2011 Fall WNFA
Homework 1 solutions

Problem 1. 20%

(a) Mean excess delay $E[\tau] = \frac{1\times0+1\times50+0.1\times75+0.01\times100}{1+1+0.1+0.01} = 27.73(ns)$

$$E[\tau^2] = \frac{1\times0^2+1\times50^2+0.1\times75^2+0.01\times100^2}{1+1+0.1+0.01} = 1498.82(ns^2)$$

RMS delay spread $\sigma_\tau = \sqrt{1498.82-(27.73)^2} = 27.02(ns)$

90% conference bandwidth = $\frac{1}{50\sigma_\tau} = 740(KHz)$

50% conference bandwidth = $\frac{1}{5\sigma_\tau} = 7.4(MHz)$

(b) Mean excess delay $E[\tau] = \frac{1\times10+0.1\times5+0.01\times0}{1+0.1+0.01} = 9.46(\mu s)$

$$E[\tau^2] = \frac{1\times10^2+0.1\times55^2+0.01\times0^2}{1+0.1+0.01} = 92.34(\mu s^2)$$

RMS delay spread $\sigma_\tau = \sqrt{92.34-(9.46)^2} = 1.69(\mu s)$

90% conference bandwidth = $\frac{1}{50\sigma_\tau} = 1.18\times10^4(Hz) = 11.8(KHz)$

50% conference bandwidth = $\frac{1}{5\sigma_\tau} = 118.3(KHz)$

Problem 2. 25%

(2-1)

$\because P_r = P_t K\left[\frac{d_0}{d}\right]^r$

$\therefore\ P_{r(dB)} - P_{t(dB)} = (10log_{10}K + r10log_{10}d_0) - 10log_{10}d = \mathbb{K} - 10log_{10}d$

MMSE error equation:

$$F(Y,\mathbb{K}) = \sum_{i=1}^{6}[M_{measured}(d_i) - M_{model}(d_i)]^2$$

Using matlab's cure fitting tool to minimize MMSE

$\mathbb{K}$ = -29.72

r = 4.04

Adding the log-normal shadowing model:

The sample variance relative to the simplified path loss model with $\mathbb{K}$ = -29.72 and r = 4.04 is

$$\sigma_{\psi dB}{}^2 = \frac{1}{6}\sum_{i=1}^{6}[M_{measured}(d_i) - M_{model}(d_i)]^2 = 9.296$$

$\sigma_{\psi dB} = 3.05(db)$=> Standard deviation for the log-normal shadowing model

(2-2)

$P_{r(dB)} - P_{t(dB)} = -29.72 - 4.04 \times 10log_{10}(2000) = -163.08(dB)$

(2-3)

Outage Probability:

$$\int_{-\infty}^{-10} \frac{1}{\sqrt{2\pi\sigma_{\psi dB}{}^2}} exp\left(\frac{-x^2}{2\sigma_{\psi dB}{}^2}\right) dx = 1 - \int_{\frac{10}{.035}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 1 - Q\left(\frac{-10}{3.05}\right) = 0.00052$$

Problem 3. 40%

Depends on individual student's answer.

Problem 4. 15%

$$\overline{P_r}(R) = P_{min} + 20dB = P_t - K - 10 \times 6log_{10}(R)$$

$$60log_{10}(R) = P_t - KP_{min} - 20dB$$

$$a = \frac{-20}{8} = -2.5, b = \frac{10 \times 6 \times 0.434}{8} = 3.255$$

By Matlab

$$C = 0.9990$$