Shape from Shading Using Near Point Light Sources*

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1 The *Approximate Solution Curve Tracing* (*ASCT*) Method

1.1 The *correct solution space (CSS)*

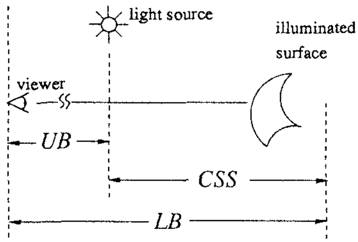

Figure 1: The *CSS* represents the *correct solution space* which all of the depth values on the illuminated object are included in. *LB* is the lower bound of the *CSS*. All of the depth values on the illuminated object will be not "deeper" than it. *UB* is the upper bound of the *CSS*. All of the depth values on the illuminated object are always "deeper" than it.

Before recovering the depth map of the illuminated object, we can assume that there exists a range to include all of the depth values of the points on the surface, as shown in Figure 1.

1.2 The *approximate solution difference* (*ASD*)

Now suppose we have two images I_1, I_2, the corresponding point light sources are (S_{x1}, S_{y1}, S_{z1}) and (S_{x2}, S_{y2}, S_{z2}), the illuminated point is (x, y, z), and the normal vector is $(p, q, -1)$. Then,

$$I_1(x,y) = k \cdot \frac{(S_{x1}-x)p + (S_{y1}-y)q - (S_{z1}-z)}{[(S_{x1}-x)^2 + (S_{y1}-y)^2 + (S_{z1}-z)^2]^{\frac{3}{2}}(p^2+q^2+1)^{\frac{1}{2}}} \quad (1)$$

$$I_2(x,y) = k \cdot \frac{(S_{x2}-x)p + (S_{y2}-y)q - (S_{z2}-z)}{[(S_{x2}-x)^2 + (S_{y2}-y)^2 + (S_{z2}-z)^2]^{\frac{3}{2}}(p^2+q^2+1)^{\frac{1}{2}}} \quad (2)$$

Substitute a value in the *CSS*, called z_{css}, for the unknown variable z in the denominator of both equations and divide Equation (1) by Equation (2). After simplifying the coefficients, a linear equation (Equation (3)) with three unknowns is obtained. These unknown values are p, q, and z.

$$A\,p + B\,q + C\,z = D \quad (3)$$

where

$$\begin{aligned} A &= (S_{x1}-x) - \alpha(S_{x2}-x) \\ B &= (S_{y1}-y) - \alpha(S_{y2}-y) \\ C &= 1-\alpha \\ D &= S_{z1} - \alpha S_{z2} \\ \alpha &= \frac{I_1 \cdot [(S_{x1}-x)^2 + (S_{y1}-y)^2 + (S_{z1}-z_{css})^2]^{\frac{3}{2}}}{I_2 \cdot [(S_{x2}-x)^2 + (S_{y2}-y)^2 + (S_{z2}-z_{css})^2]^{\frac{3}{2}}} \\ z_{css} &= \text{a value in the } CSS \end{aligned}$$

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