Digital Modulation

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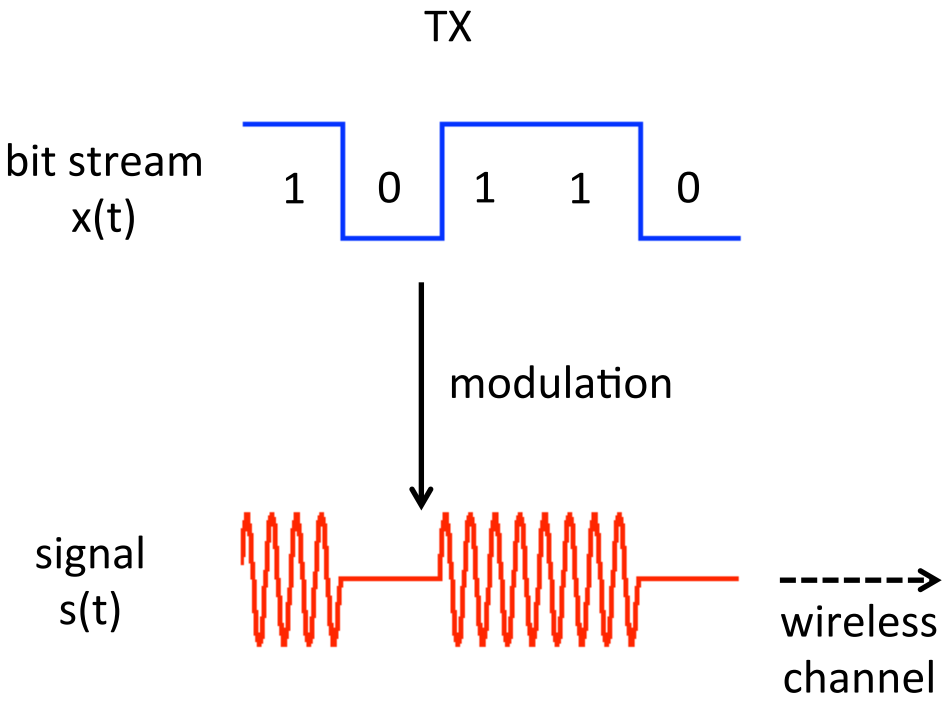
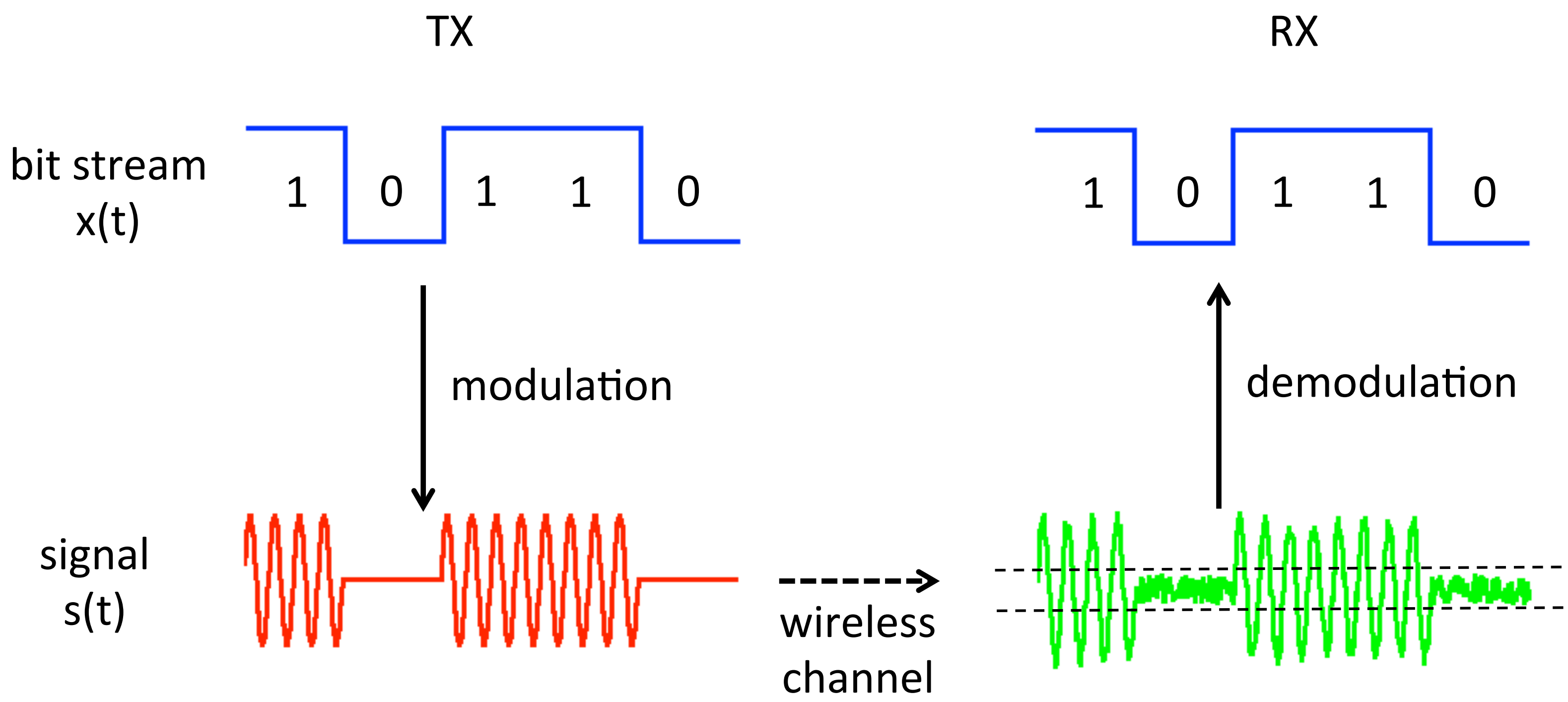
Modulation

- Map bits to signals



Demodulation

- Map signals to bits

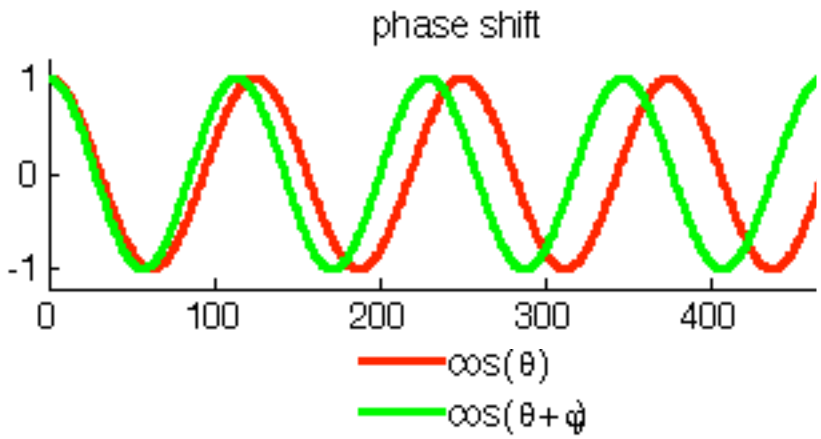


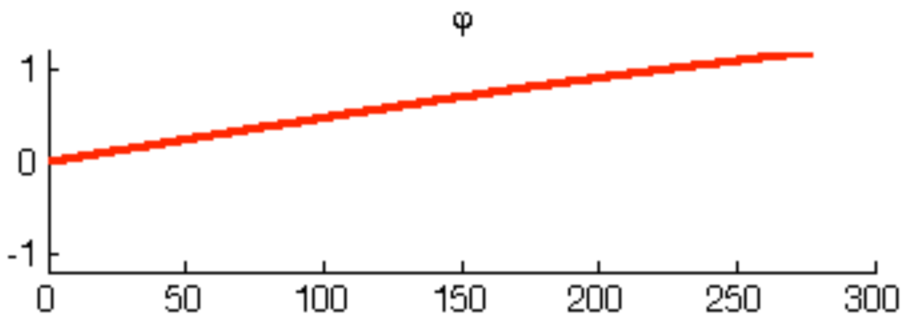
Considerations

- Data rate
  - Bits per second
- Bandwidth requirement
  - MHz
- Power efficiency
  - $\Sigma_t |s(t)|^2$
- Bit error rate
  - Related to SNR (E_b/N_0)
- Hardware cost

Sinusoid with Phase Shift

- Sinusoidal carrier with center frequency f_c
  - $s(t) = cos(2\pi f_c t)$
- Sinusoid with phase shift
  - $s(t) = cos(2\pi f_c t + \phi)$





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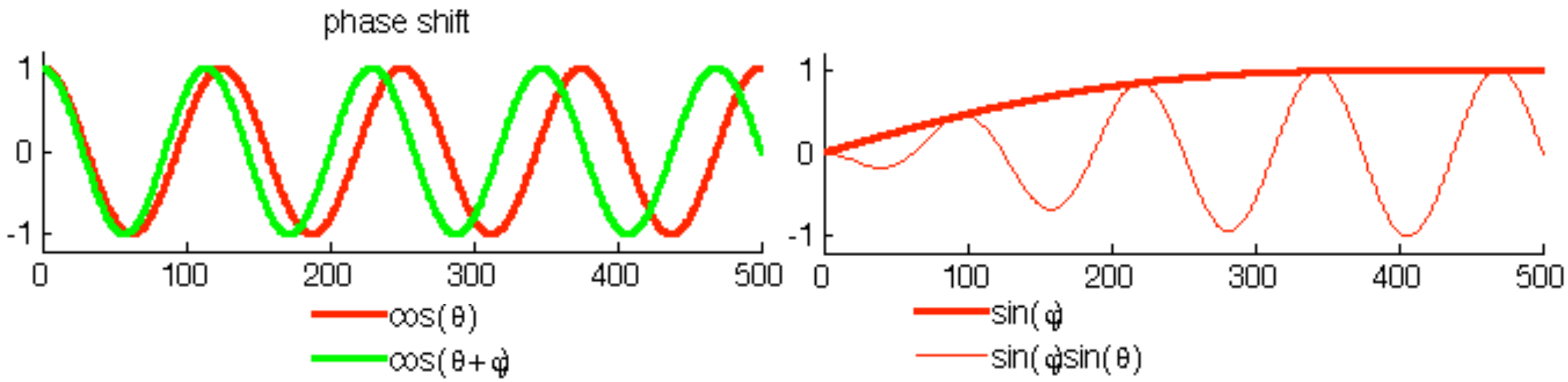
    $= cos(\phi)cos(2\pi f_c t) - sin(\phi)sin(2\pi f_c t)$



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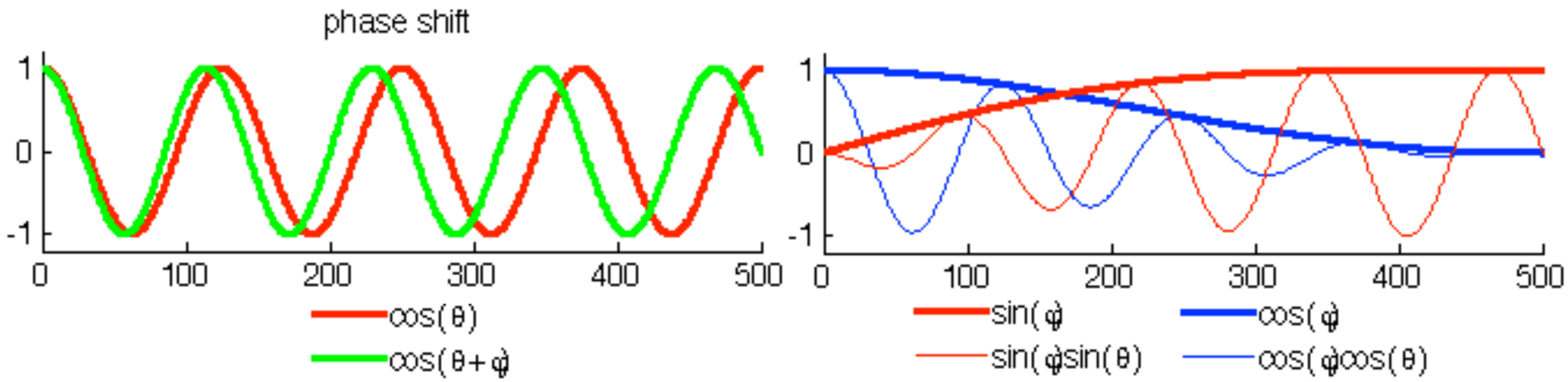

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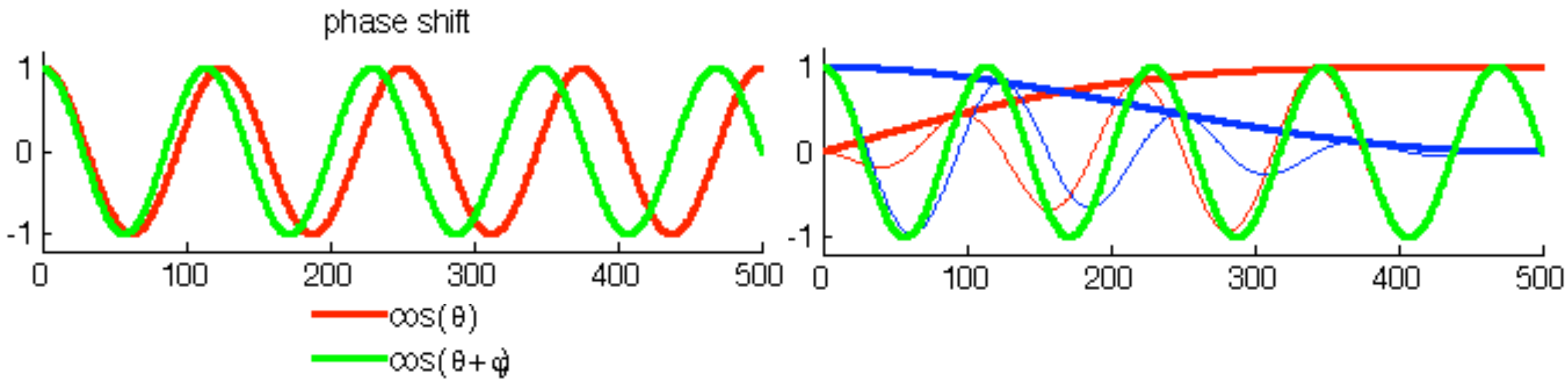

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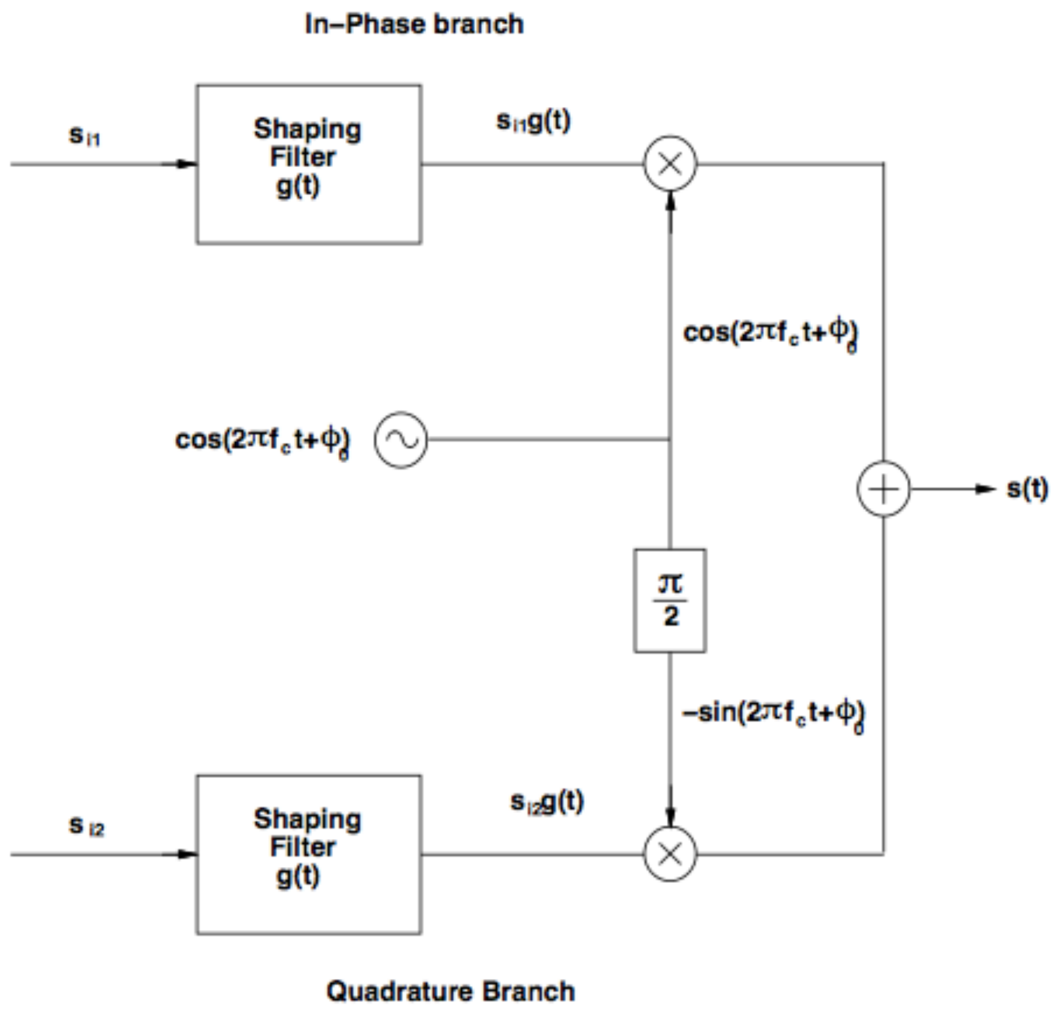
    $= cos(\phi)\cos(2\pi f_c t)-sin(\phi)sin(2\pi f_c t)$
    $= s_I * cos(2\pi f_c t) - s_Q * sin(2\pi f_c t)$
    $= s_I * cos(2\pi f_c t) - s_Q * cos(2\pi f_c t+\pi/2)$

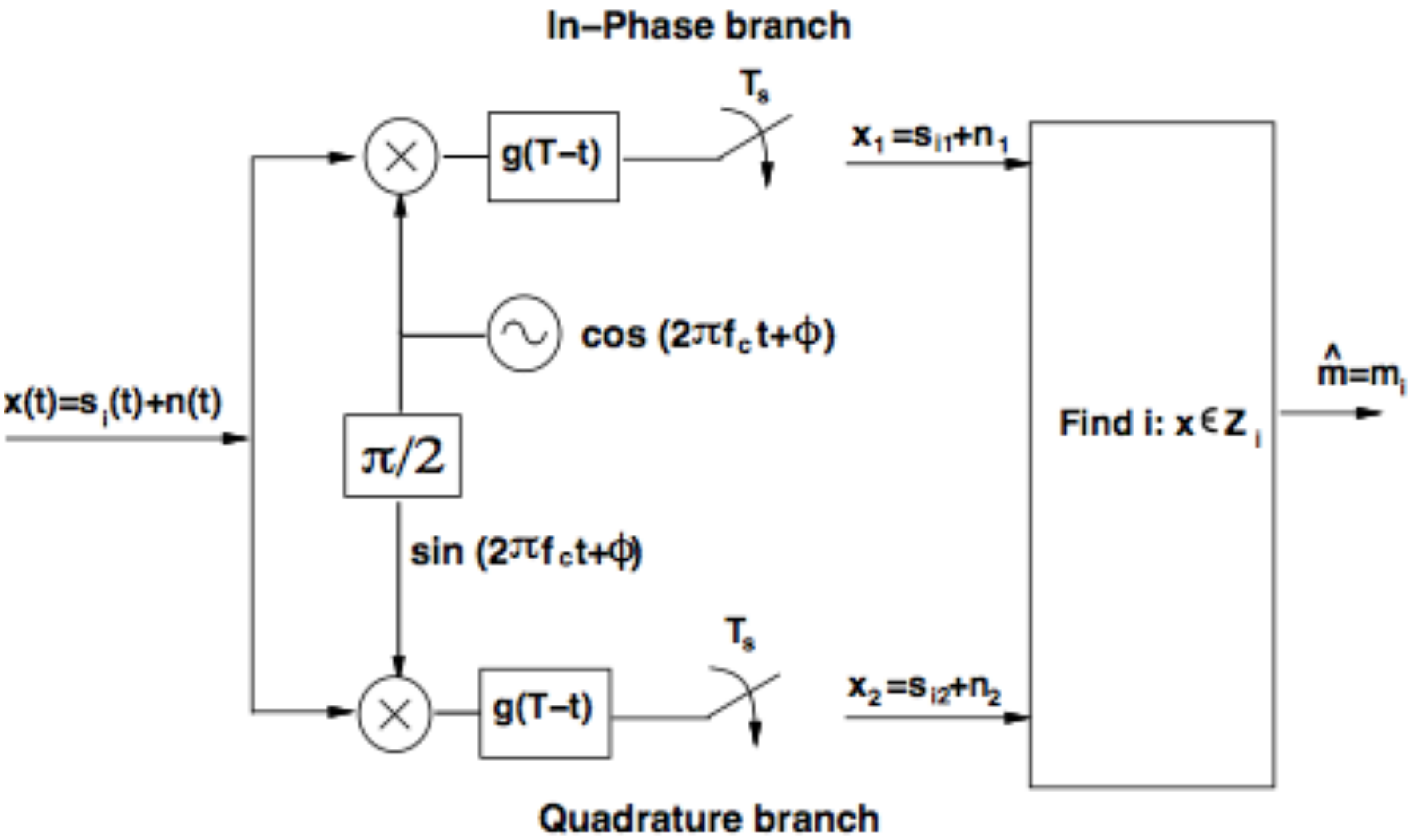
- s_I and s_Q are in-phase and quadrature components of the signal $s(t)$, respectively

Modulator



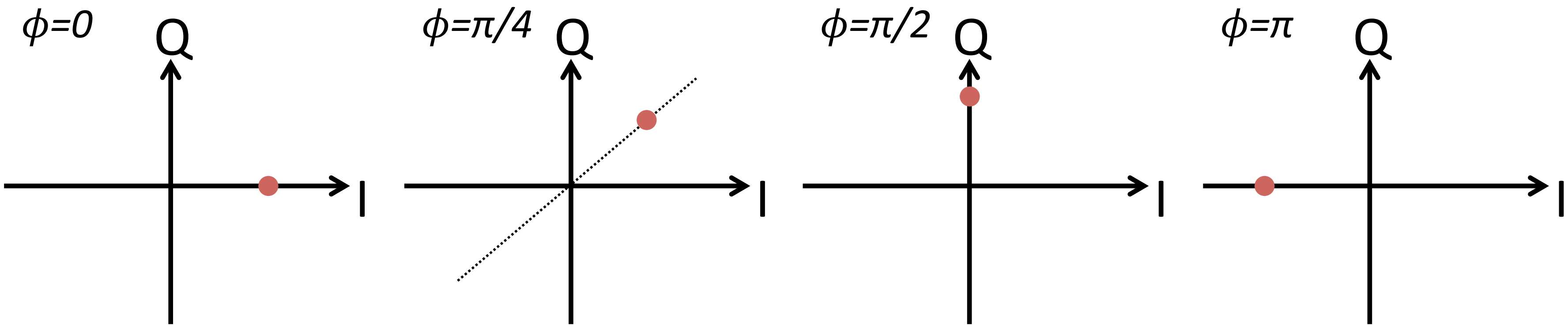
In−Phase branch
s i1
Shaping
Filter
g(t)
si1g(t)
cos(2πfct+φ0)
cos(2πfct+φ0)
s(t)
π
2
−sin(2πfct+φ0)
s i2
Shaping
Filter
g(t)
si2g(t)
Quadrature Branch

Demodulator



Constellations

- $cos(2\pi f_c t+\phi)$
  $= cos(\phi)cos(2\pi f_c t)-sin(\phi)sin(2\pi f_c t)$
  $= s_I{}^*cos(2\pi f_c t) - s_Q{}^*sin(2\pi f_c t)$
- Constellation point on I-Q plane
  - $(s_I,s_Q) = (cos(\phi), sin(\phi))$



Delay in time domain = Phase shift in frequency domain = Rotation in I-Q plane

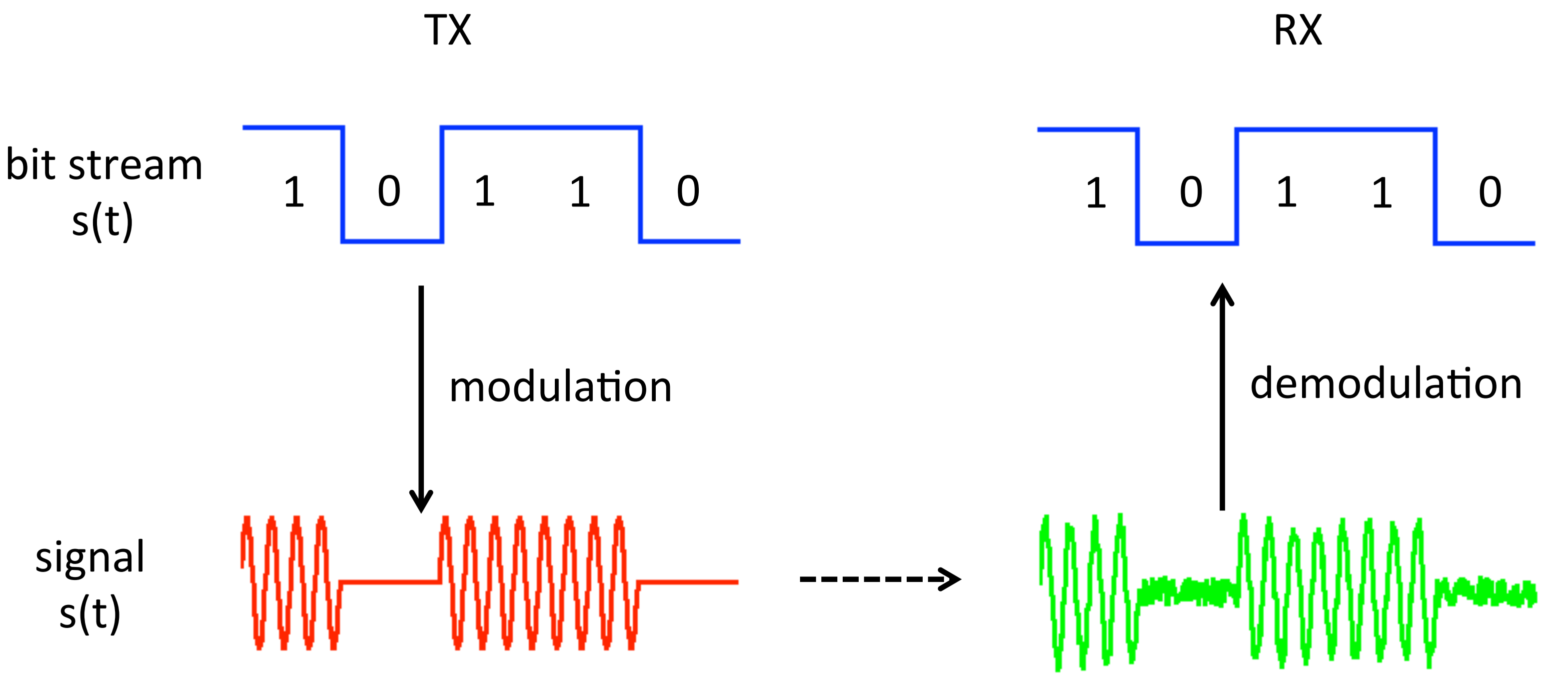
Types of Modulation

- $s(t) = A\cos(2\pi f_c t + \phi)$

- *Amplitude*
  - *ASK: Amplitude Shift Keying*
- *Frequency*
  - *FSK: Frequency Shift Keying*
- *Phase*
  - *M-PSK: Phase Shift Keying*
- *Amplitude* + *Phase*
  - *M-QAM: Quadrature Amplitude Modulation*

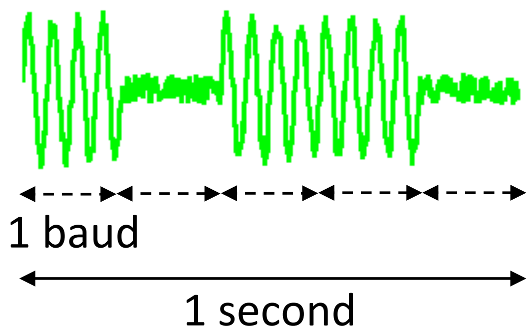
Amplitude Shift Keying (PSK)

- Represent samples using different amplitudes
  - ‘1’→A=1, ‘0’→A=0

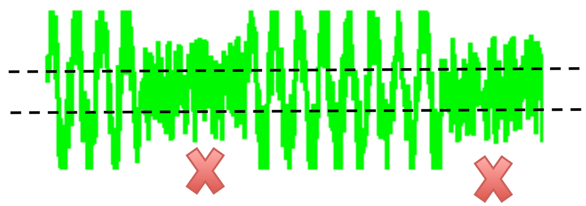


PSK

- Pros
  - Easy to implement
  - Energy efficient
  - Low bandwidth requirement
- Cons
  - Low data rate
    - bit-rate = baud rate



  - High error probability
    - Hard to pick a right threshold



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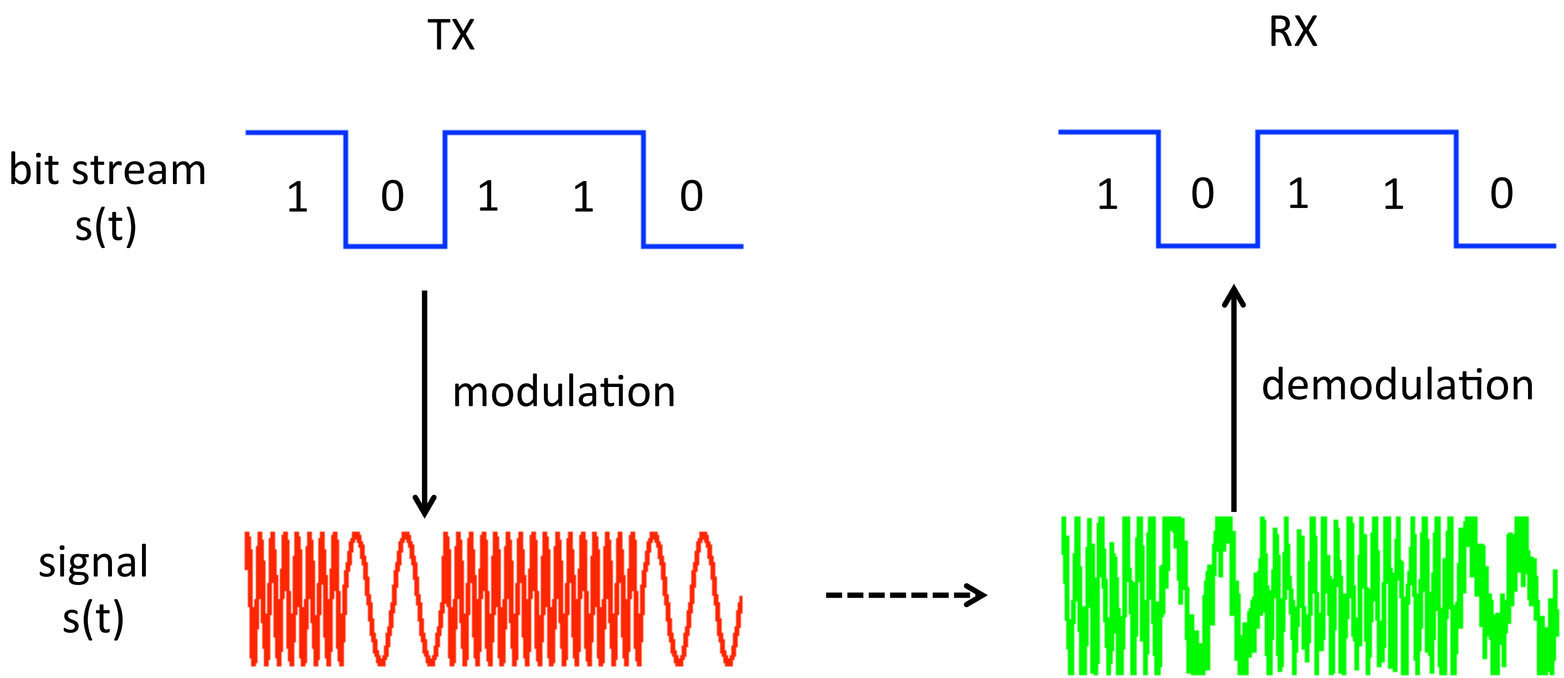
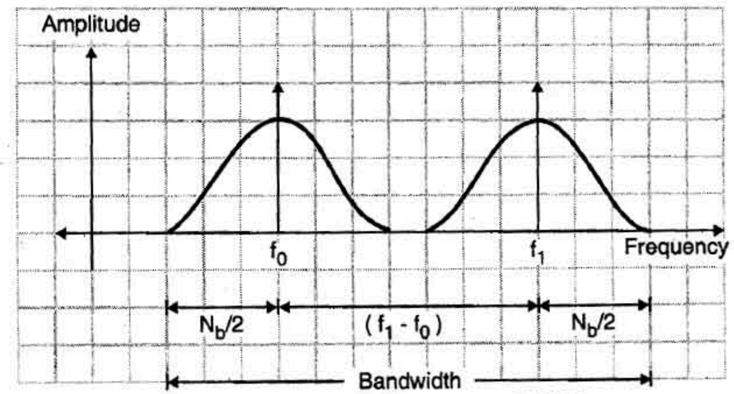
Frequency Shift Keying (FSK)

- Represent samples using different frequencies
  - '1'→$f=f_1$, '0'→$f=f_2$



FSK

- Pros
  - Easy to implement
  - Better noise immunity than ASK
- Cons
  - Low data rate
    - Bit-rate = baud rate
  - Require higher bandwidth
    - BW(min) = $N_b + N_b$



Spectrum of FSK

Types of Modulation

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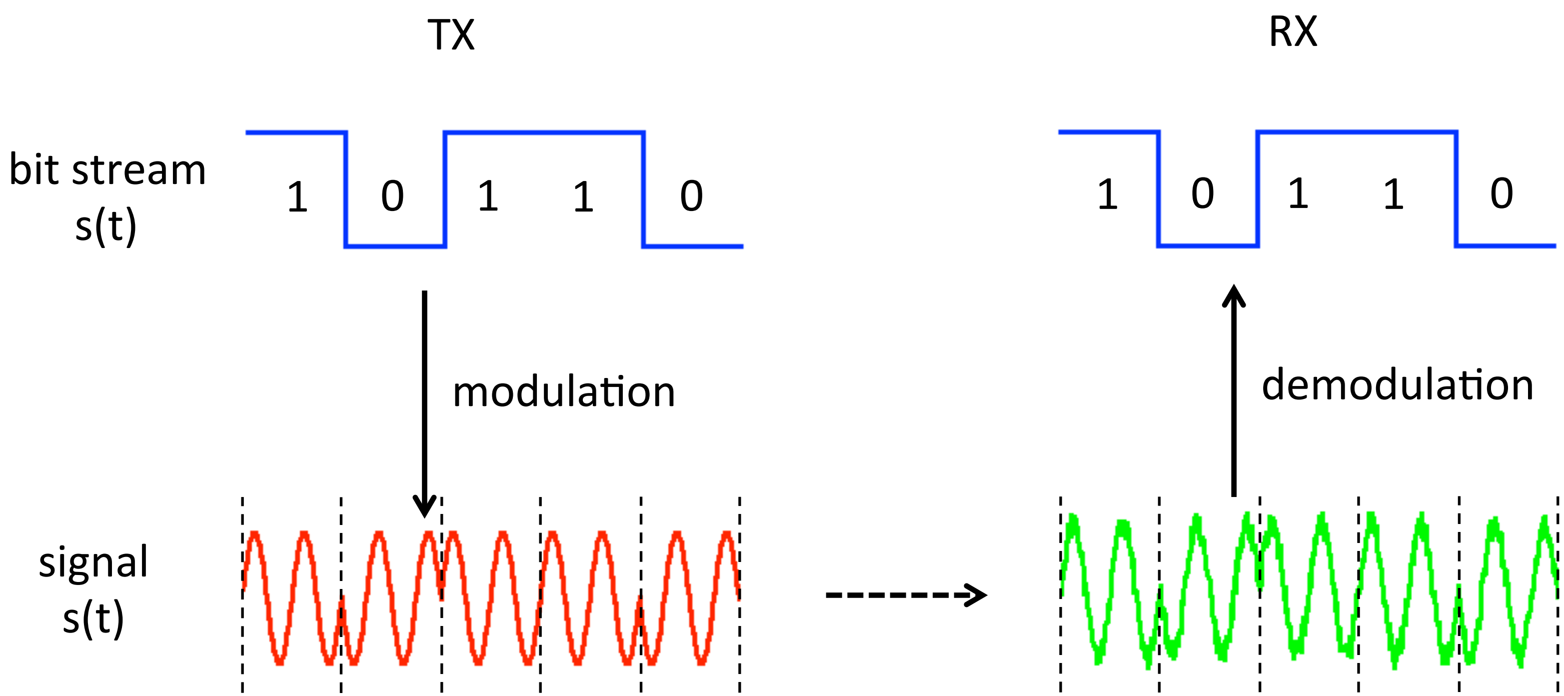
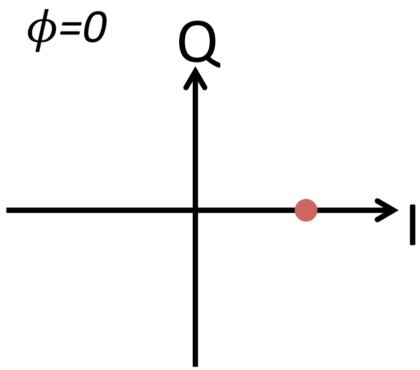
BPSK

- Represent samples using different phases
  - ‘1’→ϕ=0, ‘0’→ϕ=π

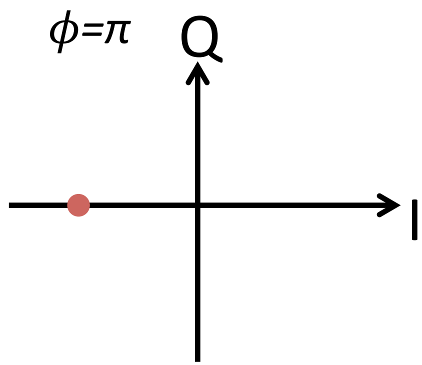


Constellation Points for BPSK

- ‘1’→ $\phi=0$
- $cos(2\pi f_c t+0)$
  $= cos(0)cos(2\pi f_c t)-sin(0)sin(2\pi f_c t)$
  $= s_I * cos(2\pi f_c t) - s_Q * sin(2\pi f_c t)$
- ‘0’→ $\phi=\pi$
- $cos(2\pi f_c t+\pi)$
  $= cos(\pi)cos(2\pi f_c t)-sin(\pi)sin(2\pi f_c t)$
  $= s_I * cos(2\pi f_c t) - s_Q * sin(2\pi f_c t)$



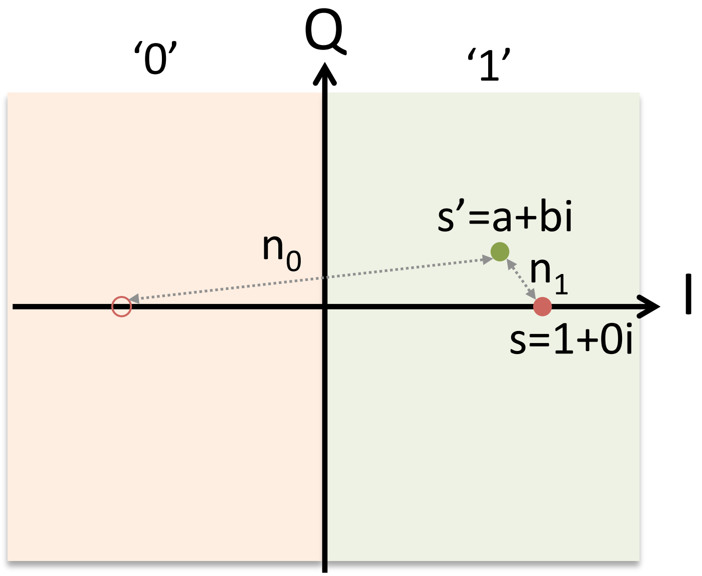
$(s_I,s_Q) = (1, 0)$
‘1’→ $1+0i$



$(s_I,s_Q) = (-1, 0)$
‘0’→ $-1+0i$

Demodulate BPSK

- Map to the closest constellation point

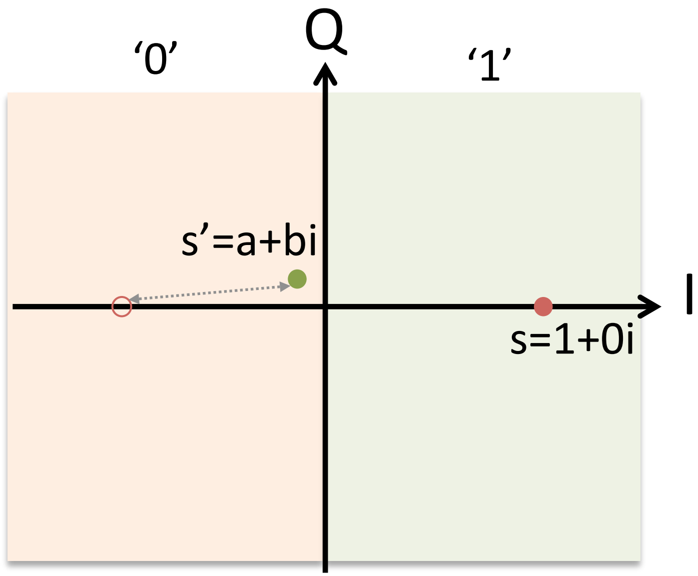


$n_1 = |s' - (1+0i)|$, $n_0 = |s' - (-1+0i)|$

Since $n_1 < n_0$, map s' to (1+0i) = '1'

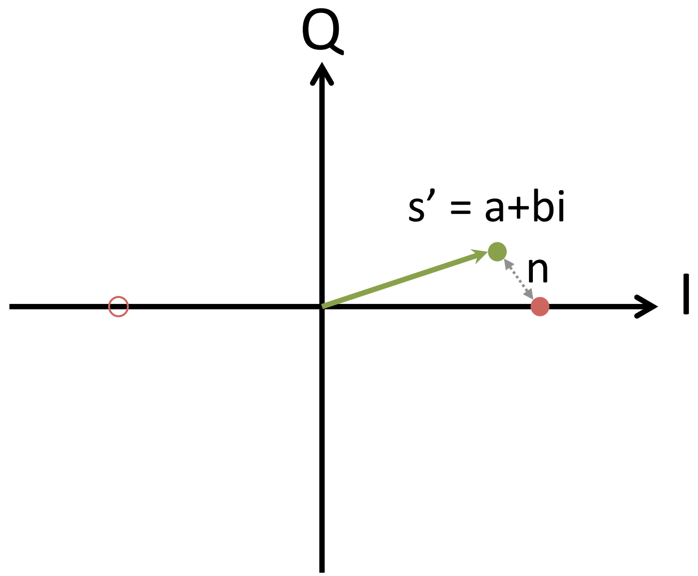
Demodulate BPSK

- Decoding error



Incorrectly map s’ to (-1+0) = ‘0’

SNR vs. BPSK BER



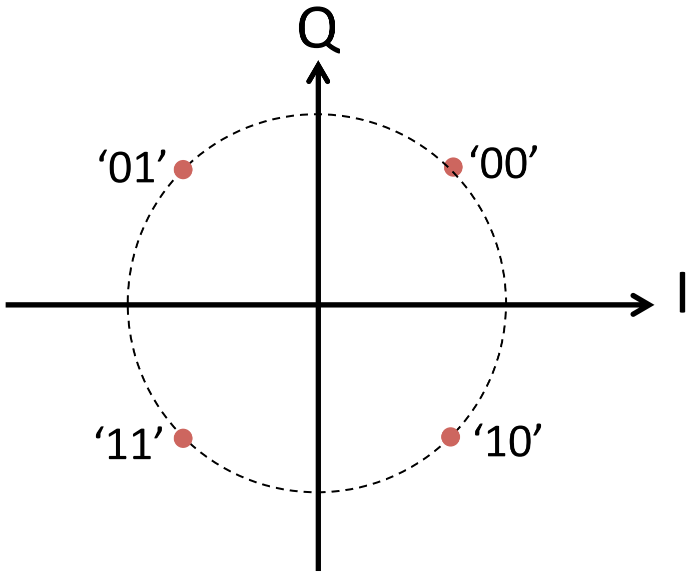
$$SNR = \frac{|s'|^2}{|n|^2} = \frac{|s'|^2}{|s' - s|^2} = \frac{|a+bi|^2}{|(a+bi)-(1+0i)|^2}$$

$$SNR_{dB} = 10\log_{10}(SNR)$$

Bit error rate: $P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

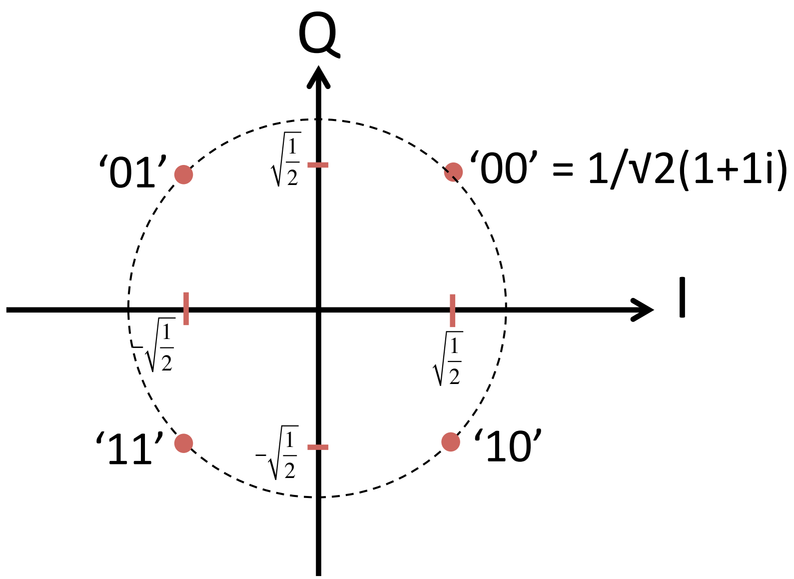
Quadrature PSK (QPSK)

- Use 2 degrees of freedom in I-Q plane
- Represent two bits as a constellation point
  - Rotate the constellations by π/2
  - Double the bit-rate
  - No free lunch: Higher error probability (Why?)



Quadrature PSK (QPSK)

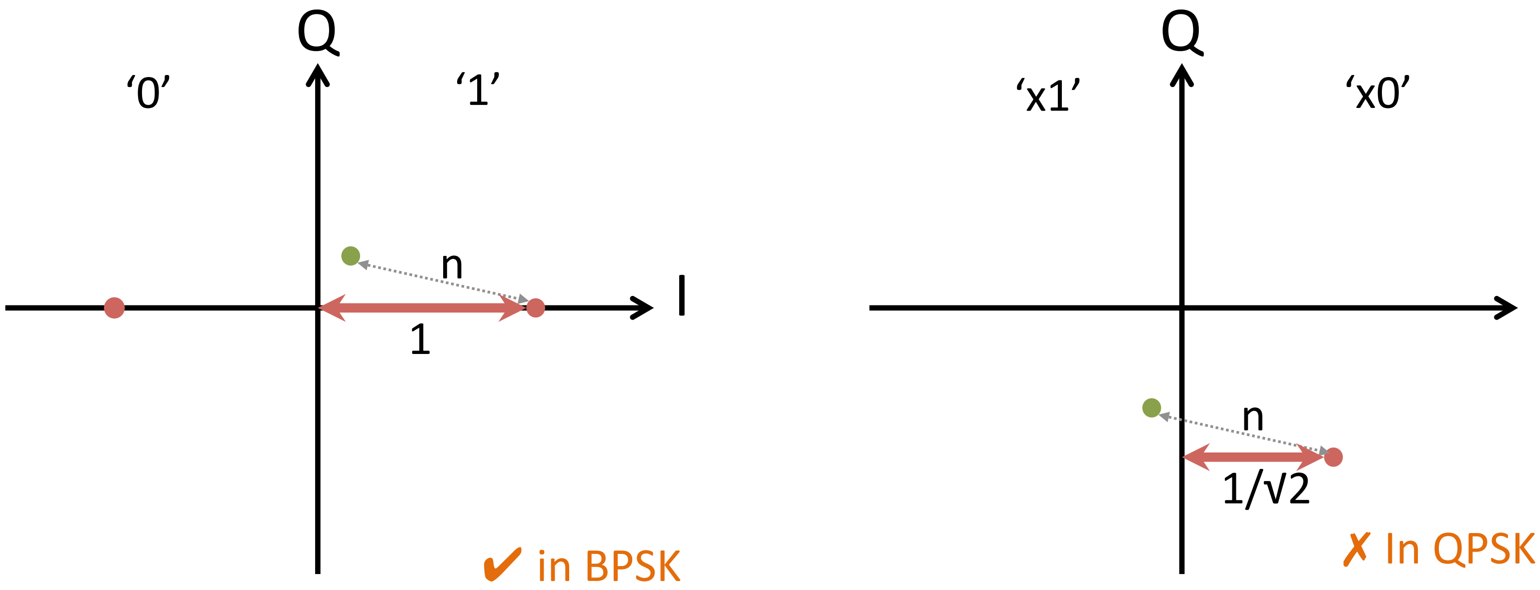
- Maximum power is bounded
  - Amplitude of each point should still be 1



Bits	Symbols
'00'	1/√2+1/√2i
'01'	-1/√2+1/√2i
'10'	1/√2-1/√2i
'11'	-1/√2-1/√2i

Higher BER in QPSK

- For a particular error *n*, the symbol could be decoded correctly in BPSK, but not in QPSK
  - Why? Each sample only gets half power.



Bit error rate: $P_b = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - \frac{1}{2}Q\sqrt{\frac{2E_b}{N_0}}\right]$

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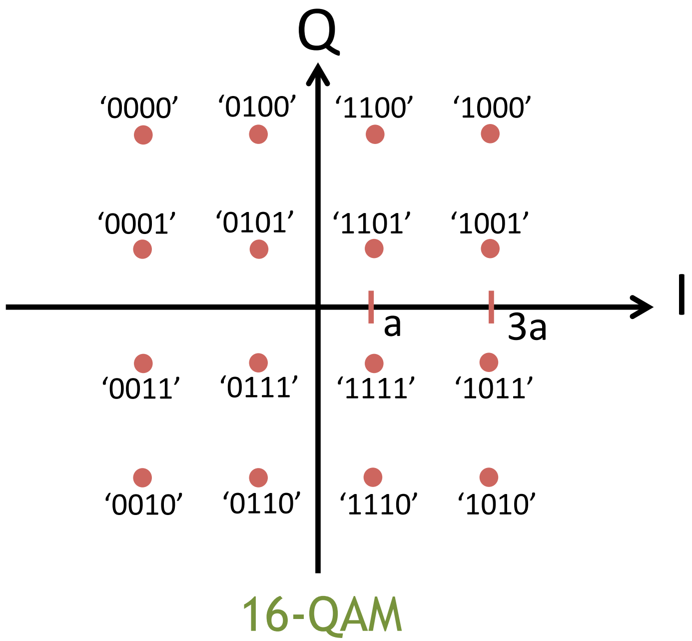
Quadrature Amplitude Modulation

- Change both amplitude and phase
- $s(t)=A\cos(2\pi f_c t+\phi)$

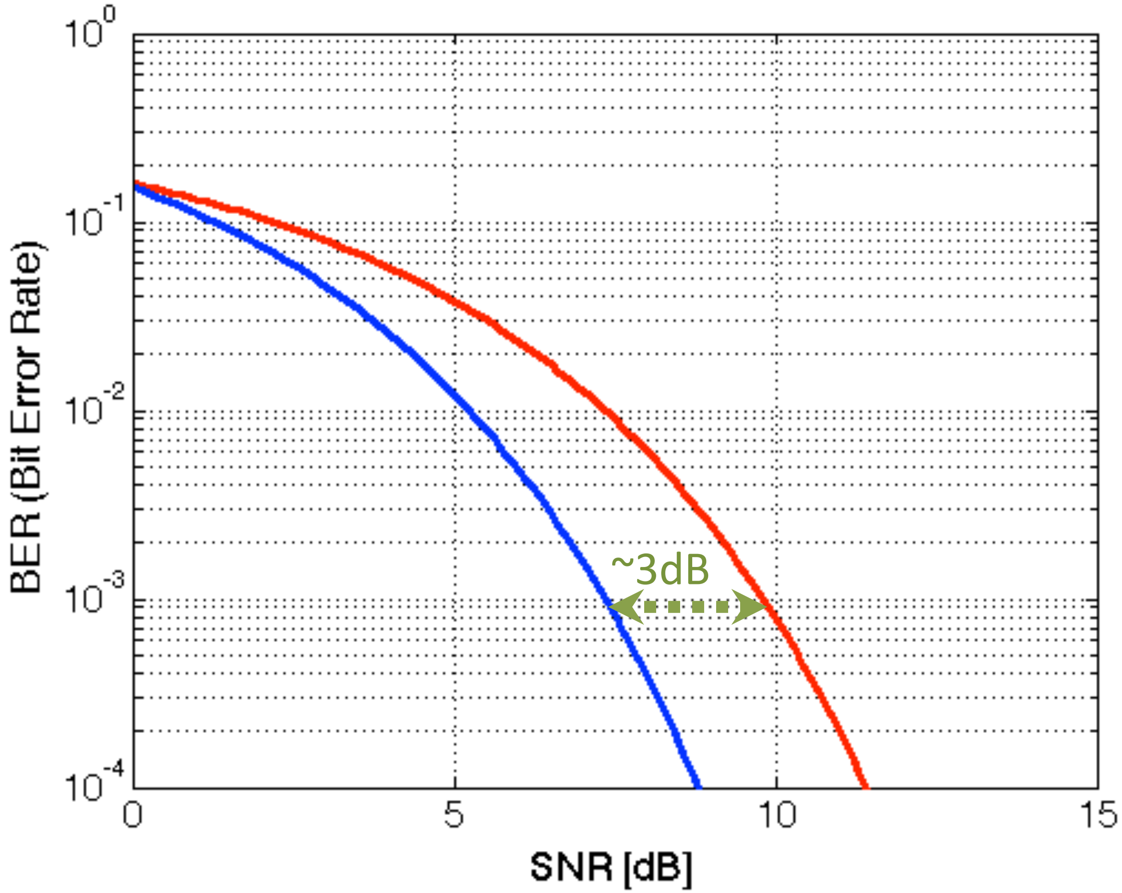


Bits	Symbols
'1000'	s_1=3a+3ai
'1001'	s_2=3a+ai
'1100'	s_3=a+3ai
'1101'	s_4=a+ai

expected power: $E\left[\left|s_i\right|^2\right]=1$

- 64-QAM: 64 constellation points, each with 8 bits

BER Comparison

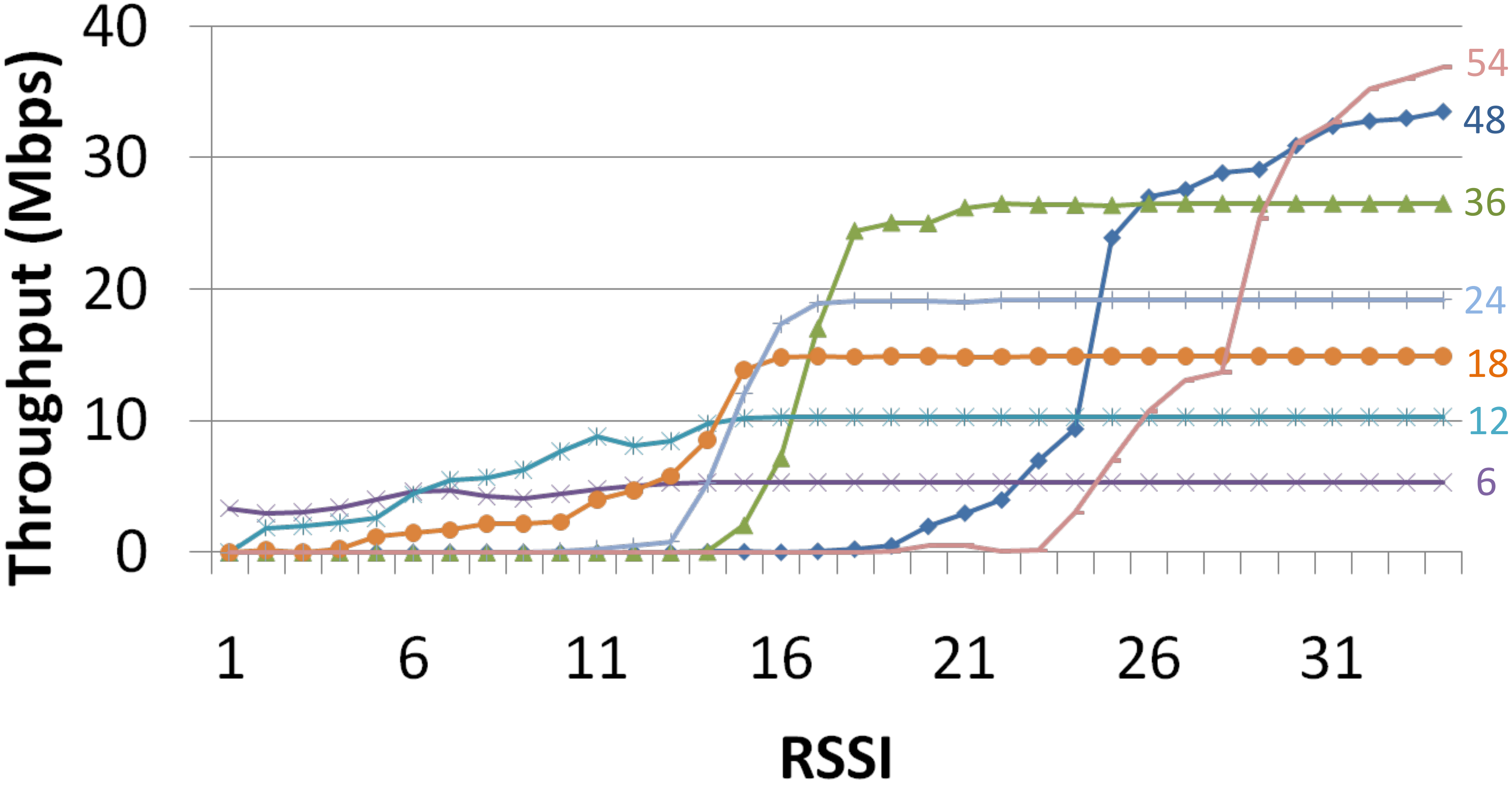


- Require extra 3dB to ensure P_b=0.001

Modulation in 802.11

- 802.11a
  - 6 mb/s: BPSK + ½ code rate
  - 9 mb/s: BPSK + ¾ code rate
  - 12 mb/s: QPSK + ½ code rate
  - 18 mb/s: QPSK + ¾ code rate
  - 24 mb/s: 16-QAM + ½ code rate
  - 36 mb/s: 16-QAM + ¾ code rate
  - 48 mb/s: 64-QAM + ⅔ code rate
  - 54 mb/s: 64-QAM + ¾ code rate
- FEC (forward error correction)
  - k/n: k-bits useful information among n-bits of data
  - Decodable if any k bits among n transmitted bits are correct

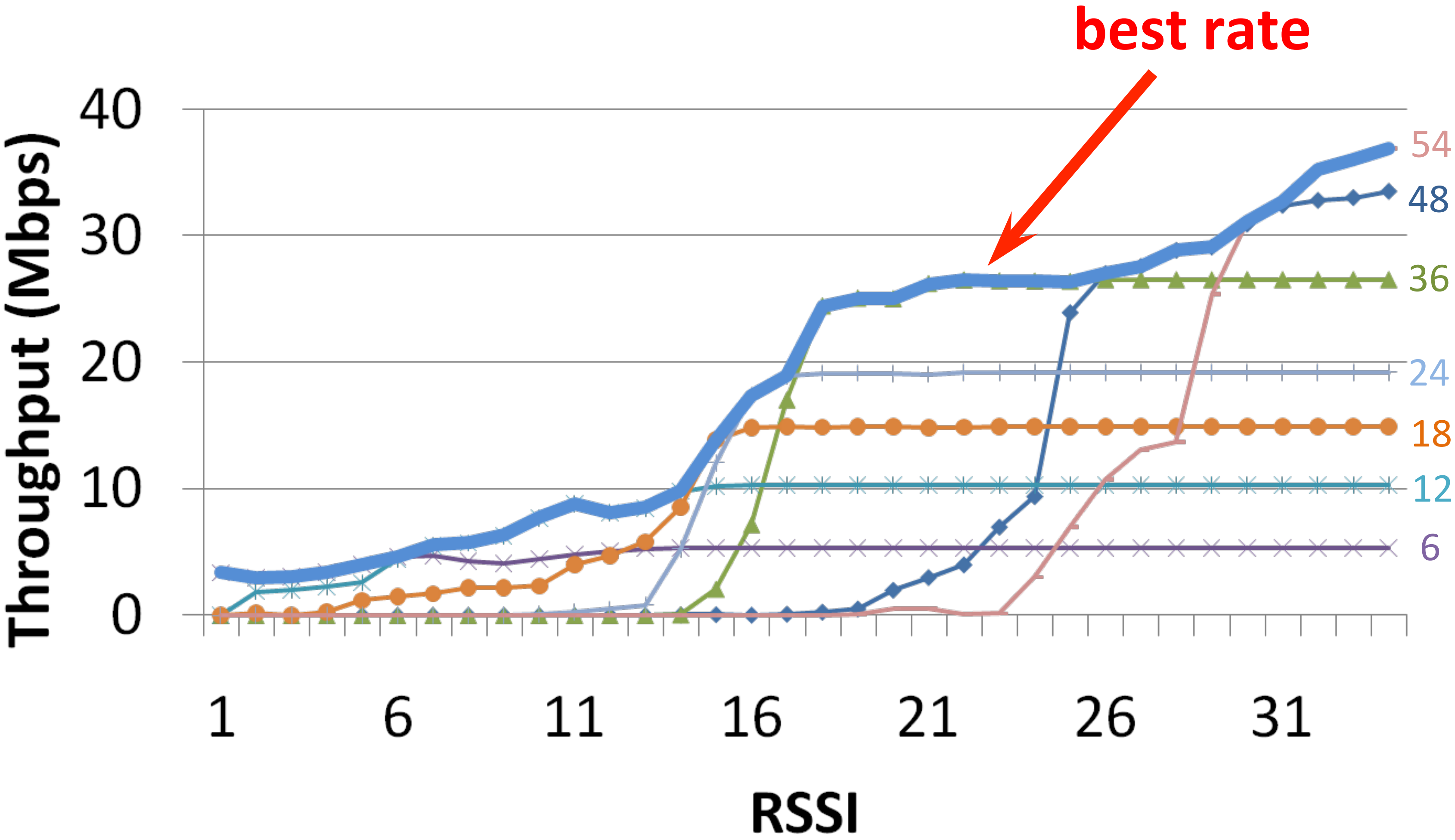
Bit-Rate Selection



$$\text{throughput}_r = (1\text{-PER}_{r,SNR}) * r = (1\text{-BER}_{r,SNR})^N * r$$

$$r^* = \text{arg max throughput}_r$$

Bit-Rate Selection



Adapt bit-rate to dynamic RSSI