

回歸分析

Linear Regression

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研究兩個樣本之間 的關係

簡單線性回歸

Simple Linear Regression

$$y = \beta x + \alpha + \varepsilon,$$

$$\varepsilon \sim N(0, \sigma_\varepsilon)$$

$$\mathbf{E}[Y = y_i \mid X = x_i] = \beta x_i + \alpha$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \hat{\beta} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \hat{\alpha} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

最小平方誤差法

Ordinary Least Square Error (OLS)

$$\varepsilon = Y - (\beta X + \alpha)$$

OLS Regression Results

```

=====
Dep. Variable:          y      R-squared:          0.903
Model:                 OLS    Adj. R-squared:     0.901
Method:                Least Squares  F-statistic:       451.9
Date:                  Wed, 11 Mar 2020  Prob (F-statistic): 6.95e-50
Time:                  16:41:04   Log-Likelihood:    -137.70
No. Observations:     100      AIC:               281.4
Df Residuals:         97       BIC:               289.2
Df Model:              2
Covariance Type:      nonrobust
=====

```

```

=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
const          1.3776      0.286        4.811      0.000        0.809      1.946
x1             -1.3230      1.323       -1.000      0.320       -3.950      1.304
x2             10.9470      1.281        8.549      0.000        8.405     13.489
=====

```

```

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Omnibus:          1.873      Durbin-Watson:      2.016
Prob(Omnibus):    0.392      Jarque-Bera (JB):   1.638
Skew:             -0.313     Prob(JB):           0.441
Kurtosis:         2.989     Cond. No.           22.5
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                                     p-value   confidence interval
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```

模型參數的估計

estimator	coef	std err	t	P> t	[0.025	0.975]
const	1.3776	0.286	4.811	0.000	0.809	1.946
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檢查殘差的分佈

Durbin-Watson: 自我相關性統計量

Jarque-Bera (JB): 偏態/峰態統計量

https://en.wikipedia.org/wiki/Coefficient_of_determination

https://en.wikipedia.org/wiki/Akaike_information_criterion

https://en.wikipedia.org/wiki/Bayesian_information_criterion

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最大概似估計

Maximum Likelihood Estimation (MLE)

https://en.wikipedia.org/wiki/Maximum_likelihood_estimation