Data Structure and Algorithm II

Homework #1

Due: 12pm, Tuesday, October 11, 2011

=== Homework submission instructions ===

- Submit the answers for writing problems (including your programming report) through the CEIBA system (electronic copy) or to the TA in R432 (hard copy). Please write down your name and school ID in the header of your documents. You also need to submit your programming assignment (problem 1) to the Judgegirl System(`http://katrina.csie.ntu.edu.tw/judgegirl/`).
- Each student may only choose to submit the homework in one way; either all as hard copies or all through CEIBA except the programming assignment. If you submit your homework partially in one way and partially in the other way, you might only get the score of the part submitted as hard copies or the part submitted through CEIBA (the part that the TA chooses).
- If you choose to submit the answers of the writing problems through CEIBA, please combine the answers of all writing problems into only one file in the doc/docx or pdf format, with the file name in the format of "hw1_[student ID].{pdf,docx,doc}" (e.g. "hw1_b99902010.pdf"); otherwise, you might only get the score of one of the files (the one that the TA chooses).

Problem 1. (30%) Given a set of n ($0 \le n \le 10000$) points $P = \{p_1, p_2, \ldots, p_n\}$ in a 2-dimensional space, where each point p_i can we represented with a 2-dimensional coordiante (x_i, y_i) and $-15000 \le x_i, y_i \le 15000$, for $i = 1$ to n. Please derive and implement an algorithm based on the divide-and-conquer strategy to determine the closest pair of points. Submit your program to the judgegirl system.

Please also submit a report in which you give a clear description of your algorithm. Write down the recurrence which represents the running time of your algorithm and solve the recurrence in the report too. Out of the 30 points of this problem, 10 points are for the report and 20 points are for the correctness of the 10 test cases on the judgegirl system.

Input: $n+1$ lines. n, the number of points in P, is in the first line. In the next n lines, each has x_i y_i to represent the coordinate of point p_i, for $i = 1, 2, ..., n$. x_i and y_i are separated by a space character (' ').

Output: 1 line. Return $\|u - v\|^2$, the square of the distance between u and v, where (u,v) is the closest pair of points in P.

Example:

Input:

5

0 2

6 67

43 71

39 107

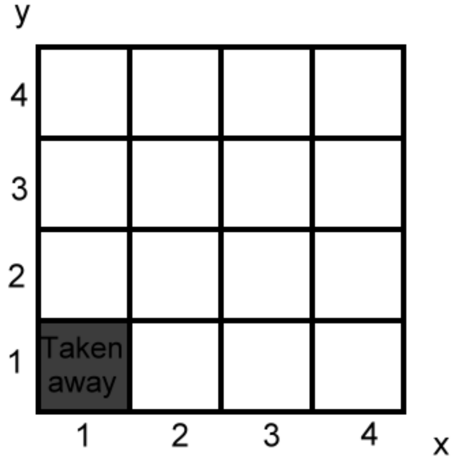
189 140

Output:

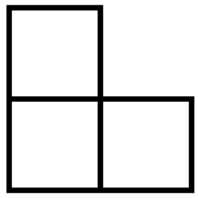
1312

Problem 2. (20%) Assume that n is a positive integer which is a power of two. We have a chessboard which is of size n x n with a randomly selected single tile taken away (see Figure 1(a)). You are asked to cover the entire chessboard with the piece shown in Figure 1(b). For example, see Figure 1(c). Note that the piece can be rotated. All the tiles on the chessboard except the one taken way have to be covered, and no tiles can be left outside of the chessboard. In this problem, we ask you to design an algorithm to solve this problem with the divide-and-conquer strategy. Analyze the running time of your algorithm by using the recurrence.

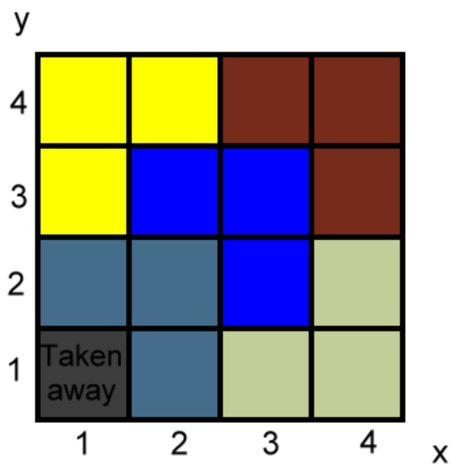
Problem 3. (16%) Consider the multiselection problem: given a set S of n elements and a set K of r ranks $k_1, k_2, ..., k_r$, find the k_1th, k_2th, ..., k_rth smallest elements. For example, if $K = \{2, 7, 9, 50\}$, the problem is to find the 2nd, 7th, 9th, and 50th smallest elements. This problem can be solved trivially in $\Theta(rn)$ by using the selection algorithm we talked about in the class (run the algorithm for r times, once for each rank k_j, $1 \leq j \leq r$. Give an $O(n \log r)$ time algorithm to solve this problem. Prove that your algorithm indeed has that running time.



(a) The chessboard with 1 tile taken away



(b) The piece to be filled in the chessboard



(c) The chessboard covered by the pieces

Figure 1: The chessboard tiling problem

Problem 4. Solve the following problems on the textbook:

1. (5%) 4.4-6 on p.93

2. (5%) 4.4-8 on p.93

3. (4%) Use the master theorem to solve the recurrence in 4-1 (c,d,e,f) on p.107.

Problem 5. (20%) Solve Problem 4-6 on p.110-111 of the textbook.