Shape Information from Shading Using Photometric Method

Sheng-Liang Kao (高聖亮) and Chiou-Shann Fuh (傅楸善)
Department of Computer Science and Information Engineering
National Taiwan University
Taipei, Taiwan
Republic of China

Abstract[1]

In this paper, a linear algorithm is proposed to recover shape information from multiple images, each of them are taken under the environment that all of the object surfaces are illuminated by a known point light source. Furthermore, for these known point light sources, the distance between any two of them are very samll when it is compared with the distance from the light source to any point on the object surface. This makes the linearization of the image intensity equation possible. At the end of this paper, some implementation results are presented.

1. Introduction

The image formation of a three-dimensional object mainly depends on its shape, its reflectance properties, and the distribution of the light sources. Therefore, in order to recover the original shape of a 3-D object, it is necessary to find out firstly those parameters that affect the format of images, such as the position of the point light source, the incident radiance and direction, the material of the object surface (e.g. Lambertian surface), the orientations of the object surfaces. After these parameters are recovered, the shape information such as the depth map can then be found out.

Shape from shading is one of the methods that use different brightness distributing on the image to discover the orientations of the object surfaces and use these orientations to approximate the depth map.

[1] This work was supported by the National Science Council of Taiwan, Republic of China under Grants NSC83-0422-E002-010 and NSC84-0422-E002-006

2. Background

Distant light source

In the previous work [1], the variety of irradiance on the object has been derived. Use the irradinace equation combined with the reflectance map, the image intensity equation has also been constructed. Given an image $I(x,y)$ and a reflectance map $R(p,q)$, the image intensity equation is

$$I(x,y) = \rho\, s_0\, R(p,q)$$

where ρ is the reflectance factor (i.e. albedo), s_0 is the incident radiance.

Consider that a Lambertian surface which appears to be of equal radiance from all viewing directions is illuminated by an ideal point light source. Then the reflectance map is

$$R(p,q) = \cos\theta_i = \frac{\mathbf{s}\cdot\mathbf{n}}{\|\mathbf{s}\|\,\|\mathbf{n}\|} \tag{1}$$

where θ_i is the angle between the surface normal vector $\mathbf{n}$ and the incident light source direction $\mathbf{s}$. Under the assumption that the single point light source is distant from the object and the direction ss is known, the above equation can be used to derive the orientation of the object surface from a single or multiple images. There are two main approaches for this, one is the partial differential equation method, the other is the photometric stereo method. Horn [1] was the first one undertaking the former study and subsequent improvements. Woolham [2] and Ikeuchi used the photometric method to determine the surface orientation.

Near light source

Equation (1) is valid under the assumption that the point light source is distant from the illuminated surface, that is, all incident light directions are the same, as shown in Figure 1[2]. This can occur when the objects are under the outdoor circumstances (e.g. the sunshine).

However, when the point light source is near the surface, as shown in Figure 2, the incident direction as well as the incident radiance is not a constant for object surfaces anymore. Furthermore, the distance between the surface point and the point light source must be taken into consideration (the consideration is ignored previously since the point light source is far from thesurface such that is far from the surface such that the differences among the distances of different surface points from the point light source can be ignored).

After some modification [3], the new intensity equation can be expressed as

$$I(x,y) = \rho\, s_0 \frac{(P_S - P_O)\cdot \mathbf{n}}{\|P_S - P_O\|^3\, \|\mathbf{n}\|}$$

where ρ, s_0, and $\mathbf{n}$ are defined in Equation (1) previously, P_S is the 3-D coordinate of the light source position, and P_O is the 3-D coordinate of the surface point on objects.

Rashid and Burger [3] have used a differential method to determine the normal vector. They assumed that the point light source was near the camera (the projection center) and small surface patches on the objects were assumed nearly planar. Using a small planar surface patch, this method could determine the surface normal vector with small error. The shortcoming of the method is that the assumption of planar surface patch is inapplicable to some surface point, such as those points that have a steep orientation.

According to the above method that recovers shape from a single image, another kind of methods are the photometric approaches. Since the intensity equation with the unknown normal vector is nonlinear, using the photometric approach, it may easily eliminate the nonlinear portion and construct a linear system of equations. To solve linear equations is usually more efficient than to solve nonlinear ones. Therefore we propose a photometric approach for the shape from shading with near point light source.

3. Approach

Concept

Fixing the camera position, the images are taken successively using a different position for the point light source for each image. In addition, The distance between any two of these positions is very small as compared with the distances from these point light sources to the illuminated surface. Then the intensity values of the image points corresponding to the same image coordinate on the images will have small differences (However, these differences are detectable). This is illustrated in Figure 3. If the distances between successive point light sources are very small, the cube of the distance from the point light source to some surface point on objects could be assumed to be the same for all light sources. Consequently, if the light source positions are known, four images taken from successive different light source positions are sufficient for solving surface normal vectors by the method described below.

[2] All figures are printed from page 4.

Algorithm

Firstly, let's make some *assumptions*.

1. The illuminated surface is Lambertian surface.
2. The positions of the light sources are known.
3. The distance between successive light sources is very small as shown in Figure 3.

Suppose that two image I_1, I_2 are taken and the corresponding light source positions are (S_{x1}, S_{y1}, S_{z1}) and (S_{x2}, S_{y2}, S_{z2}), the surface point is (x, y, z), and the normal vector is $(p, q, -1)$. Then the two image intensities can be expressed as

$I_1(x,y) =$

$$\rho\, s_0 \frac{\Delta S_{x1}\, p + \Delta S_{y1}\, q - \Delta S_{z1}}{[\Delta S_{x1}^2 + \Delta S_{y1}^2 + \Delta S_{z1}^2]^{3/2}\, (p^2+q^2+1)^{1/2}} \quad (2)$$

$I_2(x,y) =$

$$\rho\, s_0 \frac{\Delta S_{x2}\, p + \Delta S_{y2}\, q - \Delta S_{z2}}{[\Delta S_{x2}^2 + \Delta S_{y2}^2 + \Delta S_{z2}^2]^{3/2}\, (p^2+q^2+1)^{1/2}} \quad (3)$$

By *Assumption* 3, Equation (3) can be approximated by

$$\rho\, s_0 \frac{\Delta S_{x2}\, p + \Delta S_{y2}\, q - \Delta S_{z2}}{[\Delta S_{x1}^2 + \Delta S_{y1}^2 + \Delta S_{z1}^2]^{3/2}\, (p^2+q^2+1)^{1/2}} \quad (4)$$

where $\Delta S_{x1} = (S_{x1} - x)$, $\Delta S_{y1} = (S_{y1} - y)$, $\Delta S_{z1} = (S_{z1} - z)$, $\Delta S_{x2} = (S_{x2} - x)$, $\Delta S_{y2} = (S_{y2} - y)$, and $\Delta S_{z2} = (S_{z2} - z)$

From above equations,

$$\frac{(2)}{(4)} = \frac{I_1(x,y)}{I_2(x,y)} = \Delta I = \frac{\Delta S_{x1}\, p + \Delta S_{y1}\, q - \Delta S_{z1}}{\Delta S_{x2}\, p + \Delta S_{y2}\, q - \Delta S_{z2}}$$

therefore, the final simplified linear equation is

$$(\Delta S_{x1} - \Delta I\,\Delta S_{x2})\,p + (\Delta S_{y1} - \Delta I\,\Delta S_{y2})\,q + (1 - \Delta I) = \Delta I\,\Delta S_{z2} - S_{z1}$$

When four images are gotten, a system of equations with three unknown variables (p, q, z) and three linear equations can be derived. Then the normal vectors and the depth values can be recoverd by solving these equations.

4. Implementation

In the following page, the application of the algorithm to synthetic images is listed. Figure 4 is the synthetic images of a sphere. Four subimages which are marked within these four images by white frames are taken and used as the inputs of the above algorithm. Figure 5 is the range image transformed from the recovered depth map. When the range image is compared with the original depth map, we find that the error ratios are between 10% and 15% and the error ratios increase as the surface points are far from the point light source, such as those points near the boundary in the Figure 5. Since this algorithm is to solved a system of linear equations, the computing time for each image point is the same. Since the algorithm only uses the intensity of the individual image point in each image, the result is not affected by neighboring noise. The major advantage of this method is that the orientations and depth map are solved concurrently so that the error which may occur if we solve depth map from the orientations can be avoided.

5. Conclusion

Among the existing methods, the shortcoming is that the noise problem is often a major consideration since they use the neighboring pixels on the image to determine the orientation. In this method, only individual pixels in each images are considered. Although the method can solve surface orientations and depth values, serious errors exist on some points which the difference between images can't be detected. To overcome this problem, other existing methods may be used to combine with it.

References

[1] B. K. P. Horn, "Robot Vision," MIT press, Cambridge, Massachusetts, 1986.

[2] R. J. Woodham, "Photometric Method for Determining Surface Orientation from Multiple Images," *Optical Engineering*, Vol. 19, No. 1, pp. 139-144, February 1980.

[3] H.U. Rashid and P Burger, "Differential Algorithm for the Determination of Shape from Shading using a Point Light Source," *Image and Vision Computing*, Vol.10, No. 2, pp. 119-127, March 1992.

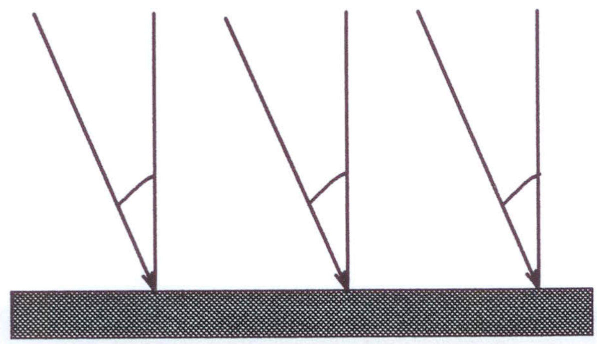

Figure 1: The incident direction for every point on the surface of the object is the the same when a distant point light source is applied

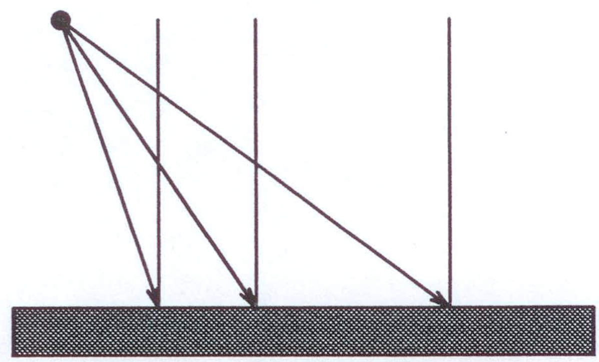

Figure 2: The incident directions will be different when the point light source is not far from the surface of the object

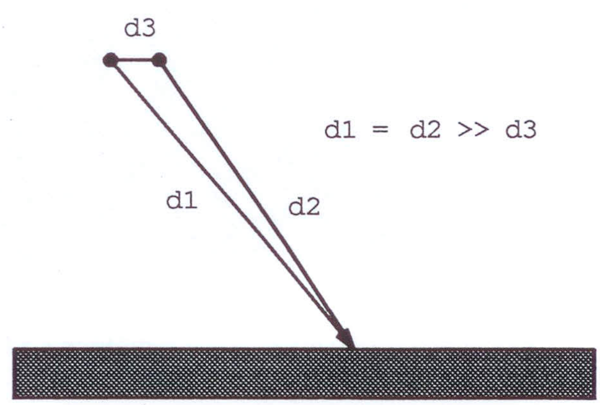

Figure 3: The distance between any two point light sources is very small as compared with the distance from any point light source to any point on the object surface

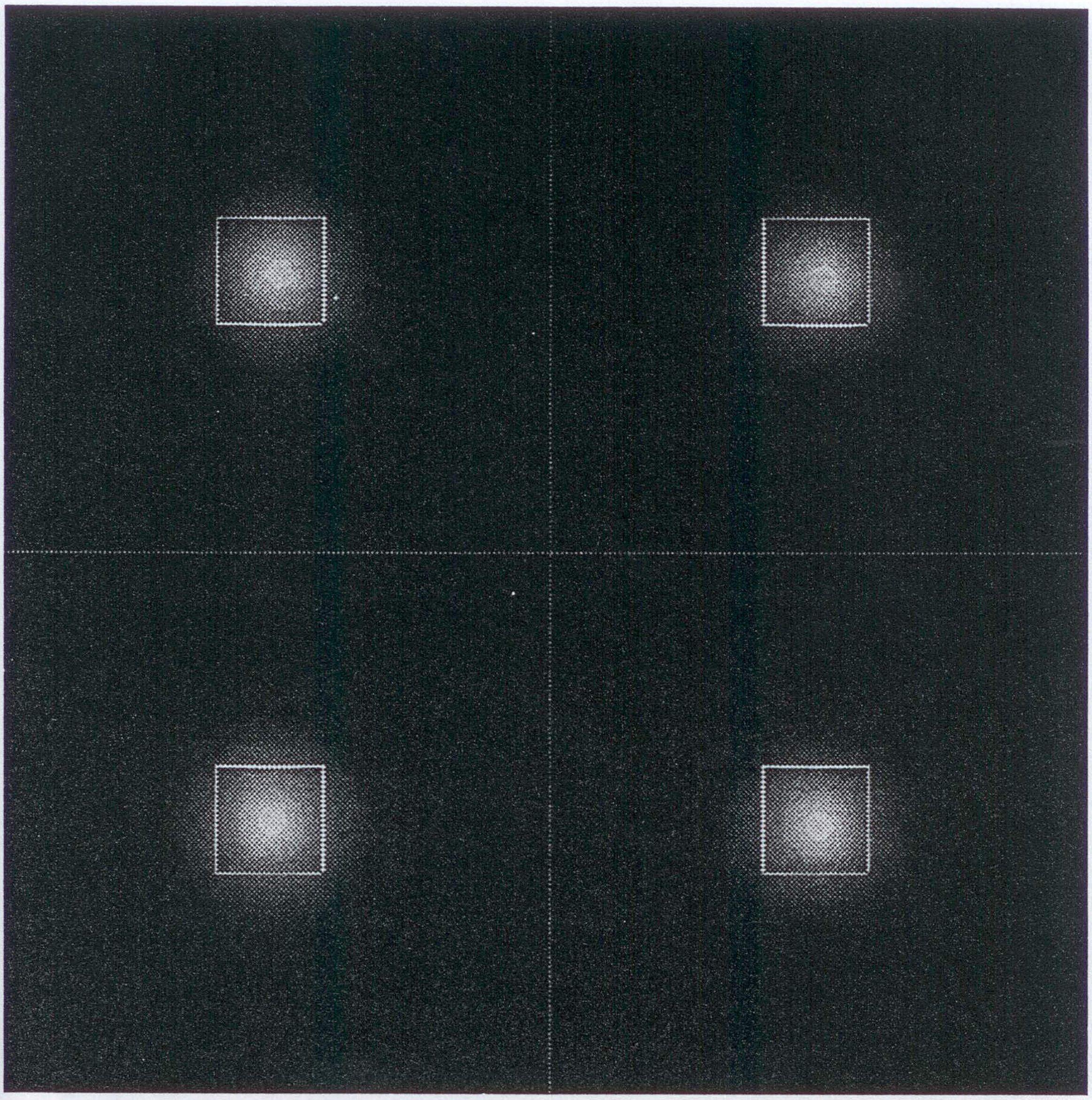

Figure 4: Four synthetic images of a sphere are taken. Those subimages which are marked by white frames are the inputs for the algorithm.

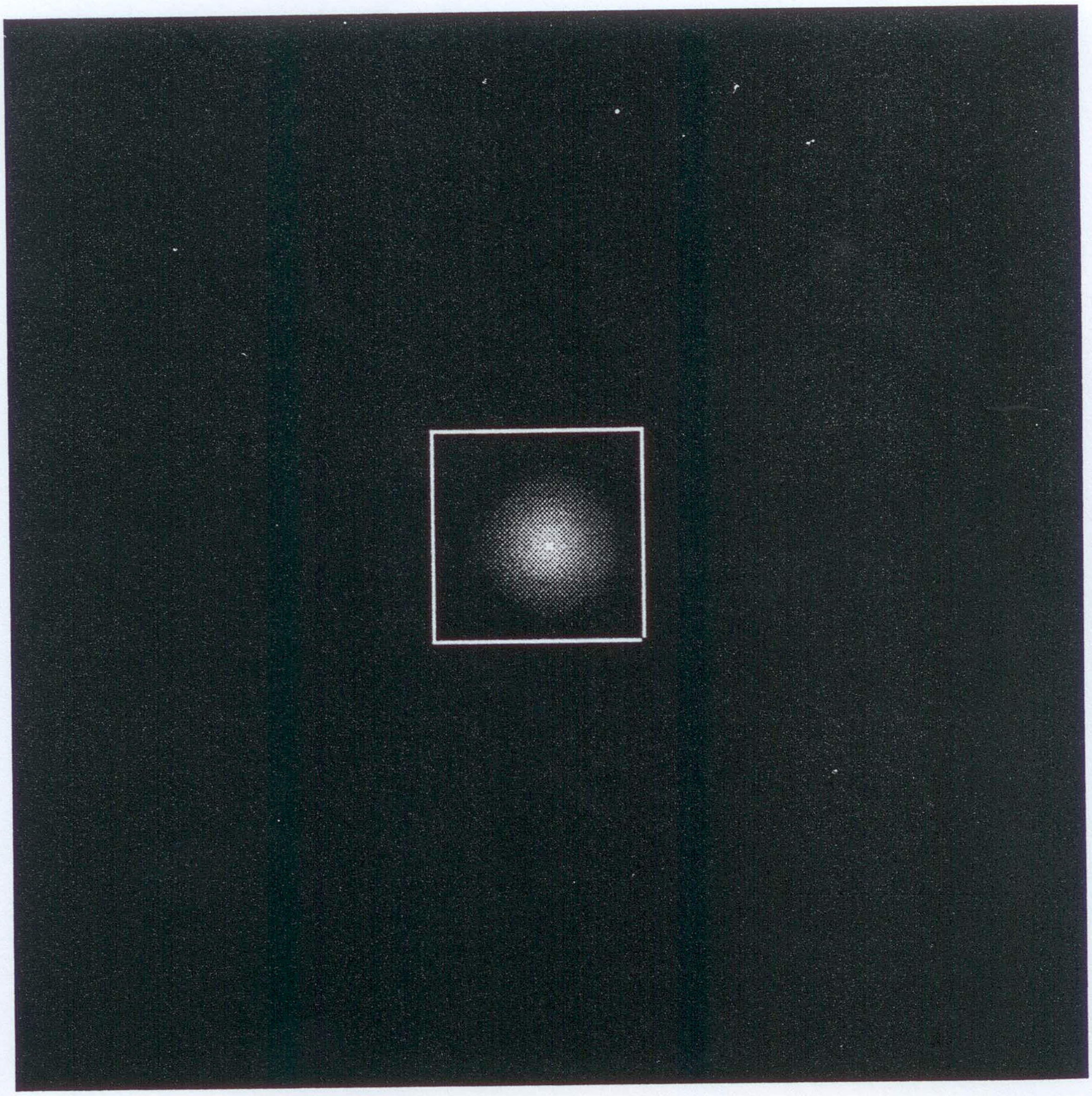

Figure 5: The range image that is transformed from the recovered depth map of above subimages.