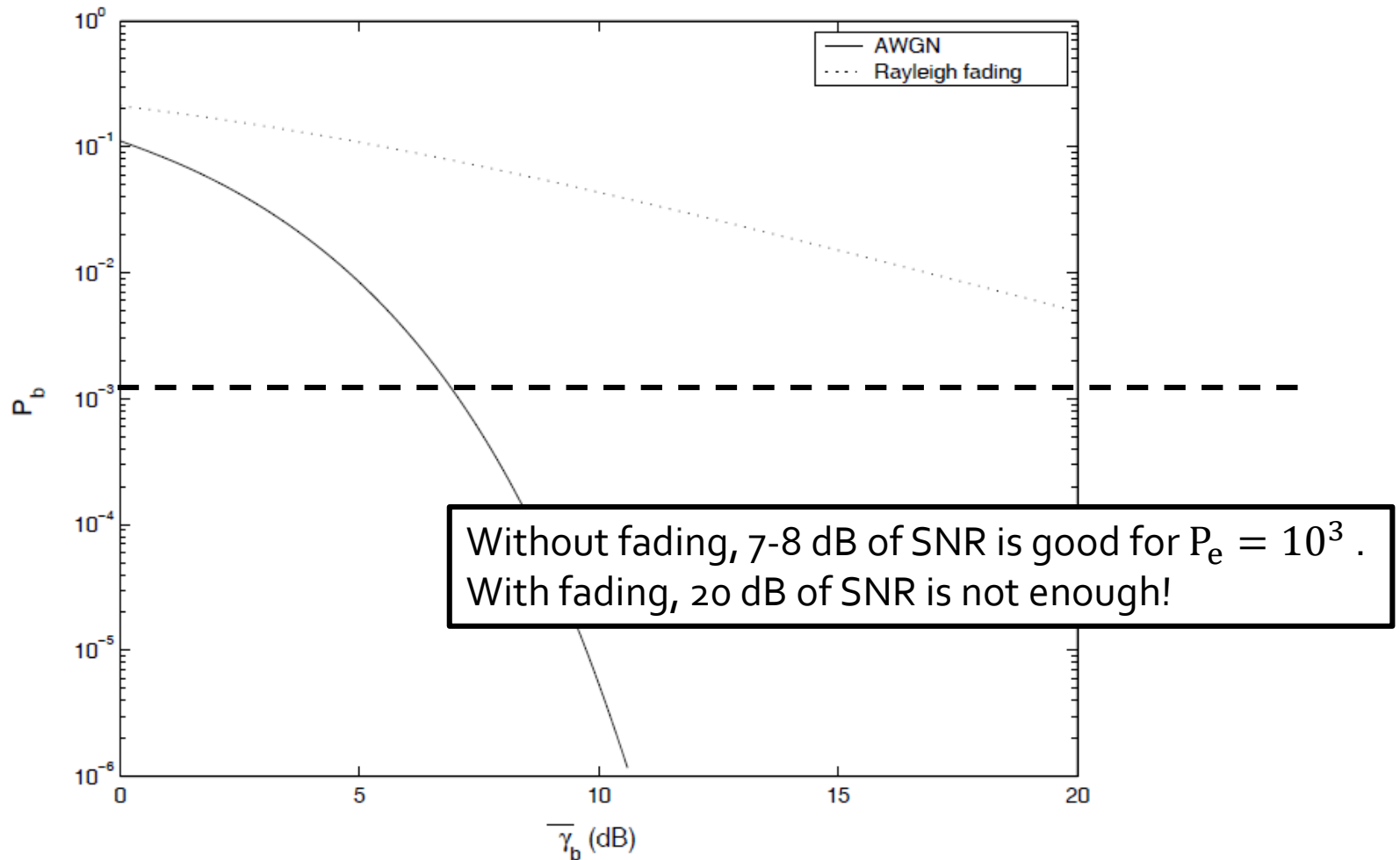


Diversity

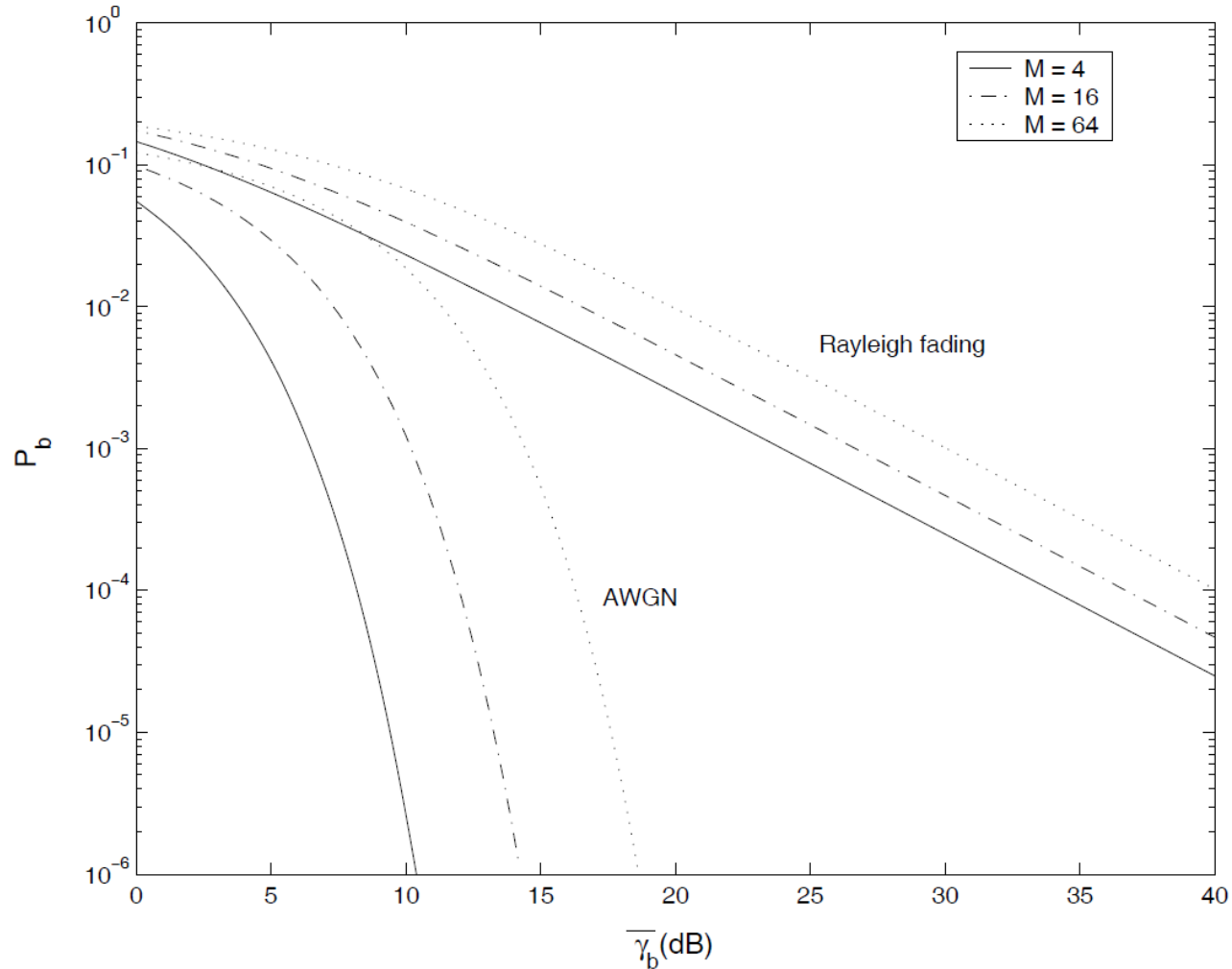
PROF. MICHAEL TSAI

2014/4/28

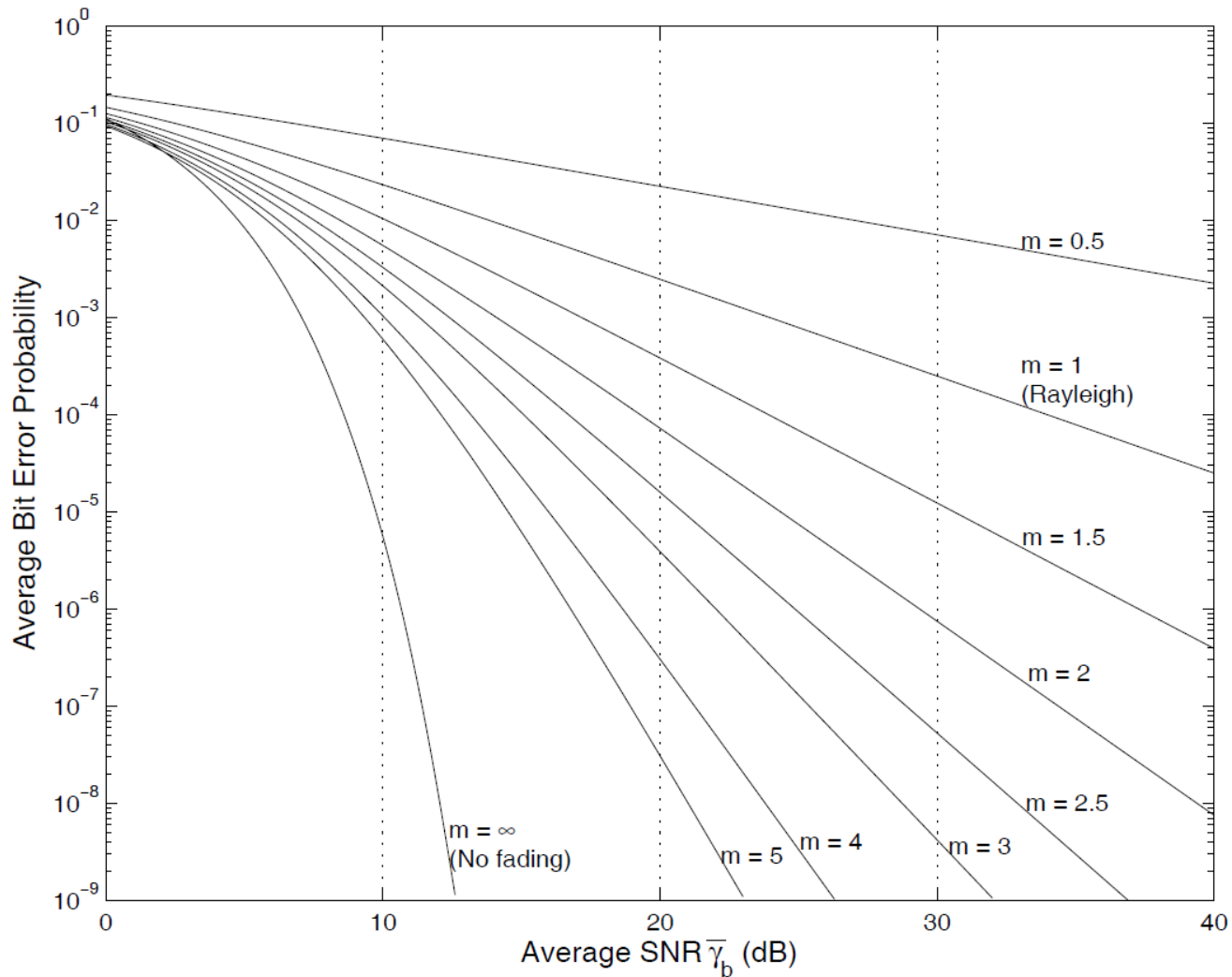
BER Performance under Fading: BPSK in Rayleigh fading



BER Performance under Fading: M-QAM in Rayleigh Fading



BER Performance under Fading: BPSK in Nakagami fading



Intuition: how does fading affect average BER?

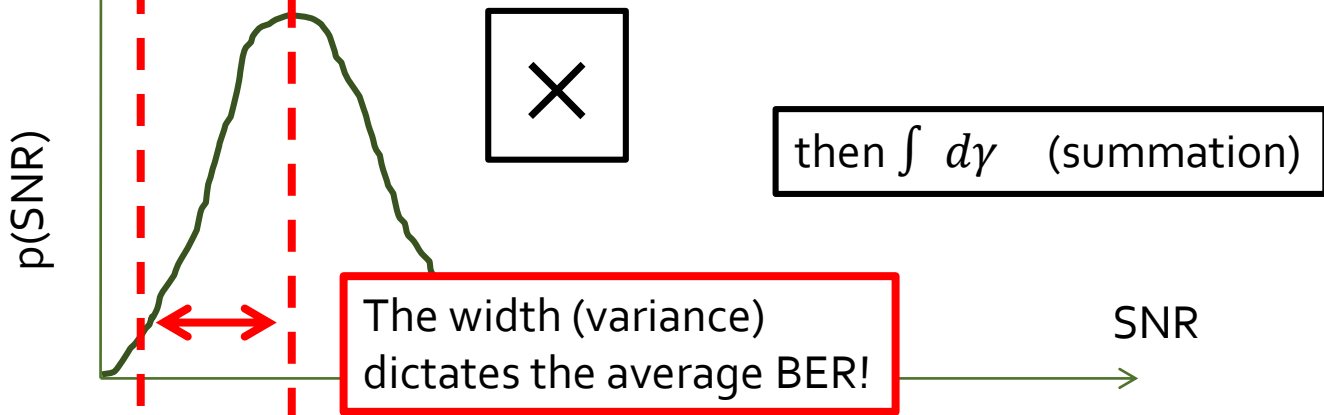
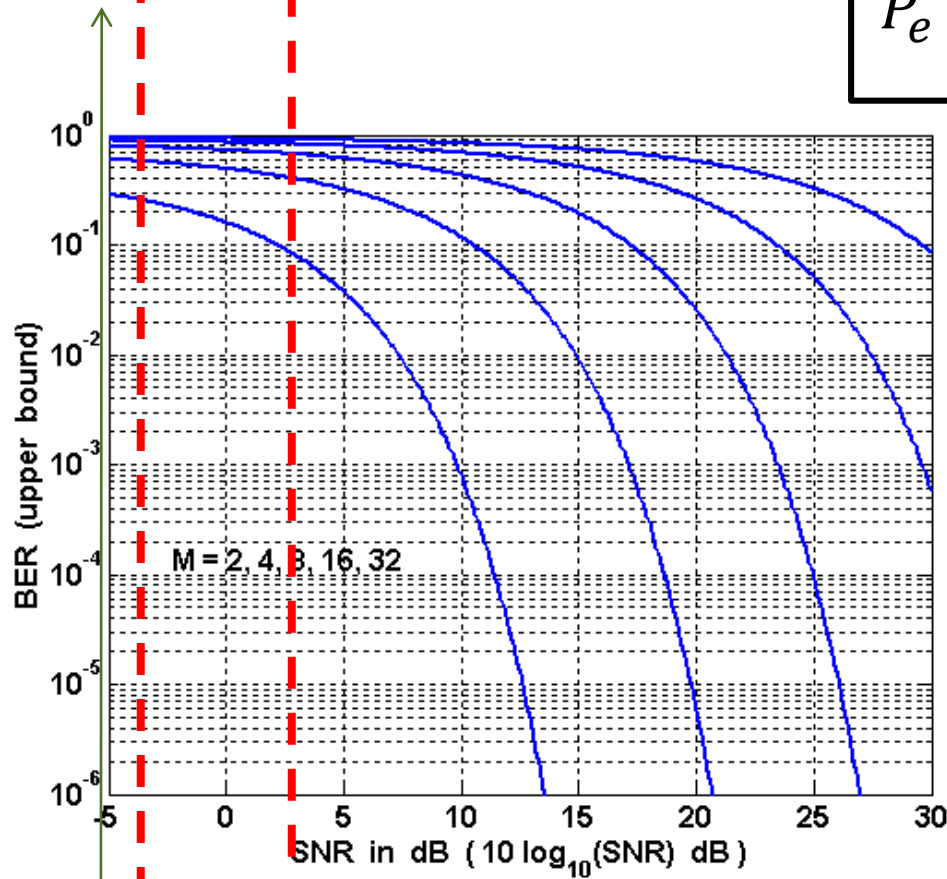
- **Average BER:**

$$\bar{P}_e = \int_0^{\infty} P_e(\gamma) p_{\gamma}(\gamma) d\gamma$$

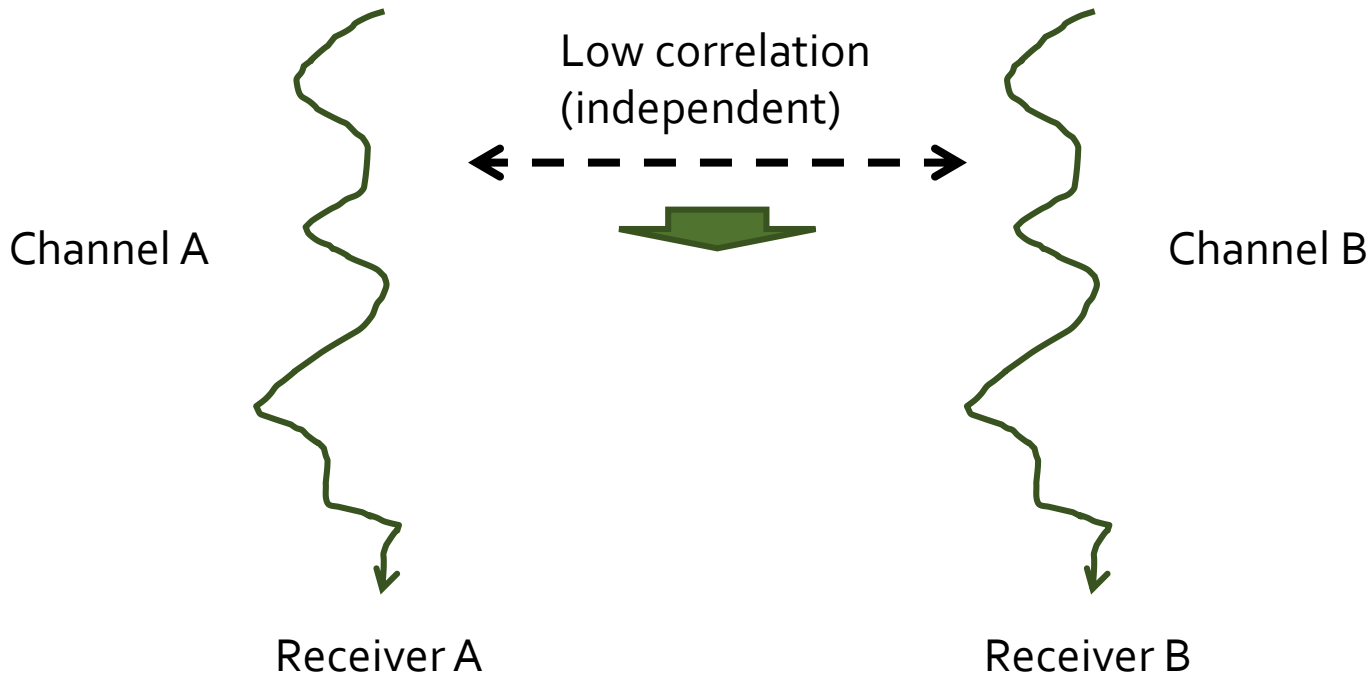
- γ : Signal-to-Noise Ratio (SNR) per bit
- $p_{\gamma}(\gamma)$: PDF of SNR (fading distribution divided by noise power)
- $P_e(\gamma)$: BER of a given SNR

Dominant Part

$$\bar{P}_e = \int_0^{\infty} P_e(\gamma) p_{\gamma}(\gamma) d\gamma$$

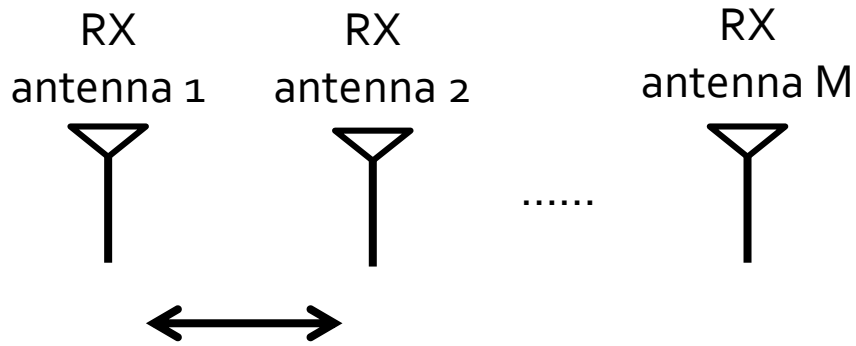


Concept: Diversity



$$\begin{aligned} & p(A \text{ has bad reception} \ \&\& \ B \text{ has bad reception}) \\ &= p(PL_A > PL_0 \ \&\& \ PL_B > PL_0) \\ &\approx p(PL_A > PL_0)p(PL_B > PL_0) \ll p(PL_A > PL_0) \end{aligned}$$

Space Diversity



Each pair separated by at least half the wavelength
(accurate version: 0.38 wavelength)



Low correlation → independent channels

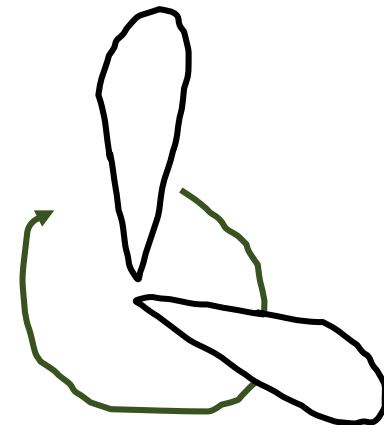
Q: What's the minimum required separation between 2 antennas? (for 802.11g and 802.11a)



A: 12.5 cm for 2.4 GHz
5.17 cm for 5.8 GHz
(which is what you see for a typical router)

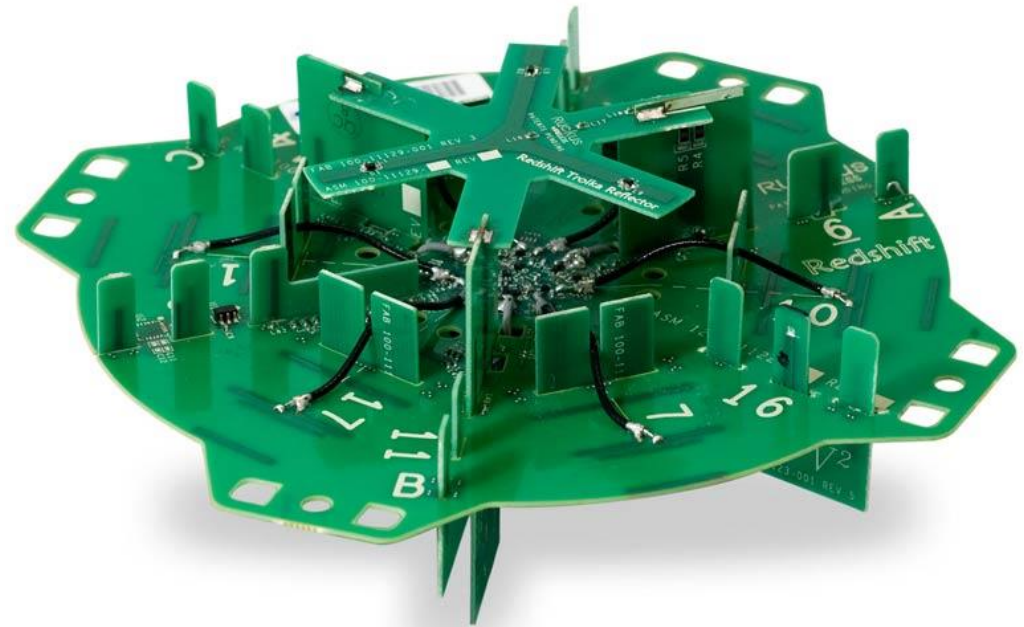
Directional (Angle) Diversity

- Split the 360 degree receiving angle into different “sectors”
- Each will receive a portion of multipath components (MPC)
 - Extreme case: if the angle of each “sector” is very very small, then you only receive one MPC → no small scale fading
 - Different sets of MPCs go through different paths → low correlation!
- **Antenna design:**
 - Multiple sectors on the same antenna (switchable multiple antennas)
 - Steerable directional antenna (mechanical)



New WiFi Access Points in the CSIE Building

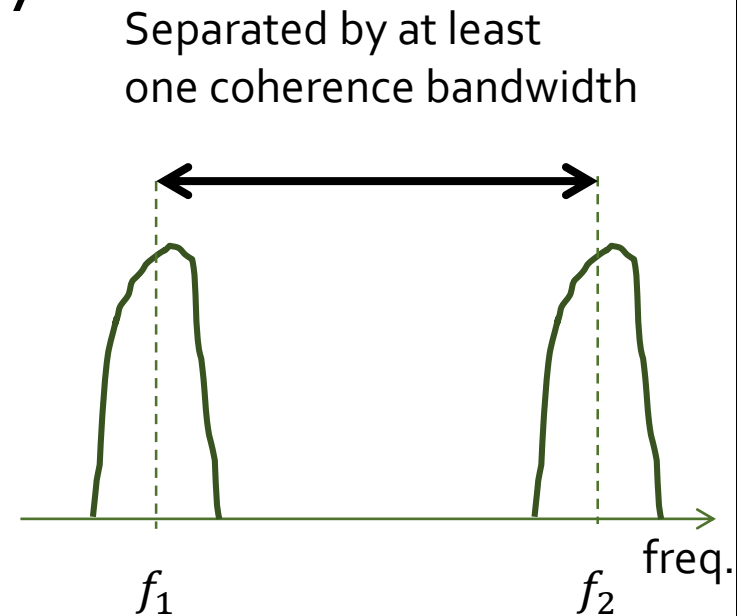
- Ruckus Zoneflex 7962
- Currently in service in the CSIE building
- 802.11 a/b/g/n
- Over 4000 unique antenna patterns
 - Many “sectors”, 3D too (from its appearance)
 - Select multiple “good” antennas for receiving
- **Can be used to reduce interference too**



Smart Antenna inside
Ruckus Zoneflex 7962

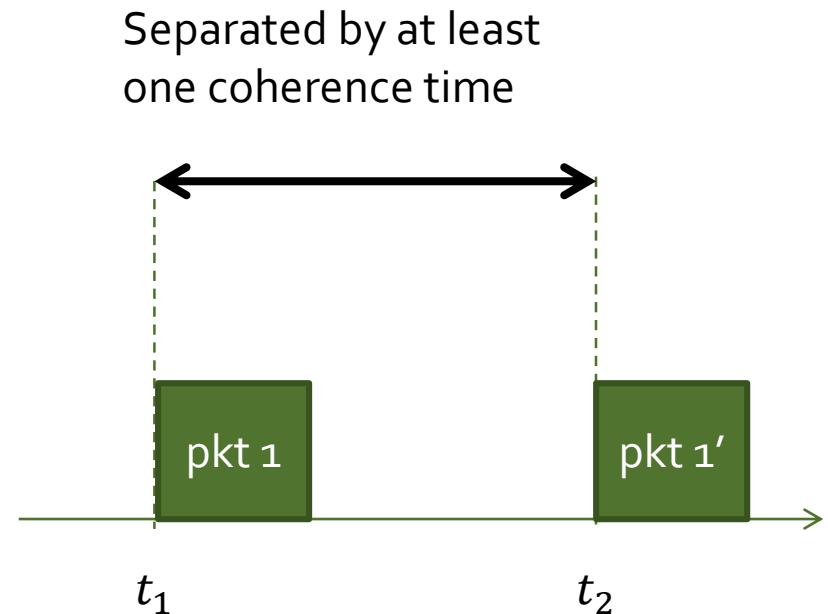
Frequency Diversity

- **Signals at two frequencies separated by at least one coherence bandwidth**
→ low correlation!
→ independent!
- **Small coherence bandwidth is sometimes good too**
 - For frequency diversity, two transmissions do not need to be too far apart in frequency
- **OFDM utilize this property too**
 - Sub-carriers separated by at least one coherence bandwidth can transmit redundant information for **diversity (reliability)**
 - Sub-carriers within the same coherence bandwidth can transmit different information for increasing the **throughput**



Time Diversity

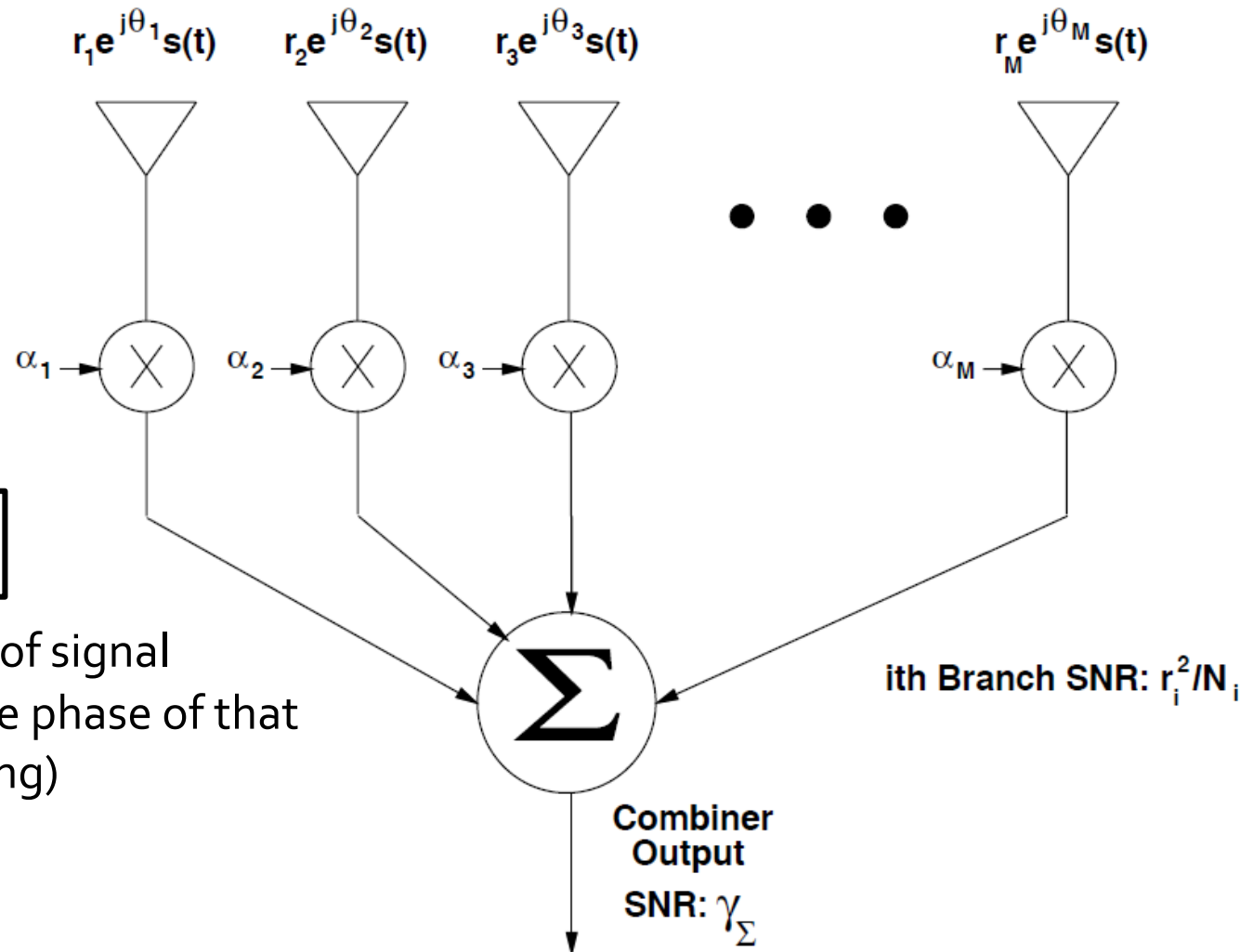
- Transmit the same packet (or a part of it) after Δt , $\Delta t > T_c$ (coherence time).
 - low correlation
 - independent
- How to do this?
 - For channel with $T_{pkt} < T_c$, coding techniques can utilize this
 - transmit redundant information in the same packet, separated by T_c .
 - Retransmission conceptually uses this too.



Some related terms

- **Micro-diversity:**
to mitigate the effects of multipath fading (small-scale fading).
- **Macro-diversity:**
to mitigate the effects of shadowing from buildings and objects (large-scale fading).
- In this lecture, we will talk about micro-diversity.

A More Formal Representation for Receiver Diversity



$$\alpha_i = a_i e^{-j\theta_i}$$

- a_i : amplification of signal
- $e^{-j\theta_i}$: remove the phase of that branch (co-phasing)

Array Gain

- **Array Gain:**
Improvements from getting the signals from multiple antennas
- **Usually refers to the gain without fading**
- **More formally, SNR of the combined signal can be calculated as:**

Setting $a_i = \frac{r_i}{\sqrt{N_0}}$, $i = 1, \dots, M$

$$\gamma_{\Sigma} = \frac{\left(\sum_{i=1}^M a_i r_i\right)^2}{N_0 \sum_{i=1}^M a_i^2} = \frac{\left(\sum_{i=1}^M \frac{E_s}{\sqrt{N_0}}\right)^2}{N_0 \sum_{i=1}^M \frac{E_s}{N_0}} = \frac{M E_s}{N_0}$$

With fading, what is the average BER?

- **Diversity gain:**
the performance advantage as a result of diversity combining (in fading).

- **Average BER:**

$$\bar{P}_e = \int_0^{\infty} P_e(\gamma) p_{\gamma\Sigma}(\gamma) d\gamma$$

- Or we can express it as

$$\bar{P}_e = c\bar{\gamma}^{-m}$$

m: the diversity order

- When $m=M$ (the number of branches), we say that the system achieves *full diversity order*.

Selection Combining (SC)

- **Concept:**
select the one branch with the best SNR and dump the rest.
- **Advantage:**
simple, no need to do co-phasing.
- **Select the highest SNR:** $\gamma_i = \frac{r_i^2}{N_i}$.
- **In practice, SNR cannot be measured.**
Since $N_i = N_0, \forall i$,
we can select the branch with the highest RSSI
instead: $r_i^2 + N_i$

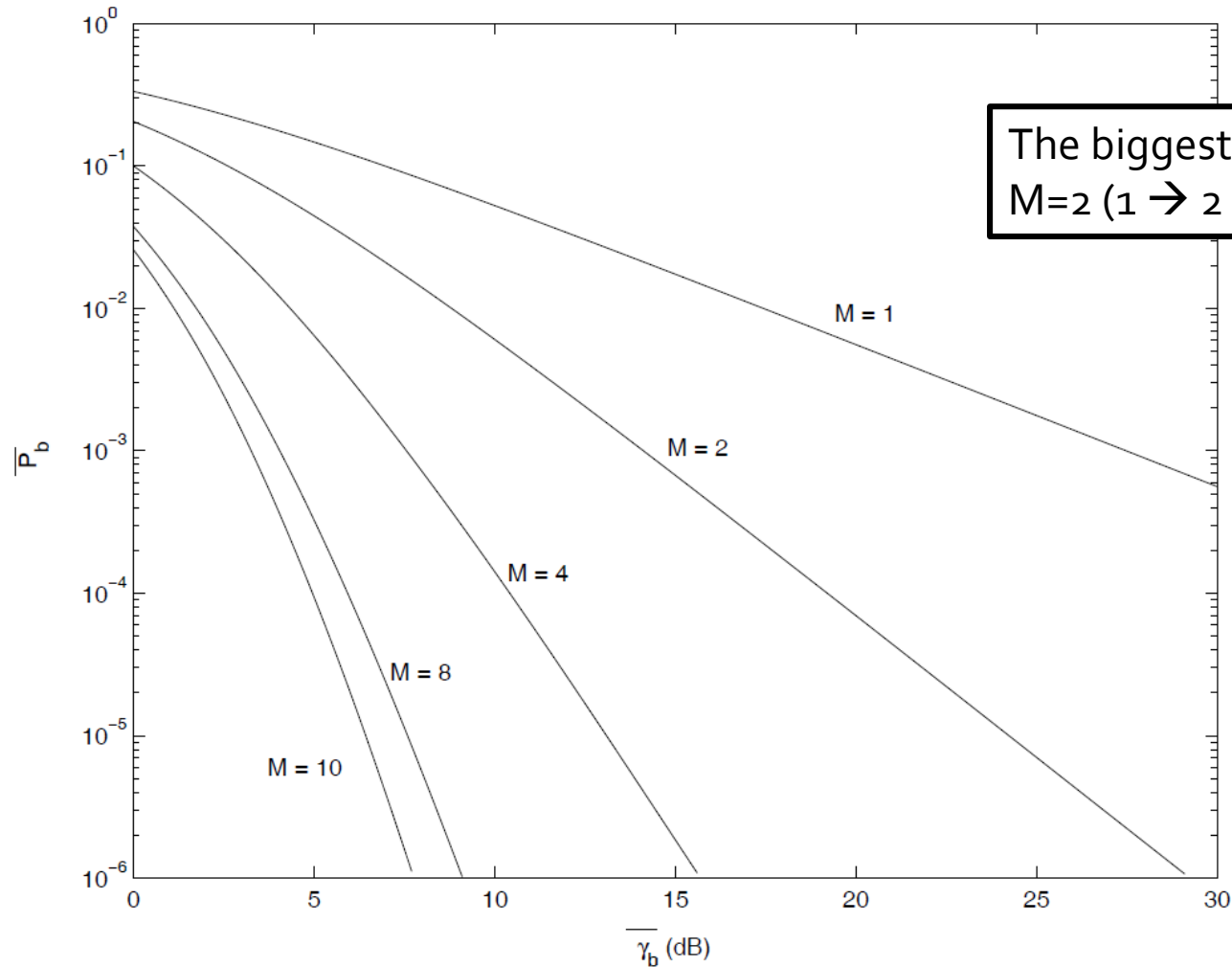
Selection Combining (SC)

- The CDF of SNR after combining:

$$\begin{aligned} P_{\gamma_{\Sigma}}(\gamma) &= p(\gamma_{\Sigma} < \gamma) \\ &= p(\max[\gamma_1, \gamma_2, \dots, \gamma_M] < \gamma) \\ &= \prod_{i=1}^M p(\gamma_i < \gamma) \end{aligned}$$

- No close form expression to obtain the average BER
→ Use simulation to obtain the result.
- Sometimes branch correlation is not 0
→ the performance will degrade
→ negligible when correlation < 0.5

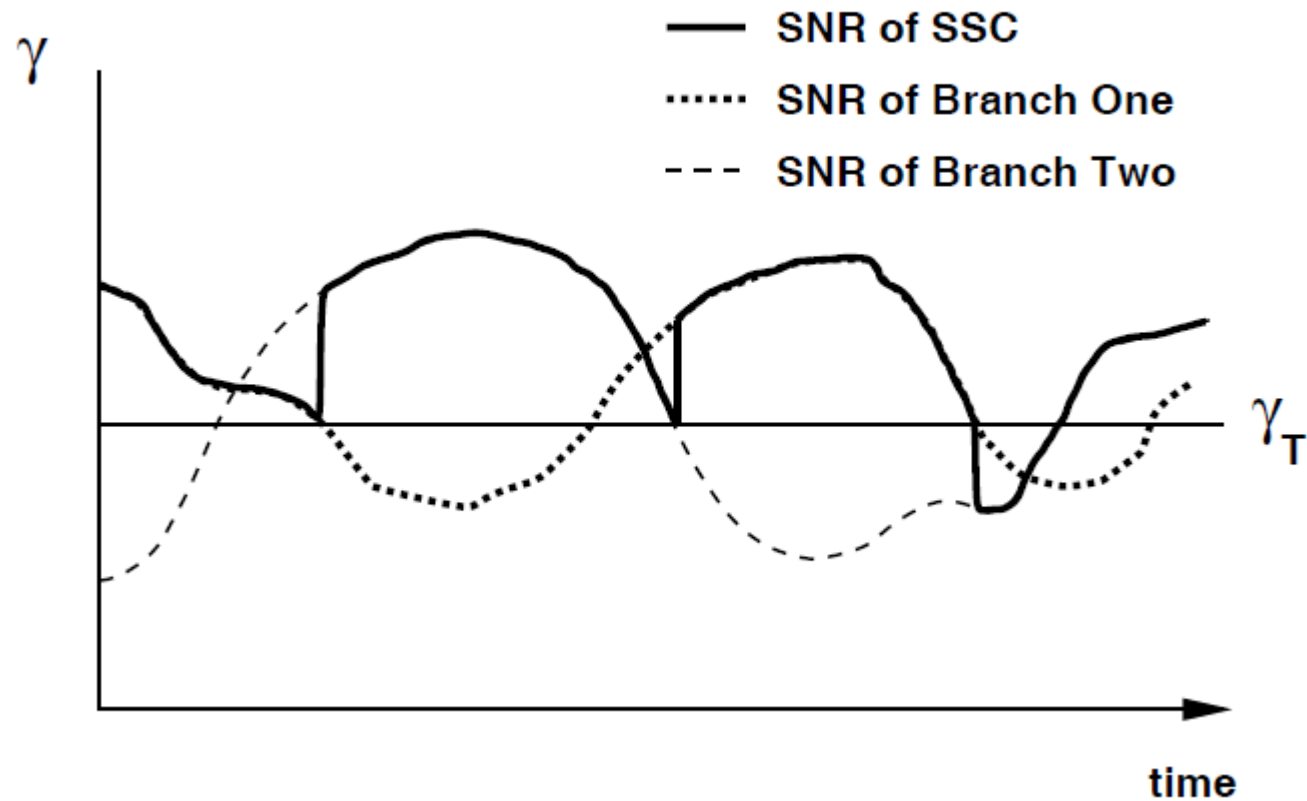
BER Performance: BPSK with SC in Rayleigh fading



Threshold Combining

- **Concept:**
Use one branch and dump the rest. When this one is not good anymore (SNR drops below a threshold), randomly select another branch.
- **Advantage:**
Even simpler, no need to monitor the SNR of all branches.
- **When there are only 2 branches, switch to the other branch when SNR is smaller than the threshold.**
 - This is called Switch-and-Stay Combining (SSC)
- **SSC has the same performance (outage probability) as SC, when setting the threshold = the minimum required SNR**

Switch-and-Stay Combining (SSC)



Maximal-Ratio Combining (MRC)

- **Concept:**
Use all branches. We amplify the branch more when its SNR is larger.
- **Advantage:**
Make use of all branches → best performance.
- **Question:**
How to set a_i so that the SNR after combining is maximized?

$$\gamma_{\Sigma} = \frac{\left(\sum_{i=1}^M a_i r_i\right)^2}{N_0 \sum_{i=1}^M a_i^2}$$

Maximal-Ratio Combining (MRC)

- Answer:

a_i^2 should be proportional to the branch SNR $\frac{r_i^2}{N_0}$.

- After optimization, it turns out that

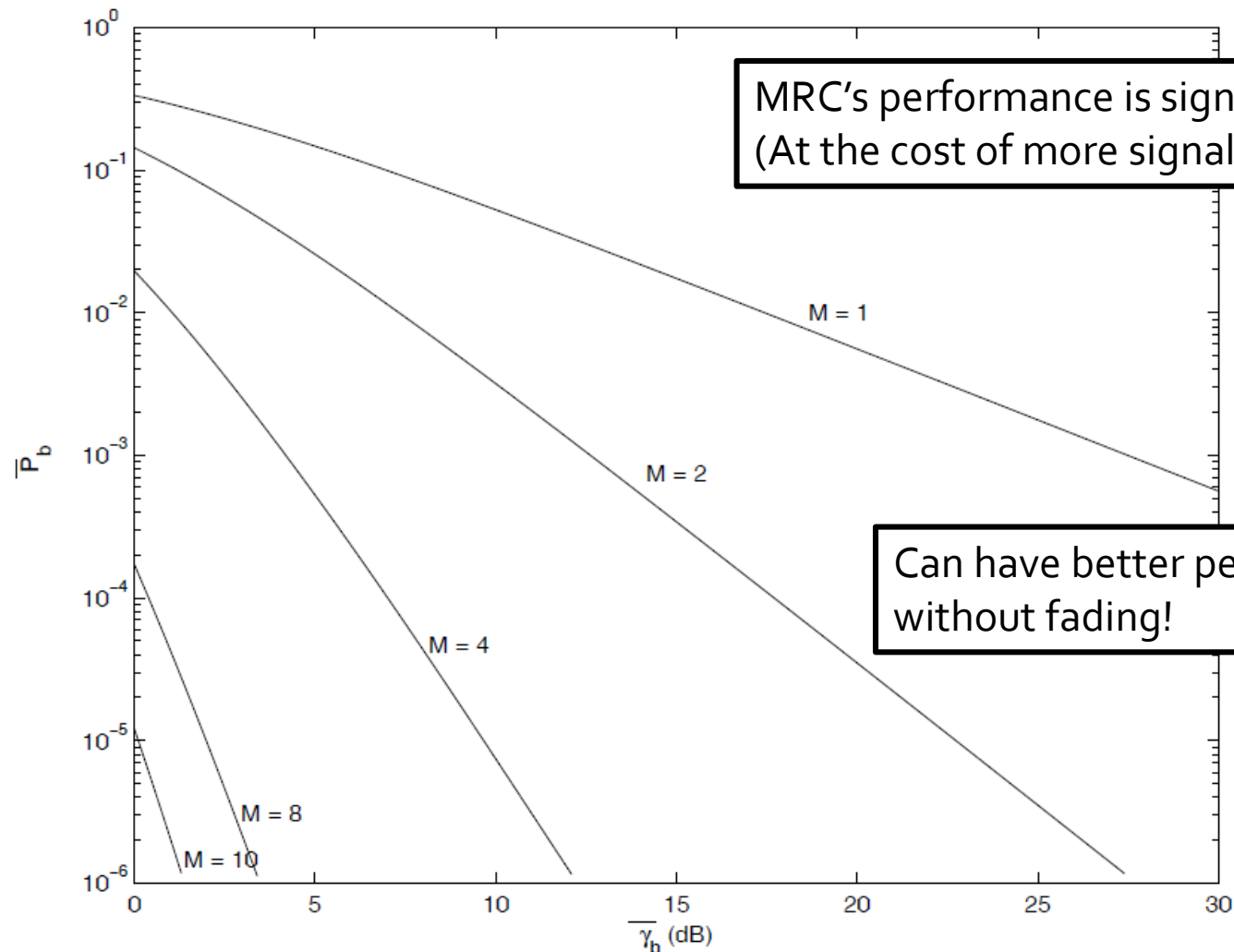
$$a_i^2 = \frac{r_i^2}{N_0}$$

- And the SNR after combining becomes

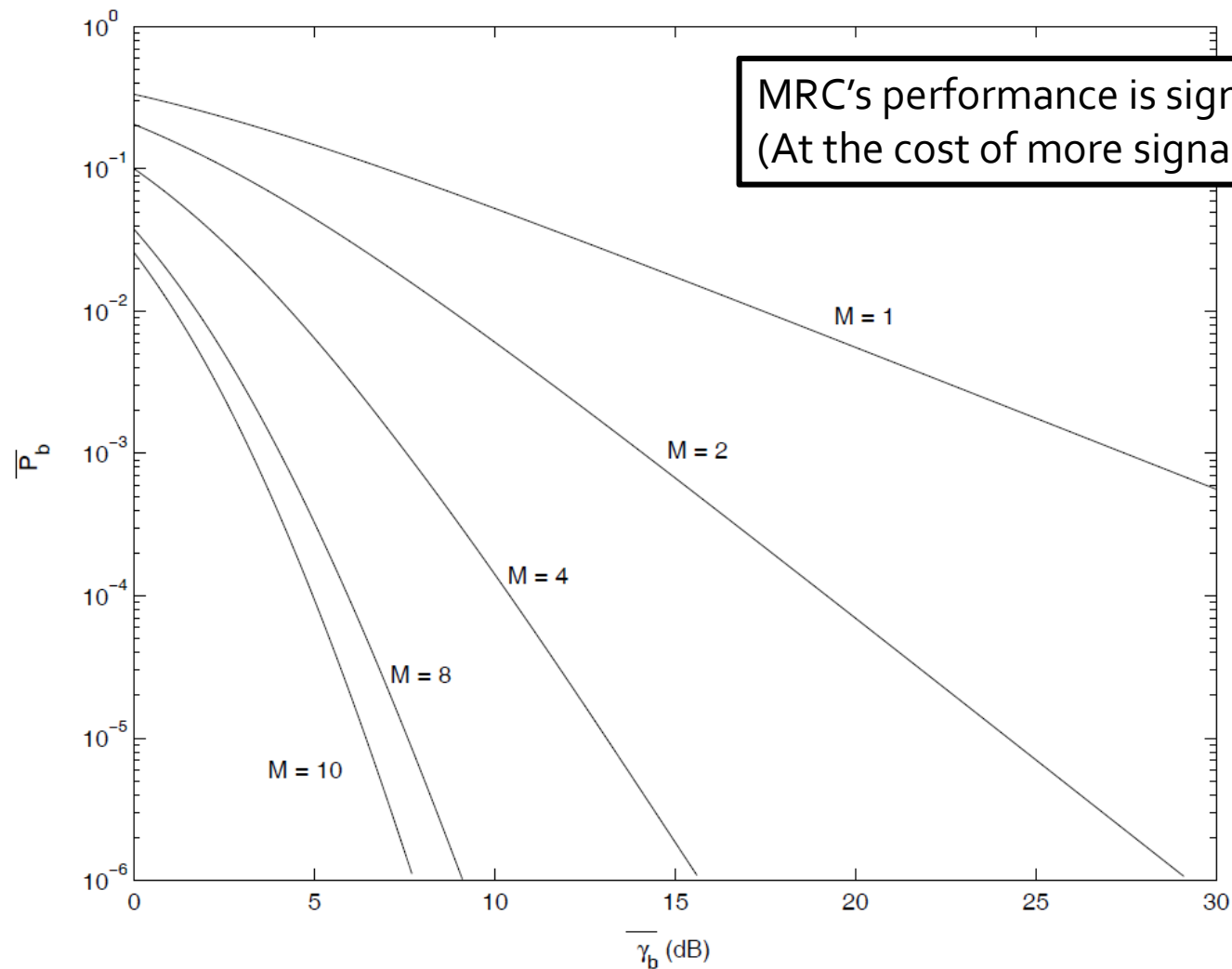
$$\gamma_{\Sigma} = \sum_{i=1}^M \frac{r_i^2}{N_0} = \sum_{i=1}^M \gamma_i$$

Note that this is linear scale, not in dB!

BER Performance: BPSK with MRC in Rayleigh fading



BER Performance: BPSK with SC in Rayleigh fading



Equal-Gain Combining (EGC)

- **Concept:**
Use all branches, but combine them with equal weight=1.
- **Advantage:**
Use the signal from all branches, but in a simpler way.
- $a_i = 1, \forall i$.
- **The SNR after combining becomes**
$$r_{\Sigma} = \frac{1}{N_0 M} \left(\sum_{i=1}^M r_i \right)^2$$
- **EGC's performance is quite close to MRC, typically only has less than dB of power penalty.**