Chapter 3 Binary Machine Vision: Region Analysis

3.1 Introduction

region properties: measurement vector, input to classifier

3.2 Thresholding

regions: produced by connected components labeling operator
region intensity histogram: gray level values for all pixels
mean gray level value: summary statistics of region's intensity

bounding rectangle: smallest rectangle circumscribes the region
area:

$$A = \sum_{(r,c) \in R} 1$$

centroid:

$$\bar{r} = \frac{1}{A} \sum_{(r,c) \in R} r$$

$$\bar{c} = \frac{1}{A} \sum_{(r,c) \in R} c$$

border pixel: has some neighboring pixel outside the region
P_4: 4-connected perimeter: if 8-connectivity for inside and outside
P_8: 8-connected perimeter: if 4-connectivity for inside and outside

$$P_4 = \{(r,c) \in R | N_8(r,c) - R \neq \emptyset\}$$

$$P_8 = \{(r,c) \in R | N_4(r,c) - R \neq \emptyset\}$$

e.g. center is in P_4 but not in P_8 for

$r\backslash c$	0	1	2
0		*	
1	*	*	*
2		*	

e.g. $P_4 = \{(1,0),(1,1),(2,1),(1,1),(1,2),(1,1),(0,1),(1,1)\}$
e.g. $P_8 = \{(1,0),(2,1),(1,2),(0,1)\}$

length of perimeter $P =< (r_0, c_0), ..., (r_{K-1}, c_{K-1}) >$, successive pixels, neighbors

$$|P| = \#\{k|(r_{k+1}, c_{k+1}) \in N_4(r_k, c_k)\}$$

$$+\sqrt{2}\#\{k|(r_{k+1}, c_{k+1}) \in N_8(r_k, c_k) - N_4(r_k, c_k)\}$$

where $k+1$ is computed modulo K i.e. $(r_0, c_0) = (r_K, c_K)$
e.g. $|P_4| = 8$, $|P_8| = 4\sqrt{2}$

mean distance μ_R from the centroid to the shape boundary

$$\mu_R = \frac{1}{K} \sum_{k=0}^{K-1} \|(r_k, c_k) - (\bar{r}, \bar{c})\|$$

e.g. $\mu_R(P_4) = 4/8 = 0.5$, $\mu_R(P_8) = 1$
standard deviation σ_R of distances from centroid to boundary

$$\sigma_R^2 = \frac{1}{K} \sum_{k=0}^{K-1} [\|(r_k, c_k) - (\bar{r}, \bar{c})\| - \mu_R]^2$$

e.g. $\sigma_R(P_4) = 0.5$, $\sigma_R(P_8) = 0$
Haralick shows that μ_R/σ_R has properties:

1. digital shape $\to$ circular, μ_R/σ_R increases monotonically
2. μ_R/σ_R similar for similar digital/continuous shapes
3. orientation (rotation) and area (scale) independent

=====marriage=====

Average gray level (intensity)

$$\mu = \frac{1}{A} \sum_{(r,c) \in R} I(r, c)$$

Gray level (intensity) variance

$$\sigma^2 = \frac{1}{A} \sum_{(r,c)\in R} [I(r,c) - \mu]^2 = [\frac{1}{A} \sum_{(r,c)\in R} I(r,c)^2] - \mu^2$$

right hand equation lets us compute variance with only one pass

microtexture properties: function of co-occurrence matrix

S: set of pixels in designated spatial relationship e.g. 4-neighbors

co-occurrence matrix P

$$P(g_1, g_2) = \frac{\#\{[(r_1, c_1), (r_2, c_2)] \in S | I(r_1, c_1) = g_1 \text{ and } I(r_2, c_2) = g_2\}}{\#S}$$

texture second moment (Haralick, Shanmugam, and Dinstein, 1973)

$$M = \sum_{g_1, g_2} P^2(g_1, g_2)$$

texture entropy

$$E = -\sum_{g_1, g_2} P(g_1, g_2) \log P(g_1, g_2)$$

texture correlation

$$\rho = \sum_{g_1, g_2} (g_1 - \mu)(g_2 - \mu) P(g_1, g_2) / \sigma^2$$

where

$$\mu = \frac{1}{2} [\sum_{g_1} \sum_{g_2} g_1 P(g_1, g_2) + \sum_{g_1} \sum_{g_2} g_2 P(g_1, g_2)]$$

$$\sigma^2 = \frac{1}{2}[\sum_{g_1}\sum_{g_2}(g_1-\mu)^2 P(g_1,g_2) + \sum_{g_1}\sum_{g_2}(g_2-\mu)^2 P(g_1,g_2)]$$

texture contrast

$$C = \sum_{g_1}\sum_{g_2} |g_1 - g_2| P(g_1, g_2)$$

texture homogeneity

$$H = \sum_{g_1}\sum_{g_2} \frac{P(g_1, g_2)}{k + |g_1 - g_2|}$$

where k is some small constant

=====New 3:10=====

3.2.1 Extremal Points

eight distinct extremal pixels: topmost right, rightmost top, rightmost bottom, bottommost right, bottommost left, leftmost bottom, leftmost top, topmost left

=====Fig. 3.1=====

different extremal points may be coincident

=====Fig. 3.2=====

association of the name of the eight extremal points with their coordinates

=====Table 3.1=====

$$r_1 = r_2 = rmin$$

$$c_1 = \min\{c|(rmin, c) \in R\}$$
$$c_2 = \max\{c|(rmin, c) \in R\}$$
$$r_3 = \min\{r|(r, cmax) \in R\}$$
$$r_4 = \max\{r|(r, cmax) \in R\}$$
$$c_3 = c_4 = cmax$$
$$r_5 = r_6 = rmax$$
$$c_5 = \max\{c|(rmax, c) \in R\}$$
$$c_6 = \min\{c|(rmax, c) \in R\}$$
$$r_7 = \max\{r|(r, cmin) \in R\}$$
$$r_8 = \min\{r|(r, cmin) \in R\}$$
$$c_7 = c_8 = cmin$$

association of the name of an external coordinate with its definition

=====Table 3.2=====

extremal points occur in opposite pairs: topmost left ↔ bottommost right, topmost right ↔ bottommost left, rightmost top ↔ leftmost bottom, rightmost bottom ↔ leftmost top

each opposite extremal point pair: defines an axis

axis properties: length, orientation

the length covered by two pixels horizontally adjacent

- 1: distance between pixel centers
- 2: from left edge of left pixel to right edge of right pixel

distance calculation: add a small increment to the Euclidean distance
M_1: axis between (r_1, c_1) and (r_5, c_5)
M_2: axis between (r_2, c_2) and (r_6, c_6)
M_3: axis between (r_3, c_3) and (r_7, c_7)
M_4: axis between (r_4, c_4) and (r_8, c_8)

$$M_1 = \sqrt{(r_1 - r_5)^2 + (c_1 - c_5)^2} + Q(\phi_1)$$
$$M_2 = \sqrt{(r_2 - r_6)^2 + (c_2 - c_6)^2} + Q(\phi_2)$$
$$M_3 = \sqrt{(r_3 - r_7)^2 + (c_3 - c_7)^2} + Q(\phi_3)$$
$$M_4 = \sqrt{(r_4 - r_8)^2 + (c_4 - c_8)^2} + Q(\phi_4)$$

orientation taken counterclockwise w.r.t. column (horizontal) axis
=====Fig. 3.3=====

$$\phi_1 = \tan^{-1} \frac{r_1 - r_5}{-(c_1 - c_5)}$$
$$\phi_2 = \tan^{-1} \frac{r_2 - r_6}{-(c_2 - c_6)}$$

$$\phi_3 = \tan^{-1} \frac{r_3 - r_7}{-(c_3 - c_7)}$$
$$\phi_4 = \tan^{-1} \frac{r_4 - r_8}{-(c_4 - c_8)}$$

orientation convention for the axes

=====Fig. 3.4=====

$$Q(\theta) = \begin{cases} \frac{1}{|\cos\theta|} & \text{if } |\theta| < 45^\circ \\ \frac{1}{|\sin\theta|} & \text{if } |\theta| > 45^\circ \end{cases}$$

length going from left edge of left pixel to right edge of right pixel

=====Fig. 3.5=====

axes paired: M_1 with M_3 and M_2 with M_4

calculation of the axis length and orientation of a linelike shape

=====Fig. 3.6=====

distance between ith and jth extremal point

$$M_{ij} = \sqrt{(r_i - r_j)^2 + (c_i - c_j)^2} + 1.12$$

average value of $Q(\theta) = 1.12$, largest error $0.294 = \sqrt{2} - 1.12$

calculations for length of sides, base, and altitude for a triangle

=====Fig. 3.7=====

geometry of the tilted rectangle

=====Fig. 3.8=====
calculation for the orientation of an example rectangle
=====Fig. 3.9=====
axes and their mates that arise from octagonal-shaped regions
=====Fig. 3.10=====

3.2.2 Spatial Moments

second-order row moment

$$\mu_{rr} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})^2$$

second-order mixed moment

$$\mu_{rc} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})(c - \bar{c})$$

second-order column moment

$$\mu_{cc} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})^2$$

3.2.3 Mixed Spatial Gray Level Moments

region properties: position, extent, shape, gray level properties second-order mixed gray level spatial moments

$$\mu_{rg} = \frac{1}{A} \sum_{(r,c)\in R} (r - \bar{r})[I(r,c) - \mu]$$

$$\mu_{cg} = \frac{1}{A} \sum_{(r,c)\in R} (c - \bar{c})[I(r,c) - \mu]$$

=====Example 3.1=====
connected components labeling of the image in Fig. 2.2
=====Fig. 3.11=====
all the properties measured from each of the regions
=====Table 3.3=====
=====joke=====

3.3 Signature Properties

vertical projection

$$P_V(c) = \#\{r | (r,c) \in R\}$$

horizontal projection

$$P_H(r) = \#\{c | (r,c) \in R\}$$

diagonal projection from lower left to upper right

$$P_D(d) = \#\{(r,c) \in R | r + c = d\}$$

diagonal projection from upper left to lower right

$$P_E(e) = \#\{(r,c) \in R | r - c = e\}$$

projections: easily obtainable in pipeline hardware
compute properties from projections
area

$$A = \sum_{(r,c)\in R} 1 = \sum_r \sum_{\{c|(r,c)\in R\}} 1 = \sum_r P_H(r)$$

rmin: top row of bounding rectangle

$$rmin = \min\{r|(r,c) \in R\} = \min\{r|P_H(r) \neq 0\}$$

rmax: bottom row of bounding rectangle

$$rmax = \max\{r|(r,c) \in R\} = \max\{r|P_H(r) \neq 0\}$$

cmin: leftmost column of bounding rectangle

$$cmin = \min\{c|(r,c) \in R\} = \min\{c|P_V(c) \neq 0\}$$

cmax: rightmost column of bounding rectangle

$$cmax = \max\{c|(r,c) \in R\} = \max\{c|P_V(c) \neq 0\}$$

row centroid

$$\bar{r} = \frac{1}{A} \sum_{(r,c)\in R} r = \frac{1}{A} \sum_r \sum_{\{c|(r,c)\in R\}} r$$

$$= \frac{1}{A}\sum_r r \sum_{\{c|(r,c)\in R\}} 1 = \frac{1}{A}\sum_r rP_H(r)$$

column centroid

$$\bar{c} = \frac{1}{A}\sum_{(r,c)\in R} c = \frac{1}{A}\sum_c \sum_{\{r|(r,c)\in R\}} c$$

$$= \frac{1}{A}\sum_c c \sum_{\{r|(r,c)\in R\}} 1 = \frac{1}{A}\sum_c cP_V(c)$$

diagonal centroid

$$\bar{d} = \frac{1}{A}\sum_d dP_D(d)$$

another diagonal centroid

$$\bar{e} = \frac{1}{A}\sum_e eP_E(e)$$

diagonal centroid related to row and column centroid

$$\bar{d} = \bar{r} + \bar{c}$$

$$\bar{e} = \bar{r} - \bar{c}$$

second row moment from horizontal projection

$$\mu_{rr} = \frac{1}{A}\sum_r (r - \bar{r})^2 P_H(r)$$

second column moment from vertical projection

$$\mu_{cc} = \frac{1}{A} \sum_c (c - \bar{c})^2 P_V(c)$$

second diagonal moment

$$\mu_{dd} = \frac{1}{A} \sum_d (d - \bar{d})^2 P_D(d)$$

second diagonal moment related to $\mu_{rc}, \mu_{rr}, \mu_{cc}$

$$\mu_{dd} = \mu_{rr} + 2\mu_{rc} + \mu_{cc}$$

second mixed moment can be obtained from projection

$$\mu_{rc} = \frac{\mu_{dd} - \mu_{rr} - \mu_{cc}}{2}$$

second diagonal moment related to $\mu_{rc}, \mu_{rr}, \mu_{cc}$

$$\mu_{ee} = \mu_{rr} - 2\mu_{rc} + \mu_{cc}$$

second mixed moment can be obtained from projection

$$\mu_{rc} = \frac{\mu_{rr} + \mu_{cc} - \mu_{ee}}{2}$$

mixed moment μ_{rc} obtained directly from μ_{dd} and μ_{ee}

$$\mu_{rc} = \frac{\mu_{dd} - \mu_{ee}}{4}$$

3.3.1 Using Signature Analysis to Determine the Center and Orientation of a Rectangle

signature analysis: important because of easy, fast implementation

surface mount device (SMD) placement: position and orientation of parts

determine center $(\Delta x, \Delta y)$ of rectangle by corner location (a_1, b_1),
side lengths w, h, orientation angle θ

$$\begin{pmatrix} x_{rot} \\ y_{rot} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x_{new} \\ y_{new} \end{pmatrix} = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \begin{pmatrix} a_1 + \frac{w}{2}\cos\theta - \frac{h}{2}\sin\theta \\ b_1 - \frac{w}{2}\sin\theta - \frac{h}{2}\cos\theta \end{pmatrix}$$

geometry for determining the translation of the center of a rectangle

=====Fig. 3.12=====

partition rectangle into six regions formed by two vertical lines
a known distance g apart and one horizontal line

=====Fig. 3.14=====

$$\Delta x = \frac{(C+D)-(A+B)}{2h}\cos\theta$$

$$\Delta y = \frac{(A+E+C)-(B+F+D)}{2w}\cos\theta$$

where rotation angle

$$\cos\theta = \frac{hg}{E+F}$$

3.3.2 Using Signature Analysis to Determine the Center of a Circle

partition the circle into four quadrants formed by two orthogonal lines which meet inside the circle

geometry for the circle, its center, and a chord

=====Fig. 3.15=====

$A + B = \pi r^2$

$r = \sqrt{\frac{A+B}{\pi}}$

$A = \frac{r^2}{2}[2\theta - \sin 2\theta]$

$\frac{2\pi A}{A+B} = 2\theta - \sin 2\theta$

$d = \sqrt{\frac{A+B}{\pi}}\cos\theta$

circle projected onto the four quadrants of the projection index image

=====Fig. 3.16=====

each quadrant area from histogram of the masked projection

$$|\Delta y| = \sqrt{\frac{A+B+C+D}{\pi}} \cos\theta_y$$

Δy positive if $A+B > C+D$, negative otherwise, where

$$\frac{2\pi(A+B)}{A+B+C+D} = 2\theta_y - \sin 2\theta_y$$

Δx positive if $B+D > A+C$, negative otherwise

$$|\Delta x| = \sqrt{\frac{A+B+C+D}{\pi}} \cos\theta_x$$

$$\frac{2\pi(B+D)}{A+B+C+D} = 2\theta_x - \sin 2\theta_x$$

3.4 Summary

region properties from connected components or signature analysis

R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, Addison Wesley, Reading, MA, 1992

4.2.2 Histogram Processing

Histogram Equalization p. 173-180
pixel transformation

$$s = T(r)$$

r, s: original, new intensity, T: transformation

- $T(r)$ single-valued, monotonically increasing
- $0 \leq T(r) \leq 255$ for $0 \leq r \leq 255$

=====Fig. 4.10=====
histogram equalization, histogram linearization

$$s_k = 255 \sum_{j=0}^{k} \frac{n_j}{n}$$

$k = 0, 1, ..., 255$, n_j: number of pixels with intensity j
n: total number of pixels
for every pixel if $I(im, i, j) = k$ then $I(imhe, i, j) = s_k$
=====Fig. 4.13=====
=====Garfield 17:3=====

Project due Oct. 26:
Write a program to do histogram equalization