Multi-Input Multi-Output Systems (MIMO)

- Channel Model for MIMO
- MIMO Decoding
- MIMO Gains
- Multi-User MIMO Systems

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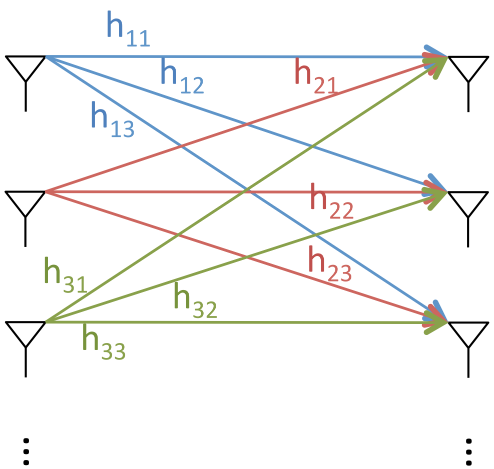
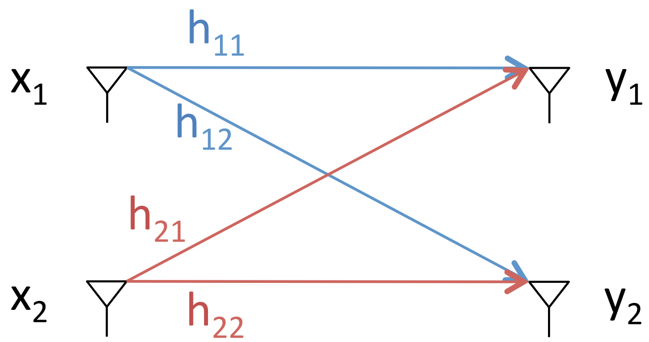
MIMO

- Each node has multiple antennas
  - Capable of transmitting (receiving) multiple streams concurrently
  - Exploit antenna diversity to increase the capacity



$$H_{N\times M} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \cdots \\ h_{21} & h_{22} & h_{23} & \cdots \\ h_{31} & h2 & h_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Channel Model (2x2)



$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$
$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

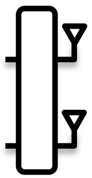
$$y = Hx + n$$

- Can be extended to N x M systems

Antenna Space

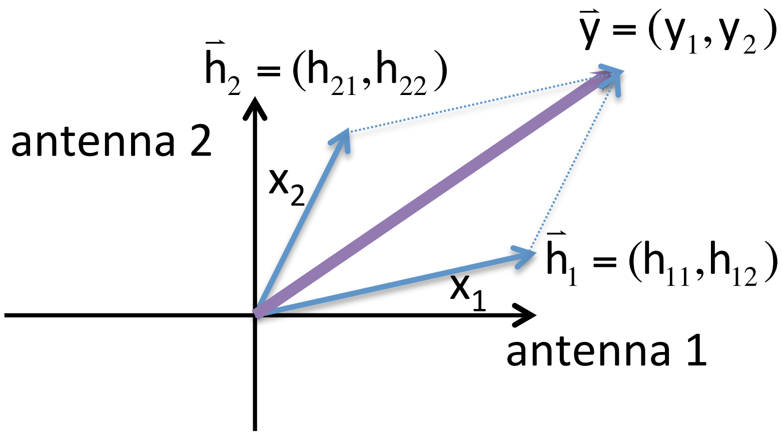
M-antenna node receives in M-dimensional space

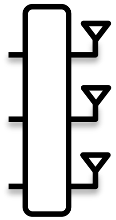
<u>2 x 2</u>

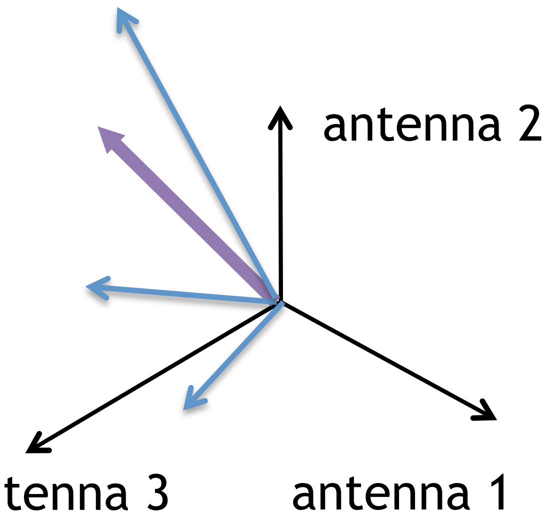


$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} x_1 + \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\vec{y} = \vec{h}_1 x_1 + \vec{h}_2 x_2 + \vec{n}$$







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Zero-Forcing Decoding (algebra)

Orthogonal vectors

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} x_1 + \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \qquad \begin{matrix} * \; h_{22} \\ * \; - h_{21} \end{matrix}$$

+)

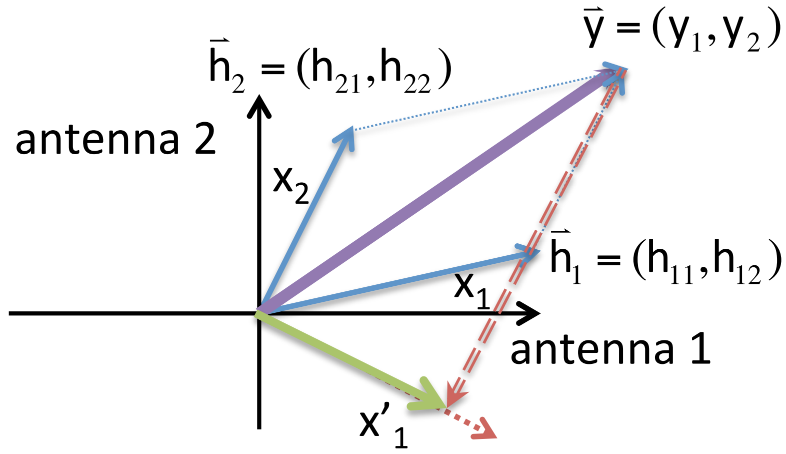
$$y_1 h_{22} - y_2 h_{21} = (h_{11} h_{22} - h_{12} h_{21}) x_1$$

$$x_1 = \frac{y_1 h_{22} - y_2 h_{21}}{h_{11} h_{22} - h_{12} h_{21}}$$

Given x_1, solve x_2 by successive interference cancellation (SIC)

To guarantee the full rank of H, antenna spacing at the transmitter and receiver must exceed half of the wavelength

Zero-Forcing Decoding (antenna space)

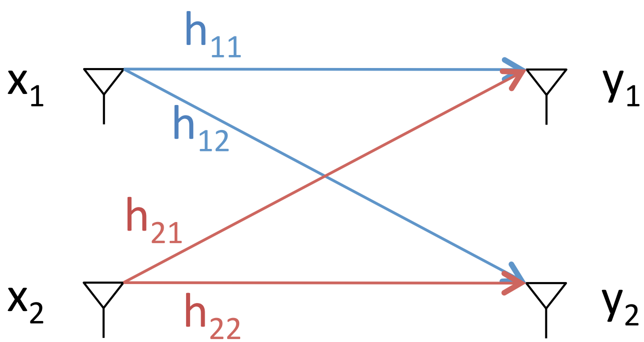


$\| x'_1 \| < \| x_1 \|$

- To decode x_1, decode vector y on the direction orthogonal to x_2
- To improve the SNR, re-encode the first detected signal, subtract it from y, and decode the second signal

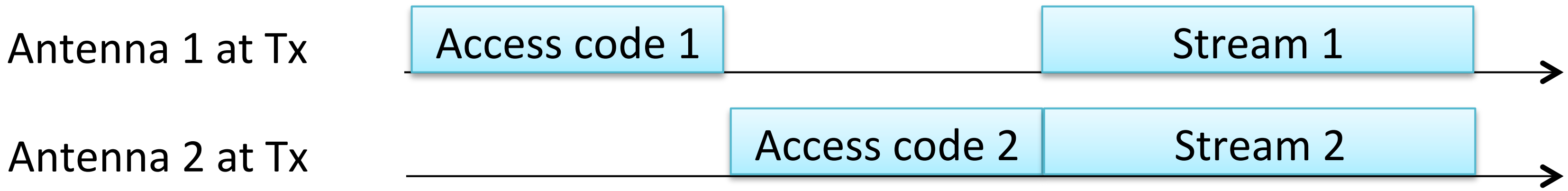
Channel Estimation

- Estimate N x M matrix H



$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$
$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

Two equations, but four unknowns



Estimate h_{11}, h_{12} Estimate h_{21}, h_{22}

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MIMO Gains

- Multiplex Gain
  - Exploit antennas to deliver multiple streams concurrently
- Diversity Gain
  - Exploit antenna diversity to increase the SNR of a single stream
  - Receive diversity and transmit diversity

Degree of Freedom

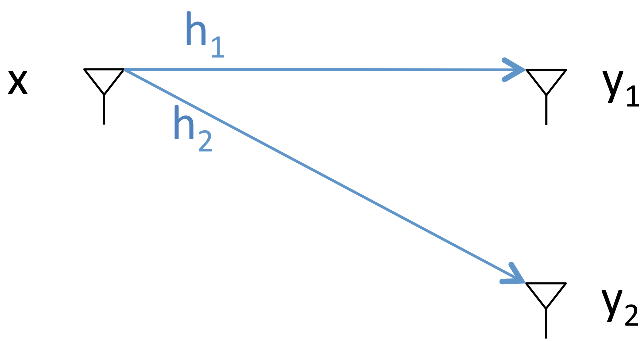
- For N x M MIMO channel
  - Degree of Freedom (DoF): min {N,M}
  - Maximum diversity: NM

Multiplexing-Diversity Tradeoff

- Tradeoff between diversity gain and multiplex gain
- Say we have a N x N system
  - Degree of freedom: N
  - The transmitter can transmit k streams concurrently, where k <= N
  - The optimal value of k is determined by the tradeoff between the diversity gain and multiplex gain

Receive Diversity

- 1 x 2 example

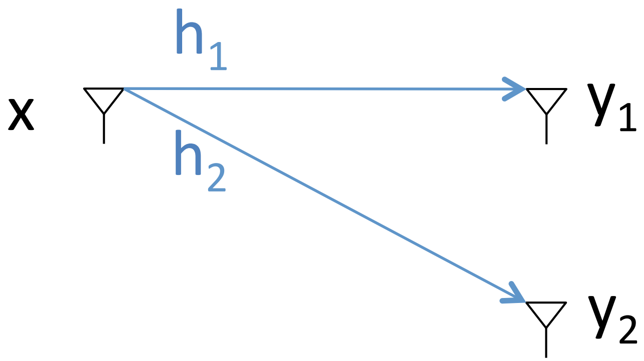


$$y_1 = h_1 x + n_1$$
$$y_2 = h_2 x + n_2$$

  - Decode the SNR of $(y_1 + y_2)$
  - Uncorrelated whit Gaussian noise with zero mean
  - Packet can be delivered through at least one of the many diver paths

Receive Diversity

- 1 x 2 example



$$SNR = \frac{P(2X)}{P(n_1 + n_2)}, \text{ where P refers to the power}$$

$$= \frac{E[(2X)^2]}{E[n_1^2 + n_2^2]}$$

$$= \frac{4E[X^2]}{2\sigma^2}, \text{ where } \sigma \text{ is the variance of AWGN}$$

$$= 2 * SNR_{\text{single antenna}}$$

- Increase SNR by 3dB
- Especially beneficial for the low SNR link

Receive Diversity
Maximal Ratio Combining (MRC)

Multiply each y with the conjugate of the channel

$$\begin{cases} y_1 = h_1 x + n_1 \\ y_2 = h_2 x + n_2 \end{cases}$$

$$\begin{cases} h_1^* y_1 = \|h_1\|^2 x + h_1^* n_1 \\ h_2^* y_2 = \|h_2\|^2 x + h_2^* n_2 \end{cases}$$

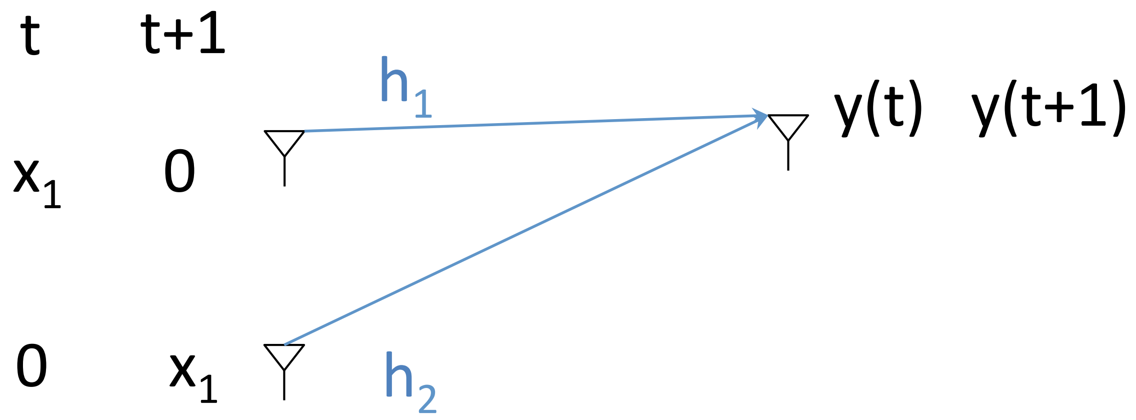
$$x = \frac{h_1^* y_1 + h_2^* y_2}{\|h_1\|^2 + \|h_2\|^2}$$

$$SNR_{MRC} = \frac{E[((\|h_1\|^2 + \|h_2\|^2)X)^2]}{E[(h_1^* n_1 + h_2^* n_2)^2]} = \frac{(\|h_1\|^2 + \|h_2\|^2)^2 E(X^2)}{(\|h_1\|^2 + \|h_2\|^2)\sigma^2} = \frac{(\|h_1\|^2 + \|h_2\|^2) E(X^2)}{\sigma^2}$$

$$SNR_{\sin gle} = \frac{E[(\|h_1\|^2 X)^2]}{E[(h_1^* n_1)^2]} = \frac{\|h_1\|^4 E(X^2)}{(\|h_1\|^2)\sigma^2} = \frac{\|h_1\|^2 E(X^2)}{\sigma^2}$$

$$\text{gain} = \frac{(\|h_1\|^2 + \|h_2\|^2)}{\|h_1\|^2}$$

Transmit Diversity



- Deliver a symbol twice in two consecutive time slots
- Repetitive code

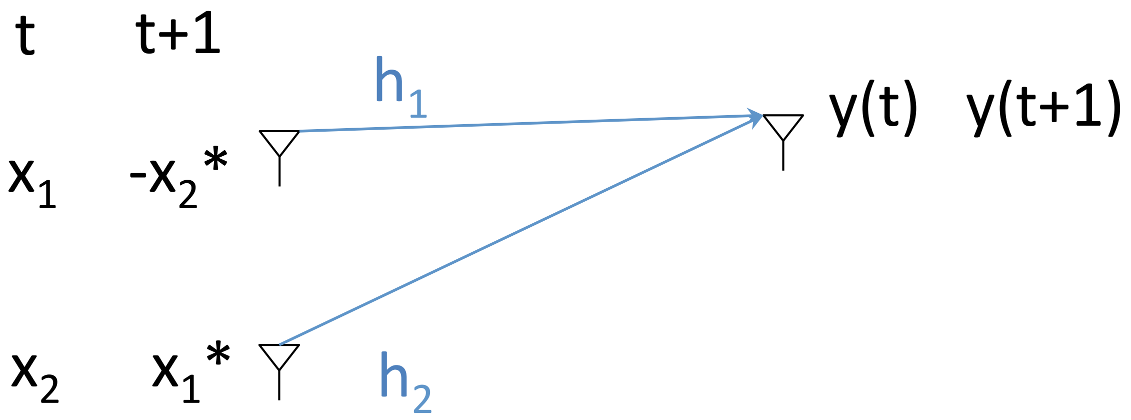
time

$$x = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}$$

space

- Diversity: 2
- Data rate: 1/2 symbols/s/Hz

Transmit Diversity



- Alamouti code (space-time block code)

$$x = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

time → ; space ↓

- Diversity: 2
- Data rate: 1 symbols/s/Hz

Transmit Diversity

- Alamouti code (space-time block code)

time →

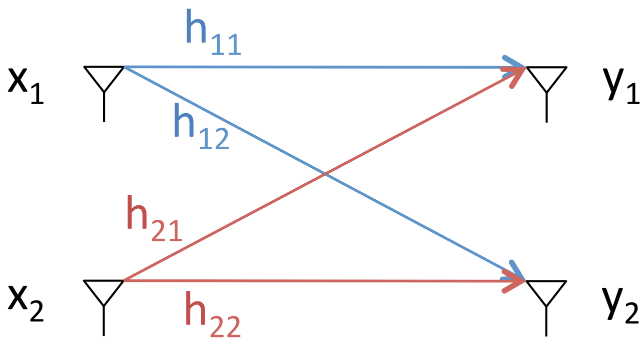
$$x = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

↓ space

$$y(t) = h_1 x_1 + h_2 x_2 + n_1$$

$$y(t+1) = h_2 x_1^* - h_1 x_2^* + n_2$$

Multiplexing-Diversity Tradeoff



Repetitive scheme

$$x = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}$$

Diversity: 4

Data rate: 1/2 sym/s/Hz

Alamouti scheme

$$x = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

Diversity: 4

Data rate: 1 sym/s/Hz

But 2x2 MIMO has 2 degrees of freedom

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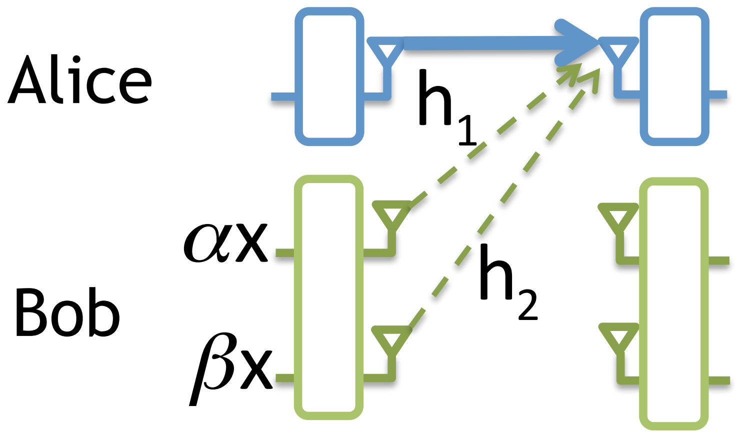
Interference Nulling



$$\Rightarrow \text{Nulling: } h_1\alpha = -h_2\beta$$

$$(h_{1a}\alpha + h_{2a}\beta)x \neq 0$$

$$(h_{1b}\alpha + h_{2b}\beta)x \neq 0$$

- Signals cancel each other at Alice's receiver
- Signals don't cancel each other at Bob's receiver
  - Because channels are different

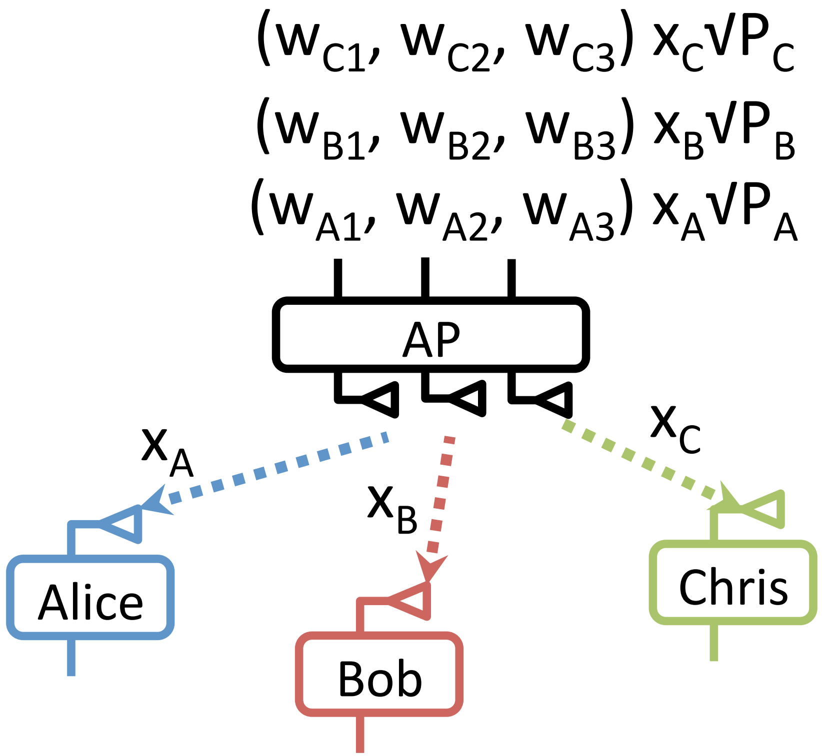
Quiz

- Say there exist a 3x2 link, which has a channel

$$H_{3\times 2} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}$$

How can a 3-antenna transmitter transmit a signal x, but null its signal at two antennas of a two-antenna receiver?

Zero-Forcing Beamforming



$$y_k = (\sqrt{P_k} h_k w_k) s_k + \sum_{j \neq k} (\sqrt{P_j} h_k w_j) s_j + n_k$$

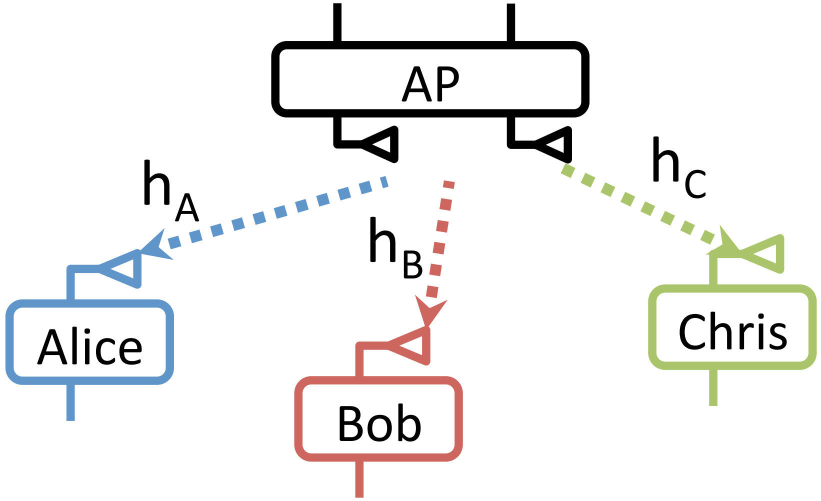
Null the interference:

$$\sum_{j \neq k} \sqrt{P_j} h_k w_j = 0$$

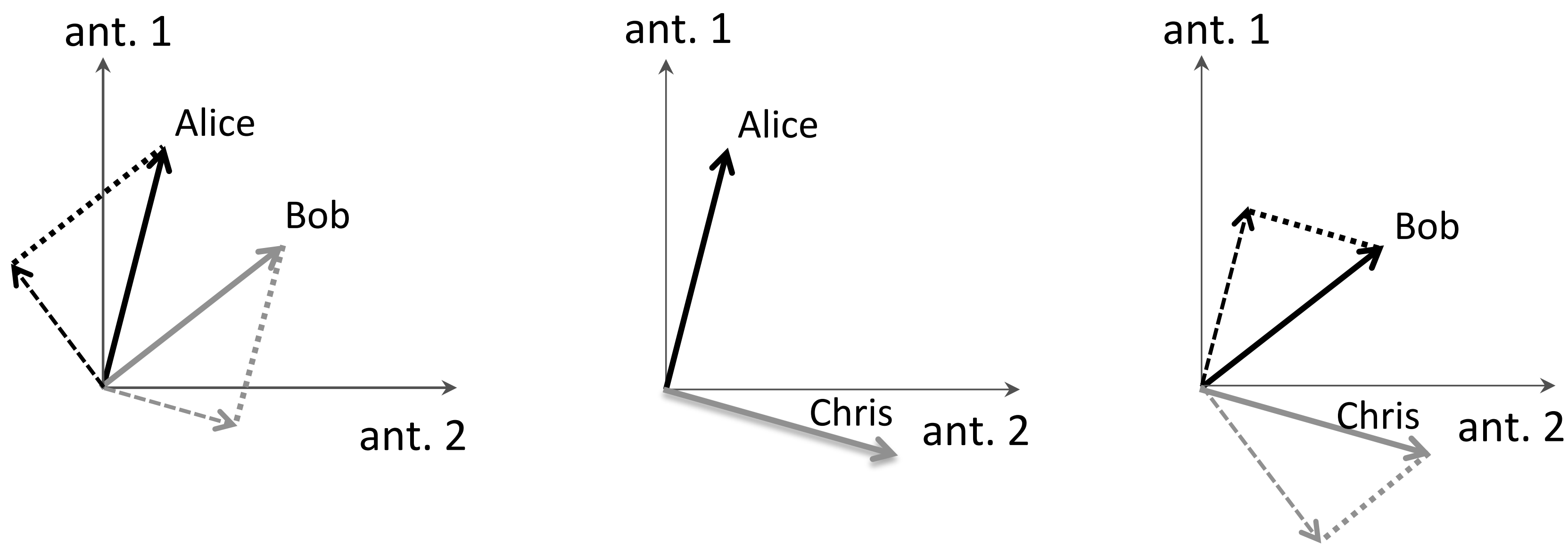
$$\mathbf{y} = \mathbf{HWPx} + \mathbf{n}$$

$$\text{Let } \mathbf{W} = \mathbf{H}^{\dagger} = \mathbf{H}^{*}(\mathbf{HH}^{*})^{-1}, \text{ then } \mathbf{y} = \mathbf{Px} + \mathbf{n}$$

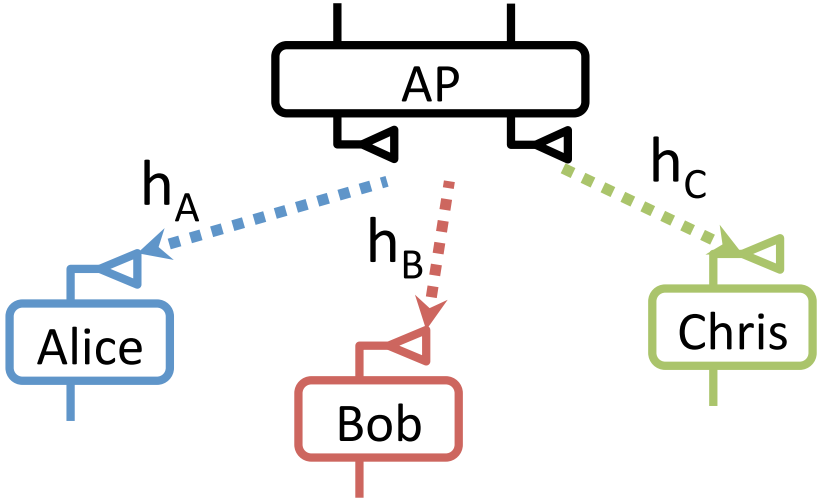
MU-MIMO Bit-Rate Selection



Select a proper rate based on SNR_{ZF}



MU-MIMO User Selection



Pairing different clients as concurrent receivers results in different sum-rates