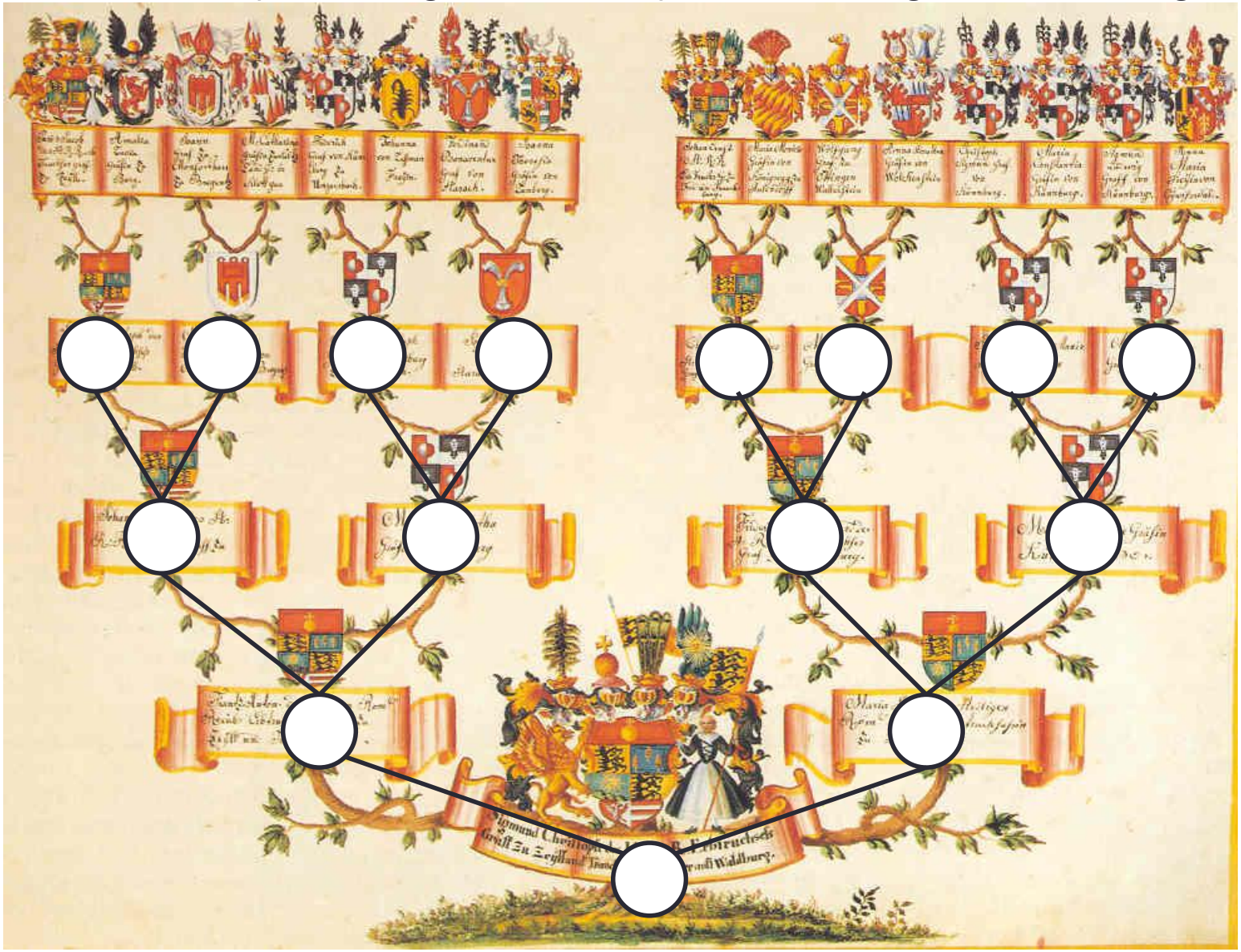
樹 1

Michael Tsai

2013/3/19

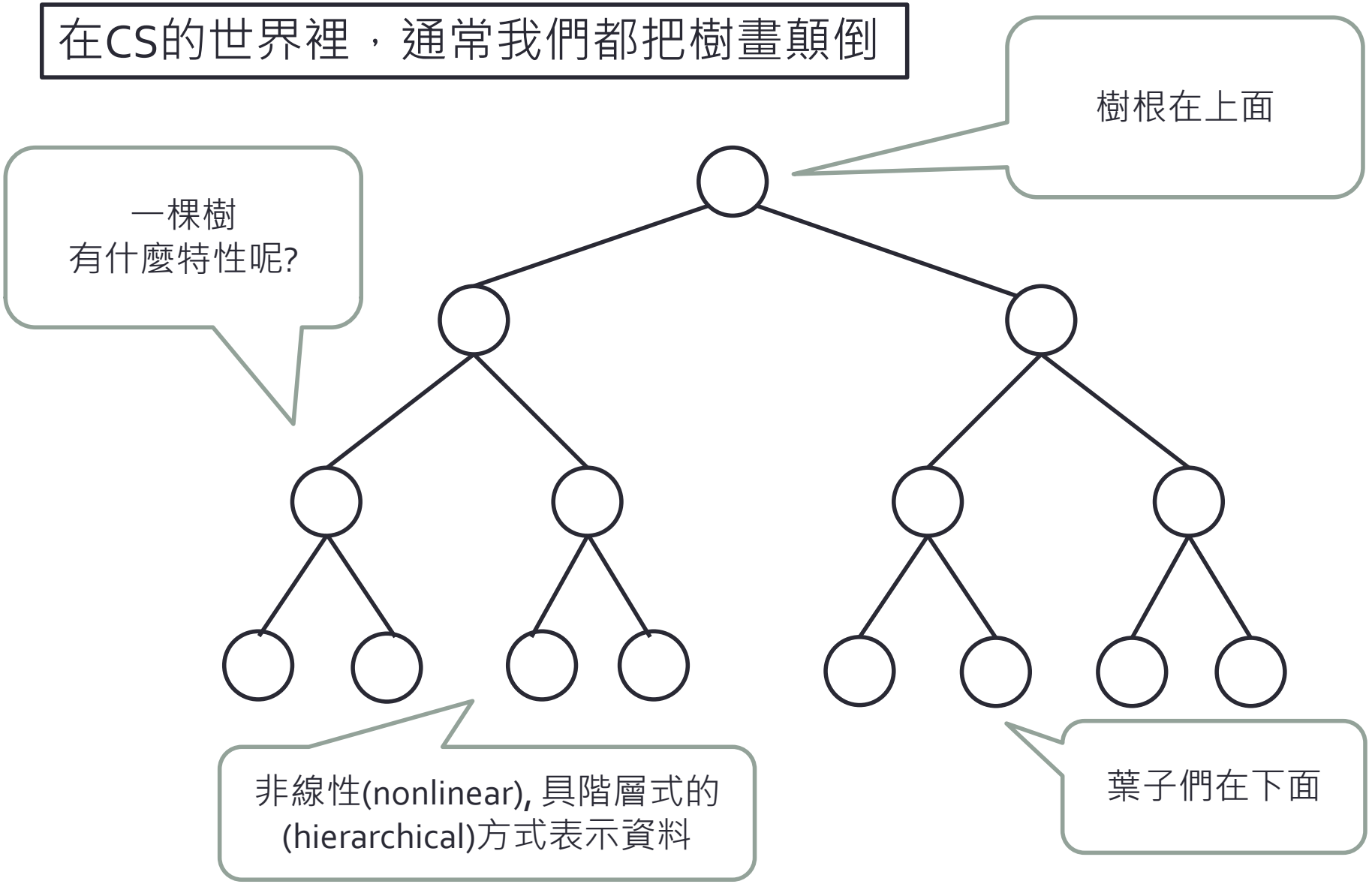
樹

The family tree of Sigmund Christoph von Waldburg-Zeil-Trauchburg



http://www.ahneninfo.com/de/ahnentafel.htm

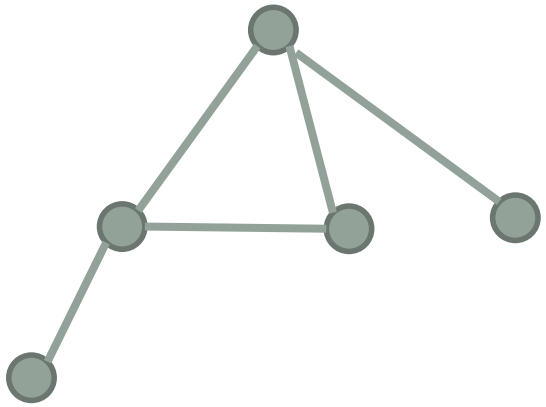
在CS的世界裡，通常我們都把樹畫顛倒



樹的定義

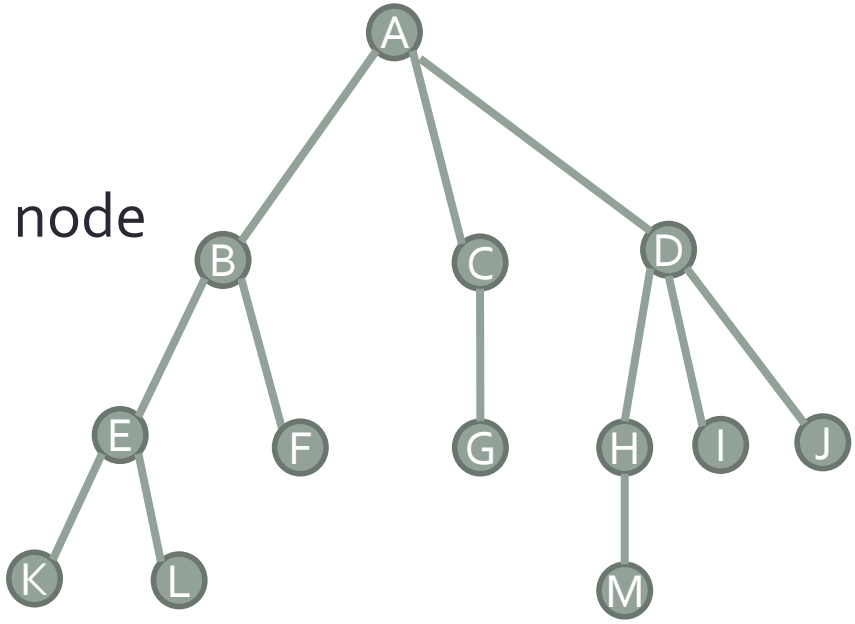
- Definition: A **tree** is a finite set of **one or more nodes** such that
- (1) There is a specially designated node called the **root**.
- (2) The remaining nodes are partitioned into $n \geq 0$ disjoint sets $T_1, T_2, \ldots, T_n$, where each of these sets is a tree.
- (3) $T_1, T_2, \ldots, T_n$ are called the **subtrees** of the root.

- 注意以上為遞迴定義
- 一個node沒有子樹的話, 是不是樹?
- 沒有node是不是樹?
- 右邊的是不是樹?
- 違反了什麼規則?



樹的字典

- **Root**
- **Node/Edge (branch)**
- **Degree (of a node):**
  The number of subtrees of a node
- **Leaf/Terminal node:**
  its degree=0
- **Parent/Children**
- **Siblings**
  they have the same parent.
- **Ancestors/Descendants**

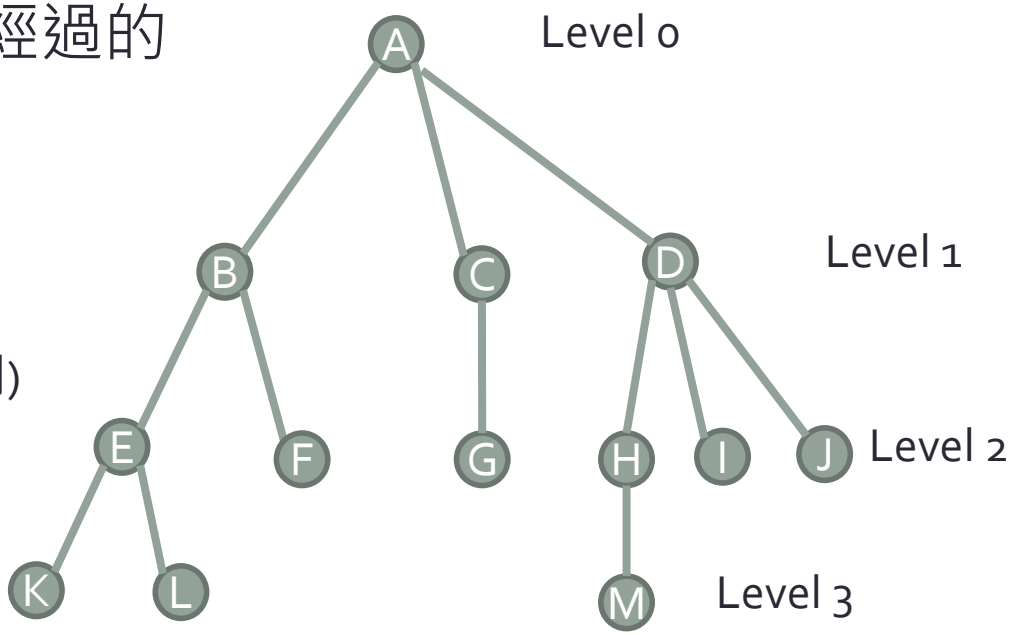


樹的字典

- **Level/depth (of a node):**
  從root走到那邊需要經過的branch數目. (root在level 0)
- **Height (of a tree):**
  tree共有幾個level.

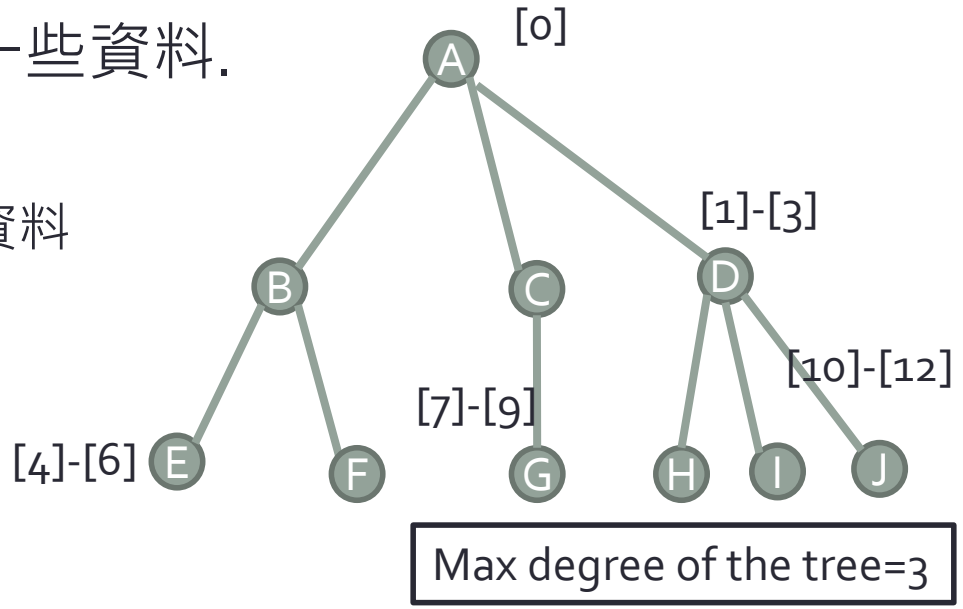
(注意Karumanchi裡面的定義略有不同)

- **Size (of a tree):**
  tree裡面總共有幾個node
- **Weight (of a tree)**:
  tree總共有幾個leaf
- **Degree (of a tree):**
  tree裡面degree最大的node的degree



怎麼在記憶體裡面記一棵樹呢? Array法

- 存什麼? 通常每個node會有一些資料.
- Array法
  - 依照level依序在array裡面儲存資料



- 在array裡面:
- 怎麼找某個node的parent?
- 怎麼找某個node的children?

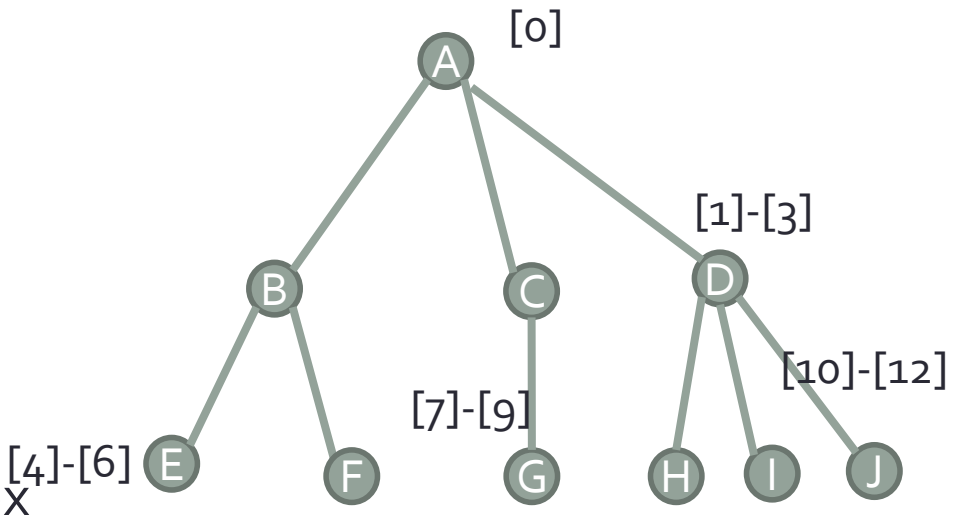
0	1	2	3	4	5	6	7	8	9	10	11	12
A	B	C	D	E	F		G			H	I	J

怎麼在記憶體裡面記一棵樹呢? Array法

- 假設degree=3
  (每個node最多有三個children)
- 怎麼找某個node的parent?
- 觀察:
  Index為i的node, 其parent之index為$\lfloor (i-1)/d \rfloor$

- 怎麼找某個node的children
- 觀察:
  Index為i的node, 其children之index為 $di+1 \sim di+d$

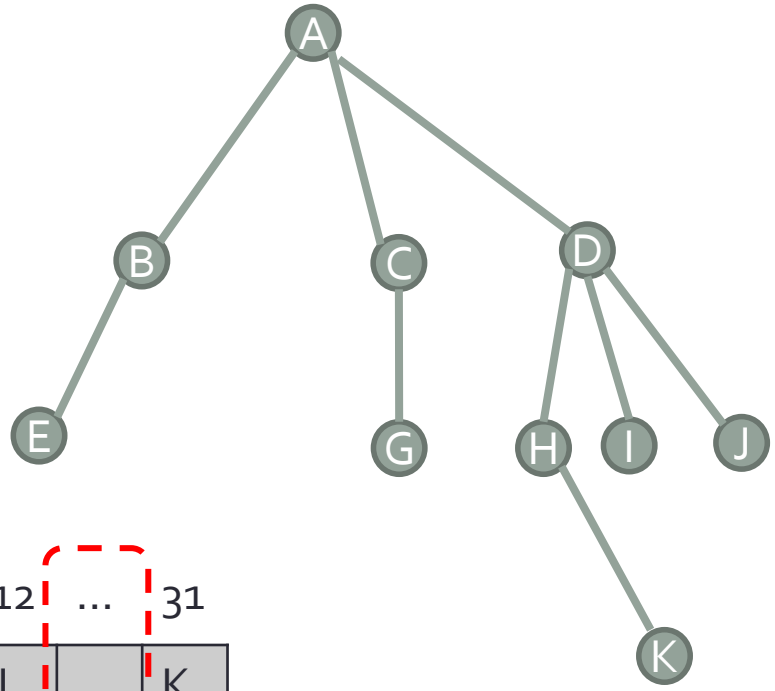
- 想想看為什麼是這樣?

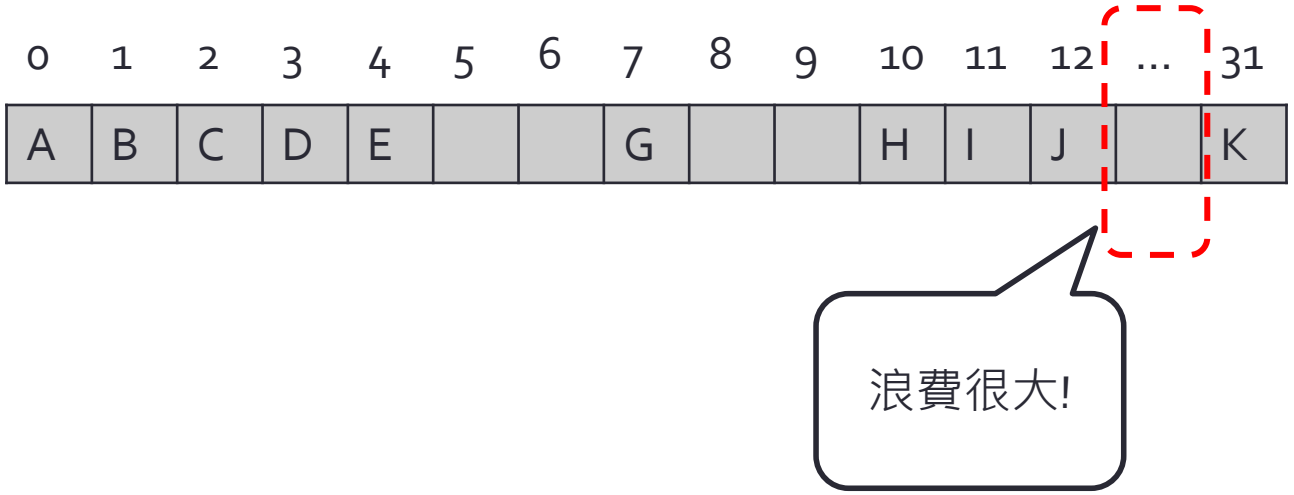


怎麼在記憶體裡面記一棵樹呢? Array法

- 壞處是什麼?

- 中間如果有許多沒有連接的地方,
- 許多node有少於d個children
- 那array中間會有很多空白



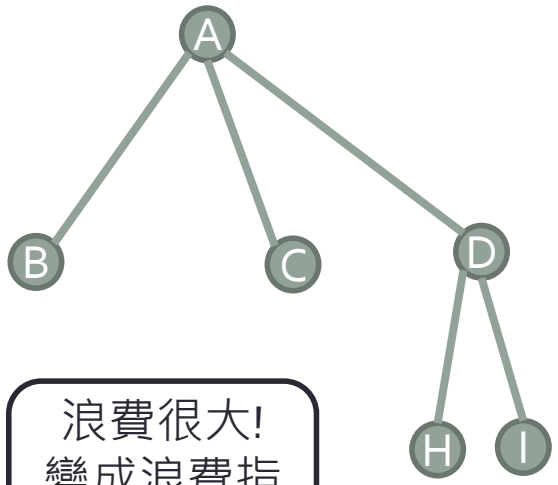


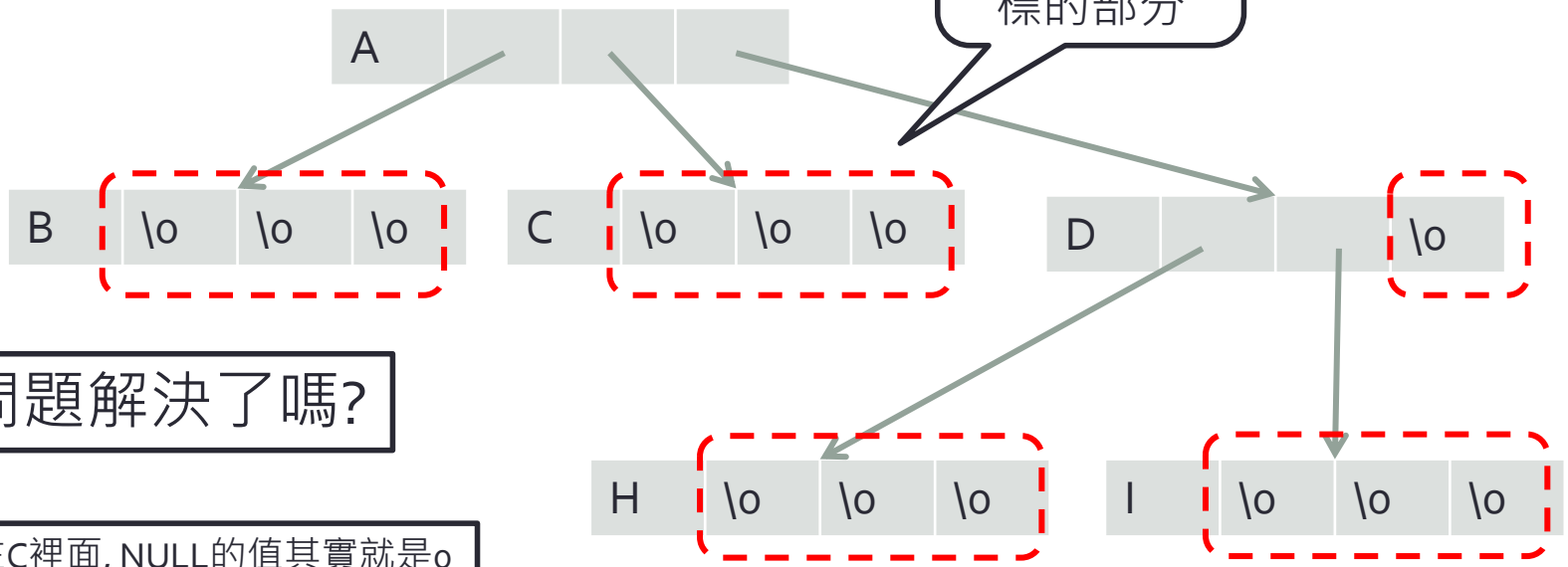
Max degree of the tree=3

怎麼在記憶體裡面記一棵樹呢?
Linked Structure法

- 假設degree=3

```
struct TreeNode{
        char data;
        struct TreeNode* child1;
        struct TreeNode* child2;
        struct TreeNode* child3;
};
```



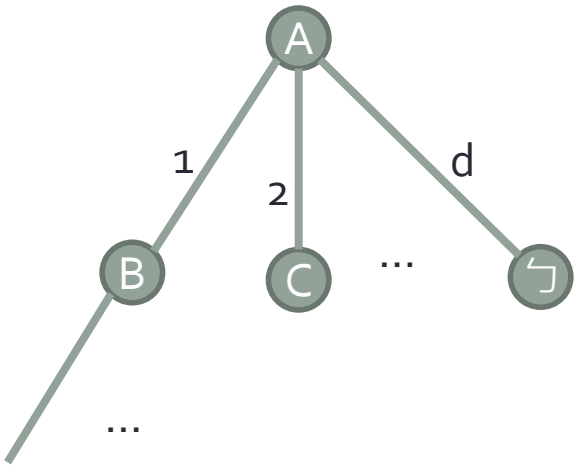


這樣問題解決了嗎?

\o代表NULL. 在C裡面, NULL的值其實就是0

怎麼在記憶體裡面記一棵樹呢?
Linked Structure法

Data	child 1	child 2	child 3	…	child k



- 假設degree of tree = d, 總共有n個nodes
- 有多少個 child欄位是NULL?
- 總共有nd個欄位
- 但是整棵樹有幾個branch?
- $n-1$個
- $nd-(n-1)=n(d-1)+1$

越小越好!

浪費很大!
變成浪費指標的部分

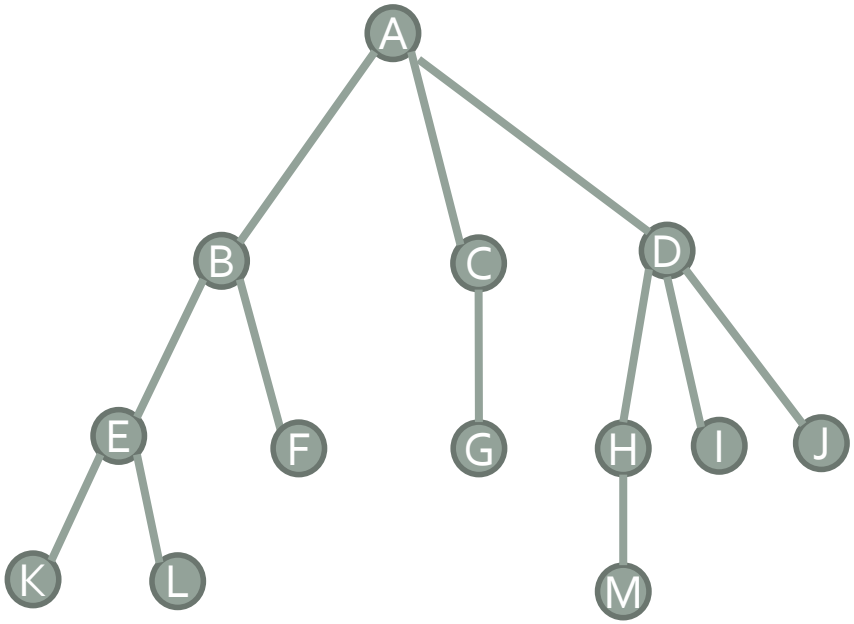
左小孩-右兄弟姊妹 表示法

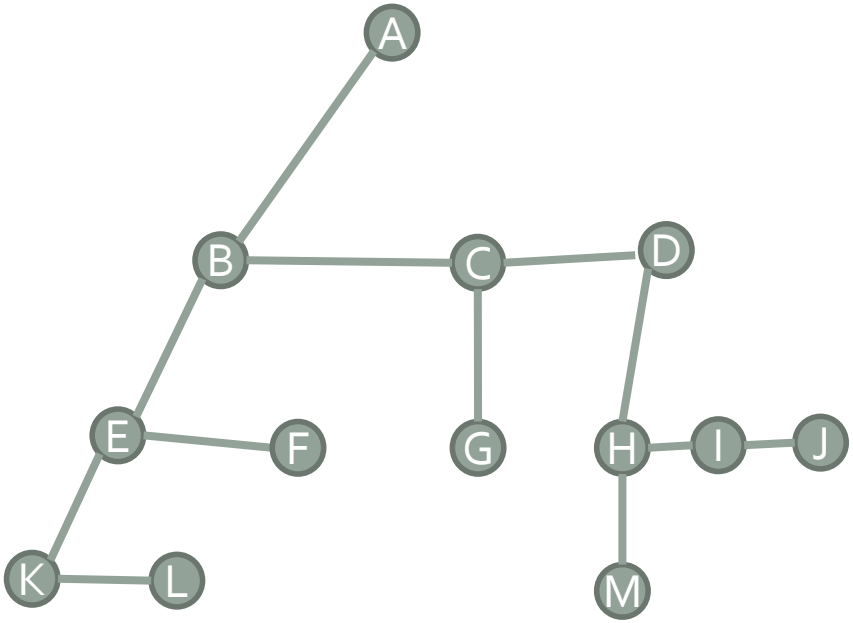
- Left child-right sibling representation

Data	left child	Right sibling

- 觀察:
- 1. 每個node最多只有一個最左邊的child (是廢話)
- 2. 每個node也最多只有一個最靠近他的右邊的sibling (也是廢話)

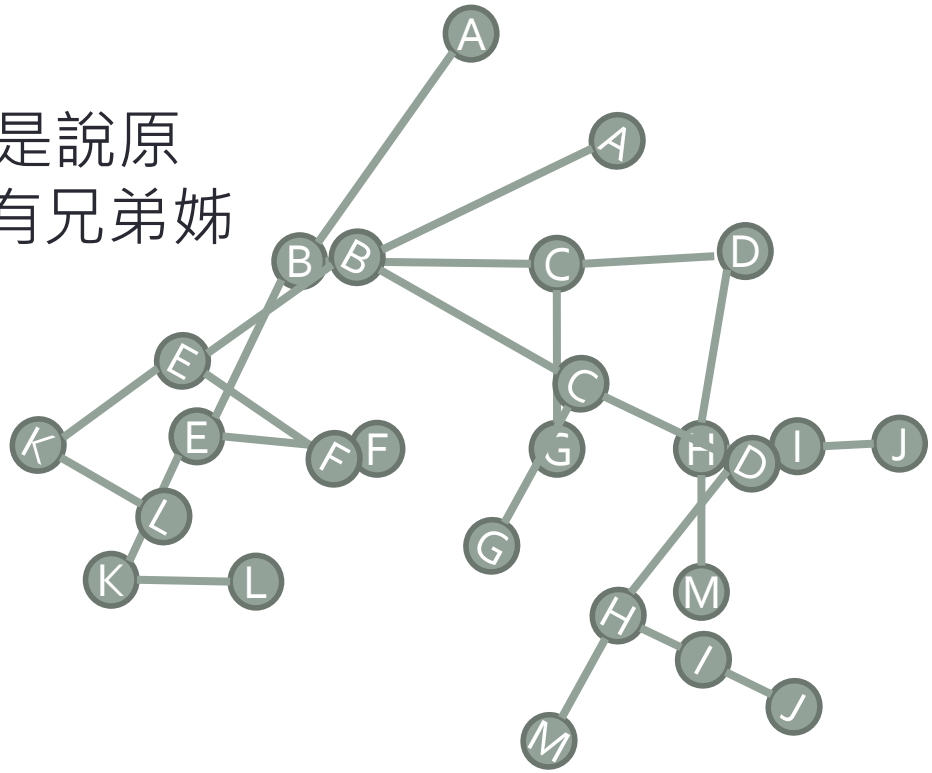
來畫一下 怎麼用LC-RS表示這棵樹?





左小孩-右兄弟姊妹 樹

- 可以變成 degree-two tree
- 也就是說, 是一種把普通的樹轉成degree-two樹的方法
- Root沒有右邊的child (也就是說原本的LC-RS樹裡面root不會有兄弟姊妹-廢話)



Binary Tree

- Definition: A **binary tree** is a finite set of nodes that is either **empty** or **consists of a root and two disjoint binary trees** called the left subtree and the right subtree.

- 注意: 可以是沒有node
- 比較: 一般tree不可以沒有node
- 注意: children在左邊或右邊是不一樣的 (有順序)
- 比較: 一般tree的children順序沒有差

一些證明

- 1. 在level i的node數目最多為$2^i, i \geq 0$ (root在level 0)
- 證明: 用歸納法
- i=0時, 為root那一層, 所以只有一個node, 也就是最多有$2^0 = 1$個node. (成立)
- 假設i=k-1的時候成立→level k-1最多有2^{k-1}個node
- 那麼i=k的時候, 最多有幾個node?
- 因為是binary tree, 所以每個node最多有兩個children
- 因此最多有$2^{k-1+1} = 2^k$node (得證)

兩些證明(誤)

- 2. 一棵height為k的binary tree, 最多有$2^k - 1$個node, $k \geq 1$.

- 證明:
- 利用1的結果
- 則總共node數目最多為
- $\sum_0^{k-1} 2^i = \frac{2^k - 1}{2-1} = 2^k - 1$. 喔耶.

三些證明(誤)

- 3. 對於任何不是空的binary tree, 假設n_0為leaf node數目, n_2為degree 2的node數目, 則$n_0 = n_2 + 1$.

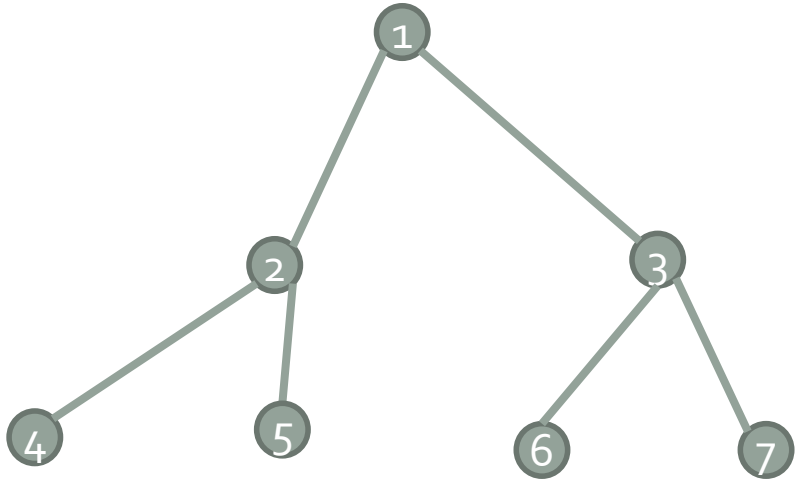
- 證明:
- 假設n為所有node數目, n_1為degree 1的node數目,
- 則$n = n_0 + n_1 + n_2$. (1)

- 假設B為branch的數目, 則$B = n_1 + 2n_2$. (2)
- 而且$n = B + 1$ (3). (只有root沒有往上連到parent的branch, 其他的node正好每個人一個)
- (2)代入(3)得 $n = n_1 + 2n_2 + 1$ (4)
- (4)減(1) 得 $n_0 = n_2 + 1$. 喔耶.

Full binary tree

- Definition: a **full binary tree** of depth k is a binary tree of depth k having $2^k - 1$ nodes, $k \geq 1$.

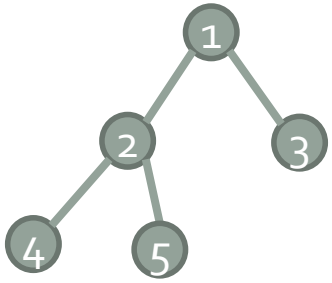
- 也就是說depth k的樹裡面最多node數目的(滿了)
- 除了leaf每個node都有兩個children



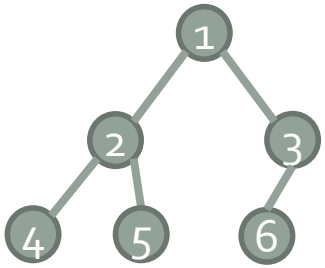
depth=3的full binary tree

Complete binary tree

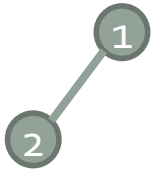
- Definition: A binary tree with n nodes and depth k is complete iff its nodes correspond to **the nodes numbered from 1 to n in the full binary tree of depth k.**
- 也就是說, 所有在level k-1和k-2(最底和倒數第二層)的leaf都沒有"缺號".



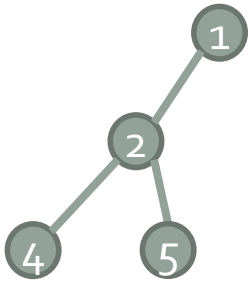
Yes



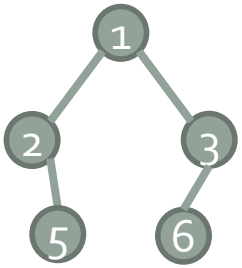
Yes



Yes



No



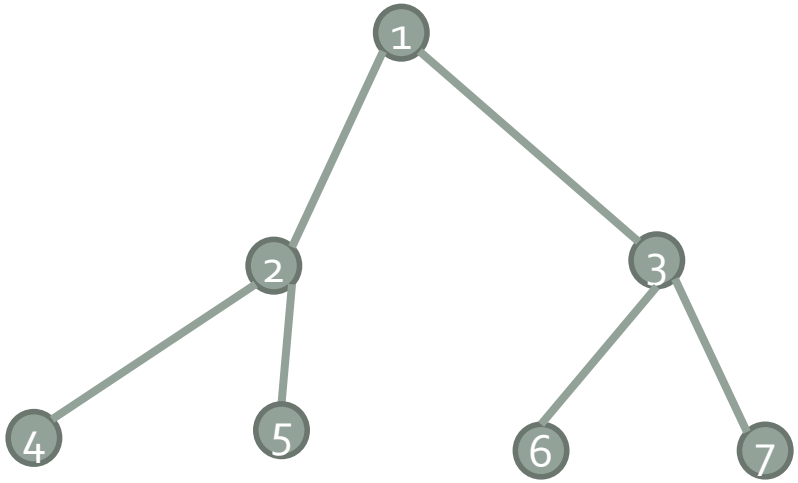
No

Complete binary tree的高度

- 如果一個complete binary tree有n個node, 那麼樹的高度為?
- Hint:高度為k 的full binary tree有$2^k - 1$個node
- Hint: 在最底層總共有多少個node?
- 答:
- 最少有幾個node?
- $2^k - 1 - 2^{k-1} + 1 = 2^{k-1}$
- 所以node個數可能為 2^{k-1}~$2^k - 1$
- $\lceil \log_2(n+1) \rceil$在各種情況都可以得到k

Binary Tree Traversal

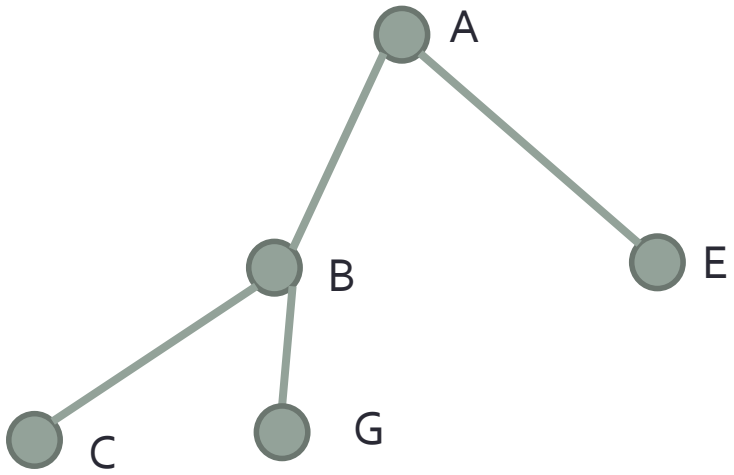
- 有一棵binary tree後, 我們要怎麼把樹的每一個node都走一遍呢?



- 到某一個node的時候, 有三件事情可以做:
- 1. 走左邊的child那邊的node們 (用L表示Left branch)
- 2. 走右邊的child那邊的node們 (用R表示Right branch)
- 3. 處理自己這個node的資料(用V表示Visit)

Binary Tree Traversal

- 如果L一定要在R之前, 那麼有三種
- VLR: preorder
- LVR: inorder
- LRV: postorder

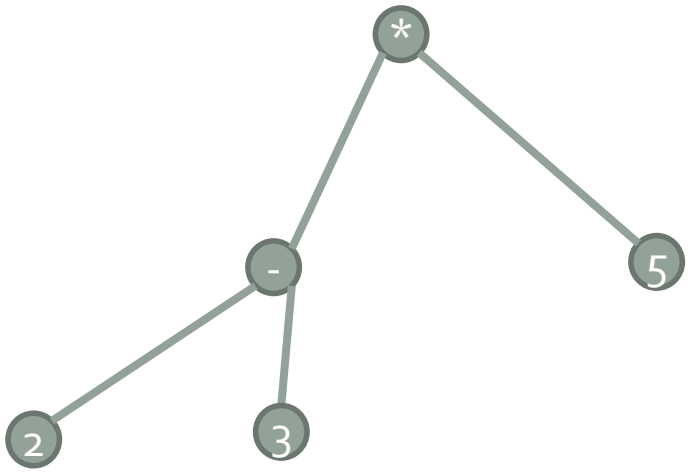


- 請同學說明preorder, inorder, postorder traversal分別順序是如何☺

Binary tree with arithmetic expression

- 每個arithmetic expression都可以建立一個expression tree

- Preorder → prefix
- Inorder → infix
- Postorder → postfix



- 請同學試試看☺
- (亂寫一個expression看看)

- Reading Assignment: Karumanchi 6.9 – How to build a expression tree?

Binary Tree Traversal

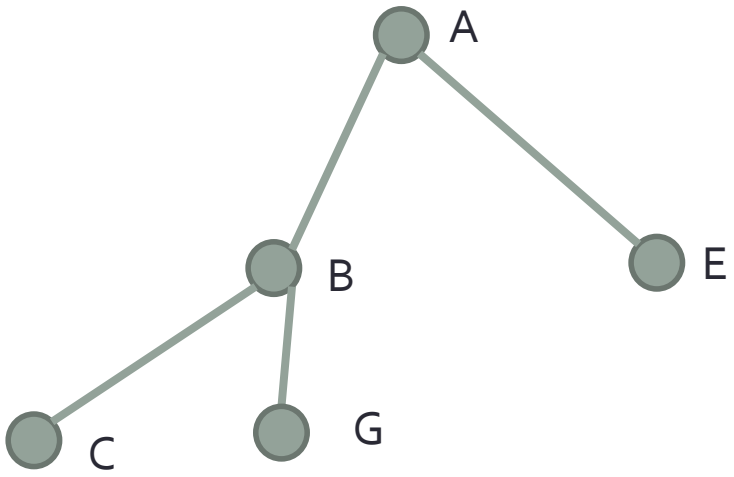
- 可以用recursive寫法來做traversal (會很簡潔):

```
void inorder(treePointer ptr) {
        inorder(ptr->leftChild);
        visit(ptr);
        inorder(ptr->rightChild);
}
```

那如果不要用recursive寫法呢?

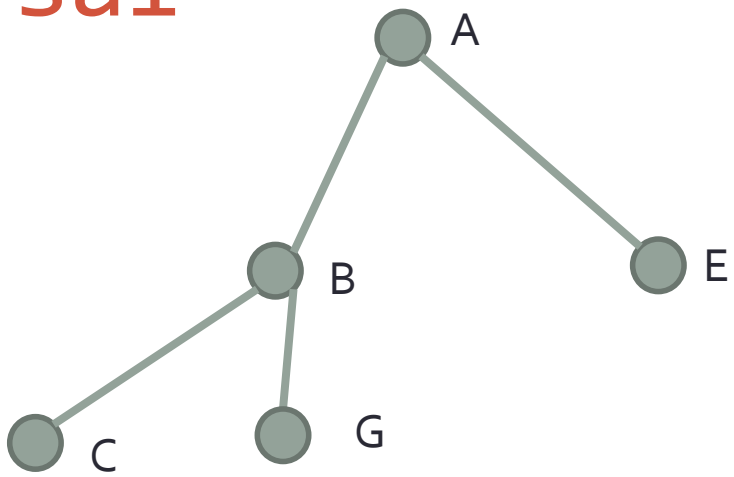
- 用Stack

```
for(;;) {
        for(;node;node=node->leftChild)
                push(node);
        node=pop();
        if (!node) break;
        printf("%s", node->data);
        node=node->rightChild;
}
```



Level-order traversal

- 如果改成用queue呢?



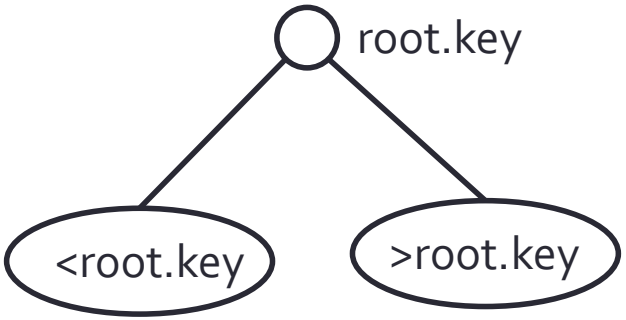

```
add(ptr);
for(;;) {
        ptr=delete();
        if (ptr) {
                printf("%s", ptr->data);
                if (ptr->leftChild)
                        add(ptr->leftChild);
                if (ptr->rightChild)
                        add(ptr->rightChild);
        } else break;
}
```

Binary search tree

- 問題: 找系上某個同學的作業成績
- 條件:
- 知道學號 (key)
- 找到學號就可以對應到儲存資料的地方(search)
- 常常有人進來(add node)
- 常常有人出去(delete node)

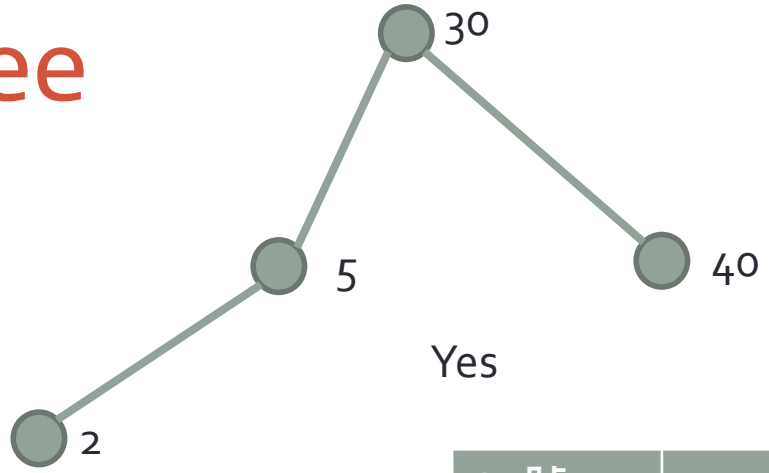
Binary search tree

- Definition: A binary search tree is a binary tree. It may be empty. If it is not empty then it satisfies the following properties:
- 1. The root has a key.
- 2. The keys (if any) in the **left** subtree are **smaller** than the key in the root
- 3. The keys (if any) in the **right** subtree are **larger** than the key in the root
- 4. The left and right subtrees are **also binary search trees**
- (隱藏) All keys are distinct.

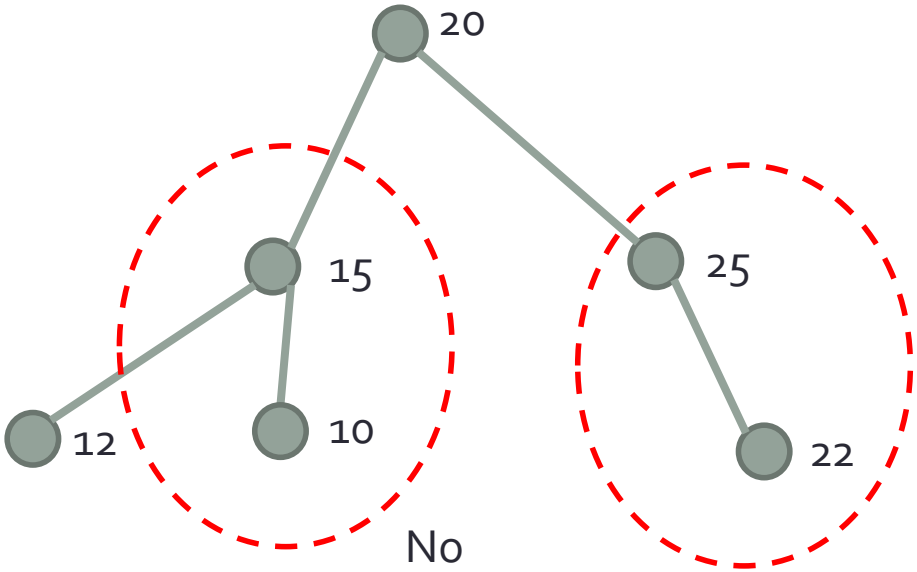


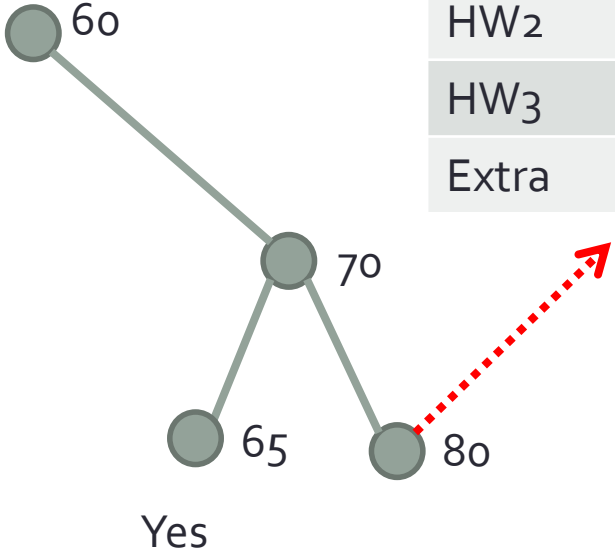
Binary search tree

- 這些是不是binary search tree?
- 找到以後要做什麼?



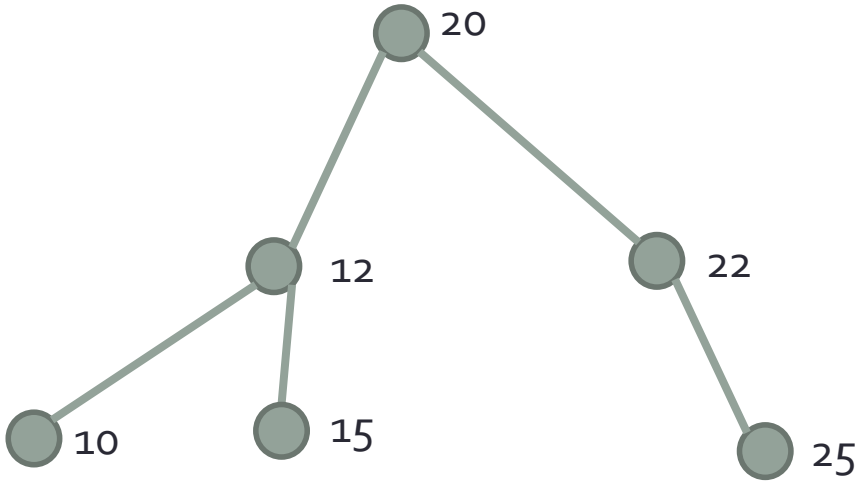
80號	
HW1	65
HW2	65
HW3	空
Extra	-20000





如何尋找?

- 假設要找10

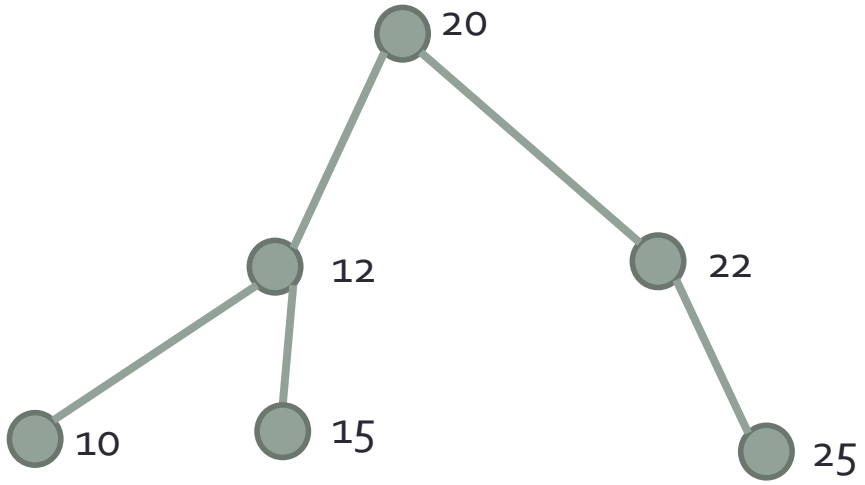



```
struct TreeNode* find(struct TreeNode* root, int data)
{
  if (root==NULL) return NULL;
  if (data==root->data) return root;
  if (data<root->data) return find(root->left,data);
  return find(root->right, data);
}
```

- 簡單. 那time complexity = O(??)
- 答: O(h), h: height of the tree.
- Worst case: $O(n)$ Average case: $O(\log_2 n)$

Binary Search Tree的Algorithm常常是以上的型態:
(1)如果key跟現在的node一樣, 那麼就做一些處理後return結果.
(2)如果key比較大或比較小, 分別call自己的分身去處理右邊或左邊的subtree

其他使用方法

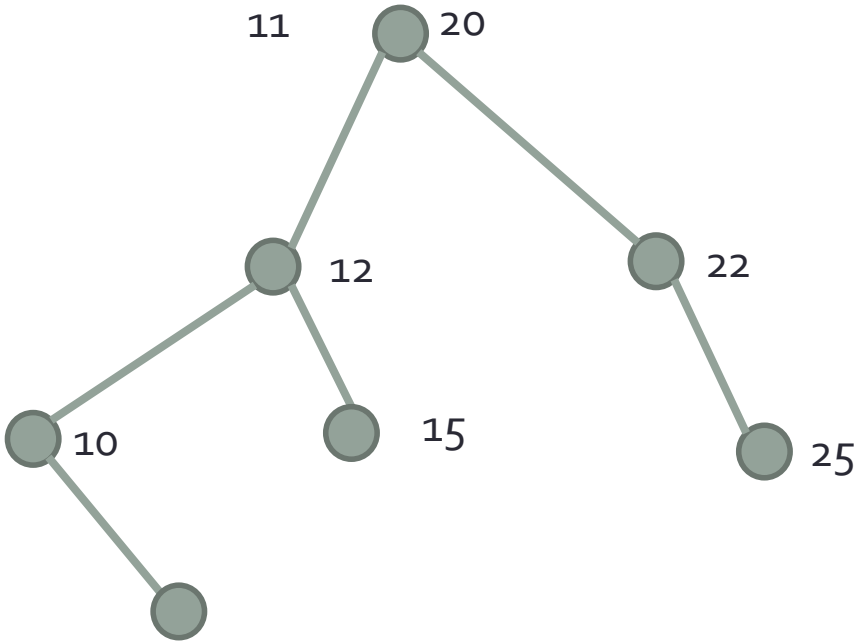


- Q: 怎麼找到Binary Search Tree裡面最大(小)的元素?
- A: 一直往右(左)邊的child走去, 直到碰到NULL為止. (Karumanchi p.149)

- Q: 怎麼把Binary Search Tree裡面所有的元素依序列出?
- A: 做Binary Search Tree的Inorder Traversal!

如何插入新的node?

- 先找有沒有一樣的 (隱藏版rule: binary search tree裡面不可以有一樣的key)
- 找不到的話, 插在最後"找不到"的位置
- 插入: **11**



```
struct BinarySearchTreeNode *Insert(struct BinarySearchTreeNode *root,
int data) {
        if (root==NULL) {
                root=(struct BinarySearchTreeNode*)malloc(sizeof(struct
BinarySearchTreeNode));
                if (root==NULL) {
                        printf("Error\n");
                        exit(-1);
                }
                root->data=data;
                root->left=NULL;
                root->right=NULL;
        }else{
                if (data<root->data)
                        root->left=Insert(root->left,data);
                else if (data>root->data)
                        root->right=Insert(root->right,data);
        }
        return root;
}
```

找到NULL表示已經到樹的leaf了, 而且沒有找到→插在這個位置

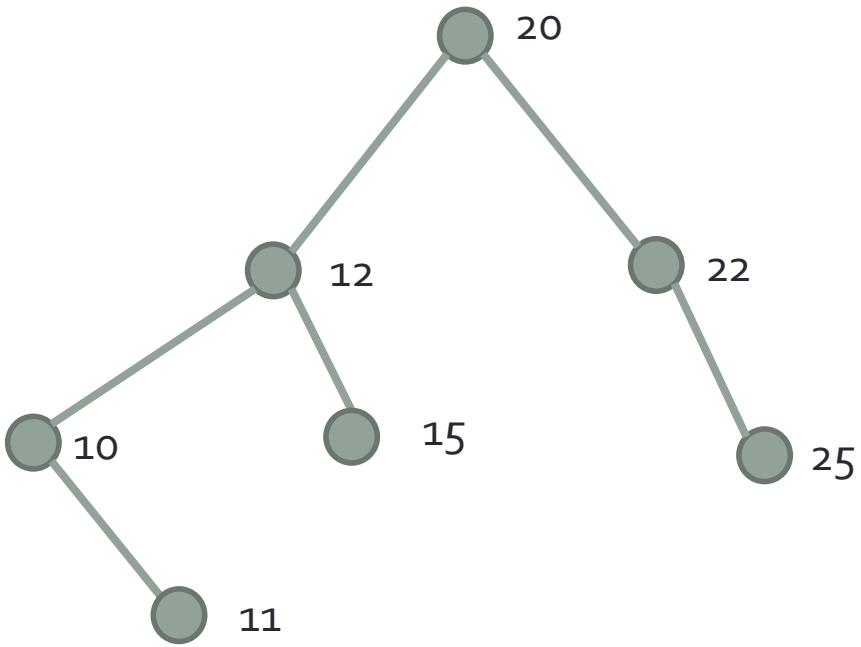
如果比較大或比較小，交給自己的分身去處理

回傳給上一層

如何刪掉一個node?

- 首先要先找到那個node
- 接著, 有各種不同情形:

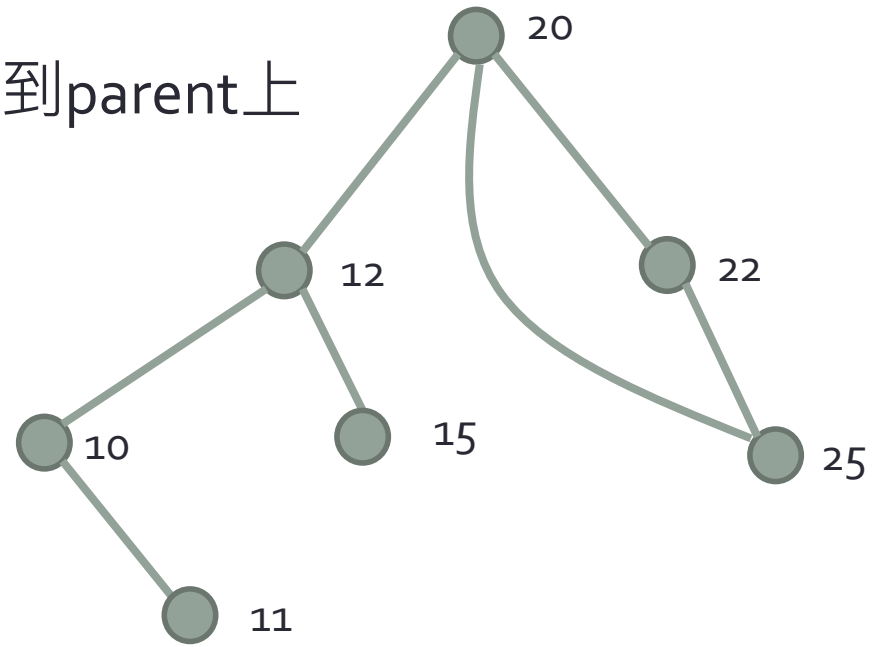
- 如果沒有手(degree=0)
- 直接拿掉



如何刪掉一個node?

- 如果只有一隻手 (degree=1)
- 則把那個唯一的child拿上來接到parent上

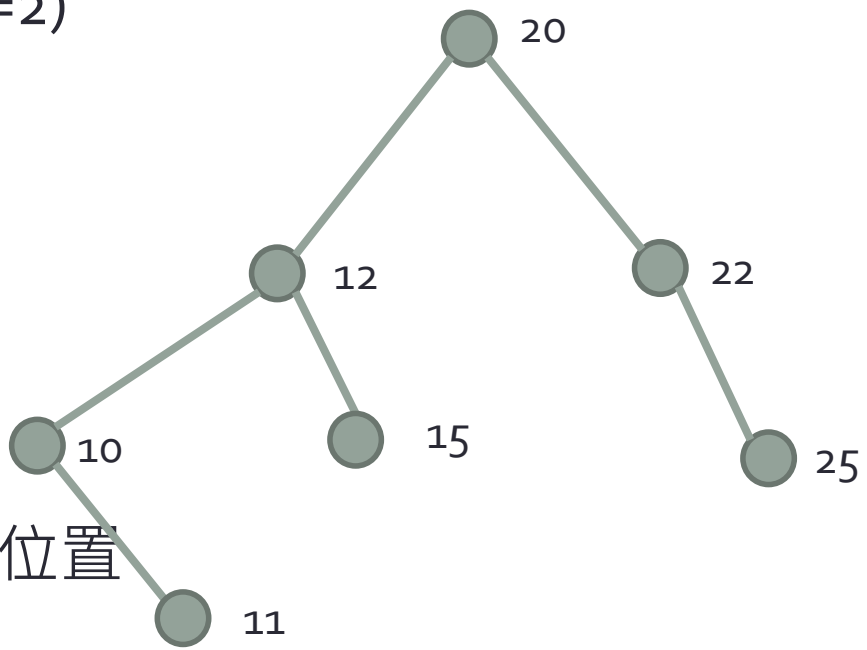
- 例如: 拿掉25



- 問題: 要怎麼接?
- 怎麼記得parent是誰?
- (回傳給return給上一層去設定, slide #37)
- (Karumanchi p.152)

如何刪掉一個node?

- 如果兩手都有東西呢? (degree=2)
- 例如刪掉12
- 找左邊child底下最大的
- (或者是右邊child底下最小的)
- 刪掉它並把它移到原本刪掉的位置
- 問題: 那如果那個最大的底下還有child呢?



答案: 最多只會有一個child, 所以很好處理

```
struct TreeNode *delete(struct TreeNode *root, int data) {
        TreeNode * temp;
        if (root==NULL) {
                printf("error\n");
                return NULL;
        } else if (data < root->data)
                root->left=delete(root->left, data);
        else if (data > root->data)
                root->right=delete(root->right, data);
        else { // data == root->data
                if (root->left && root->right) { //two children
                        temp=findmax(root->left);
                        root->data=temp->data;
                        root->left=delete(root->left,root->data);
                }else{ // one child or no child
                        temp=root;
                        if (root->left==NULL)
                                root=root->right;
                        if (root->right==NULL)
                                root=root->left;
                        free(temp);
                }
        return root;    }
```

如果比較大或比較小，交給自己的分身去處理

回傳給上一層, 如果把現在這個node殺掉的話, 則可以回傳給上一層的pointer接上

如果一樣的話，在這邊處理.

Today's Reading Assignments

本次的reading assignment重要性依序為:

1. Karumanchi 6.1-6.7
2. Karumanchi 6.11或Cormen 12.1-12.3
3. Karumanchi Problem [6-7], [10-11],14,30,31
4. Karumanchi 6.9
5. Karumanchi Problem [49-52,59]