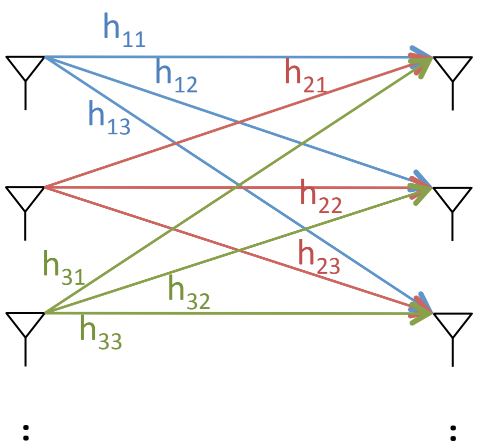
Multi-Input Multi-Output Systems (MIMO)

- Channel Model for MIMO
- MIMO Decoding
- MIMO Gains
- Multi-User MIMO Systems

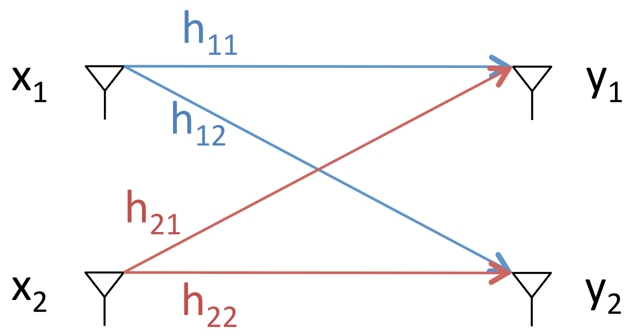
MIMO

- Each node has multiple antennas
  - Capable of transmitting (receiving) multiple streams concurrently
  - Exploit antenna diversity to increase the capacity



$$H_{N\times M} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \cdots \\ h_{21} & h_{22} & h_{23} & \cdots \\ h_{31} & h2 & h_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Channel Model (2x2)



$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$

$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

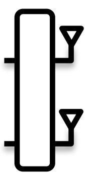
$$y = Hx + n$$

- Can be extended to N x M systems

Antenna Space

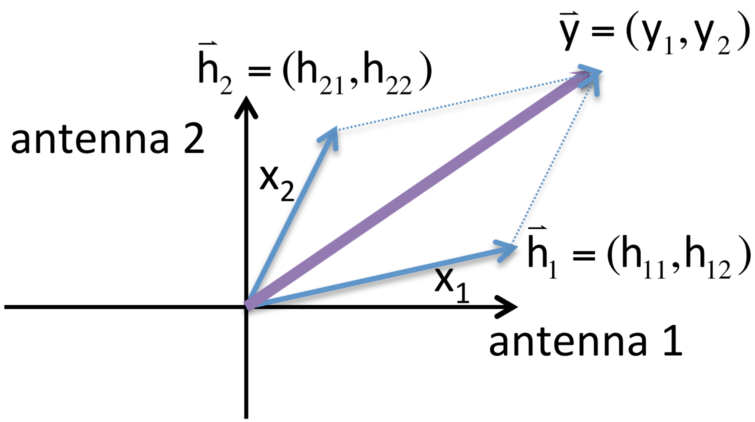
M-antenna node receives in M-dimensional space

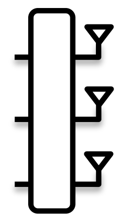
<u>2 x 2</u>

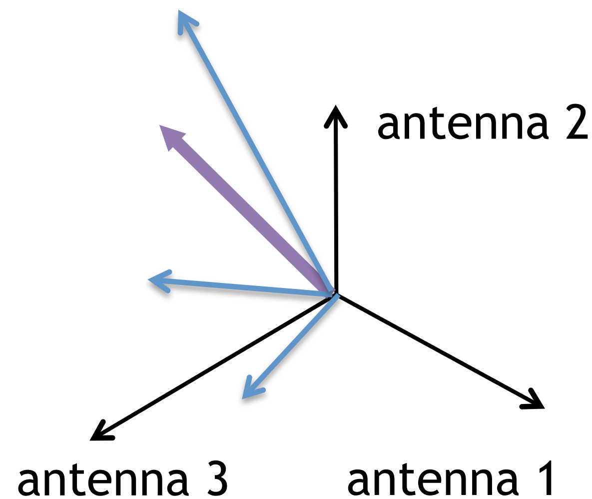


$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} x_1 + \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\vec{y} = \vec{h}_1 x_1 + \vec{h}_2 x_2 + \vec{n}$$







MIMO Decoding (algebra)

Orthogonal vectors

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{11} \\ h_{12} \end{pmatrix} x_1 + \begin{pmatrix} h_{21} \\ h_{22} \end{pmatrix} x_2 + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \qquad \begin{matrix} * \; h_{22} \\ * \; - h_{21} \end{matrix}$$

+)

$$y_1 h_{22} - y_2 h_{21} = (h_{11} h_{22} - h_{12} h_{21}) x_1$$

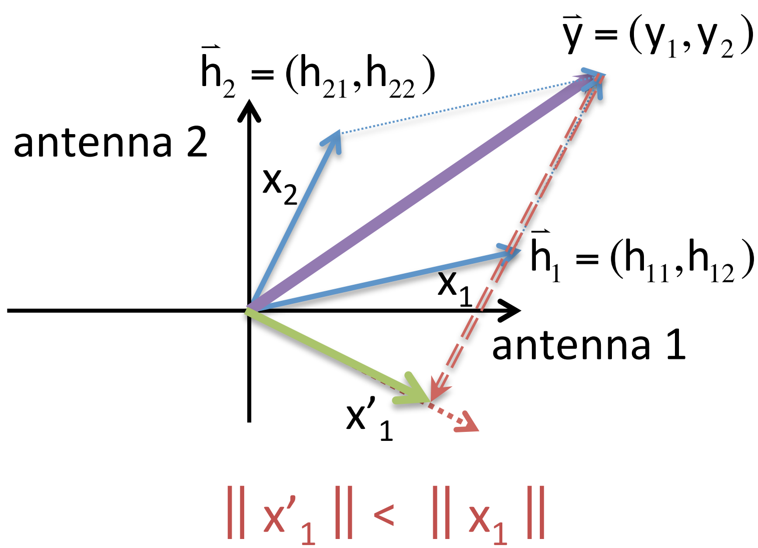
$$x_1 = \frac{y_1 h_{22} - y_2 h_{21}}{h_{11} h_{22} - h_{12} h_{21}}$$

Given x_1, solve x_2

To guarantee the full rank of H, antenna spacing at the transmitter and receiver must exceed half of the wavelength

MIMO Decoding (antenna space)

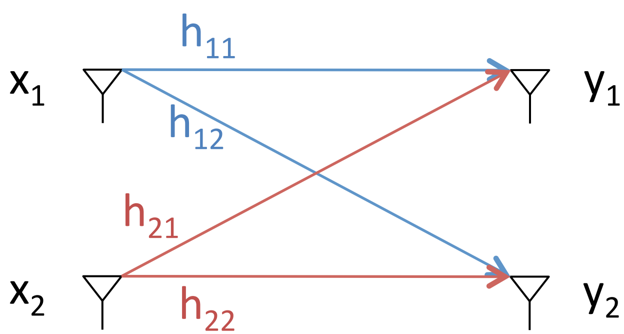
- Zero forcing



- To decode x_1, decode vector y on the direction orthogonal to x_2
- To improve the SNR, re-encode the first detected signal, subtract it from y, and decode the second signal

Channel Estimation

- Estimate N x M matrix H



$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$
$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

Two equations, but four unknowns

Estimate h_{11}, h_{12} Estimate h_{21}, h_{22}

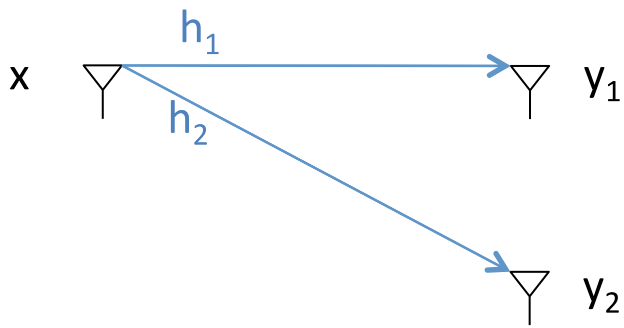
MIMO Gains

- Multiplex Gain
  - Exploit antenna diversity to deliver multiple streams concurrently

- Diversity Gain
  - Exploit antenna diversity to increase the SNR of a single stream

Diversity Gain

- 1 x 2 example



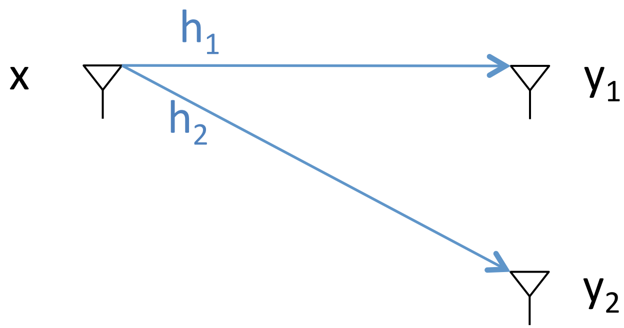
$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

- Decode the SNR of $(y_1 + y_2)$
- Uncorrelated whit Gaussian noise with zero mean
- Packet can be delivered through at least one of the many diver paths

Diversity Gain

- 1 x 2 example



$$y_1 = h_1 x + n_1$$
$$y_2 = h_2 x + n_2$$

$$\begin{aligned} SNR &= \frac{P(2X)}{P(n_1 + n_2)}, \text{ where P refers to the power} \\ &= \frac{E[(2X)^2]}{E[n_1^2 + n_2^2]} \\ &= \frac{4E[X^2]}{2\sigma^2}, \text{ where } \sigma \text{ is the variance of AWGN} \\ &= 2 * SNR_{\text{single antenna}} \end{aligned}$$

- Increase SNR by 3dB
- Especially beneficial for the low SNR link

Diversity Gain

Multiply each y with the conjugate of the channel

$$\begin{cases} y_1 = h_1 x + n_1 \\ y_2 = h_2 x + n_2 \end{cases} \qquad \begin{cases} h_1^* y_1 = \|h_1\|^2 x + h_1^* n_1 \\ h_2^* y_2 = \|h_2\|^2 x + h_2^* n_2 \end{cases}$$

$$\mathrm{SNR}_{diversity} = \frac{E[((\|h_1\|^2 + \|h_2\|^2)X)^2]}{E[(h_1^* n_1 + h_2^* n_2)^2]} = \frac{(\|h_1\|^2 + \|h_2\|^2)^2 E(X^2)}{(\|h_1\|^2 + \|h_2\|^2)\sigma^2} = \frac{(\|h_1\|^2 + \|h_2\|^2) E(X^2)}{\sigma^2}$$

$$\mathrm{SNR}_{single} = \frac{E[((\|h_1\|^2 + \|h_2\|^2)X)^2]}{E[(h_1^* n_1 + h_2^* n_2)^2]} = \frac{\|h_1\|^4 E(X^2)}{(\|h_1\|^2)\sigma^2} = \frac{\|h_1\|^2 E(X^2)}{\sigma^2}$$

$$\mathrm{gain} = \frac{(\|h_1\|^2 + \|h_2\|^2)}{\|h_1\|^2}$$

Trade off

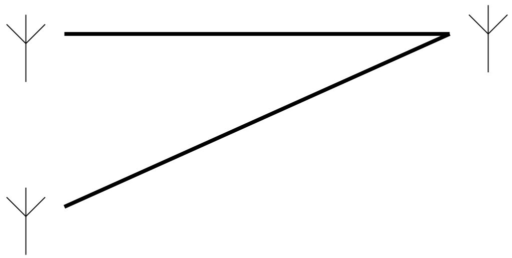
- Between diversity gain and multiplex gain

- Say we have a N x N system
  - Degree of freedom: N
  - The transmitter can transmit k streams concurrently, where k <= N
  - The optimal value of k is determined by the tradeoff between the diversity gain and multiplex gain

Degree of Freedom

- For N x M MIMO channel
  - Degree of Freedom (DoF): min {N,M}
  - Maximum diversity: NM

Space-Time Code Examples: 2×1 Channel



Repetition Scheme:

time

$$\mathbf{X} = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}$$

space

diversity: 2

data rate: 1/2 sym/s/Hz

Alamouti Scheme:

time

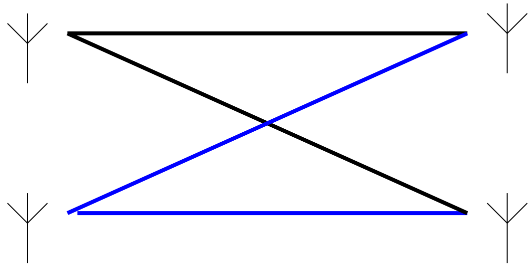
$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

space

diversity: 2

data rate: 1 sym/s/Hz

Space-Time Code Examples: 2×2 Channel



Repetition Scheme:

time

$$\mathbf{X} = \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}$$

space

diversity: 4

data rate: 1/2 sym/s/Hz

Alamouti Scheme:

time

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

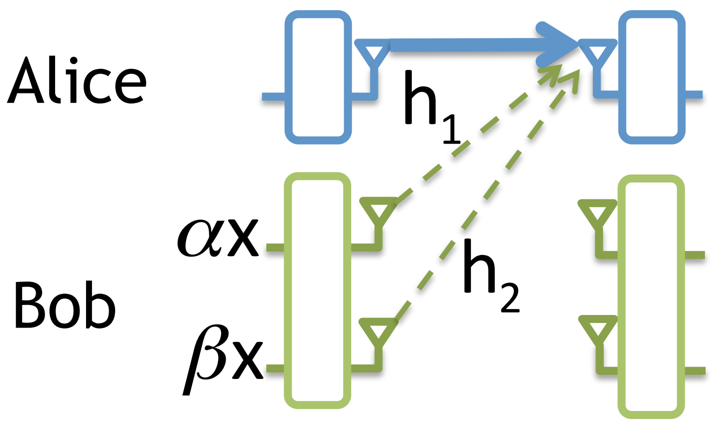
space

diversity: 4

data rate: 1 sym/s/Hz

But the 2×2 channel has 2 degrees of freedom!

Interference Nulling



$$h_1\alpha x + h_2\beta x = 0$$

$$\Rightarrow \text{Nulling}: \; h_1\alpha = -h_2\beta$$

$$(h_{1a}\alpha + h_{2a}\beta)x \neq 0$$

$$(h_{1b}\alpha + h_{2b}\beta)x \neq 0$$

- Signals cancel each other at Alice’s receiver
- Signals don’t cancel each other at Bob’s receiver
  - Because channels are different

Homework

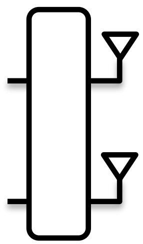
- Say there exist a 3x2 link, which has a channel

$$H_{3\times 2} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}$$

How can a three-antenna transmitter transmit a signal x, but null its signal at two antennas of a two-antenna receiver?

Interference Alignment

2-antenna receiver

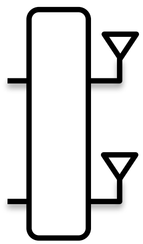


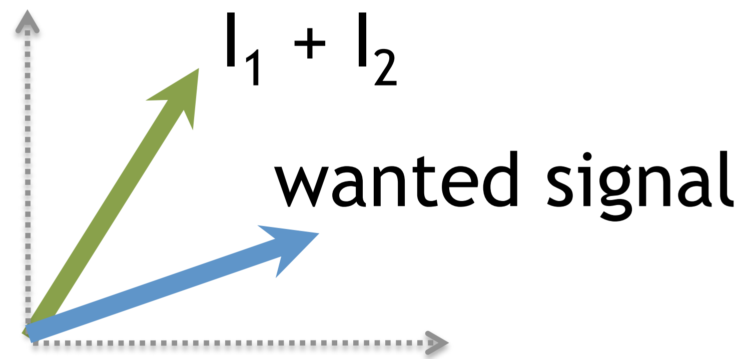
N-antenna node can only decode N signals

If I_1 and I_2 are aligned,

Interference Alignment

2-antenna receiver





N-antenna node can only decode N signals

If I_1 and I_2 are aligned,

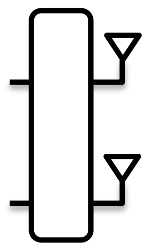
→ appear as one interferer

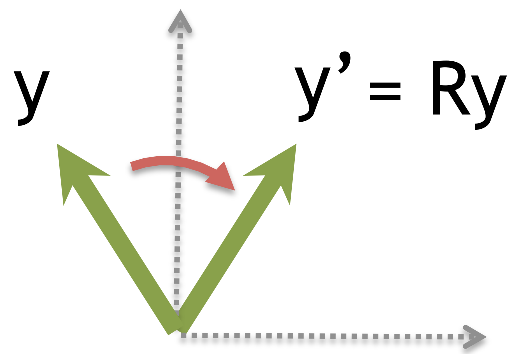
→ 2-antenna receiver can decode the wanted signal

Rotate Signal

1. Transmitter can rotate the received signal

2-antenna receiver





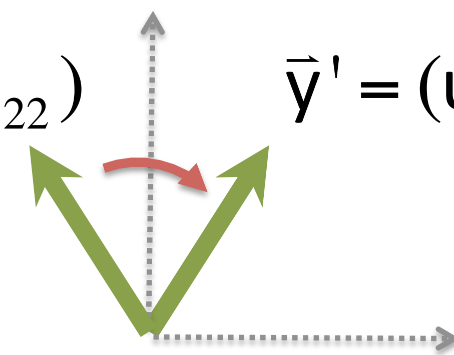
To rotate received signal y to y' = Ry, transmitter multiplies its transmitted signal by the same rotation matrix R

Rotate Signal

$$y_1 = (h_{11}\alpha + h_{21}\beta)x$$

$$y_2 = (h_{12}\alpha + h_{22}\beta)x$$

$$\vec{y} = (h_{11} + h_{21}, h_{12} + h_{22})$$



$$\vec{y}\,' = (u, v)$$

$$(h_{11}\alpha + h_{21}\beta) = u$$

$$(h_{12}\alpha + h_{22}\beta) = v$$

How to align the signal along the interference?

→ Find the direction of the interference and rotate the signal to that direction