Diversity

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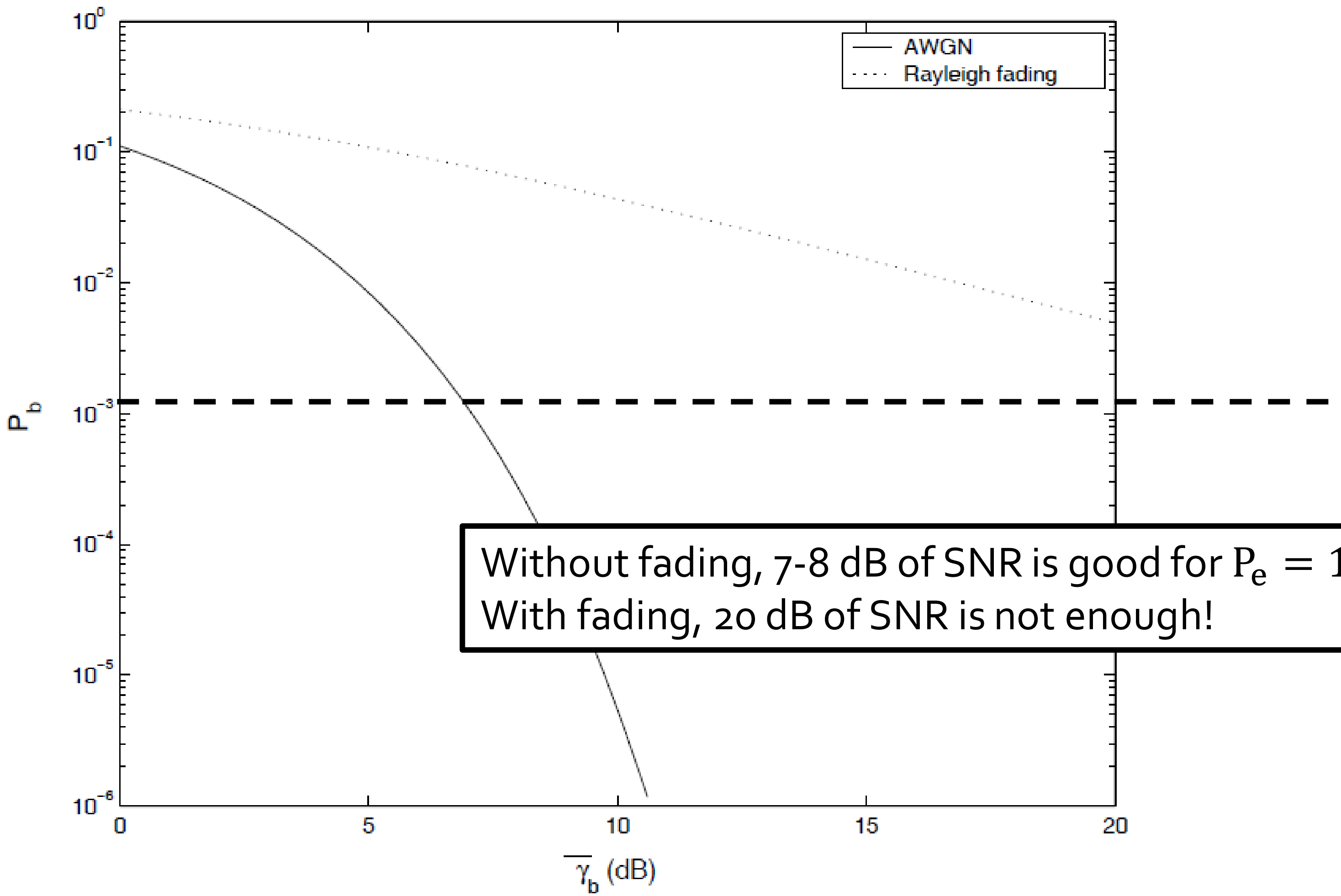
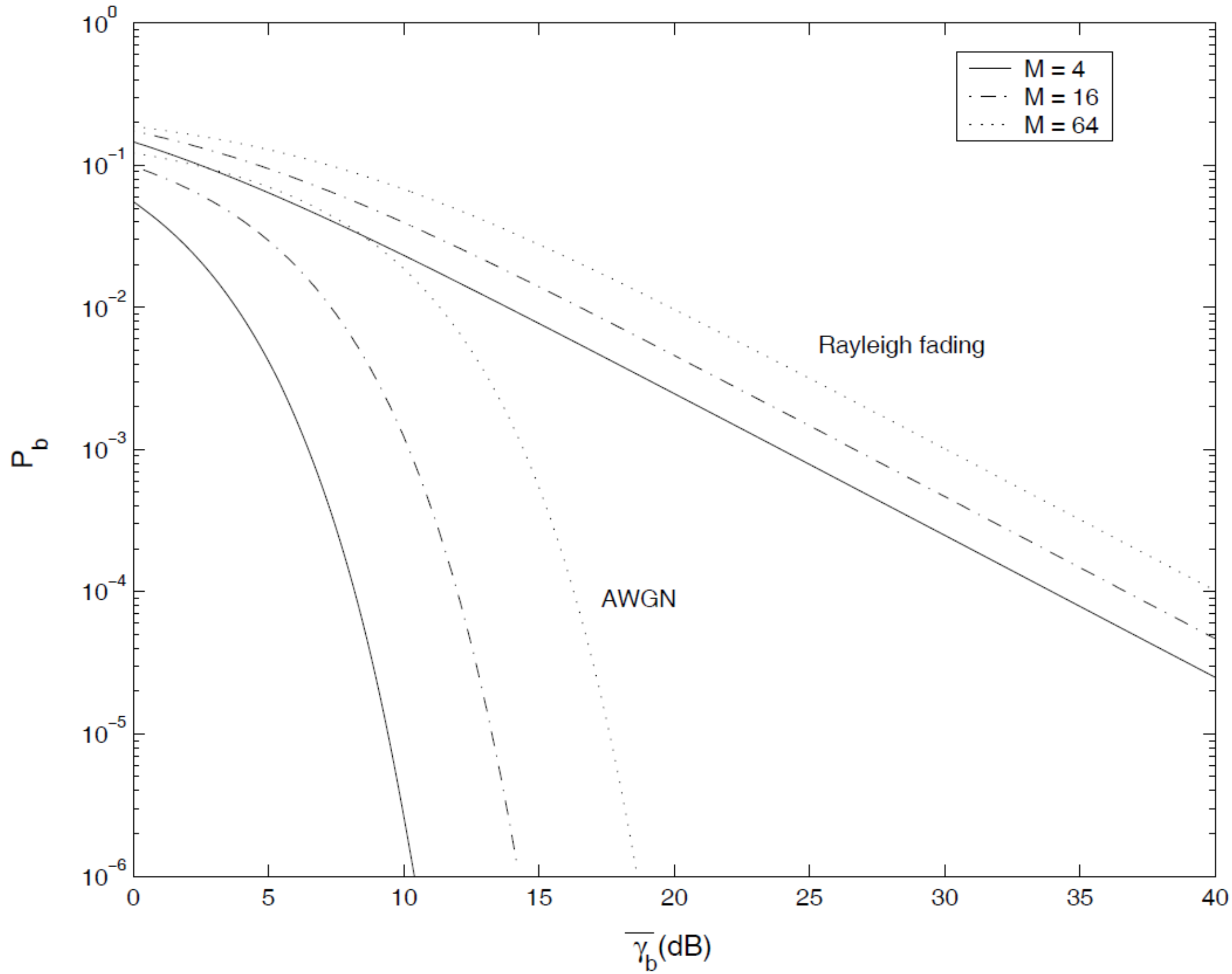
BER Performance under Fading: BPSK in Rayleigh fading

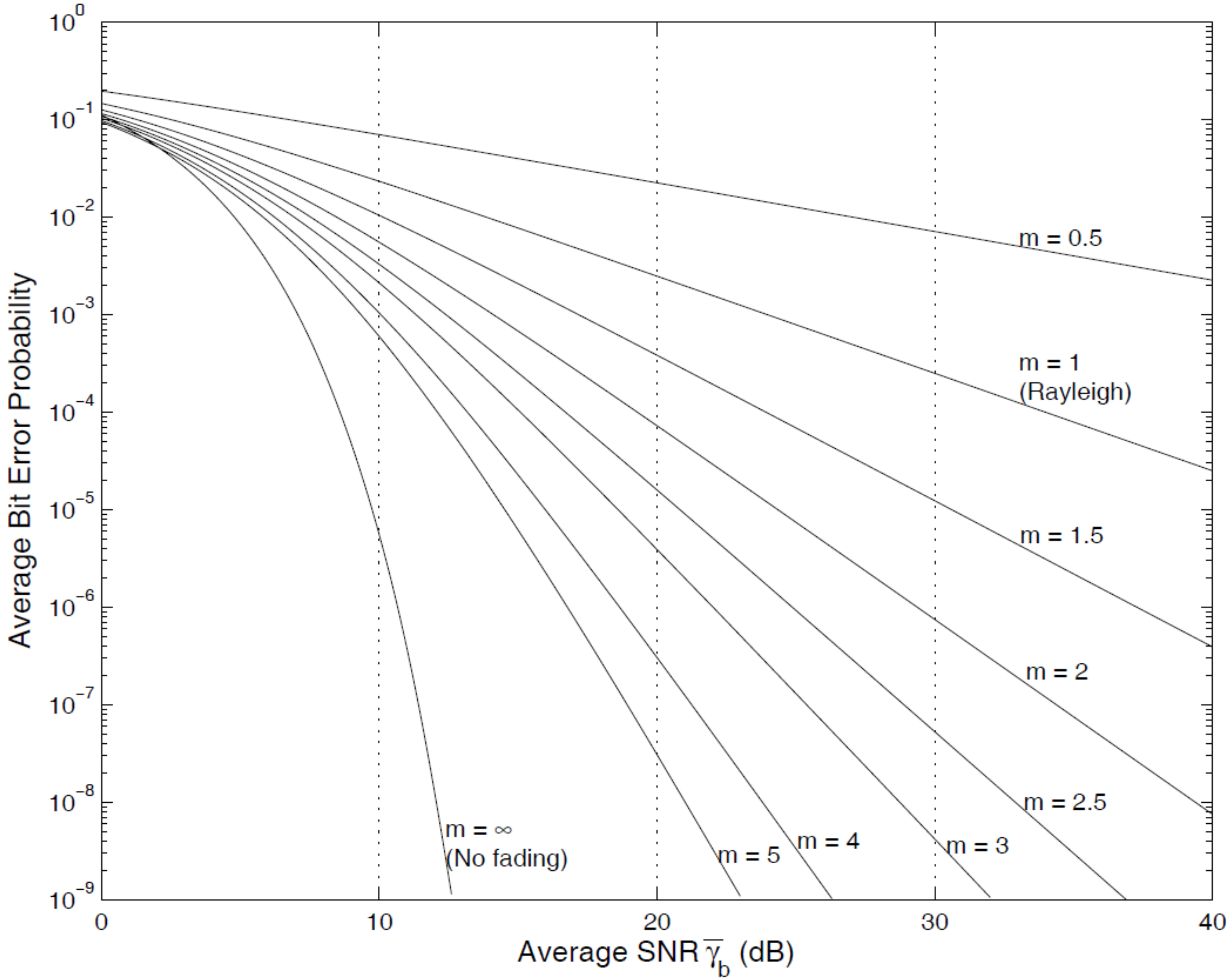


Without fading, 7-8 dB of SNR is good for $\mathrm{P_e} = 10^3$.
With fading, 20 dB of SNR is not enough!

BER Performance under Fading: M-QAM in Rayleigh Fading



BER Performance under Fading: BPSK in Nakagami fading

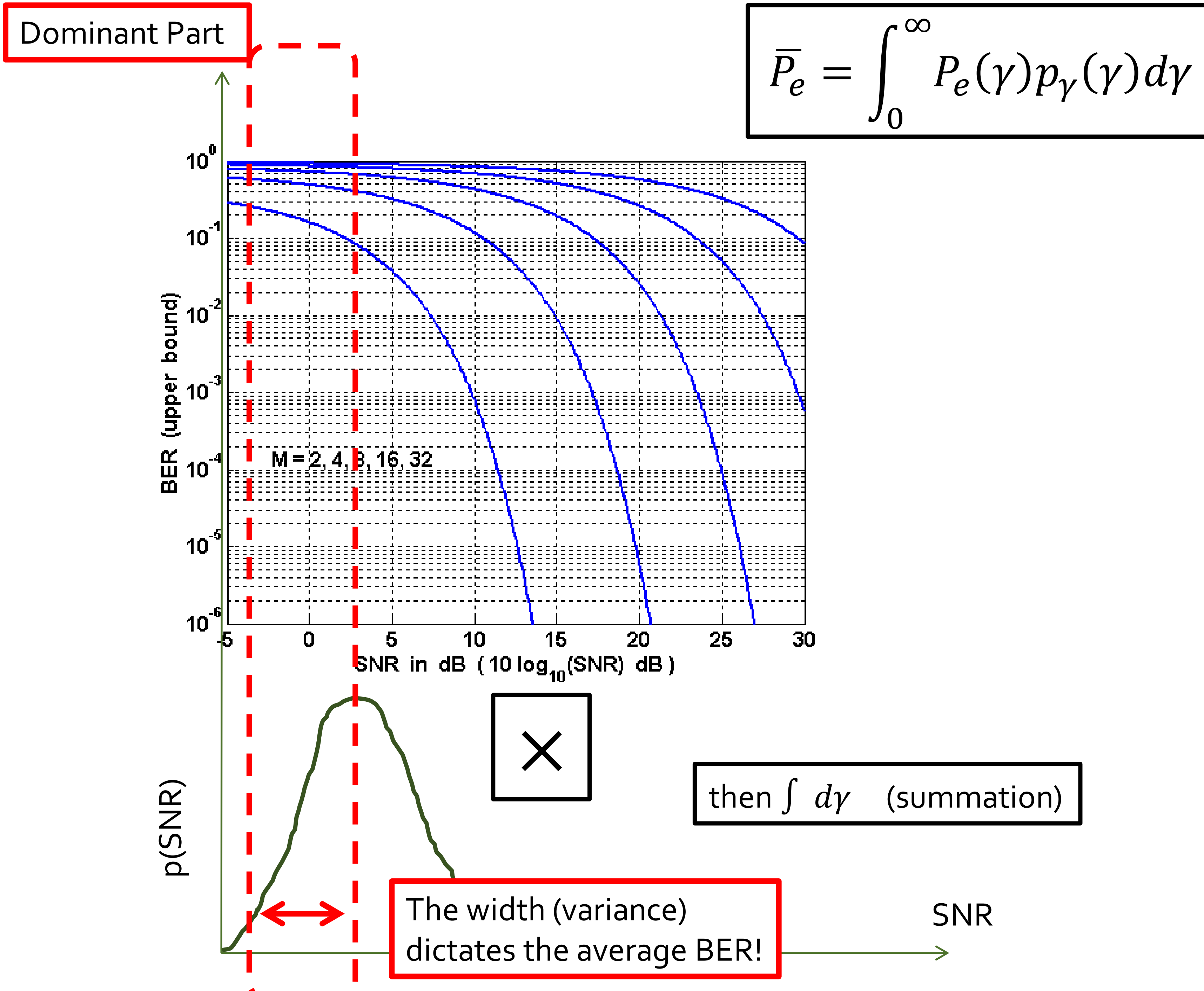


Intuition: how does fading affect average BER?

- **Average BER:**

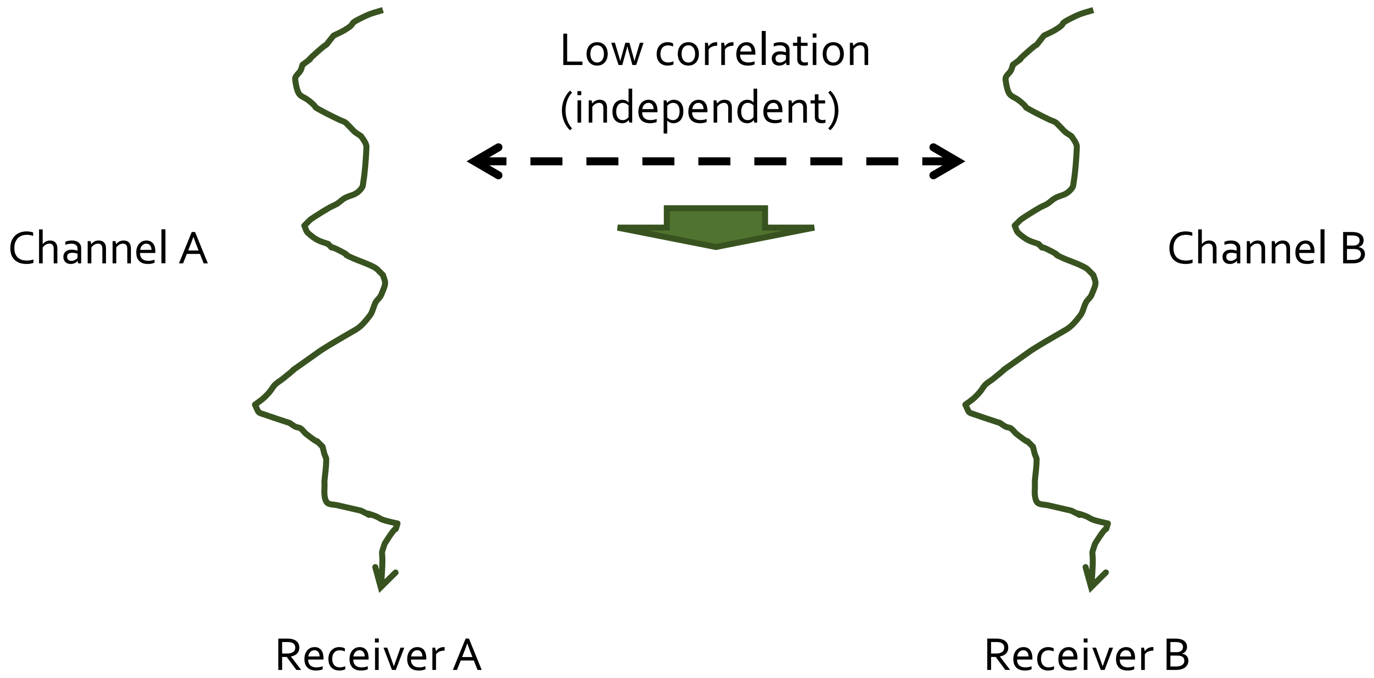
$$\bar{P_e} = \int_0^\infty P_e(\gamma) p_\gamma(\gamma) d\gamma$$

  - γ: Signal-to-Noise Ratio (SNR) per bit
  - $p_\gamma(\gamma)$: PDF of SNR (fading distribution divided by noise power)
  - $P_e(\gamma)$: BER of a given SNR



Dominant Part
$$\overline{P_e} = \int_0^\infty P_e(\gamma) p_\gamma(\gamma) d\gamma$$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
BER (upper bound)
M = 2, 4, 8, 16, 32
-5
0
5
10
15
20
25
30
SNR in dB ($10 \log_{10}$(SNR) dB)
×
then $\int d\gamma$ (summation)
p(SNR)
The width (variance)
dictates the average BER!
SNR

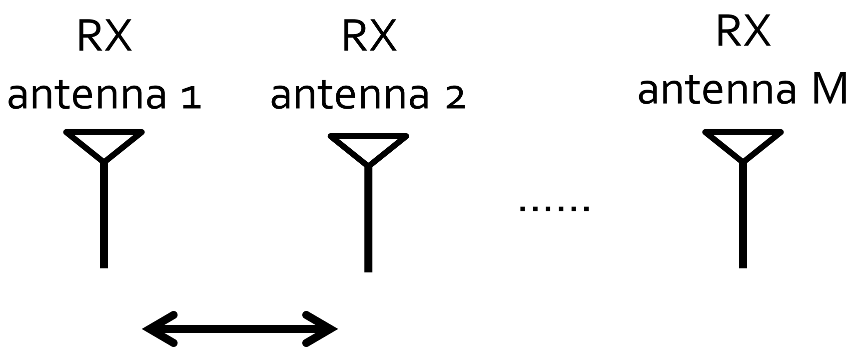
Concept: Diversity



$$p(A\ has\ bad\ reception\ \&\&\ B\ has\ bad\ reception)$$
$$= p(PL_A > PL_0\ \&\&\ PL_B > PL_0)$$
$$\approx p(PL_A > PL_0)p(PL_B > PL_0) \ll p(PL_A > PL_0)$$

Space Diversity

Q: What's the minimum required separation between 2 antennas? (for 802.11g and 802.11a)



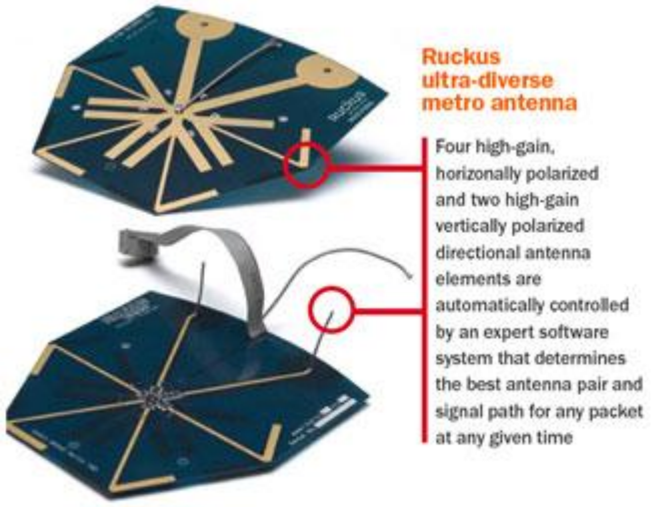
Each pair separated by at least half the wavelength (accurate version: 0.38 wavelength)

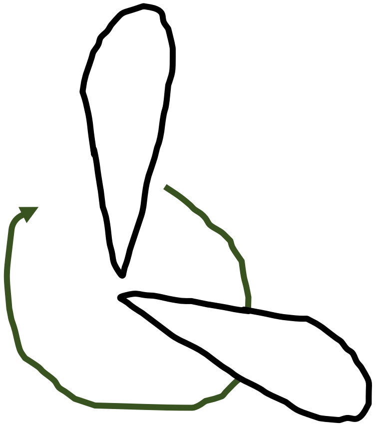
Low correlation → independent channels

A: 12.5 cm for 2.4 GHz
5.17 cm for 5.8 GHz
(which is what you see for a typical router)

Directional (Angle) Diversity

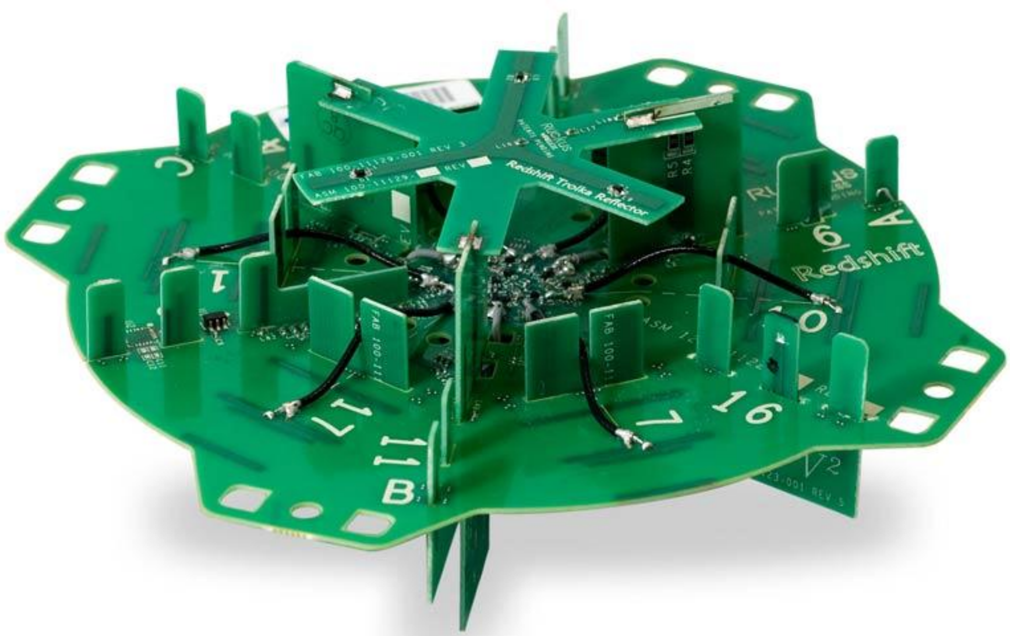
- **Split the 360 degree receiving angle into different "sectors"**
- **Each will receive a portion of multipath components (MPC)**
  - Extreme case: if the angle of each "sector" is very very small, then you only receive one MPC → no small scale fading
  - Different sets of MPCs go through different paths → low correlation!
- **Antenna design:**
  - Multiple sectors on the same antenna (switchable multiple antennas)
  - Steerable directional antenna (mechanical)





New WiFi Access Points in the CSIE Building

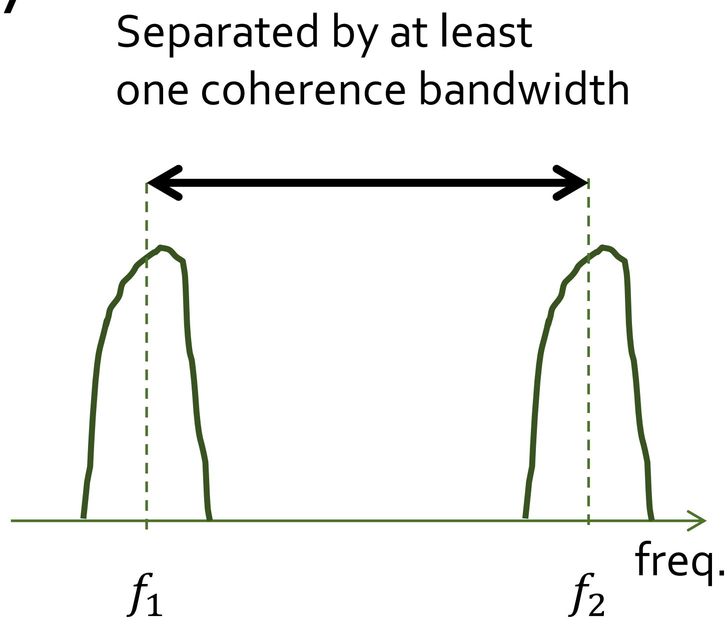
- **Ruckus Zoneflex 7962**
- **Will replace the 5-year old "CSIE" Proxim AP4000 (~20 of them)**
- **802.11 a/b/g/n**
- **Over 4000 unique antenna patterns**
  - Many "sectors", 3D too (from its appearance)
  - Select multiple "good" antennas for receiving
- **Can be used to reduce interference too**



Smart Antenna inside
Ruckus Zoneflex 7962

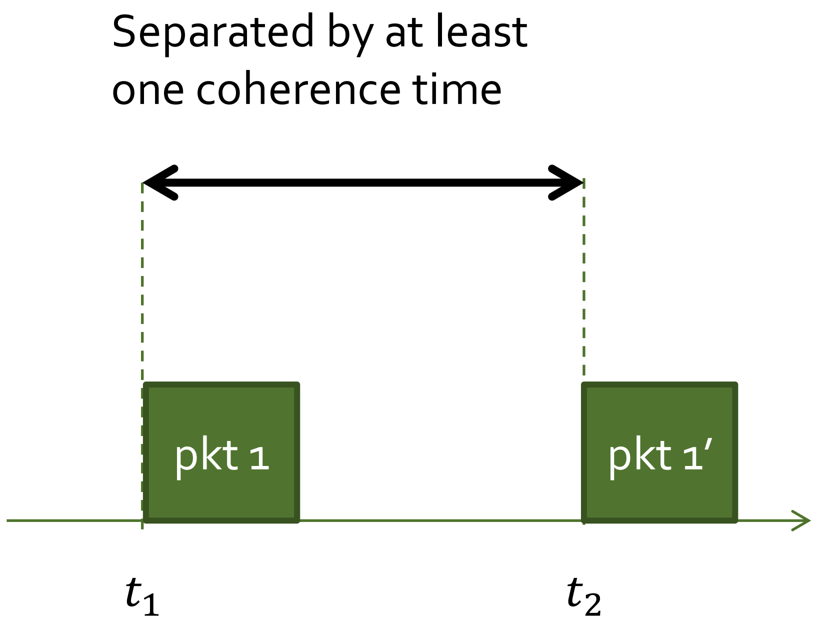
Frequency Diversity

- **Signals at two frequencies separated by at least one coherence bandwidth**
  → low correlation!
  → independent!
- **Small coherence bandwidth is sometimes good too**
  - For frequency diversity, two transmissions do not need to be too far apart in frequency
- **OFDM utilize this property too**
  - Sub-carriers separated by at least one coherence bandwidth can transmit redundant information for **diversity (reliability)**
  - Sub-carriers within the same coherence bandwidth can transmit different information for increasing the **throughput**



Time Diversity

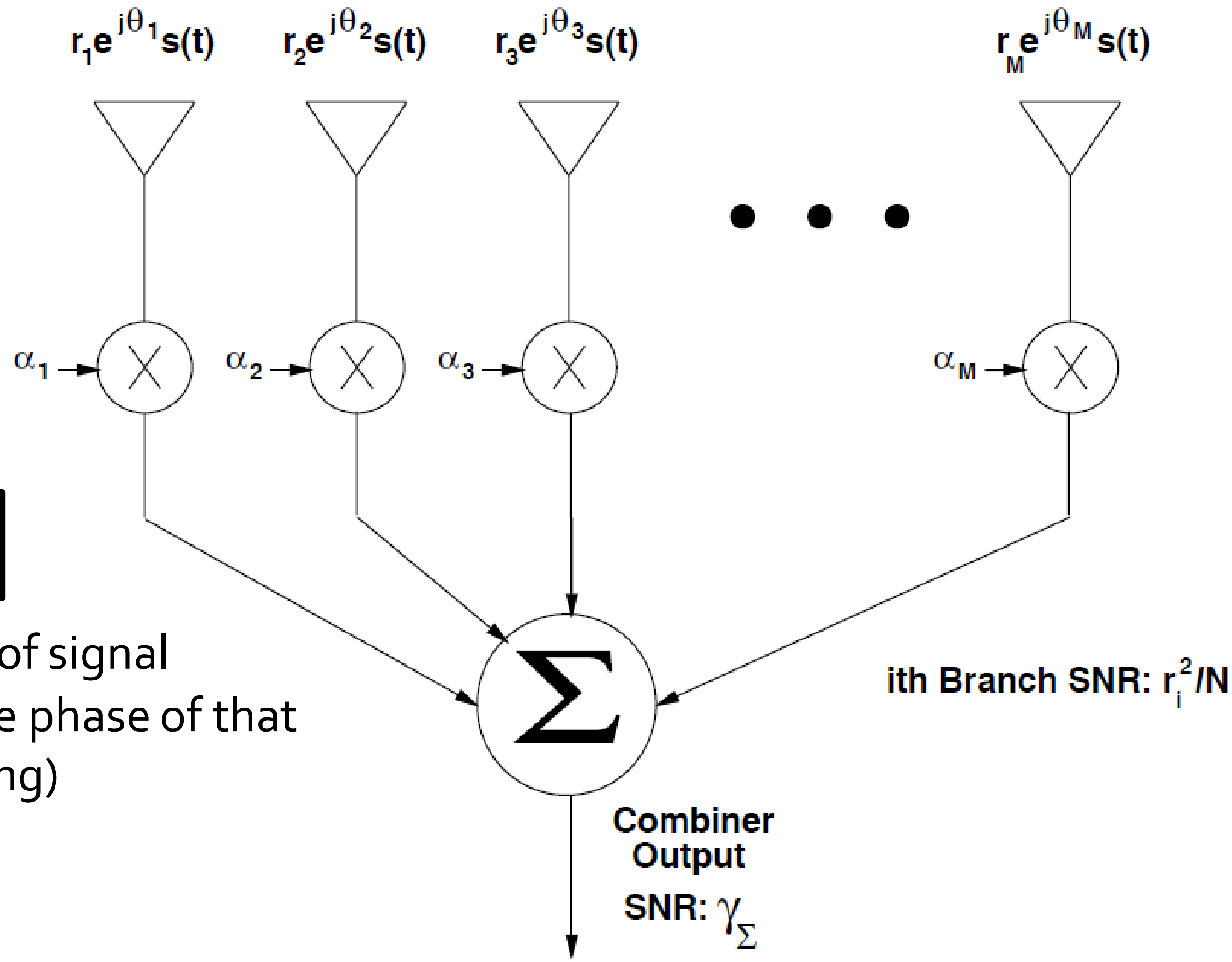
- **Transmit the same packet (or a part of it) after Δt, $\Delta t > T_c$ (coherence time). →low correlation →independent**
- **How to do this?**
  - For channel with $T_{pkt} < T_c$, coding techniques can utilize this →transmit redundant information in the same packet, separated by T_c.
  - Retransmission conceptually uses this too.



Some related terms

- **Micro-diversity:**
  to mitigate the effects of multipath fading (small-scale fading).
- **Macro-diversity:**
  to mitigate the effects of shadowing from buildings and objects (large-scale fading).
- In this lecture, we will talk about micro-diversity.
- In lab4, you can experiment with both with more emphasis on macro-diversity.

A More Formal Representation for Receiver Diversity



$$\alpha_i = a_i e^{-j\theta_i}$$

- a_i: amplification of signal
- $e^{-j\theta_i}$: remove the phase of that branch (co-phasing)

Array Gain

- **Array Gain:**
  Improvements from getting the signals from multiple antennas
- **Usually refers to the gain without fading**
- **More formally, SNR of the combined signal can be calculated as:**

$$\text{Setting } a_i = \frac{r_i}{\sqrt{N_0}}, i = 1, \ldots, M$$

$$\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{N_0 \sum_{i=1}^{M} a_i^2} = \frac{\left(\sum_{i=1}^{M} \frac{E_s}{\sqrt{N_0}}\right)^2}{N_0 \sum_{i=1}^{M} \frac{E_s}{N_0}} = \frac{\mathrm{ME_s}}{\mathrm{N_0}}$$

With fading, what is the average BER?

- **Diversity gain:**
  the performance advantage as a result of diversity combining (in fading).
- **Average BER:**

$$\overline{P_e} = \int_0^{\infty} P_e(\gamma) p_{\gamma\Sigma}(\gamma) d\gamma$$

- **Or we can express it as**

$$\overline{P_e} = c\bar{\gamma}^{-m}$$

m: the diversity order

- **When m=M (the number of branches), we say that the system achieves *full diversity order*.**

Selection Combining (SC)

- **Concept:**
  select the one branch with the best SNR and dump the rest.
- **Advantage:**
  simple, no need to do co-phasing.
- **Select the highest SNR:** $\gamma_i = \frac{r_i^2}{N_i}$.
- **In practice, SNR cannot be measured.**
  Since $N_i = N_0, \forall i$,
  we can select the branch with the highest RSSI instead: $r_i^2 + N_i$

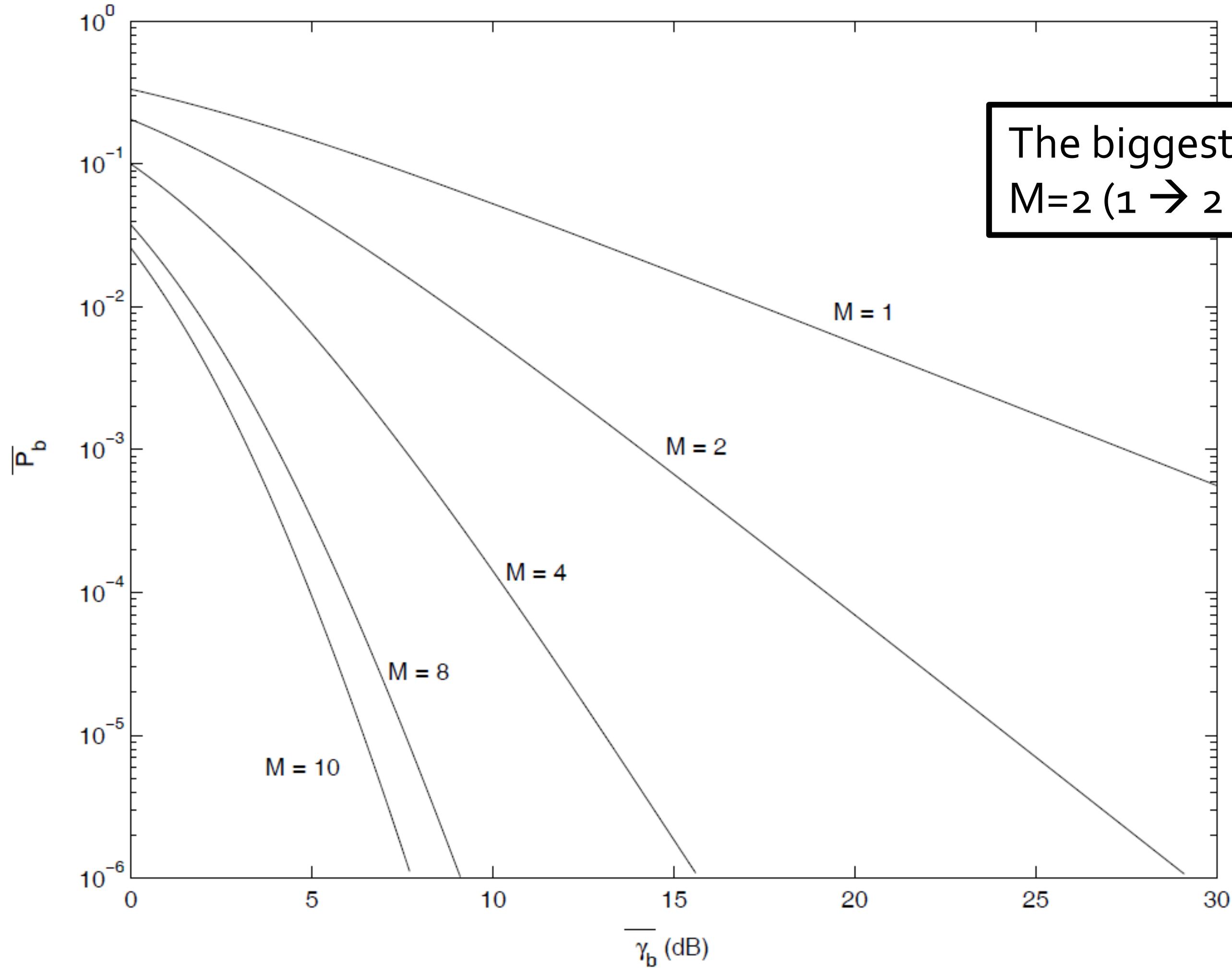
Selection Combining (SC)

- **The CDF of SNR after combining:**

$$P_{\gamma_\Sigma}(\gamma) = p(\gamma_\Sigma < \gamma)$$
$$= p(\max[\gamma_1, \gamma_2, \ldots, \gamma_M] < \gamma)$$
$$= \prod_{i=1}^{M} p(\gamma_i < \gamma)$$

- **No close form expression to obtain the average BER**
  → Use simulation to obtain the result.
- **Sometimes branch correlation is not 0**
  →the performance will degrade
  →negligible when correlation < 0.5

BER Performance: BPSK with SC in Rayleigh fading

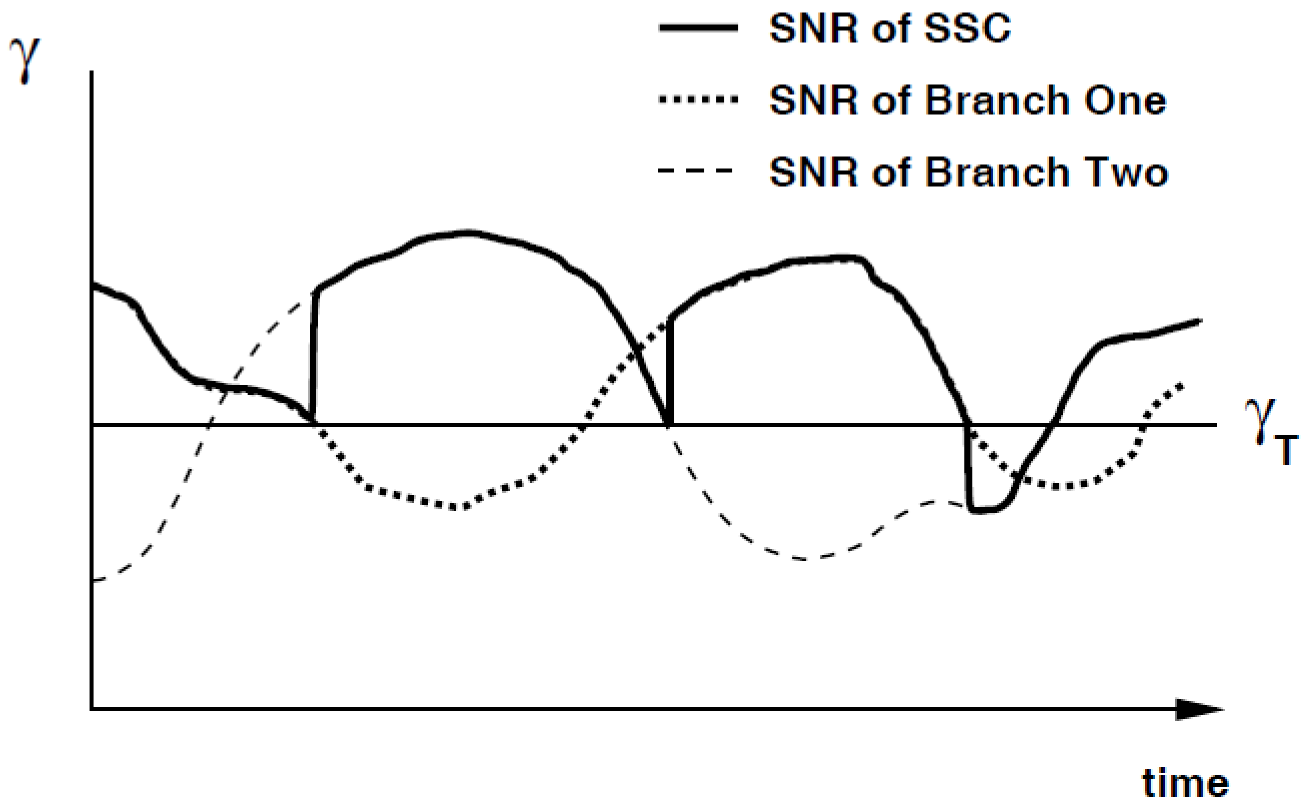


The biggest gain is from M=1 to M=2 (1 → 2 antennas)

Threshold Combining

- **Concept:**
  Use one branch and dump the rest. When this one is not good anymore (SNR drops below a threshold), randomly select another branch.
- **Advantage:**
  Even simpler, no need to monitor the SNR of all branches.
- **When there are only 2 branches, switch to the other branch when SNR is smaller than the threshold.**
  - This is called Switch-and-Stay Combining (SSC)
- **SSC has the same performance (outage probability) as SC, when setting the threshold = the minimum required SNR**

Switch-and-Stay Combining (SSC)



Maximal-Ratio Combining (MRC)

- **Concept:**
  Use all branches. We amplify the branch more when its SNR is larger.
- **Advantage:**
  Make use of all branches → best performance.
- **Question:**
  How to set a_i so that the SNR after combining is maximized?

$$\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{N_0 \sum_{i=1}^{M} a_i^2}$$

Maximal-Ratio Combining (MRC)

- **Answer:**

  $\boldsymbol{a_i^2}$ **should be proportional to the branch SNR** $\boldsymbol{\frac{r_i^2}{N_0}}$.

- **After optimization, it turns out that**

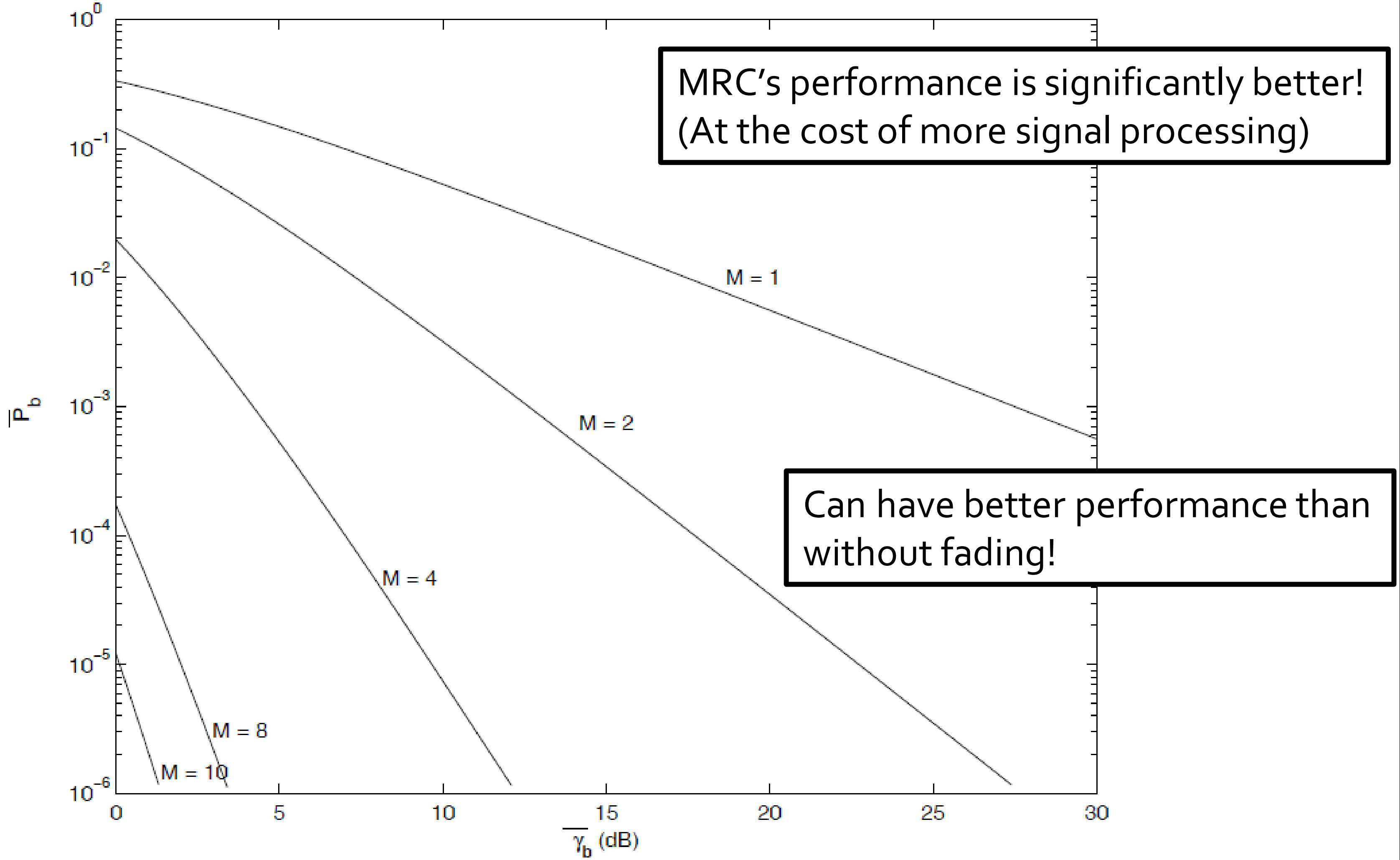
$$a_i^2 = \frac{r_i^2}{N_0}$$

- **And the SNR after combining becomes**

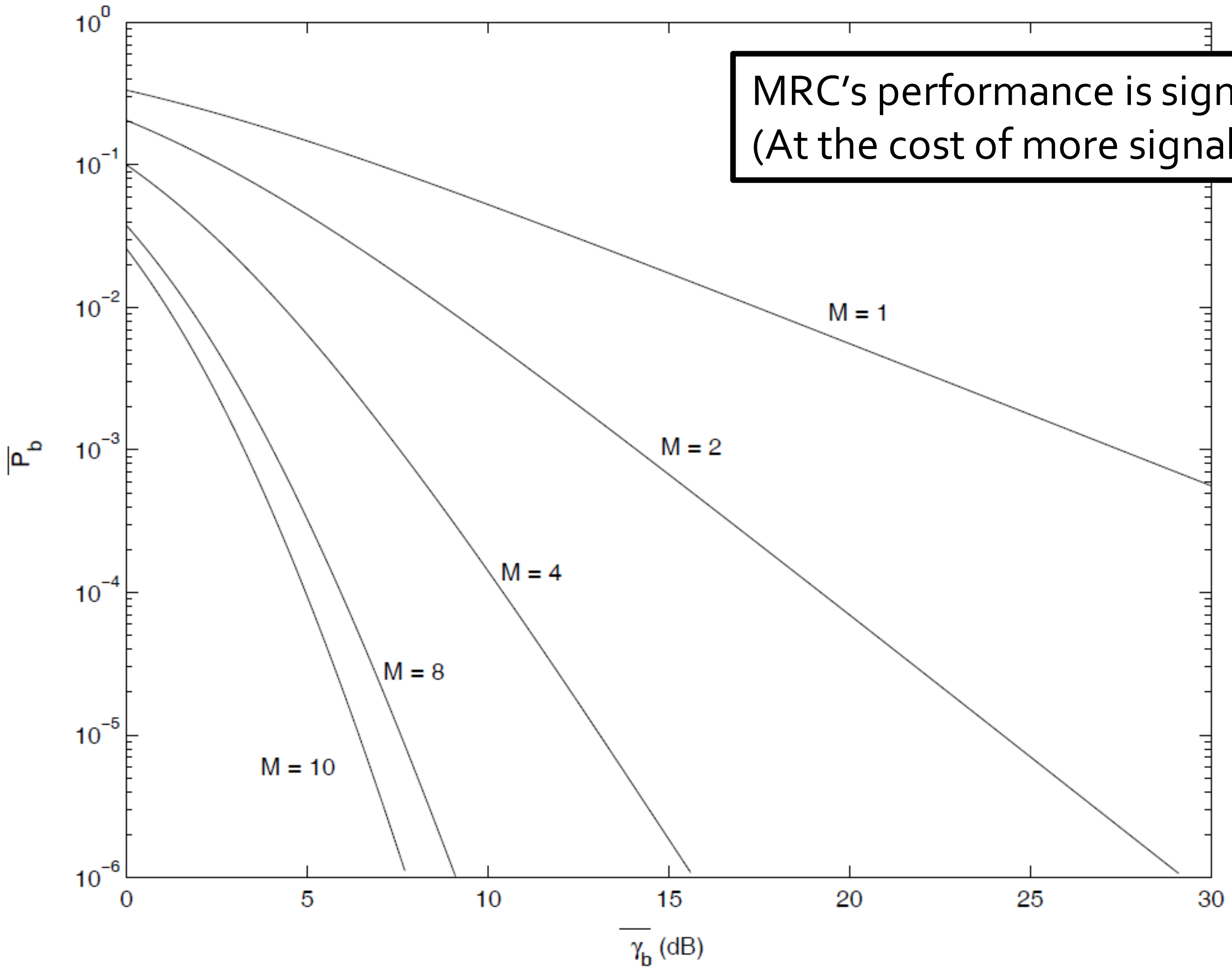
$$\gamma_\Sigma = \sum_{i=1}^{M} \frac{r_i^2}{N_0} = \sum_{(i=1)}^{M} \gamma_i$$

Note that this is linear scale, not in dB!

BER Performance: BPSK with MRC in Rayleigh fading



BER Performance: BPSK with SC in Rayleigh fading



MRC's performance is significantly better!
(At the cost of more signal processing)

Equal-Gain Combining (EGC)

- **Concept:**
  Use all branches, but combine them with equal weight=1.
- **Advantage:**
  Use the signal from all branches, but in a simpler way.
- $\boldsymbol{a_i = 1, \forall i}$.
- **The SNR after combining becomes**

$$r_{\Sigma} = \frac{1}{N_0 M}\left(\sum_{i=1}^{M} r_i\right)^2$$

- **ERC's performance is quite close to MRC, typically only has less than dB of power penalty.**