# Java Programming 

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Java 405<br>Spring 2024

```
class Lecture2 {
    "Data types, Variables, and Operators"
}
// Keywords:
byte, short, int, long, char, float, double, boolean, true, false,
import, new
```


## Example

Given the circle radius, say 10 , determine the area.

- Input: how to store the value of the circle radius?
- Algorithm: how to compute the resulting area?
- Output: how to show the result?

```
public class ComputeAreaDemo {
    public static void main(String[] args) {
        // INPUT
        int r = 10;
        // ALGORITHM
        double A = r * r * 3.14;
        // OUTPUT
        System.out.println(A);
    }
}
```

- In Line 6, we declare the variable $r$ an integer (int) with its initial value 10 .
- In Line 9, we store the circle area in the variable $A$ which is decimal (double).
- The keywords int and double are two of primitive types.


## Simple Analog: Variable $\approx$ Box



## Variable Declaration

- First, we name the variable, say price.
- We then determine a proper type for price, for example,

| 1 | $\ldots$ // ignore the common part; the same applies hereinafter |
| :--- | :--- | :--- |
| 2 |  |
| 3 |  |
| 4 | int price; // price is a variable declared an integer type |
| 5 | $\ldots$ |

- The rule of variable declaration looks like data-type variable-name;
- For example, String[] args in the main method.
- This rule is similar to $\mathrm{C}, \mathrm{C}++$, and $\mathrm{C} \#$.


## Naming Rules

- The naming rule excludes the following cases:
- cannot start with a digit;
- cannot be any reserved word (see the next page);
- cannot include any blank between letters;
- cannot contain operators, like $+,-, *, /$.
- Note that Java is case-sensitive, for example, the letter A is different from the letter a.
- These rules are also applicable to methods, classes, etc.
- These rules, again, are similar to $\mathrm{C}, \mathrm{C}++$, and $\mathrm{C} \#$.


## Reserved Words ${ }^{1}$

| abstract | double | int | super |
| :--- | :--- | :--- | :--- |
| assert | else | interface | switch |
| boolean | enum | long | synchronized |
| break | extends | native | this |
| byte | final | new | throw |
| case | finally | package | throws |
| catch | float | private | transient |
| char | for | protected | try |
| class | goto | public | void |
| const | if | return | volatile |
| continue | implements | short | while |
| default | import | static |  |
| do | instanceof | strictfp* |  |

- Coverage: 44 / $50=88 \%$.

[^0]
## Things behind Variable Declaration

- Variable declaration asks to allocate a proper memory space to the variable (box).
- The size of the allocated space depends on its data type.
- We count the space size in bits or bytes.
- A bit presents a binary digit.
- 1 byte is equal to 8 bits.
- For example, an int value occupies 32 bits (or 4 bytes) in the memory.


## Variable Name as Alias of Memory Address


memory

- Literals that start with $0 x$ are hexadecimal (hex) integers. ${ }^{2}$
- Hex numbers are widely used to represent, say addresses and colors. ${ }^{3}$
${ }^{2}$ See https://en.wikipedia.org/wiki/Hexadecimal.
${ }^{3}$ Try https://htmlcolorcodes.com/.


## Data Types

- Every variable needs a type.
- Also, every statement (or expression) has a final type.
- The notion of data types is vital to programming languages.
- I would say that, the role of data types acts like the physics law in the universe.
- Java is a static-typed language, similar to C, C++, and C\#.
- A variable is available after declaration and cannot changed in runtime.
- We now proceed to introduce the two categories of data types: primitive types, and reference types.


## Type System: Overview



## Digression: Binary System ${ }^{5}$

- We have been familiar with the decimal system. (Why?)
- Computers know only the binary system because of its nature: only two states, on and off. ${ }^{4}$
- However, both systems are equivalent except that they differ in representations.
- For example,

$$
999_{10}=9 \times 10^{2}+9 \times 10^{1}+9 \times 10^{0} .
$$

- Similarly,

$$
111_{2}=1 \times 2^{2}+1 \times 2^{1}+1 \times 2^{0}=7_{10} .
$$

- In most cases, we don't need to deal with binary codes directly because we are using high-level languages.

[^1]
## Integers

| Name | Bits | Range | Approx. Range |
| :---: | :---: | :---: | :---: |
| byte | 8 | 0 to 255 | $<=255$ |
| short | 16 | -32768 to 32767 | $\pm 3 \times 10^{4}$ |
| int | 32 | -2147483648 to 2147483647 | $\pm 2 \times 10^{9}$ |
| long | 64 | -9223372036854775808 to 9223372036854775807 | $\pm 9 \times 10^{18}$ |

- The range is limited to its finite size of storage.
- If a value is out of the feasible range, an overflow occurs.
- The int type is the most used unless otherwise noted.
- If you want to write down a long-type literal, say 9876543210, you should write 9876543210 L , where the suffix $L$ indicates the long type.


## Floating-Point Numbers

| Name | Bits | Range |
| :---: | :---: | :---: |
| float | 32 | $1.4 e-045$ to $3.4 e+038$ |
| double | 64 | $4.9 e-324$ to $1.8 e+308$ |

- The notation $e($ or $E$ ) represents the scientific notation, based 10.
- For example, $1 e 2=100$ and $-1.8 e-3=-0.0018$.
- We use floating-point numbers when evaluating expressions that require fractional precision, say sqrt() and $\log ()$.
- In this sense, integers seem redundant because floating-point numbers could represent integers and also decimals.
- However, the floating-point system can only approximate the real-number arithmetic! (Why?)


## Machine Epsilon ${ }^{7}$

```
System.out.println(0.5 - 0.1 - 0.1 - 0.1 - 0.1 - 0.1);
// Output? Why?
```

- We relieve the machine epsilon by proper algorithm design.
- In critical applications, we even avoid to use the floating-point numbers but integers. ${ }^{6}$

[^2]
## Another Example

```
System.out.println(3.14 + 1e20 - 1e20); // Output?
System.out.println(3.14 + (1e20 - 1e20)); // Output?
```

- Floating-point arithmetic (FP) ${ }^{8}$ is arithmetic using formulaic representation of real numbers as an approximation to support a trade-off between range and precision. ${ }^{9}$

[^3]
## IEEE Floating-Point Representation ${ }^{10}$

$$
x=(-1)^{s} \times M \times 2^{E}
$$

- The sign bit $s$ determines whether the number is negative $(s=1)$ or positive $(s=0)$.
- The mantissa $M$ is a fractional binary number that ranges either between 1 and $2-\varepsilon$, or between 0 and $1-\varepsilon$.
- The exponent $E$ weights the value by a (possibly negative) power of 2 .


## Illustration



- That is why we call a double value.
- Double values have at least 16 significant digits in decimal!


## Assignments

- The equal sign $(=)$ is used as the assignment operator.
- An assignment statement designates a value to the variable.

```
1
```

```
int x, y; // Variable declaration.
```

int x, y; // Variable declaration.
x = 0; // Assign 0 to x.
x = 0; // Assign 0 to x.
y = x + 1; // y = 1 (trivial?)
y = x + 1; // y = 1 (trivial?)
x = x + 1; // Is this weird?

```
x = x + 1; // Is this weird?
```

- Direction: from the right-hand side to the left-hand side. ${ }^{11}$
- Copy a value from the right-hand side (value or expression) to the space indicated by the variable in the left-hand side.
- You cannot write codes like $1=x$ because 1 cannot be resolved to a memory space.
${ }^{11}$ The variable $x$ can be a $l$-value and $r$-value, but 1 and other numbers can be only $r$-value but not l -value. See Value.


## Two-Before Rule

```
int x;
x = 0;
x = x + 1;
```

- Rule 1: a variable must be declared before any assignment.
- Rule 2: a variable must be initialized with a value before being used.


## Arithmetic Operators

| Operator | Operation | Example | Result |
| :---: | :---: | :---: | :---: |
| + | Addition | $12+34$ | 46 |
| - | Subtraction | $56-78$ | -22 |
| $*$ | Multiplication | $90 * 12$ | 1080 |
| $/$ | Division | $3.0 / 2.0$ | 1.5 |
| $\%$ | Remainder | $20 \% 3$ | 2 |

- What if $3 / 2$ ?
- The result depends on the types of its operands!


## Concept Check



- What is the output?


## Two of Program Stages

- Compile time (or compilation period):
- memory allocation for $x$,
- constant literals (in this case 1, 2),
- linking the println method, etc.
- Run time (or execution period):
- execution of arithmetic operation
- output the result, etc.


## Compatibility and Type Conversion

- If a type is compatible to another, then the compiler will perform the implicit conversion.
- For example, the integer 1 is compatible to a double value 1.0.
- Clearly, Java is a weakly-typed language. ${ }^{12}$
- However, there is no automatic conversion from double to int. (Why?)
- To do so, you must use a cast, which performs an explicit conversion.
- Similarly, a long value is not compatible to int.


## Casting

```
int x = 1;
double y = x; // Compatible; implicitly converted.
x = y; // Not allowed unless casting.
x = (int) y; // Succeeded!!
```

- Note that the Java compiler does only type checking but no real execution before compilation.
- In other words, the actual values of $x$ and $y$ are unknown until the program is executed.


## Compatibility and Type Conversion (Concluded)

- Small-size types $\rightarrow$ large-size types.
- Small-size types $\nleftarrow$ large-size types (need a cast).
- Simple types $\rightarrow$ complicated types.
- Simple types $\leftarrow$ complicated types (need a cast).


## Text: Characters \& Strings

- Each character is encoded in a sequence of 0's and 1's.
- For example, ASCII. (See the next page.)
- The char type denotes characters, which are represented in Unicode, a 16 -bit unsigned value. ${ }^{13}$
- However, we often use String to present texts, as shown before.
- As an analogy, a molecule (string) consists of atoms (characters). ${ }^{14}$
${ }^{13}$ Unicode defines a fully international character set that can represent all of the characters found in all human languages.
${ }^{14} \mathrm{~A}$ String object comprises characters equipped with plentiful tools.


## ASCII (7-bit version)

| Hex | Dec | Char |  | Hex | Dec | Char | Hex | Dec | Char | Hex | Dec | Char |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0x00 | 0 | NULL | null | 0x20 | 32 | Space | 0x40 | 64 | @ | 0x60 | 96 |  |
| $0 \times 01$ | 1 | SOH | Start of heading | $0 \times 21$ | 33 | ! | 0x41 | 65 | A | 0x61 | 97 | a |
| 0x02 | 2 | STX | Start of text | 0x22 | 34 | " | 0x42 | 66 | B | 0x62 | 98 | b |
| $0 \times 03$ | 3 | ETX | End of text | 0x23 | 35 | \# | 0x43 | 67 | C | 0x63 | 99 | c |
| $0 \times 04$ | 4 | EOT | End of transmission | 0x24 | 36 | \$ | 0x44 | 68 | D | 0x64 | 100 | d |
| 0x05 | 5 | ENQ | Enquiry | 0x25 | 37 | \% | 0x45 | 69 | E | 0x65 | 101 | e |
| 0x06 | 6 | ACK | Acknowledge | 0x26 | 38 | \& | 0x46 | 70 | F | 0x66 | 102 | f |
| 0x07 | 7 | BELL | Bell | 0x27 | 39 | ' | 0x47 | 71 | G | 0x67 | 103 | g |
| $0 \times 08$ | 8 | BS | Backspace | 0x28 | 40 | ( | 0x48 | 72 | H | 0x68 | 104 | h |
| 0x09 | 9 | TAB | Horizontal tab | 0x29 | 41 | ) | 0x49 | 73 | I | 0x69 | 105 | i |
| 0x0A | 10 | LF | New line | 0x2A | 42 | * | 0x4A | 74 | J | 0x6A | 106 | j |
| $0 \times 0 \mathrm{~B}$ | 11 | VT | Vertical tab | 0x2B | 43 | + | 0x4B | 75 | K | 0x6B | 107 | k |
| 0x0C | 12 | FF | Form Feed | 0x2C | 44 | , | 0x4C | 76 | L | 0x6C | 108 | 1 |
| 0x0D | 13 | CR | Carriage return | 0x2D | 45 | - | 0x4D | 77 | M | 0x6D | 109 | m |
| 0x0E | 14 | SO | Shift out | 0x2E | 46 | - | 0x4E | 78 | N | 0x6E | 110 | n |
| 0x0F | 15 | SI | Shift in | 0x2F | 47 | 1 | 0x4F | 79 | 0 | 0x6F | 111 | - |
| $0 \times 10$ | 16 | DLE | Data link escape | 0x30 | 48 | 0 | $0 \times 50$ | 80 | P | 0x70 | 112 | p |
| $0 \times 11$ | 17 | DC1 | Device control 1 | 0x31 | 49 | 1 | 0x51 | 81 | Q | 0x71 | 113 | q |
| $0 \times 12$ | 18 | DC2 | Device control 2 | 0x32 | 50 | 2 | $0 \times 52$ | 82 | R | 0x72 | 114 | r |
| $0 \times 13$ | 19 | DC3 | Device control 3 | 0x33 | 51 | 3 | $0 \times 53$ | 83 | S | 0x73 | 115 | s |
| 0×14 | 20 | DC4 | Device control 4 | 0×34 | 52 | 4 | 0x54 | 84 | T | 0x74 | 116 | t |
| $0 \times 15$ | 21 | NAK | Negative ack | 0x35 | 53 | 5 | 0x55 | 85 | U | 0x75 | 117 | u |
| $0 \times 16$ | 22 | SYN | Synchronous idle | $0 \times 36$ | 54 | 6 | 0x56 | 86 | V | 0x76 | 118 | v |
| $0 \times 17$ | 23 | ETB | End transmission block | 0x37 | 55 | 7 | 0x57 | 87 | W | 0x77 | 119 | w |
| $0 \times 18$ | 24 | CAN | Cancel | 0x38 | 56 | 8 | $0 \times 58$ | 88 | X | 0×78 | 120 | X |
| $0 \times 19$ | 25 | EM | End of medium | 0x39 | 57 | 9 | 0x59 | 89 | Y | 0x79 | 121 | y |
| $0 \times 1 \mathrm{~A}$ | 26 | SUB | Substitute | 0x3A | 58 | : | 0x5A | 90 | Z | 0x7A | 122 | z |
| 0x1B | 27 | FSC | Escape | 0x3B | 59 | ; | 0x5B | 91 | [ | 0x7B | 123 | \{ |
| $0 \times 1 \mathrm{C}$ | 28 | FS | File separator | 0x3C | 60 | $<$ | 0x5C | 92 | 1 | 0x7C | 124 |  |
| $0 \times 1 \mathrm{D}$ | 29 | GS | Group separator | 0x3D | 61 | = | 0x5D | 93 | ] | 0x7D | 125 | \} |
| 0x1E | 30 | RS | Record separator | 0x3E | 62 | > | 0x5E | 94 | $\wedge$ | 0x7E | 126 | ~ |
| $0 \times 1 \mathrm{~F}$ | 31 | US | Unit separator | 0x3F | 63 | ? | 0x5F | 95 |  | 0x7F | 127 | DEL |

## Example

```
char c = 'a'; // A char value should be single-quoted.
System.out.println(c + 1); // Output 98!! (why?)
System.out.println((char)(c + 1)); // Output b.
String s = "Java"; // A string should be double-quoted.
System.out.println(s + 999); // Output Java999.
```

- We may apply arithmetic operators to characters, say Line 4 for some purposes. ${ }^{15}$
- In Line 7, the result of applying the + operator to string is totally different from Line 3 \& 4. (Why?)
${ }^{15}$ For example, https://en.wikipedia.org/wiki/Cryptography.


## Boolean Values ${ }^{17}$

- Programs are expected to do decision making by itself, say self-driving cars. ${ }^{16}$
- Java provides the boolean-type flow controls (branching and iteration).
- The boolean type allows only two values: true and false.
- Note that boolean values cannot be cast to non-boolean type, and vice versa. (Why?)
${ }^{16}$ See https://www.google.com/selfdrivingcar/.
${ }^{17}$ George Boole (1815-1864) is the namesake of the branch of algebra known as Boolean algebra. See https://en.wikipedia.org/wiki/George_Boole.


## Relational Operators

| Operator | Name |
| :---: | :---: |
| $<$ | less than |
| $<=$ | less than or equal to |
| $>$ | greater than |
| $>=$ | greater than or equal to |
| $==$ | equal to |
| $!=$ | not equal to |

- Relational operators take two operands and return a boolean value.
- Note that the mathematical equality operator is $==$, not $=$ (assignment).


## Example

```
int x = 2;
System.out.println(x > 1); // Output true.
System.out.println(x < 1); // Output false.
System.out.println(x == 1); // Output false.
System.out.println(x != 1); // Output true.
System.out.println(1 < x < 3); // Sorry?
```

- In Line $7,1<x<3$ is syntactically wrong.
- You need to split a complex statement into several basic statements and joint them by proper logical operators.
- For example, $1<x<3$ should be

$$
1<x \& \& x<3
$$

where \&\& represents the AND operator.

## Conditional Logical Operators ${ }^{18}$

| Operator | Name |
| :---: | :---: |
| $!$ | NOT |
| $\& \&$ | AND |
| $\\|$ | OR |
| $\wedge$ | EXCLUSIVE-OR |

- We often use XOR to denote the exclusive-or operator.
${ }^{18}$ The bit-wise operators are ignored in my course because most of Java programmers do not use those directly. See Bitwise and Bit Shift Operators if necessary.


## Truth Table

- Let $X$ and $Y$ be two boolean variables.
- The truth table for logical operators is shown below:

| X | Y | I X | $\mathrm{X} \& \mathrm{Y}$ | $\mathrm{X} \\| \mathrm{Y}$ | $\mathrm{X} \wedge Y$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | F | T | T | F |
| T | F | F | F | T | T |
| F | T | T | F | T | T |
| F | F | T | F | F | F |

## Life Applications Using Boolean Logic

- Basic instructions, such as arithmetic operators, are implemented by Boolean logic.
- For example, 1-bit adder can be implemented by using the XOR operator. ${ }^{19}$ (Try!)
- Can you image that the combination of these very basic elements ( 0,1, AND, OR, NOT) with jumps produces so-called Artificial Intelligence (AI) like AlphaGo which beat human beings in 2016 and ChatGPT which starts a new era in the end of 2022?
"Logic is the anatomy of thought."
- John Locke (1632-1704)
"This sentence is false."
- anonymous
"I know that I know nothing."
- Plato
(In Apology, Plato relates that Socrates accounts for his seeming wiser than any other person because he does not imagine that he knows what he does not know.)


## Arithmetic Compound Assignment Operators

- For simplicity, let $x$ and $k$ be any number.

| Operator | Description |
| :--- | :--- |
| $x++$ | Increment by one |
| $x+=k$ | Cumulative increment by $k$ |
| $x-=k$ | Cumulative subtraction by $k$ |
| $x *=k$ | Cumulative multiplication by $k$ |
| $x /=k$ | Cumulative division by $k$ |
| $x \%=k$ | Cumulative modulus by $k$ |
| $x--$ | Decrement by one |

## Example: Integers



## Example: Characters and Strings

- Some of the aforesaid operators are also applicable to char values and String objects.
- For example,



## Discussion: $++x$ vs. $x++$

```
int }x=0
int y = ++x;
System.out.println(y); // Output 1.
System.out.println(x); // Output 1.
int w = 0;
int z = w++;
System.out.println(z); // Output 0.
System.out.println(w); // Output 1.
```

- $x++$ first returns the old value of $x$ and then increments itself.
- Instead, $++x$ first increments itself and then returns the new value of $x$.
- We will use these notations very often.


## Operator Precedence ${ }^{20}$

| Precedence | Operator |
| :---: | :---: |
| $\downarrow$ | ```var++ and var-- (Postfix) + , - (Unary plus and minus), ++var and --var (Prefix) (type) (Casting) ! (Not) *, /, \% (Multiplication, division, and remainder) ,+- (Binary addition and subtraction) <, <=, >, >= (Comparison) \(==\), ! (Equality) \(\wedge\) (Exclusive OR) \&\& (AND) \|| (OR) \(=,+=,-=,{ }^{*}=, /=, \%=\) (Assignment operator)``` |

## Tip: Using Parentheses

- The program always evaluates the expression inside of parentheses first.
- If necessary, using parentheses in expressions could change the natural order of precedence among the operators.


## Scanner: Example of Reference Types

- To reuse your program, it is inconvenient to modify and recompile the source code for every radius.
- Reading inputs from the user's keyboard in the console is the easiest way to interact with programs.
- Java provides the Scanner object with easy-to-use input methods.
- Note that System.in refers to the standard input device, by default, the keyboard.


## Example

```
import java.util.Scanner;
    // Create Scanner object to receive data from keyboard.
    Scanner input = new Scanner(System.in);
    // INPUT
    System.out.println("Enter r?");
    int r = input.nextInt(); //
    // ALGORITHM
    double A = r * r * 3.14;
    // OUTPUT
    System.out.println(A);
    input.close(); // Cleanup: reclaim the resource.
```


## Discussions (1/2)

- In Line 1, we include the Scanner class, which belongs to the java.util package, by using the import statement.
- We put these import statements in the beginning of the file.
- Note that we can't leave these import statements in any class.
- In Line 4, the new operator followed by Scanner is to create a Scanner object.
- This object works as an agent between the keyboard and your program.
- In Line 9, the nextInt method of Scanner is used to convert the input to an int value.


## Discussions (2/2): General Concepts

- All runtime objects are created dynamically and resided in the heap. (See the figure in the next page.)
- Before manipulating the Scanner object, its address is assigned to the variable input, which is allocated in the stack.
- Hence input is called a reference to the Scanner object. ${ }^{21}$
- Clearly, the memory contains human data and also references (i.e., memory addresses).
${ }^{21}$ If you have programming experiences in $\mathrm{C} / \mathrm{C}++$, then this reference is similar to the concept of pointers.


## Illustration: Simplified Memory Model



## Methods Provided by Scanner ${ }^{22}$

| Method | Description |
| :--- | :--- |
| nextByte() | reads an integer of the byte type. |
| nextShort() | reads an integer of the short type. |
| nextInt() | reads an integer of the int type. |
| nextLong() | reads an integer of the long type. |
| nextFloat() | reads a number of the float type. |
| nextDouble() | reads a number of the double type. |
| next() | reads a string that ends before a whitespace character. |
| nextLine() | reads a line of text (i.e., a string ending with the Enter key pressed). |

${ }^{22}$ See Table 2-1 in YDL, p. 38.

## Exercise: Body Mass Index (BMI)

Write a program to take user name, height (in cm ), weight (in kgw) as input, and then output the user name attached with his/her BMI, which is

$$
\mathrm{BMI}=\frac{\text { weight }}{\text { height }^{2}} .
$$

- Be careful about unit conversion!

```
Scanner input = new Scanner(System.in);
// INPUT
System.out.println("Enter your name?");
String name = input.nextLine();
System.out.println("Enter your height (cm)?");
double height = input.nextDouble();
System.out.println("Enter your weight (kgw)?");
double weight = input.nextDouble();
// ALGORITHM
double bmi = 10000 * weight / height / height;
// OUTPUT: name (bmi)
System.out.println(name + " (" + bmi + ")");
```

- Make sure that you understand Line 18.


## Exercise: Two Descriptive Statistics

Write a program to take 3 numbers as user's input and output the arithmetic average with its standard deviation.

- Let $a, b, c$ be the double variables.
- Then its standard deviation is

$$
\sqrt{\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{3}}
$$

where $x_{i}=\{a, b, c\}$ and $\bar{x}=(a+b+c) / 3$.

- You may use two of Math methods: ${ }^{23}$ Math.pow(double $x$, double $y$ ) for $x^{y}$ and Math.sqrt(double $x$ ) for $\sqrt{x}$.

```
// INPUT
Scanner input = new Scanner(System.in);
System.out.println("a = ?");
double a = input.nextDouble();
System.out.println("b = ?");
double b = input.nextDouble();
System.out.println("c = ?");
double c = input.nextDouble();
input.close();
// ALGORITHM
double mean = (a + b + c) / 3;
double std = Math.sqrt((Math.pow(a - mean, 2) +
                                    Math.pow(b - mean, 2) +
                                    Math.pow(c - mean, 2)) / 3);
// OUTPUT
System.out.println("Mean = " + mean);
System.out.println("Std = " + std);
```


[^0]:    ${ }^{1}$ Based on JDK8. You can check the language changes here.

[^1]:    ${ }^{4}$ How about the quantum computers? Spin up and down. See Qubit. ${ }^{5}$ See How Exactly Does Binary Code Work? by José Américo NLF Freitas.

[^2]:    ${ }^{6}$ Also read https://news.cnyes.com/news/id/3680649.
    ${ }^{7}$ See Machine Epsilon and https://0.30000000000000004acom/

[^3]:    ${ }^{8}$ See https://en.wikipedia.org/wiki/Floating-point_arithmetic.
    ${ }^{9}$ You may also read this article What Every Computer Scientist Should Know About Floating-Point Arithmetic.

