

# Basics 1

2015/03/06

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# Harmonic Signals and Exponentials

$$s = \cos(\omega t)$$

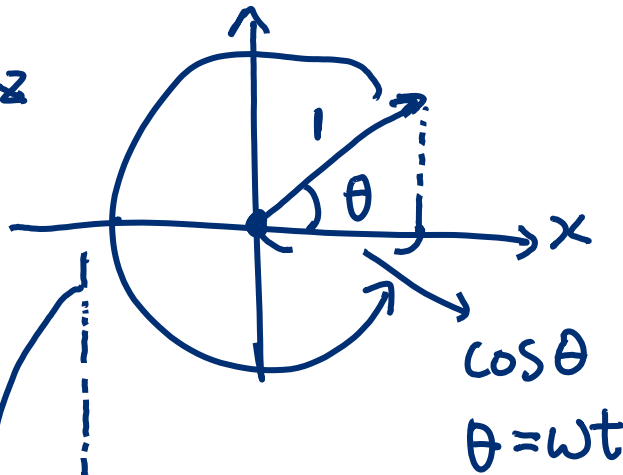
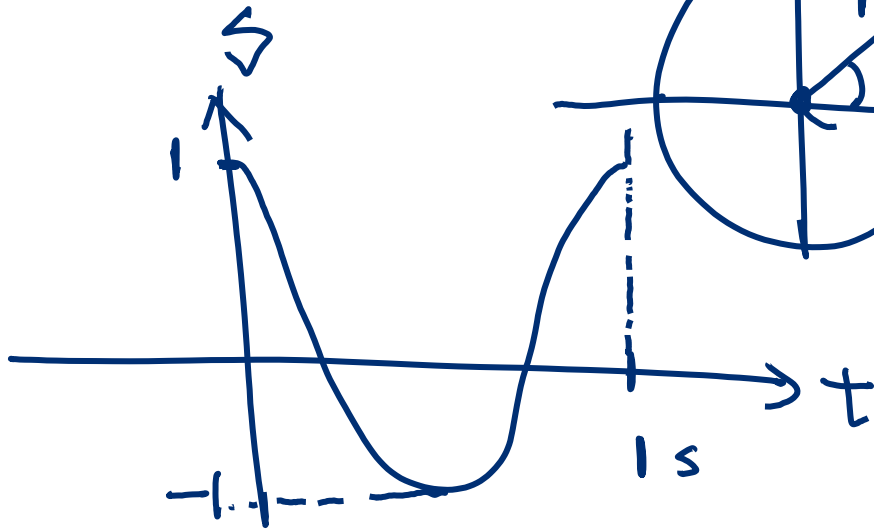
$$= \cos(2\pi f_1 t)$$

$$\boxed{\omega_1 = 2\pi} \quad f_1 = 1$$

1 Hz

$\omega$ : angular frequency

單位時間內轉多少 rad



$f$ : frequency

單位時間

轉多少圈 ( $\sqrt{2\pi}$ )

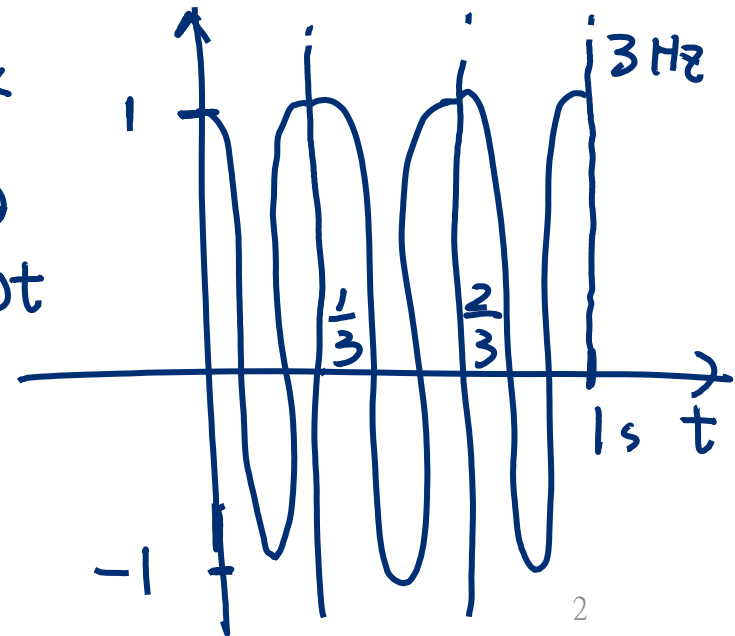
$$\cos(2\pi f t)$$

$\omega$

$$\omega_2 = 6\pi \quad 2\pi f_2$$

$f_2 = 3$

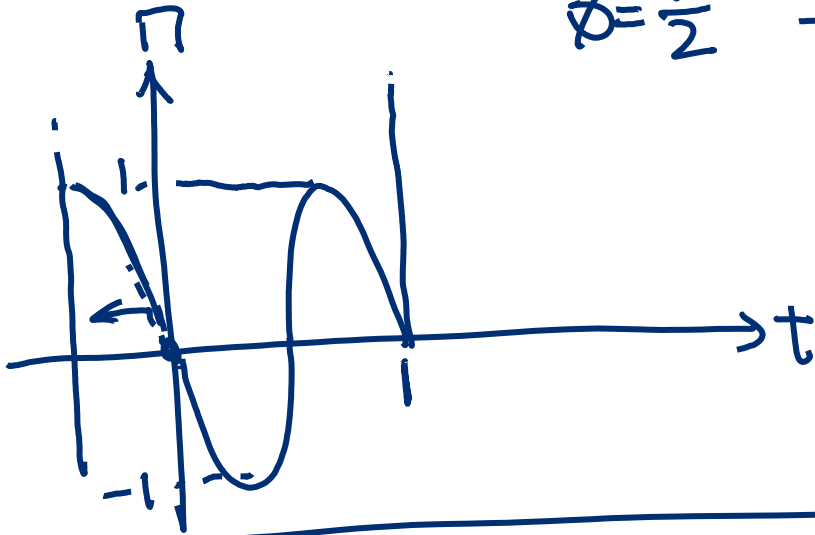
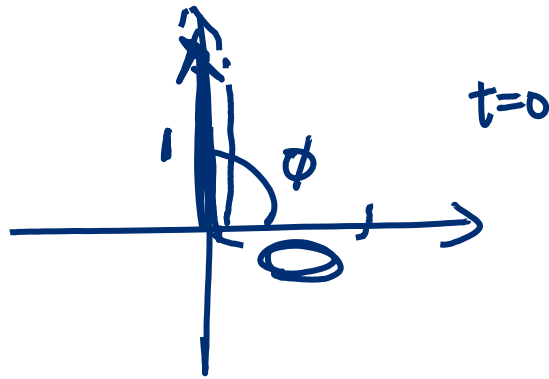
$$s = \cos(\omega_2 t)$$



$$f=1$$

$$\pi = \cos(2\pi f t + \phi)$$

$$\phi = \frac{\pi}{2}$$



$\xi$

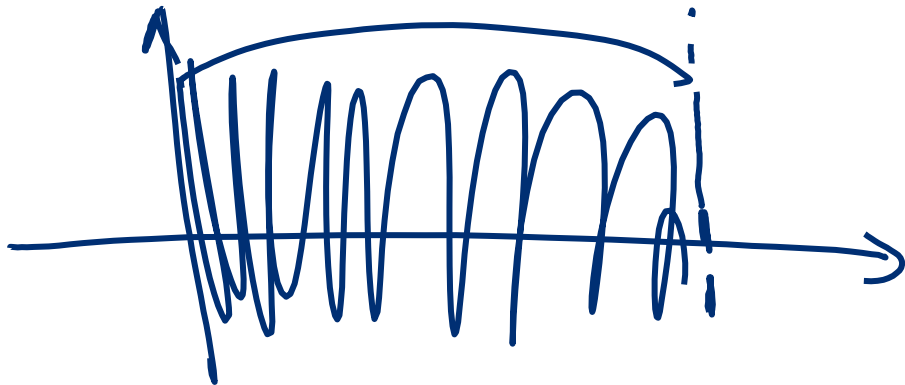
$f_c$

2.46 Hz

$Q = 10^9$

$\cos(2\pi f_c t + \theta)$

2400,000,000 Hz



# Complex Numbers

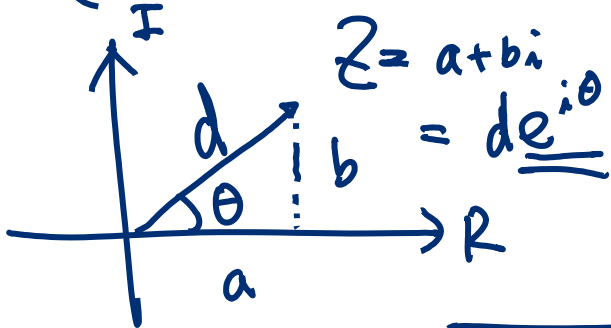
$$i = j = \sqrt{-1}$$

$$z = a + bi$$

$$z^* = a - bi$$

$$z_1 + z_2 = (a + bi) + (c + di)$$

$$= (a + c) + (b + d)i$$



$$d = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$z = d e^{i\theta}$$

$$= d \cos \theta + di \sin \theta$$

$$a = d \cos \theta$$

$$b = d \sin \theta$$

$$z_1 = d_1 e^{i\theta_1} \quad z_2 = d_2 e^{i\theta_2}$$

$$z_1 \cdot z_2 = d_1 d_2 e^{i\theta_1} e^{i\theta_2} = d_1 d_2 e^{i(\theta_1 + \theta_2)}$$

$$e^{i(\omega t + \phi)} = \cos(\omega t + \phi) + i \sin(\omega t + \phi)$$

$$= \boxed{e^{i\omega t}} \boxed{e^{i\phi}}$$

$$= \cos(\omega t) \cos \phi - \sin(\omega t) \sin \phi + i \sin(\omega t) \cos \phi + i \cos(\omega t) \sin \phi$$

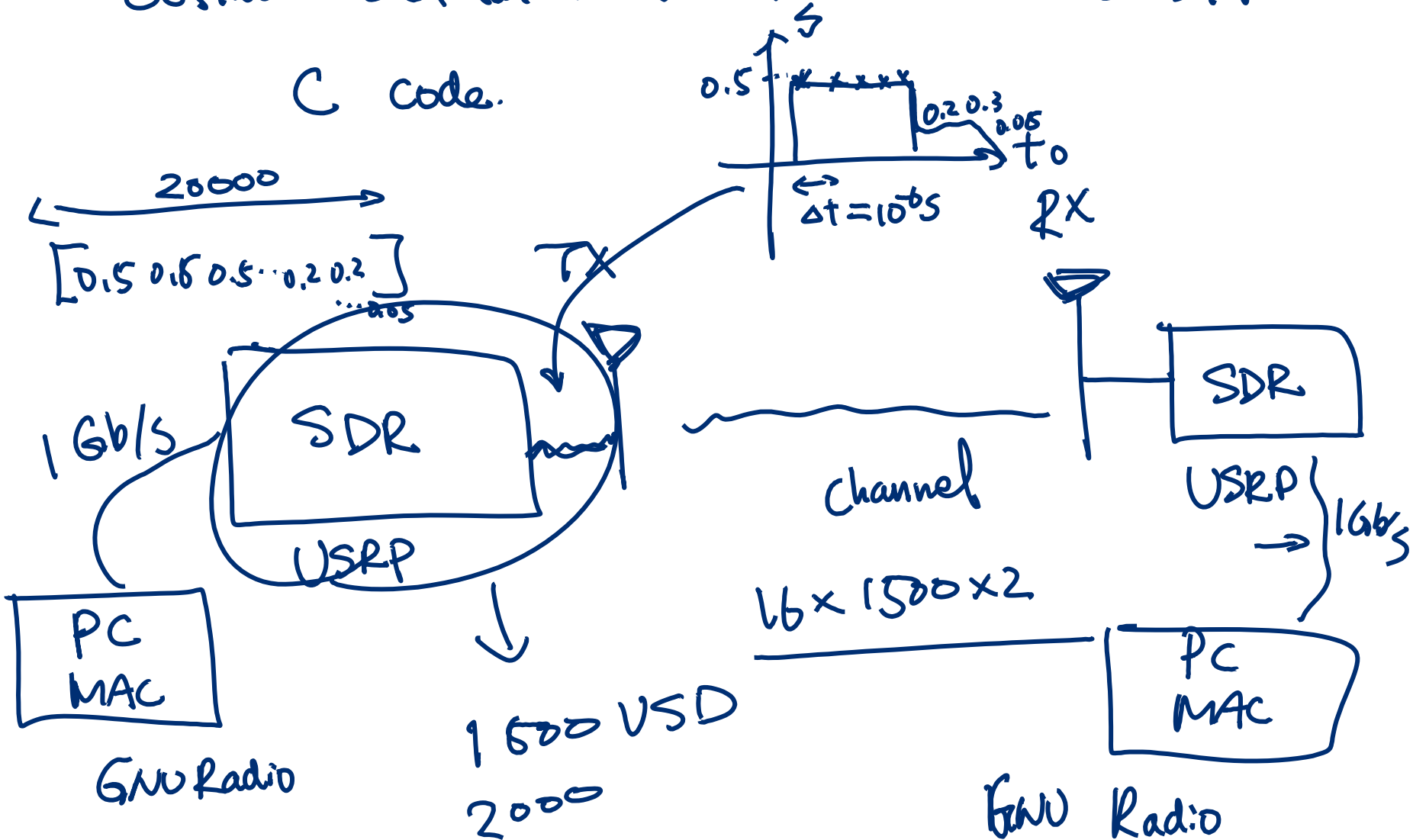
$$= \left[ \cos(\omega t) + i \sin(\omega t) \right] \cdot \left[ \cos \phi + i \sin \phi \right]$$

=

# Lab 3.

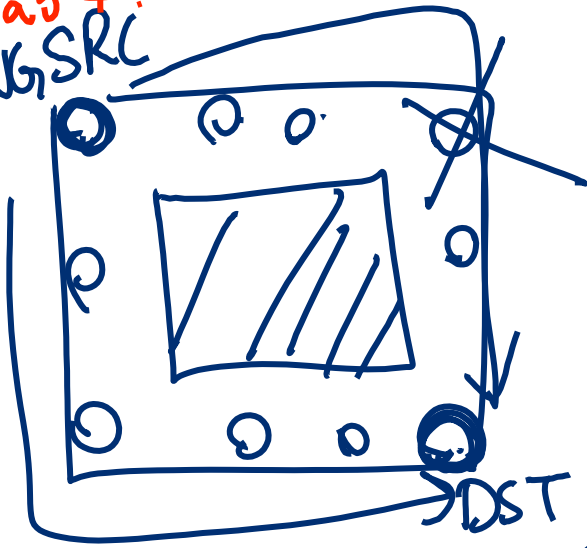
## Software Defined Radio + WiFi + OFDM

C code.



Lab 4.

PING SRC

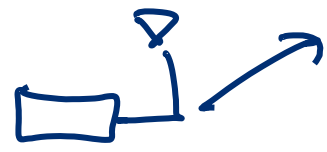
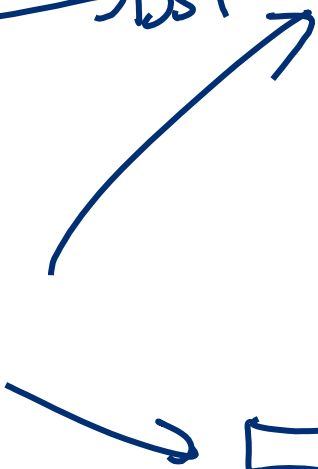
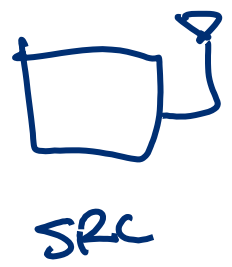


250 Kb/s

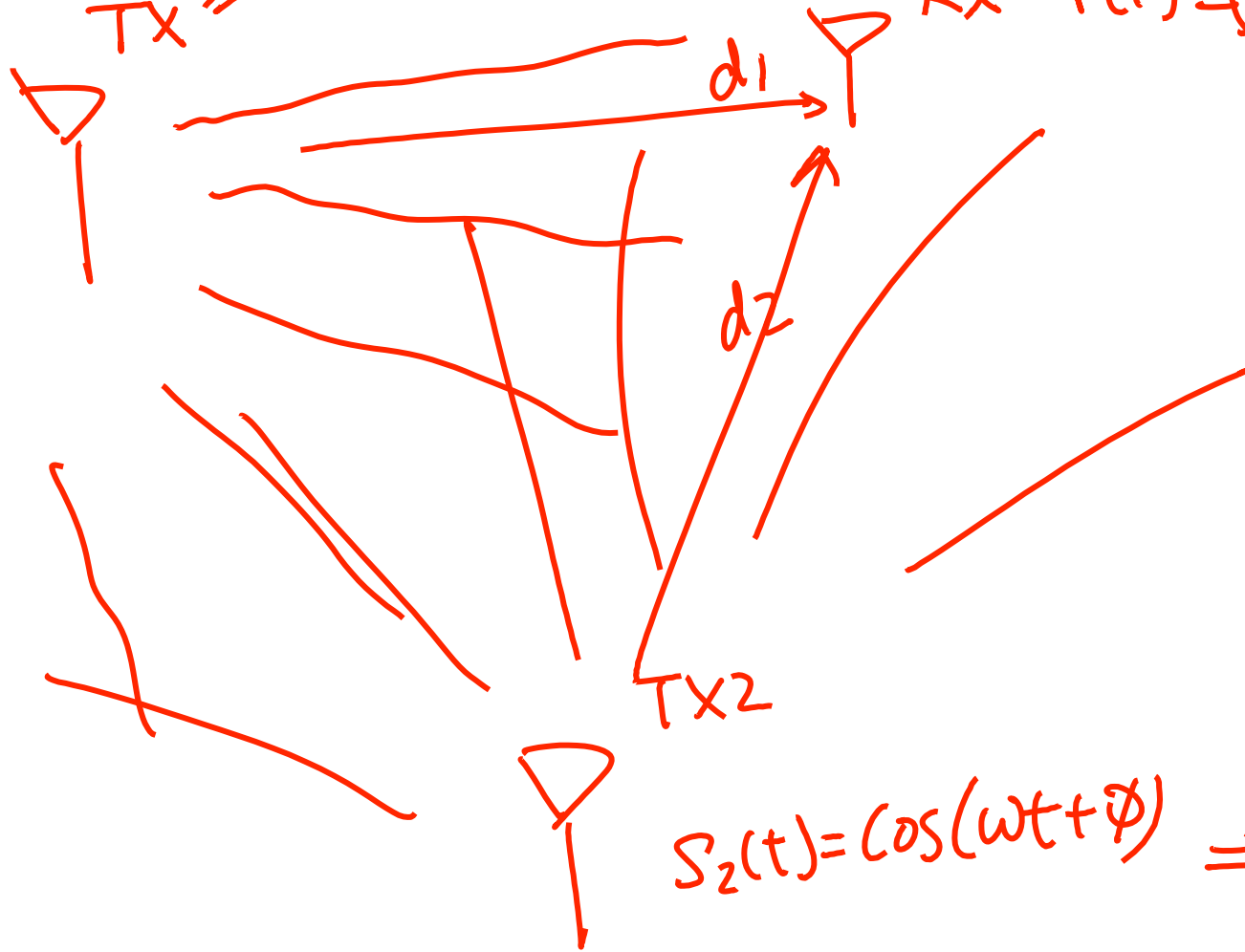
802.15.4

(ZigBee)

2.4 GHz



Multiplexing  $S_1(t) = \cos(\omega t + \phi)$   
 TX = Transmitter RX = Receiver



$r(t) = S_1(t) + S_2(t)$

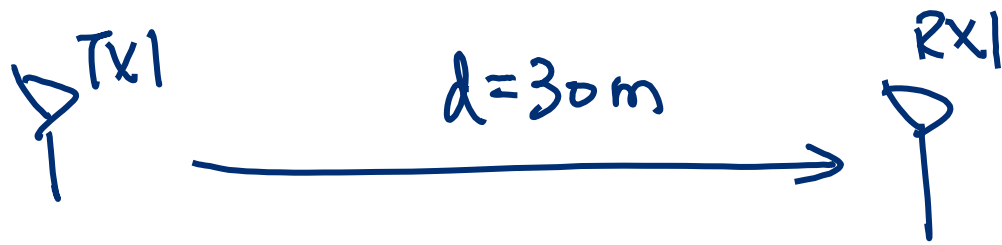
" + "

↓

0



Wireless  
 $S_2(t) = \cos(\omega t + \phi) \Rightarrow$  Broadcast  
 " + "



$$S_1(t) = \cos(\omega t)$$

$$r_1(t) = \cos(\omega t + \boxed{\phantom{00}})$$

① Propagation Delay

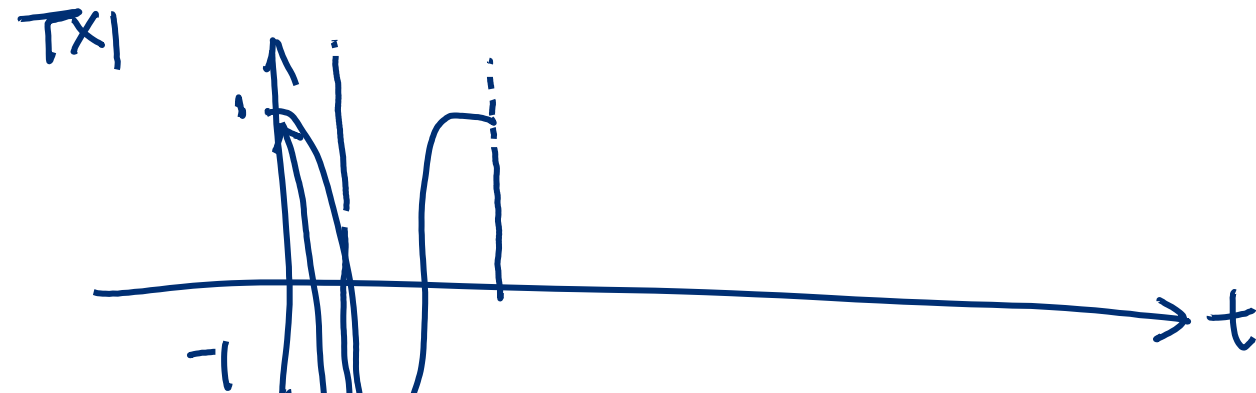
$$c = 3 \times 10^8 \text{ m/s}$$

$$\Delta t = \frac{d}{c} = \frac{10 \times 3}{3 \times 10^8}$$

$$= 10^{-7} \text{ s}$$

$$= 100 \text{ ns}$$

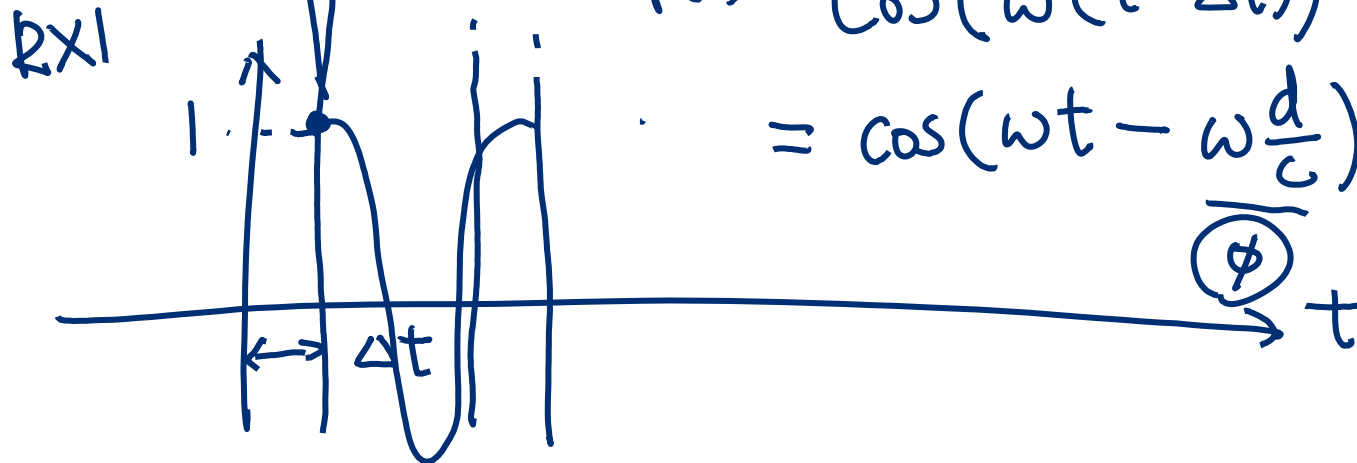
$$1 \text{ ns} = 10^{-9} \text{ s}$$



$$r(t) = \cos(\omega(t - \Delta t))$$

$$= \cos(\omega t - \omega \frac{d}{c})$$

$$\phi$$







②

$a$ : Channel  
loss

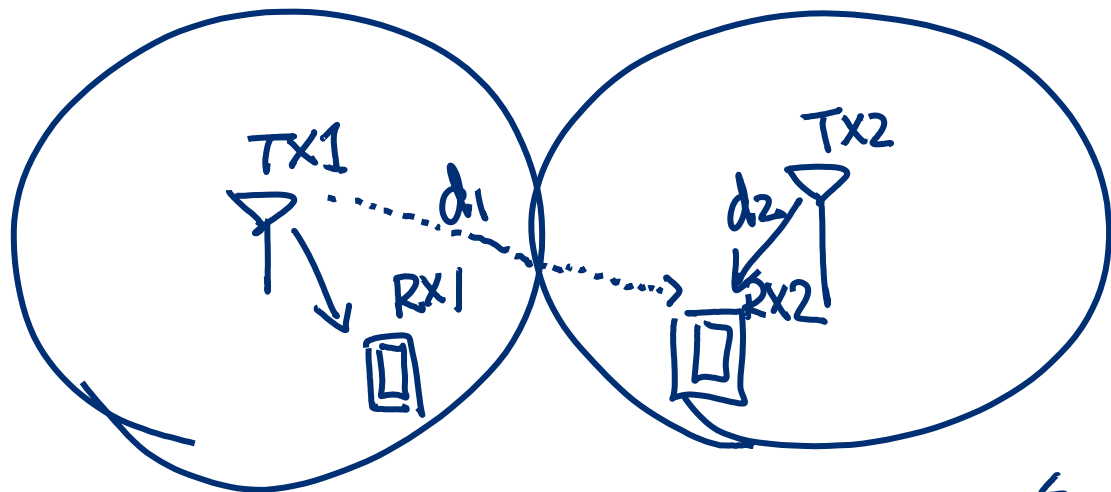
$$s(t) = \sqrt{P} \cos(\omega t)$$

$$r(t) = \frac{\sqrt{P}}{a} \cos(\omega t + \phi)$$

$$a \propto d^r$$

# 1. Spatial Multiplexing (Directional)

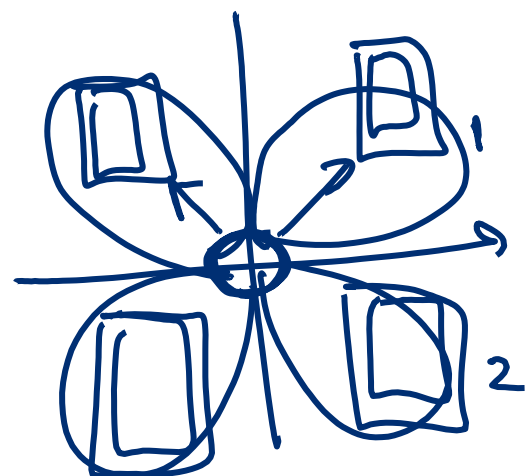
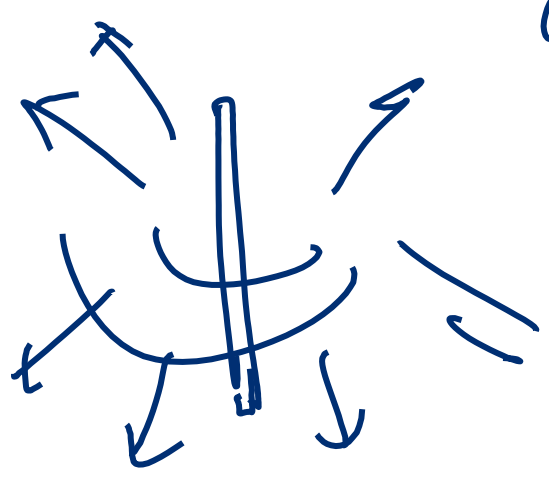
Spatial Reuse



$d_1 \gg d_2$

$$r_2(t) = \frac{s}{d_1} S_1(t) + \frac{s}{d_2} S_2(t)$$

↗ 0

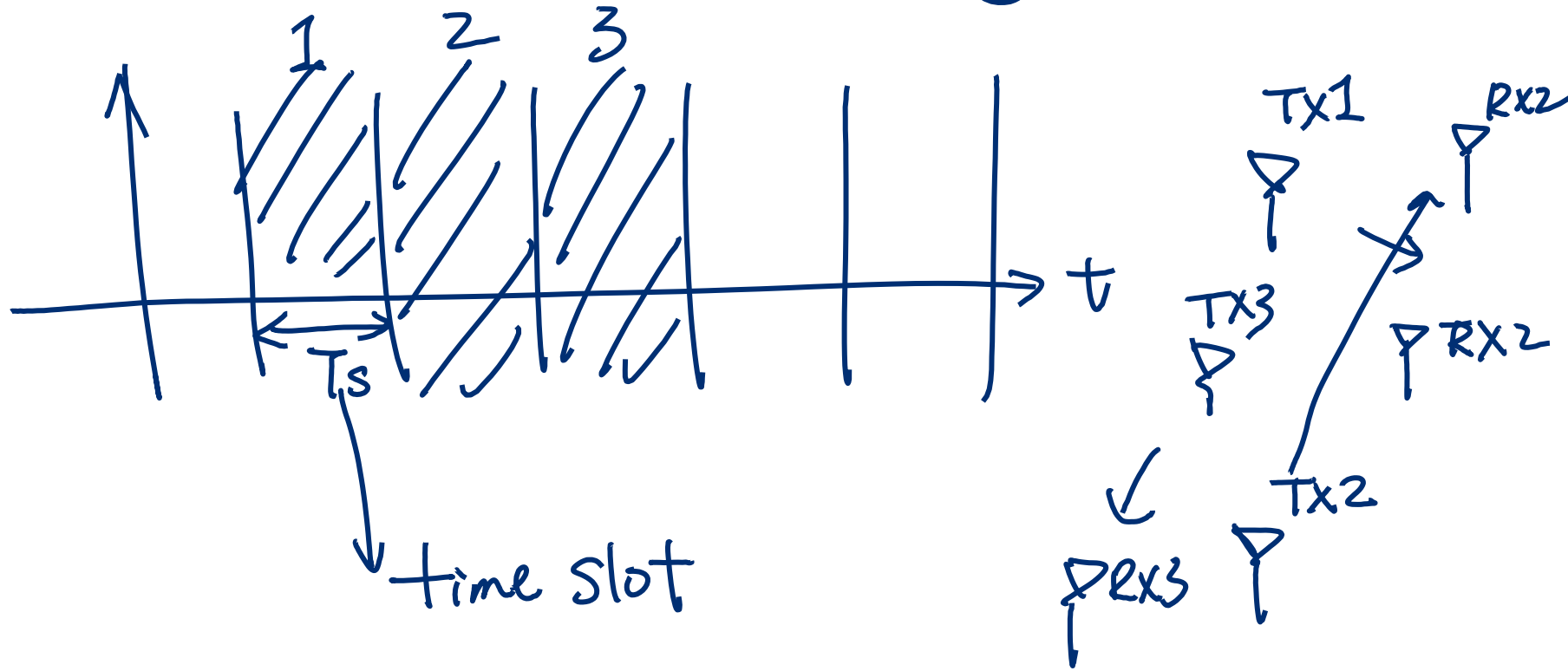


$d_1 \gg d_2$

$$r_2(t) = S_1(t) + S_2(t)$$

↗ ↘

## 2. Time-Division Multiplexing



## 3. Code Division Multiplexing

CDMA

# 4. Frequency Multiplexing

$$S_1(t) = \cos(\omega_c t)$$

$\omega_c$   
 $f_c$

c: carrier wave

$$\int_0^T r(t) \times \underline{\cos(\omega_c t)} dt = \int_0^T \cos(\omega_1 t) \cos(\omega_c t) + \cos(\omega_2 t) \cos(\omega_c t) dt$$

$$S_1(t) + S_2(t)$$

$$= \cos(\omega_1 t) + \cos(\omega_2 t)$$

$$\begin{aligned} & \cos(\omega_1 t) \cos(\omega_2 t) \\ &= \frac{1}{2} \cos\left(\frac{2\omega_1}{2} t\right) \\ & \quad + \frac{1}{2} \cos\left(\frac{(\omega_1 - \omega_2)}{2} t\right) \end{aligned}$$

①  $\underline{\omega_1 = \omega_2}$

②  $\underline{\omega_1 \neq \omega_2}$

$$\int_0^T \cos(\vec{\omega} t) dt \approx 0$$