 x_1 + h_2 x_2 + n_1
\]

\[
y(t+1) = h_2 x_1^* - h_1 x_2^* + n_2
\]
$$h_1^y y_1(t) = h_1^y x_1^y + h_2^y h_1^x x_2^y + h_1^y n_1^y$$

$$y^y(t+1) = -h_1^y x_2 + h_2^y x_1 + n_2^y$$

$$h_2^y y_2^y(t+1) = -h_1^y h_2 x_2 + |h_2|^2 x_1 + h_2^y n_2^y$$

$$h_1^y y_1(t) + h_2^y y_2^y(t+1)$$

$$= (h_1^2 + h_2^2) x_1 + (h_1^y + h_2^y) n$$

$$\text{SNR: } \frac{(h_1^2 + h_2^2) \mathbb{E}[x_i^2]}{\sigma^2}$$
Multiplexing-Diversity Tradeoff

![Diagram showing signal transmission](image)

**Repetitive scheme**

\[ x = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix} \]

Diversity: 4
Data rate: 1/2 sym/s/Hz

**Alamouti scheme**

\[ x = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \]

Diversity: 4
Data rate: 1 sym/s/Hz

But 2x2 MIMO has 2 degrees of freedom
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**Interference Nulling**

- Signals cancel each other at Alice’s receiver
- Signals don’t cancel each other at Bob’s receiver
  - Because channels are different

\[ h_1 \alpha + h_2 \beta = 0 \]
Quiz

• Say there exist a 3x2 link, which has a channel

\[
H_{3\times2} = \begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22} \\
    h_{31} & h_{32}
\end{bmatrix}
\]

How can a 3-antenna transmitter transmit a signal \( x \), but null its signal at two antennas of a two-antenna receiver?
Zero-Forcing Beamforming

\[ y_k = (\sqrt{P_k} h_k w_k) s_k + \sum_{j \neq k} (\sqrt{P_j} h_j w_j) s_j + n_k \]

\[ y = H WPx + n \]

Let \( W = H^\dagger = H^* (HH^*)^{-1} \), then \( y = Px + n \)
Interference Alignment

2-antenna receiver

N-antenna node can only decode N signals

If $I_1$ and $I_2$ are aligned,
Interference Alignment

If $I_1$ and $I_2$ are aligned,

→ appear as one interferer

→ 2-antenna receiver can decode the wanted signal

N-antenna node can only decode N signals
1. Transmitter can rotate the received signal

To rotate received signal $y$ to $y' = Ry$, 

- Rotate Signal

2-antenna receiver
Rotate Signal

\[ y_1 = (h_{11} \alpha + h_{21} \beta) x \]

\[ y_2 = (h_{12} \alpha + h_{22} \beta) x \]

\[ \tilde{y} = (h_{11} + h_{21}, h_{12} + h_{22}) \]

\[ \tilde{y}' = (u, v) \]

\[ (h_{11} \alpha + h_{21} \beta) = u \]

\[ (h_{12} \alpha + h_{22} \beta) = v \]

How to align the signal along the interference?

→ Find the direction of the interference and rotate the signal to that direction
MU-MIMO Bit-Rate Selection

Select a proper rate based on $\text{SNR}_{ZF}$

Select a proper rate based on $\text{SNR}_{ZF}$
MU-MIMO User Selection

Pairing different clients as concurrent receivers results in different sum-rates.