Small-Scale Fading I

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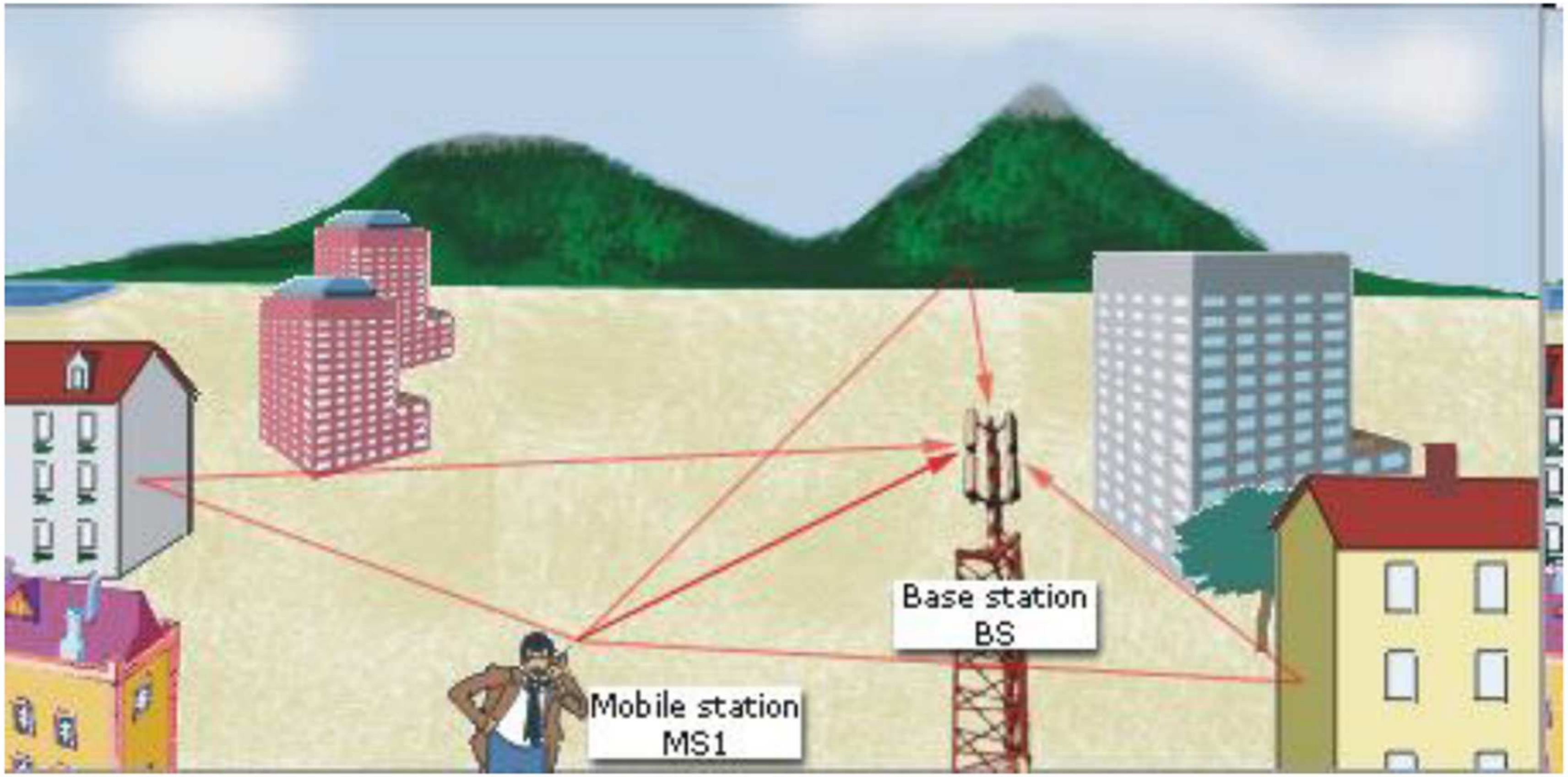
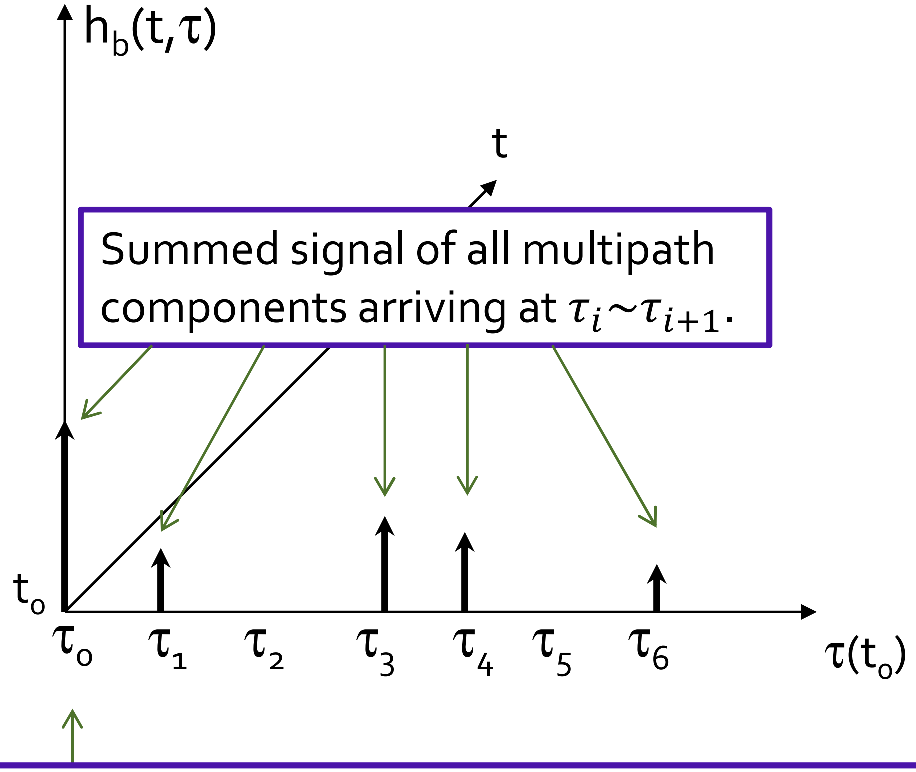
Multipath Propagation



RX just sums up all Multi Path Component (MPC).

Multipath Channel Impulse Response

An example of the time-varying discrete-time impulse response for a multipath radio channel



Summed signal of all multipath components arriving at $\tau_i \sim \tau_{i+1}$.

$\tau_0 = 0$, and represents the time the first signal arrives at the receiver.

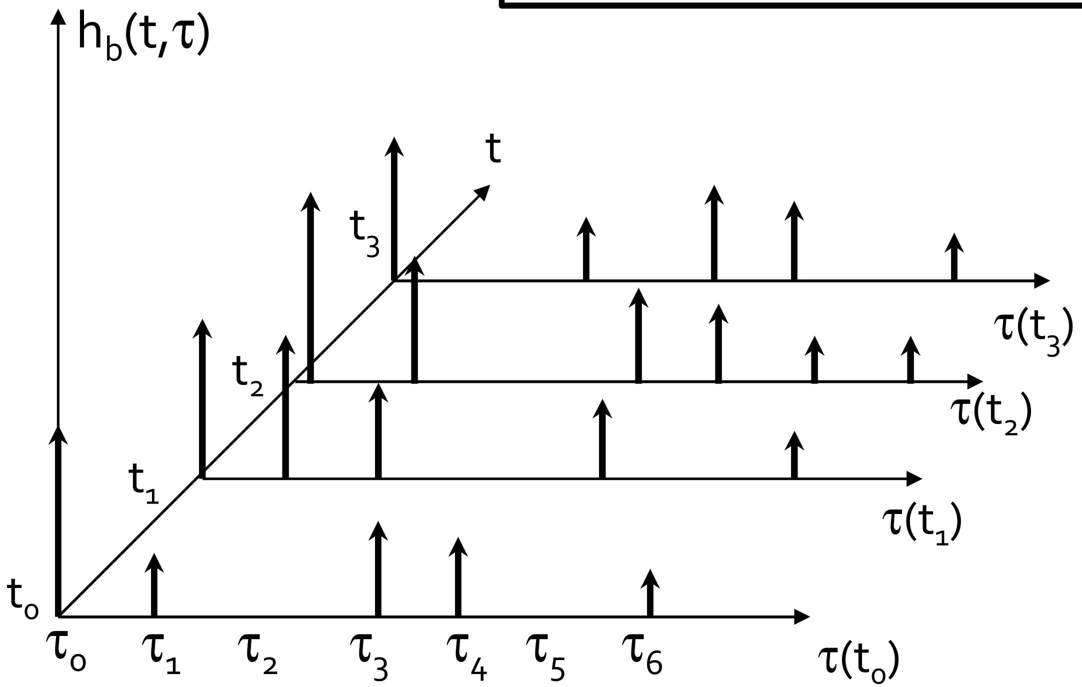
Excess delay: the delay with respect to the first arriving signal (τ)

Maximum excess delay: the delay of latest arriving signal

The channel impulse response when $t = t_0$ (what you receive at the receiver when you send an impulse at time t_0)

Time-Variant Multipath Channel Impulse Response

Because the transmitter, the receiver, or the reflectors are moving, the impulse response is time-variant.



Multipath Channel Impulse Response

- **The channels impulse response is given by:**

Summation over all MPC

Additional phase change due to reflections

$$h_b(t,\tau)=\sum_{i=0}^{N-1}a_i(t,\tau)\exp\left[-j\{2\pi f_c\tau(t_i)+\phi_i(t,\tau)\}\right]\delta(t-\tau_i(t))$$

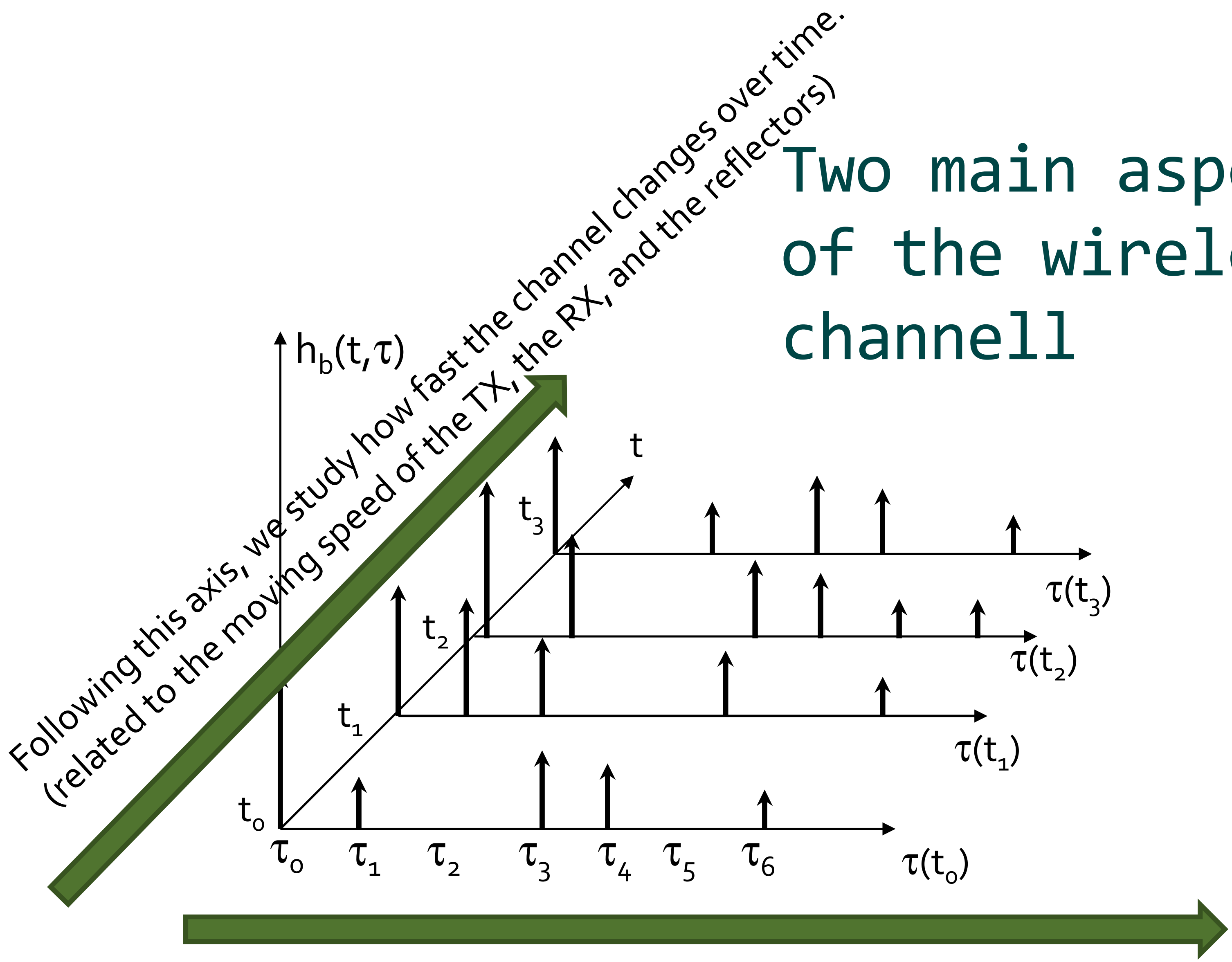
Amplitude change (mainly path loss)

Phase change due to different arriving time

- **If assumed time-invariant (over a small-scale time or distance):**

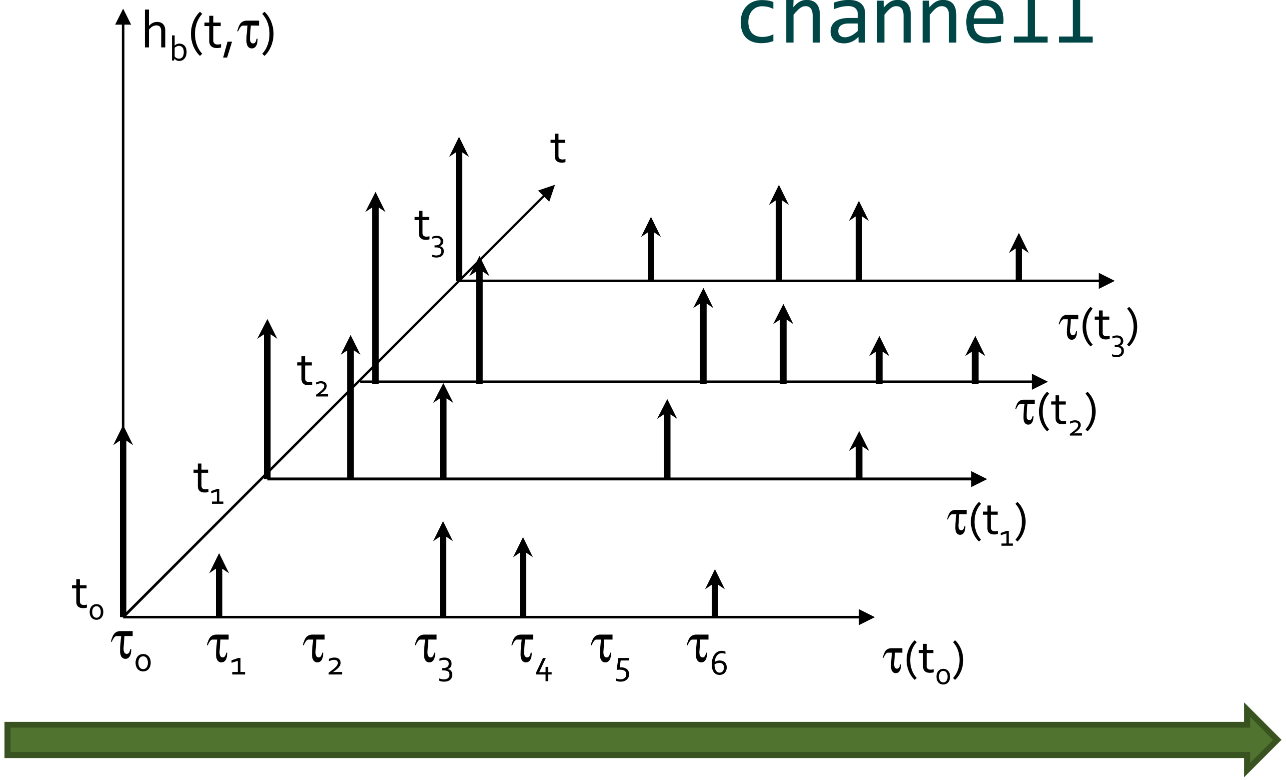
$$h_b(\tau)=\sum_{i=0}^{N-1}a_i\exp\left[-j\theta_i\right]\delta(\tau-\tau_i)$$

Two main aspects of the wireless channell



Following this axis, we study how "spread-out" the impulse response are.
(related to the physical layout of the TX, the RX, and the reflectors at a single time point)

Two main aspects of the wireless channell



Following this axis, we study how “spread-out” the impulse response are. (related to the physical layout of the TX, the RX, and the reflectors at a single time point)

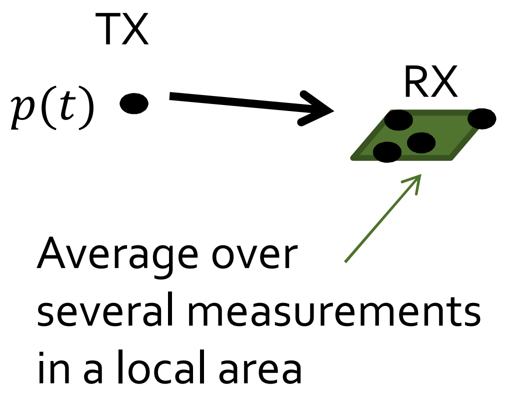
Power delay profile

- **To predict $h_B(\tau)$ a probing pulse p(t) is sent s.t.**

$$p(t) \approx \delta(t-\tau)$$

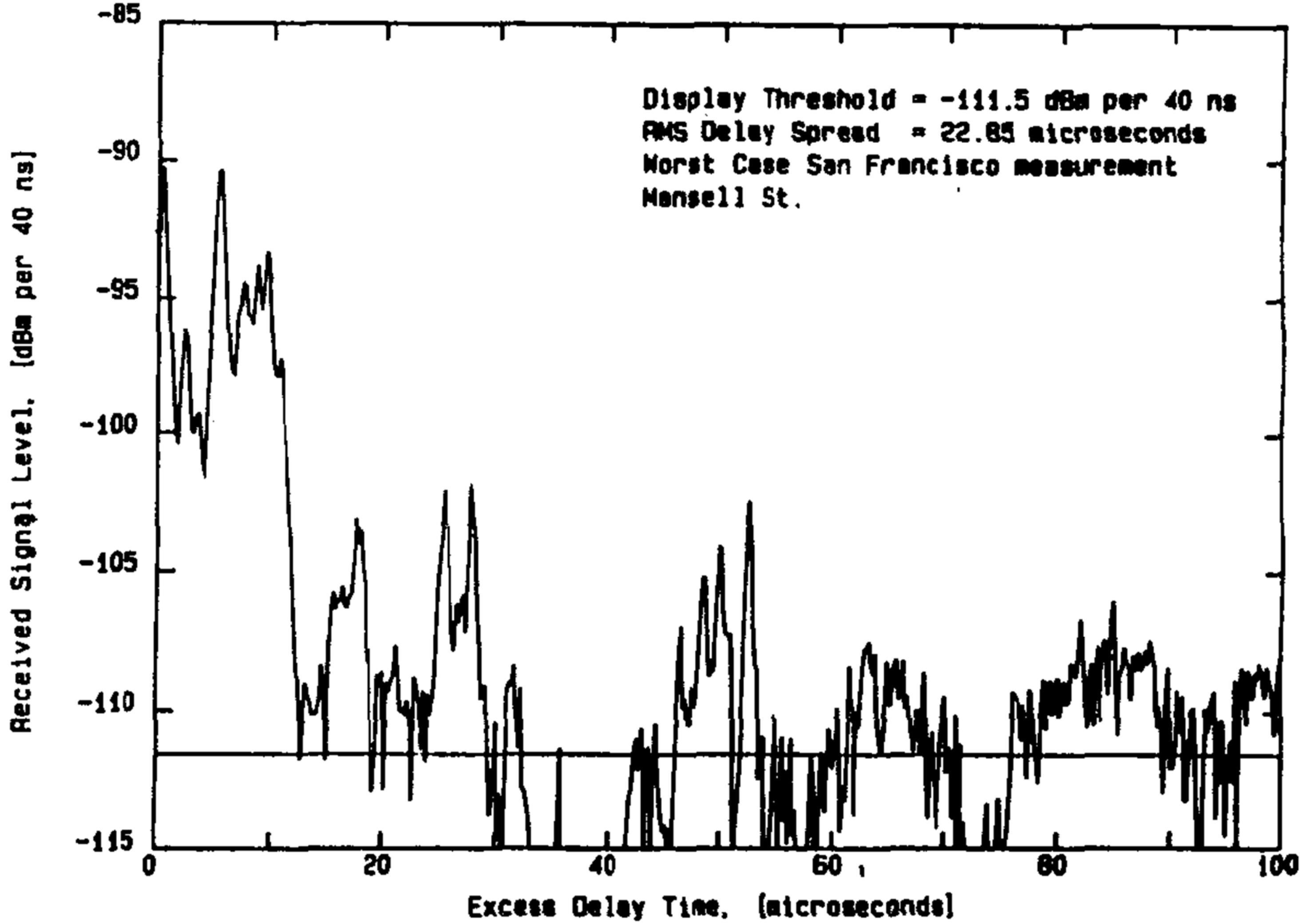
- **Therefore, for small-scale channel modeling, POWER DELAY PROFILE is found by computing the spatial average of $|h_B(t;\tau)|^2$ over a local area.**

$$P(t;\tau) \approx k|h_b(t;\tau)|^2$$



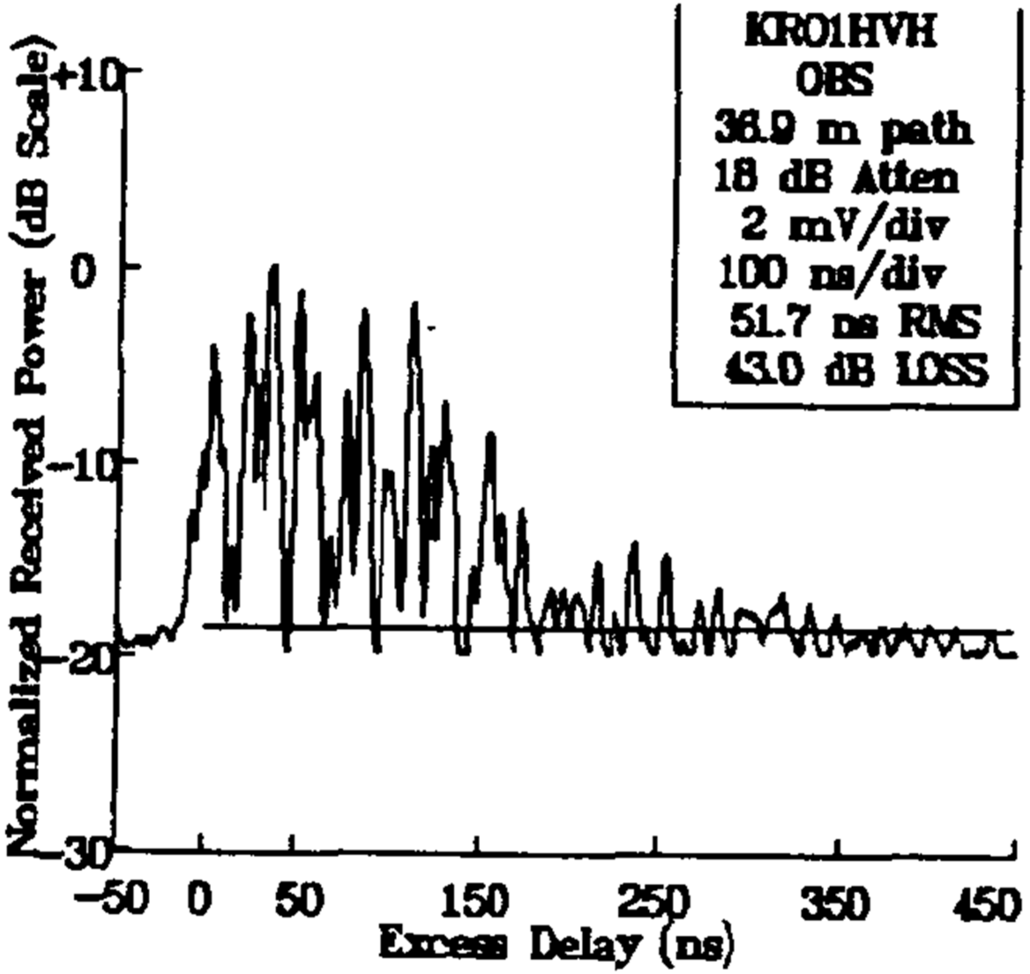
Example: power delay profile

From a 900 MHz cellular system in San Francisco



Example: power delay profile

In side a grocery store at 4 GHz



Time dispersion parameters

- **Power delay profile is a good representation of the average "geometry" of the transmitter, the receiver, and the reflectors.**
- **To quantify "how spread-out" the arriving signals are, we use time dispersion parameters:**

Already talked about this

  - Maximum excess delay: the excess delay of the latest arriving MPC
  - Mean excess delay: the "mean" excess delay of all arriving MPC
  - RMS delay spread: the "standard deviation" of the excess delay of all arriving MPC

Time dispersion parameters

- **Mean Excess Delay**

First moment of the power delay profile

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- **RMS Delay Spread**

Square root of the second moment of the power delay profile

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

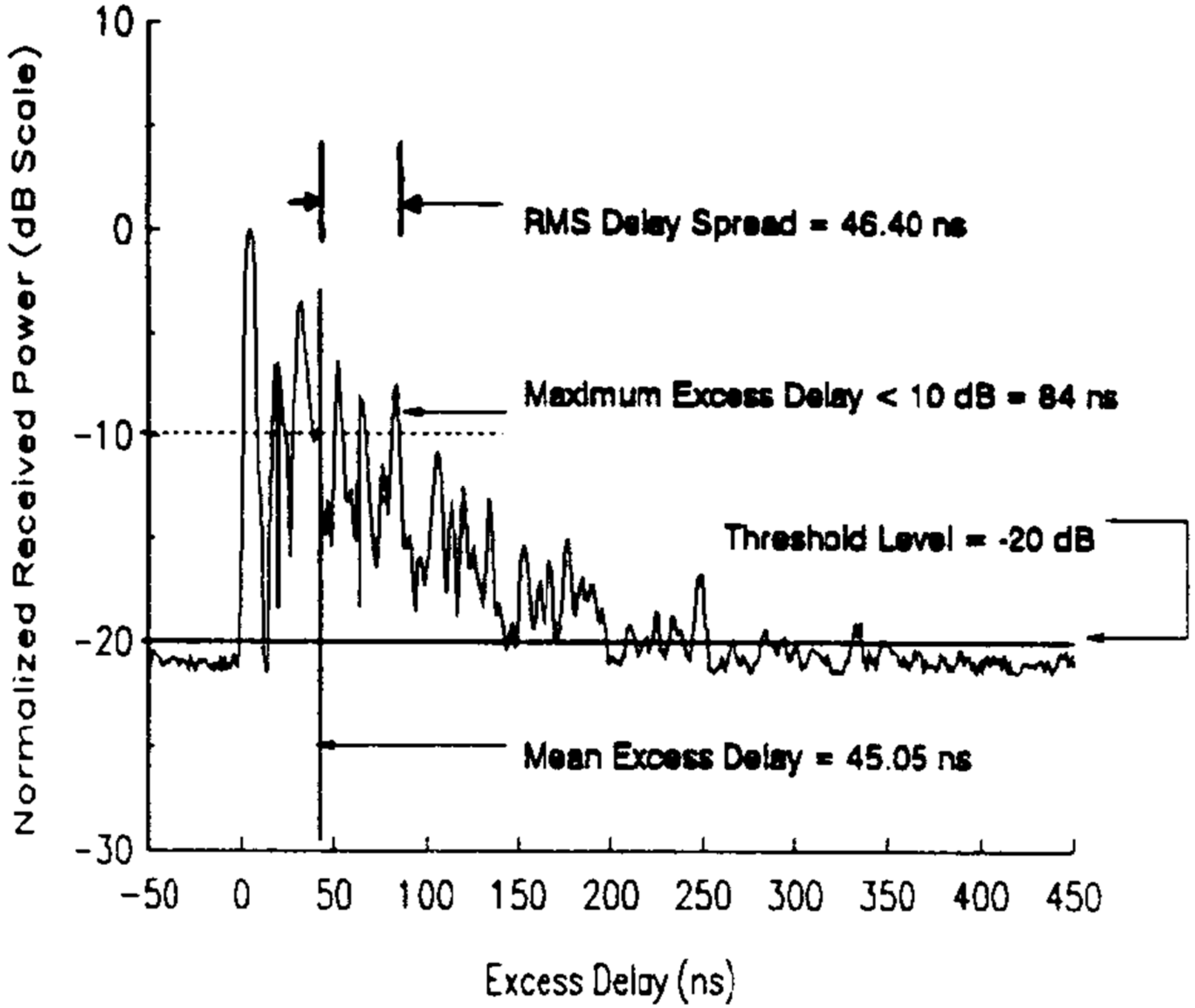
$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

Second moment of the power delay profile

Time dispersion parameters

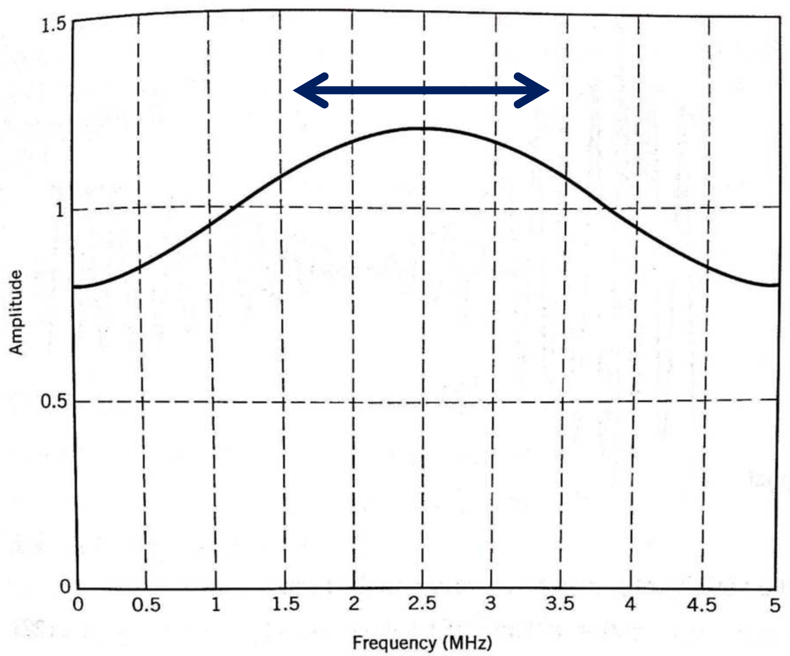
- **Maximum Excess Delay:**
  - Original version: the excess delay of the **latest** arriving MPC
  - In practice: the latest arriving could be smaller than the noise
  - No way to be aware of the “latest”
- **Maximum Excess Delay (practical version):**
  - The time delay during which multipath energy falls to X dB below the maximum.
- **This X dB threshold could affect the values of the time-dispersion parameters**
  - Used to differentiate the noise and the MPC
  - Too low: noise is considered to be the MPC
  - Too high: Some MPC is not detected

Example: Time dispersion parameters



Coherence Bandwidth

- **Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat"**
  → a channel passes all spectral components with approximately equal gain and linear phase.



Recall this:
Transfer function

Coherence Bandwidth

- **Bandwidth over which Frequency Correlation function is above 0.9**

$$B_c \approx \frac{1}{50\sigma_\tau}$$

- **Bandwidth over which Frequency Correlation function is above 0.5**

$$B_c \approx \frac{1}{5\sigma_\tau}$$

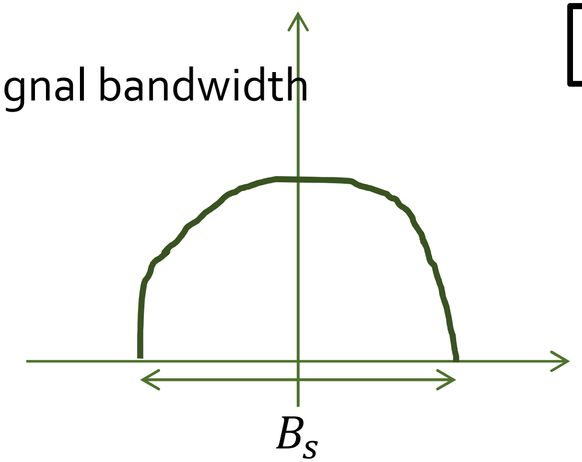
Those two are approximations derived from empirical results.

Typical RMS delay spread values

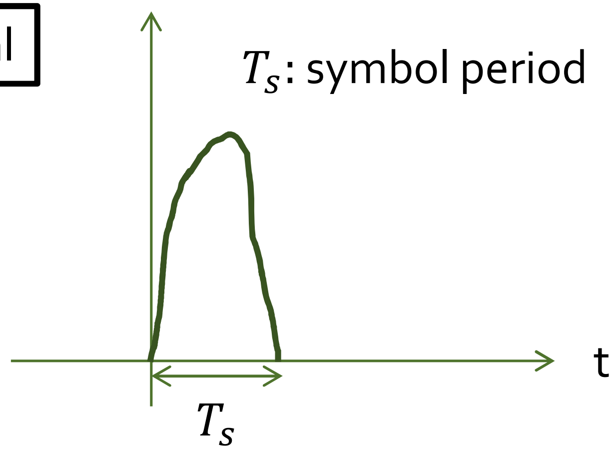
Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10-25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200-310 ns	Averaged typical case	[Cox72]
Suburban	910	1960-2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10-50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70-94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

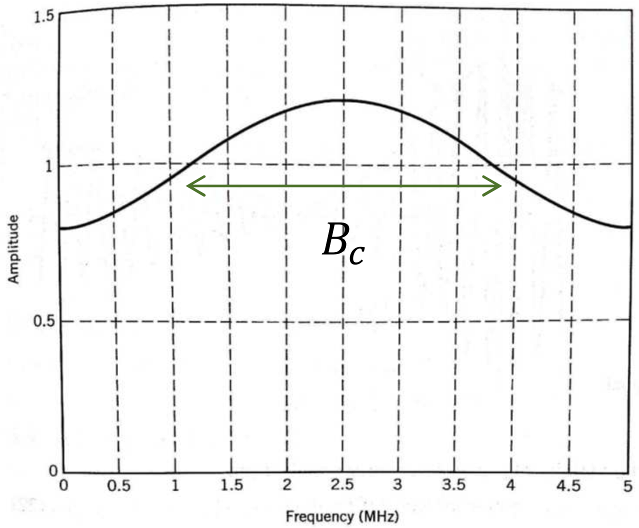
Signal Bandwidth & Coherence Bandwidth

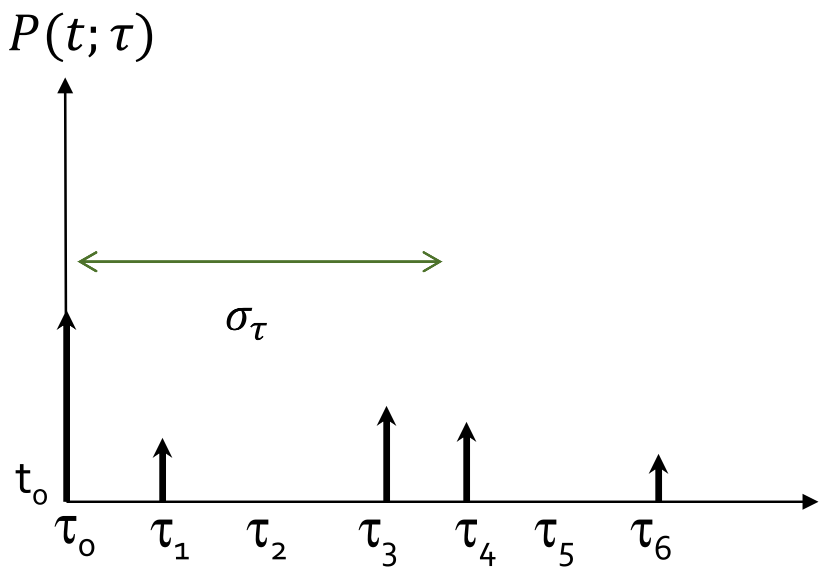
Transmitted Signal



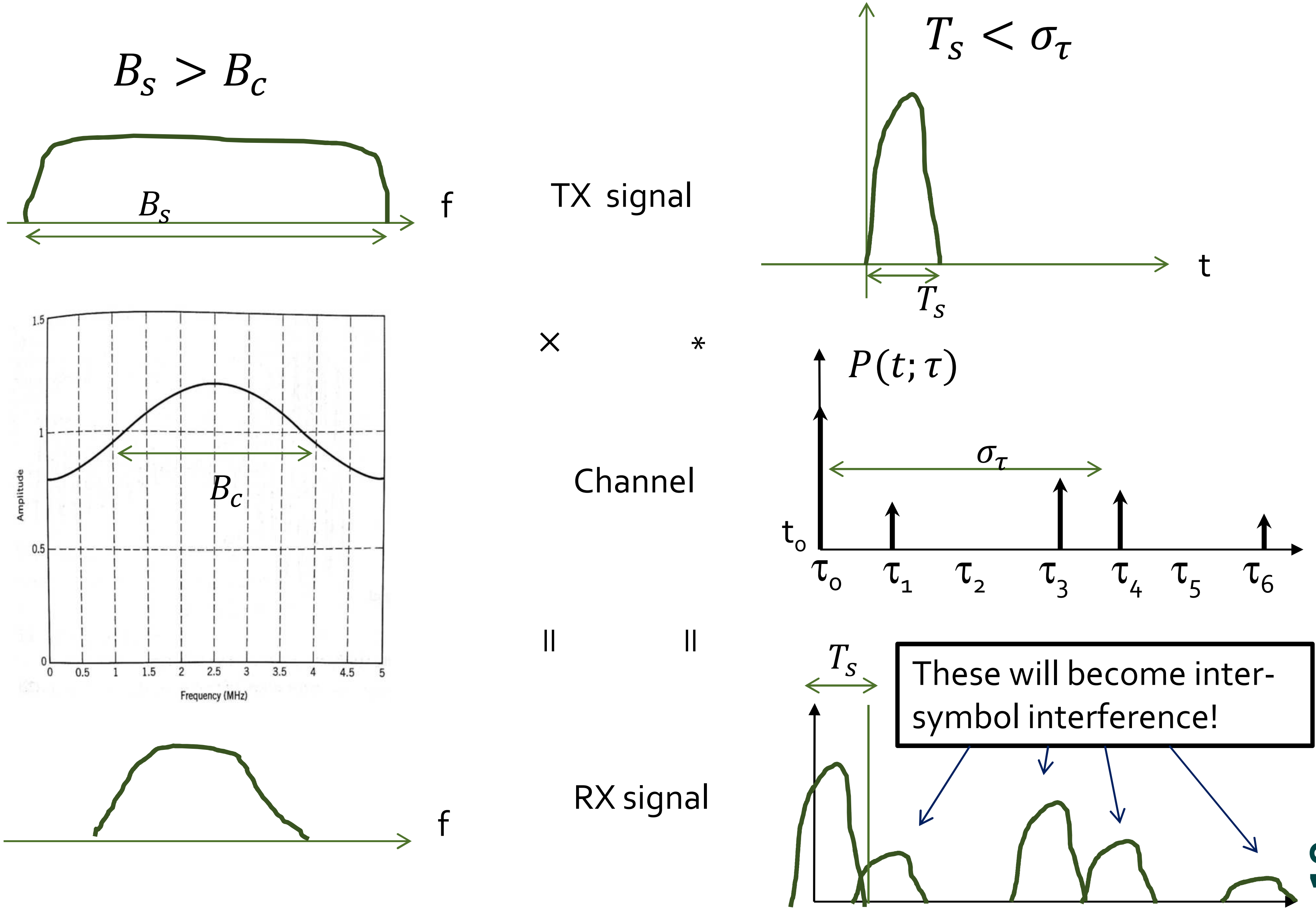
$$T_s \approx \frac{1}{B_s}$$





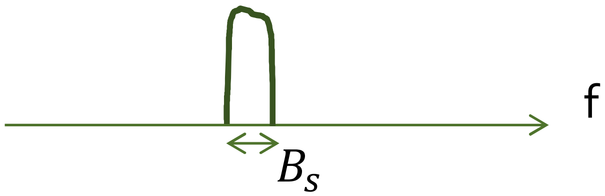


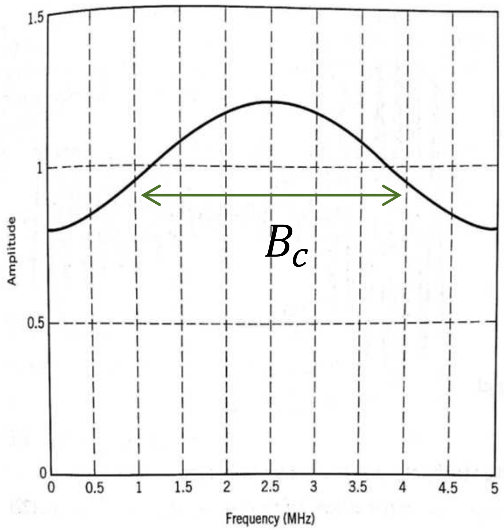
Frequency-selective fading channel

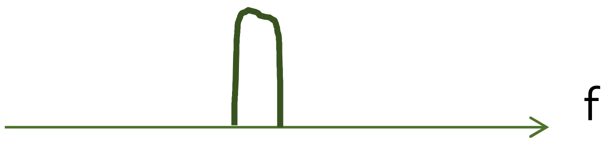


Flat fading channel

$B_s < B_c$







TX signal

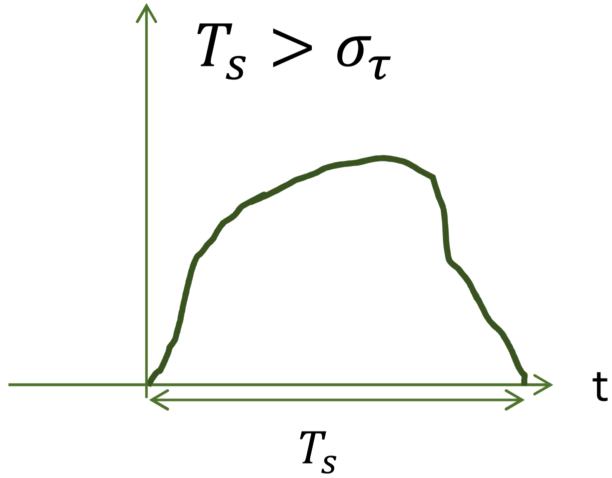
× *

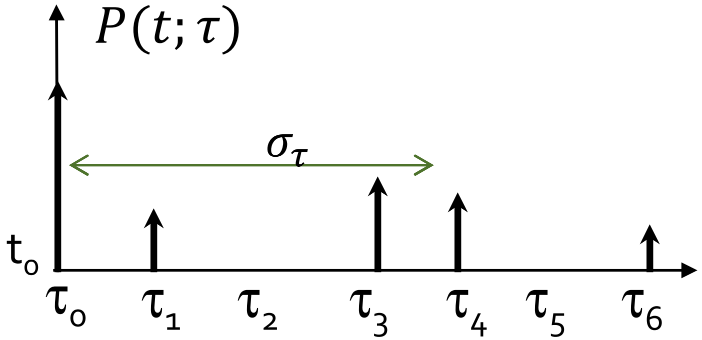
Channel

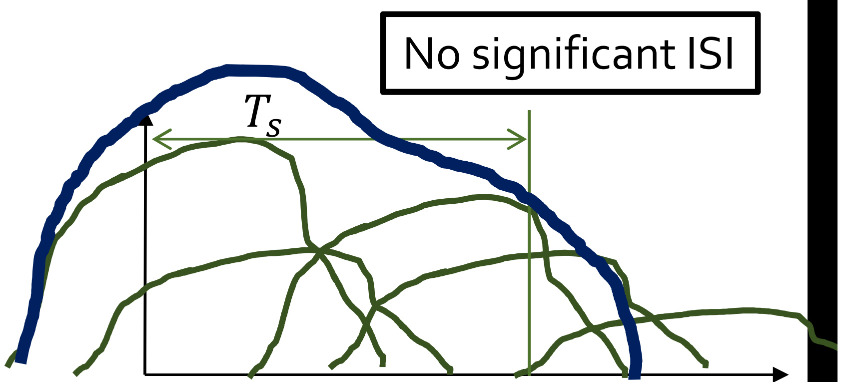
= =

RX signal

$T_s > \sigma_\tau$



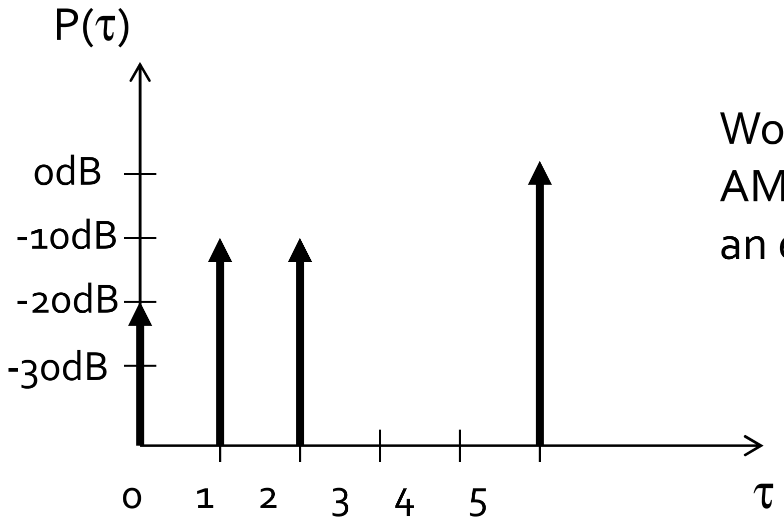




Equalizer 101

- **An equalizer is usually used in a frequency-selective fading channel**
  - When the coherence bandwidth is low, but we need to use high data rate (high signal bandwidth)
- **Channel is unknown and time-variant**
  - Step 1: TX sends a known signal to the receiver
  - Step 2: the RX uses the TX signal and RX signal to estimate the channel
  - Step 3: TX sends the real data (unknown to the receiver)
  - Step 4: the RX uses the estimated channel to process the RX signal
  - Step 5: once the channel becomes significantly different from the estimated one, return to step 1.

Example



Would this channel be suitable for AMPS or GSM without the use of an equalizer?

$$\text{Mean Excess Delay} = \bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} = \frac{5(1)+2(0.1)+1(0.1)+0(0.01)}{1+0.1+0.1+0.01} = 4.38\mu s$$

$$\overline{\tau^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} = \frac{(1)5^2+(0.1)2^2+(0.1)1^2+(0.01)0^2}{1+0.1+0.1+0.01} = 21.07\mu s^2$$

Example

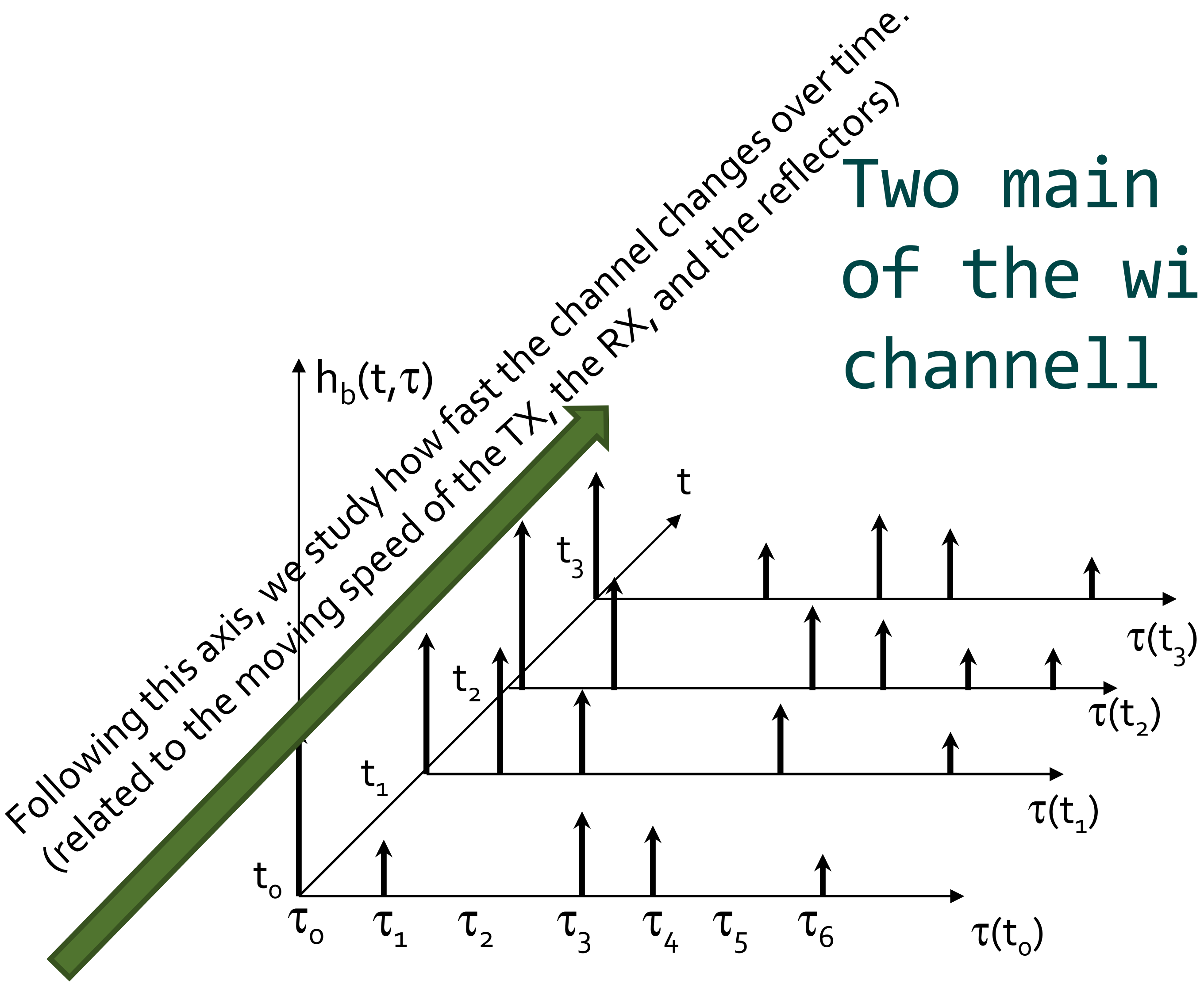
- **Therefore:**

$$\text{RMS Delay Spread} = \sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2} = \sqrt{21.07 - (4.38)^2} = 1.37\mu s$$

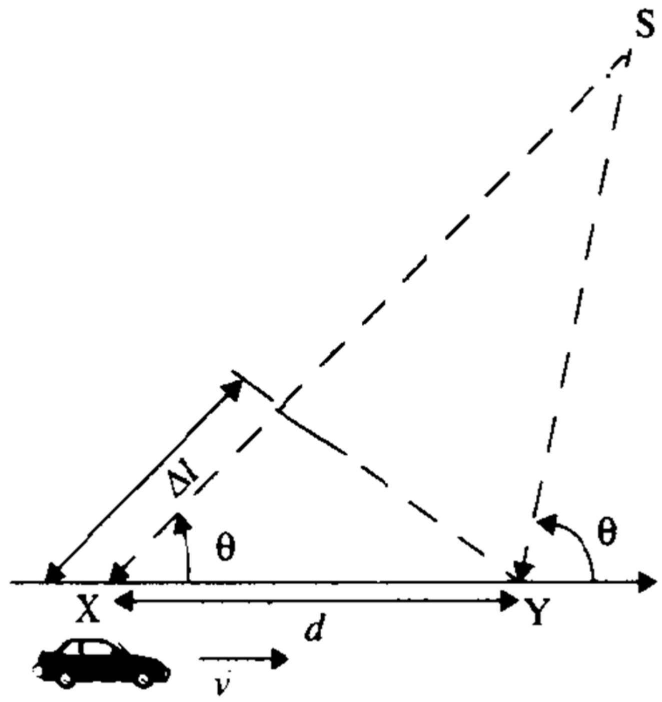
$$\text{Coherence Bandwidth} = B_C = \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu)} = 146KHz$$

- **Since B_C > 30KHz, AMPS would work without an equalizer.**
- **GSM requires 200 KHz BW > B_C → An equalizer would be needed.**

Two main aspects of the wireless channell



Doppler Effect



- Difference in path lengths $\Delta l = d\, cos\theta = v\Delta t\, cos\theta$
- Phase change $\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda} cos\theta$
- Frequency change, or Doppler shift,

$$f_d = \frac{1}{2\pi}\ \frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda} cos\theta$$

Example

$$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}cos\theta$$

- **Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving**

  1. directly toward the transmitter.
  2. directly away from the transmitter
  3. in a direction which is perpendicular to the direction of arrival of the transmitted signal.

- **Ans:**

  - Wavelength=$\lambda = \frac{c}{f_c} = \frac{3\times 10^8}{1850\times 10^6} = 0.162\ (m)$
  - Vehicle speed $v = 60mph = 26.82\ \left(\frac{m}{s}\right)$

  1. $f_d = \frac{26.82}{0.162}\cos(0) = 160\ (Hz)$
  2. $f_d = \frac{26.82}{0.162}\cos(\pi) = -160\ (Hz)$
  3. Since $\cos\left(\frac{\pi}{2}\right) = 0$, there is no Doppler shift!

Doppler Effect

- **If the car (mobile) is moving toward the direction of the arriving wave, the Doppler shift is positive**
- **Different Doppler shifts if different θ (incoming angle)**
- **Multi-path: all different angles**
- **Many Doppler shifts → Doppler spread**