Small-Scale Fading I
Multipath Propagation

RX just sums up all Multi Path Component (MPC).
Multipath Channel Impulse Response

An example of the time-varying discrete-time impulse response for a multipath radio channel.

The channel impulse response when \( t = t_0 \) (what you receive at the receiver when you send an impulse at time \( t_0 \)).

Excess delay: the delay with respect to the first arriving signal (\( \tau \)).

Maximum excess delay: the delay of latest arriving signal.

\( \tau_0 = 0 \), and represents the time the first signal arrives at the receiver.
Because the transmitter, the receiver, or the reflectors are moving, the impulse response is time-variant.
Multipath Channel Impulse Response

- The channels impulse response is given by:

\[ h_b(t, \tau) = \sum_{i=0}^{N-1} a_i(t, \tau) \exp\left[-j\{2\pi f_c \tau(t_i) + \phi_i(t, \tau)\}\right] \delta(t - \tau_i(t)) \]

Summation over all MPC  
Additional phase change due to reflections  
Amplitude change (mainly path loss)  
Phase change due to different arriving time

- If assumed time-invariant (over a small-scale time or distance):

\[ h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp[-j \theta_i] \delta(\tau - \tau_i) \]
Following this axis, we study how “spread-out” the impulse response are. (related to the physical layout of the TX, the RX, and the reflectors at a single time point)

Two main aspects of the wireless channel
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Power delay profile

- To predict $h_B(\tau)$ a probing pulse $p(t)$ is sent s.t.

$$p(t) \approx \delta(t - \tau)$$

- Therefore, for small-scale channel modeling, POWER DELAY PROFILE is found by computing the spatial average of $|h_B(t; \tau)|^2$ over a local area.

$$P(t; \tau) \approx k |h_B(t; \tau)|^2$$

Average over several measurements in a local area.
Example: power delay profile

From a 900 MHz cellular system in San Francisco
Example: power delay profile

Inside a grocery store at 4 GHz
Time dispersion parameters

- Power delay profile is a good representation of the average “geometry” of the transmitter, the receiver, and the reflectors.
- To quantify “how spread-out” the arriving signals are, we use time dispersion parameters:
  - Maximum excess delay: the excess delay of the latest arriving MPC
  - Mean excess delay: the “mean” excess delay of all arriving MPC
  - RMS delay spread: the “standard deviation” of the excess delay of all arriving MPC

Already talked about this
Time dispersion parameters

- **Mean Excess Delay**
  \[
  \tau = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}
  \]
  First moment of the power delay profile

- **RMS Delay Spread**
  \[
  \sigma_{\tau} = \sqrt{\tau^2 - (\bar{\tau})^2}
  \]
  Square root of the second moment of the power delay profile

\[
\tau^2 = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}
\]
Second moment of the power delay profile
Time dispersion parameters

• **Maximum Excess Delay:**
  • Original version: the excess delay of the latest arriving MPC
  • In practice: the latest arriving could be smaller than the noise
  • No way to be aware of the “latest”

• **Maximum Excess Delay (practical version):**
  • The time delay during which multipath energy falls to X dB below the maximum.

• **This X dB threshold could affect the values of the time-dispersion parameters**
  • Used to differentiate the noise and the MPC
  • Too low: noise is considered to be the MPC
  • Too high: Some MPC is not detected
Example: Time dispersion parameters

- **RMS Delay Spread**: 46.40 ns
- **Maximum Excess Delay < 10 dB**: 84 ns
- **Threshold Level**: -20 dB
- **Mean Excess Delay**: 45.05 ns
Coherence Bandwidth

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered “flat”
  → a channel passes all spectral components with approximately equal gain and linear phase.

Recall this: Transfer function
Coherence Bandwidth

• Bandwidth over which Frequency Correlation function is above 0.9

\[ B_c \approx \frac{1}{50\sigma_\tau} \]

• Bandwidth over which Frequency Correlation function is above 0.5

\[ B_c \approx \frac{1}{5\sigma_\tau} \]

Those two are approximations derived from empirical results.
## Typical RMS delay spread values

<table>
<thead>
<tr>
<th>Environment</th>
<th>Frequency (MHz)</th>
<th>RMS Delay Spread ($\sigma_z$)</th>
<th>Notes</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>910</td>
<td>1300 ns avg. 600 ns st. dev. 3500 ns max.</td>
<td>New York City</td>
<td>[Cox75]</td>
</tr>
<tr>
<td>Urban</td>
<td>892</td>
<td>10-25 $\mu$s</td>
<td>Worst case San Francisco</td>
<td>[Rap90]</td>
</tr>
<tr>
<td>Suburban</td>
<td>910</td>
<td>200-310 ns</td>
<td>Averaged typical case</td>
<td>[Cox72]</td>
</tr>
<tr>
<td>Suburban</td>
<td>910</td>
<td>1960-2110 ns</td>
<td>Averaged extreme case</td>
<td>[Cox72]</td>
</tr>
<tr>
<td>Indoor</td>
<td>1500</td>
<td>10-50 ns 25 ns median</td>
<td>Office building</td>
<td>[Sal87]</td>
</tr>
<tr>
<td>Indoor</td>
<td>850</td>
<td>270 ns max.</td>
<td>Office building</td>
<td>[Dev90a]</td>
</tr>
<tr>
<td>Indoor</td>
<td>1900</td>
<td>70-94 ns avg. 1470 ns max.</td>
<td>Three San Francisco buildings</td>
<td>[Sei92a]</td>
</tr>
</tbody>
</table>
Signal Bandwidth & Coherence Bandwidth

$B_s$: signal bandwidth

$T_s$: symbol period

$T_s \approx \frac{1}{B_s}$

$P(t; \tau)$

$B_c$: coherence bandwidth

$\sigma_{\tau}$
Frequency-selective fading channel

\[ B_S > B_c \]

TX signal

Channel

RX signal

These will become inter-symbol interference!

\[ T_S < \sigma_{\tau} \]
Flat fading channel

\[ B_S < B_c \]

TX signal

Channel

RX signal

No significant ISI

\[ T_S > \sigma_T \]
Equalizer 101

• An equalizer is usually used in a frequency-selective fading channel
  • When the coherence bandwidth is low, but we need to use high data rate (high signal bandwidth)
• Channel is unknown and time-variant
  • Step 1: TX sends a known signal to the receiver
  • Step 2: the RX uses the TX signal and RX signal to estimate the channel
  • Step 3: TX sends the real data (unknown to the receiver)
  • Step 4: the RX uses the estimated channel to process the RX signal
  • Step 5: once the channel becomes significantly different from the estimated one, return to step 1.
Would this channel be suitable for AMPS or GSM without the use of an equalizer?

Mean Excess Delay $\bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)} = \frac{5(1) + 2(0.1) + 1(0.1) + 0(0.01)}{1 + 0.1 + 0.1 + 0.01} = 4.38 \mu s$

$\bar{\tau}^2 = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} = \frac{(1)5^2 + (0.1)2^2 + (0.1)1^2 + (0.01)0^2}{1 + 0.1 + 0.1 + 0.01} = 21.07 \mu s^2$
Example

• Therefore:

\[
\text{RMS Delay Spread} = \sigma_\tau = \sqrt{\tau^2 - (\bar{\tau})^2} = \sqrt{21.07 - (4.38)^2} = 1.37\,\mu s
\]

\[
\text{Coherence Bandwidth} = B_C = \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\,\mu)} = 146\,\text{KHz}
\]

• Since \( B_C > 30\,\text{KHz} \), AMPS would work without an equalizer.

• GSM requires 200 KHz BW > \( B_C \rightarrow \) An equalizer would be needed.
Two main aspects of the wireless channel

Following this axis, we study how fast the channel changes over time.

(related to the moving speed of the TX, the RX, and the reflectors)
Doppler Effect

- Difference in path lengths $\Delta l = d \cos \theta = v \Delta t \cos \theta$
- Phase change $\Delta \phi = \frac{2\pi \Delta l}{\lambda} = \frac{2\pi v \Delta t}{\lambda} \cos \theta$
- Frequency change, or Doppler shift,

$$f_d = \frac{1}{2\pi} \frac{\Delta \phi}{\Delta t} = \frac{v}{\lambda} \cos \theta$$
Example

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving

1. directly toward the transmitter.
2. directly away from the transmitter
3. in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Ans:

- Wavelength \( \lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{1850 \times 10^6} = 0.162 \) (m)

- Vehicle speed \( v = 60 \text{mph} = 26.82 \text{ } \left( \frac{\text{m}}{\text{s}} \right) \)

1. \( f_d = \frac{26.82}{0.162} \cos(0) = 160 \) (Hz)
2. \( f_d = \frac{26.82}{0.162} \cos(\pi) = -160 \) (Hz)
3. Since \( \cos \left( \frac{\pi}{2} \right) = 0 \), there is no Doppler shift!
Doppler Effect

- If the car (mobile) is moving toward the direction of the arriving wave, the Doppler shift is positive
- Different Doppler shifts if different $\theta$ (incoming angle)
- Multi-path: all different angles
- Many Doppler shifts $\rightarrow$ Doppler spread