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# What Distinguish One from Its Peers in Social Networks?

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**Abstract** Being able to discover the uniqueness of an individual is a meaningful task in social network analysis. This paper proposes two novel problems in social network analysis: how to identify the uniqueness of a given query vertex, and how to identify a group of vertices that can mutually identify each other. We further propose intuitive yet effective methods to identify the uniqueness identification sets and the mutual identification groups of different properties. We further conduct an extensive experiment on both real and synthetic datasets to demonstrate the effectiveness of our model.

**Keywords** social query · node identification · social networks

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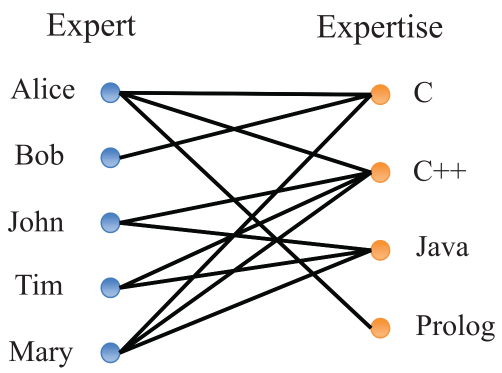

**Fig. 1** Example of a bipartite expertise network.

## 1 Introduction

In a heterogeneous social network, each entity is assigned a type (or label) to describe its category. A node type can be a place, a person, an organization, etc. For example, consider an expertise bipartite network, where each node is an entity of either the type expert or expertise, as illustrated in Figure 1. "Ego" is a node of focus while the ego-centric view of a node generally contains the neighborhood information of this node centralized as an ego. In this work, ego is considered as a query node of interests. It is often interesting and useful to distinguish an ego of some type from the rest of the entities of the same type. How can one expert be distinguished from another? In Figure 1, Alice can be uniquely identified from all the other experts because she possesses the skill Prolog that nobody else does. Mary can be uniquely identified from the other experts using the set of expertise C, C++, and Java because she is the only person that owns all three skills. In other words, we call {C, C++, Java} the *uniqueness identification set* of Mary.

Distinguishing a node from its peers in a social network is highly useful in many applications. In the above example, using the knowledge that Prolog uniquely identifies Alice from all the other experts, any consulting projects requiring the expertise on Prolog needs to involve Alice. To take the full advantage of Mary's expertise, Mary should be assigned first to those projects requiring expertise on C, C++, and Java together. Similarly, we may also distinguish different expertise by finding a unique set of experts that have the interested expertise.

Distinguishing a node from its peers in a social network is far from trivial. Not every node can be uniquely identified *only* by its *one-degree* neighbors. For example, in Figure 1, Tim and John cannot be distinguished from one another using only their expertise, since their expertise sets, {C++, Java}, are identical. To identify John, we have to use John's 2-hop neighbors including C++, Java, and Tim. The identification set for John essentially indicates, "Besides Tim, John is the only person who knows C++ and Java". Such information about John is useful – it provides a unique expertise set of John as well as his alternatives in the network.

Additionally, we may use the identification information to find interesting "communities" in a social network. For example, consider the sub-graph that contains the 5 nodes John, Tim, Mary, C++, and Java. Each of these nodes can be identified by the rest of the nodes (or a subset of them). For instance, the node C++ can be identified by the set Tim, Java, which can be read as "C++ is the only language other than Java that Tim knows". Similarly, the node Java can be identified by the sets {Mary, C++} and {John, Mary, C++}. In other words, the induced graph on {John, Tim, Mary, C++, Java} is a closure in identification, which we call a *mutual identification group*. Intuitively, in a mutual identification group $S$, each member can be identified by some other members of $S$. Such a closure group discloses some interesting information – John, Mary and Tim may replace each other as they are the only experts who master both C++ and Java.

Essentially, by designating any node as an ego, we can use its identification sets and mutual identification groups to conduct ego-centric analysis. Such analysis enables valuable applications including social entity search engines and substitution recommendation systems. For a social entity search engine, existing work such as [9,13] did not emphasize on choosing neighboring nodes to display, instead they opted to show all the neighbors of the ego up to certain degree. In contrast, our goal is to leverage the social network itself and report the uniqueness identification set for a query entity/ego to highlight its unique information, which helps users quickly catch the main differences among this entity and others of the same type from a large amount of redundant information. Such a mechanism facilitates better visualization and summarization for the task of social relationship search of an entity. Moreover, with the mutual identification group of each query entity/ego, we can build a substitution recommendation system that finds an alternative for a query entity that is out of stock or unavailable. That is, items that can be mutually identified by the same set of objects can be regarded as serving similar roles in a network, and therefore can be regarded as a surrogate for each other. Consider a movie network as an example, which contains relationship between movies, actors, directors, etc. For a movie, an entity search engine can report its unique identification set containing the minimal set of information of its actors, director, or places that distinguish this movie from others, rather than simply shows its neighborhood graph. Moreover, a mutual identification group may capture a set of movies, actors and places that are closely related to each other. If some movie is sold out, the substitution system may recommend other movies in the same uniqueness identification group. We will show several cases in Section 6.3.

As astute readers might point out, we may find more than one identification set for each ego we are interested in a network. For example, to identify Mary we may choose its neighbor Java. We then find that both Tim and John have the same expertise and thus are structure equivalent to Mary given their mutual neighbor Java. The final identification set of Mary includes Tim, John, and Java. On the other hand, we may also choose both C and C++ to distinguish Mary from other experts. This time, only Alice has both expertise and the resulting uniqueness identification set of Mary is {Alice, C, C++}. Compared to the former uniqueness identification set of Mary, the latter one is preferred since fewer experts are involved and the uniqueness of Mary possessing the combination of expertise C and C++ is highlighted. Similar concept of using as few entities as possible when choosing the mutual identification group should be followed.

Motivated by the above identification needs, in this paper, we propose a novel ego-centric data mining task as follows. In a network where each node *is associated with a unique type*, we want to achieve the following two goals: 1) Given an ego, find a uniqueness identification set (UID) that distinguishes the ego from its peers, *which are nodes of the same type.* As there can be more than one uniqueness identification set of an ego, we aim to find one containing minimum number of structurally equivalent nodes to distinguish the ego from others of the same type. When two UIDs have the same number of structurally equivalent nodes, we choose the one that can be identified with a smaller number of neighbors; 2) Given an ego, find a mutual identification group (MUID) where the uniqueness of each node in the group can be identified by the rest of the nodes, or a subset of them. Similarly, when more than one mutual identification group is available, we aim to find the one with fewer structurally equivalent neighbor nodes to distinguish the nodes. We explore different evaluation criteria for the two tasks.

There are four main contributions in this paper. First, we define two novel data mining problems for social network analysis, namely mining uniqueness identification sets and mutual identification groups. Second, we introduce a series of simple yet effective methods, *1-Hop+*, *One-Neighbor*, and *Multiple-Neighbor*, to tackle the problems from different angles. We prove that our methods are guaranteed to identify a node set that can identify the uniqueness of the query node. Third, we evaluate the three methods systematically on 3 real data sets and 3 synthetic data sets. The results and follow up analysis show the advantages of these methods. Finally, we analyze some of the interesting mutual identification groups from a real-world movie dataset, and provide explanation about them. The analysis justifies the usefulness of our approach.

The rest of the paper is organized as follows. We review the related work briefly in Section 2. We define the problems of uniqueness identification set of a vertex in a network and the mutual identification group in Section 3. We present three effective methods to solve the corresponding problems in Section 4. We report the experiment results in Section 5 and conclude the paper in Section 6.

## 2 Related Works

Our work is related to the existing studies on social networks in three aspects: social network search/extraction, community queries, and social network anonymization.

The work on social entity search/extraction focuses on extracting social relationship among a specific set of people from available open resources such as the Web. For example, Zhu et al. [13] developed an entity relationship extraction framework for relation extraction from the Web data, and a search engine, Renlifang, was built to report the entity relationship graph of some query persons, locations, or organizations. Tang et al. [9] developed Arnetminer, an academic search system that aims to automatically extract the researcher profile including the co-authorship relation graph from the Web. Those studies focus on extracting social networks. In contrast to the above works, we do not rely on parsing and extracting the information from external sources, but focus on extracting the mutual identification group from a given social network.

The works on group or community queries, [8,4,6,11], focuses on selecting a set of nodes or searching a specific community according to the given query nodes or some constraints. Given a set of query nodes in a graph, Sozio and Gionis [8] searched a community containing the nodes by finding a densely connected sub-graph. They proposed a greedy algorithm with heuristics to find the optimum solution under the monotone constraints on the density measure. Lappas et al. [4] tackled a team formation problem, which seeks a group of suitable people (each as a node in a graph) with different skills (as attributes of each node) to complete a task with certain skill requirement. The output results are determined by minimizing two types of communication cost between the selected people. Li and Shan [6] further generalized the problem in [4] by associating each required skill with a specific number of experts. They proposed a density-based measure for selecting the seed node and a grouping-based approach to find the team for generalized tasks. Similar to the previous two works, given a query initiator, Yang et al. [11] found a group of people that are available to attend activities while satisfying the acquaintanceship and social-temporal constraints. The problem was formulated as integer linear programming problem. With two efficient algorithms proposed to find the optimal solution. Based on a specified ego node, Li and Lin [5] reported an egocentric abstraction graph to summarize the features of the given ego node using an unsupervised learning mechanism. Although all of the above works extract a query-based social graph, their goals are very different from ours, which is finding a group of nodes that can uniquely identify the query node.

Our work is also related to the problem of graph anonymization. Specifically, the *k-anonymization* of social net-works [12] alters a given social network such that the 1-neighborhood of each node is isomorphic to those of at least $k-1$ other nodes. To achieve the goal, the critical step is to determine whether the 1-neighborhood of a node can identify the node with probability higher than $1/k$. However, in *k-anonymization*, the search is constrained to only 1-

**Table 1** Symbols and terminology.

| **Symbol** | **Definition** |
|---|---|
| $G = (V, E)$ | A graph consists of vertices V and edges E. |
| $N(v)$ | The first order neighbor set of vertex $v$. |
| $N_2(v)$ | The second order neighbor set of vertex $v$. |
| $type(v)$ | The vertex type of $v$. |
| $UI$ | Uniqueness identification. See Definition 2. |
| UID | Uniqueness identification set. See Definition 1. |
| MUID | Mutual identification group. See Definition 3. |
| $SE(v, M)$ | The structure equivalent set of node $v$ given a set $M$. See Definition 1. |

neighborhood, while our work does not have such constraint. Moreover, the goals in social network anonymization and ours are fundamentally different.

To our knowledge, we are the first to identify and tackle the problem of finding identification sets and mutual identification groups. Our work is clearly different from community detection [3, 7] in a social network, which aims to divide an entire social network into a set of disjoint or overlapping partitions.

## 3 Preliminaries

In Section 3.1, we provide the formal definitions of the problems. We then prove that the problem of finding the optimal UID is NP-hard.

### 3.1 Problem Definition

We are given an undirected, simple, and labeled graph $G(V, E)$ and a query node $v \in V$ whose $1^{st}$ and $2^{nd}$ order neighbor set is denoted as $N(v)$ and $N_2(v)$, respectively. Each vertex in $G$ is labeled with a type denoted by $type(v)$. Table 1 summarizes the symbols and terminology used in this paper.

**Definition 1** In a graph $G(V, E)$, given a vertex $v \in V$ and a set of nodes $M \subseteq N(v)$, a node $u \in V$ is said to be *structure equivalent*(abbreviated as SE) to $v$ given $M$, denoted by $u \in SE(v, M)$, if $type(u) = type(v)$ and $M \subseteq N(u)$. $SE(v, M)$ is the set of nodes structure equivalent to $v$ given $M$. Please note that $SE(v, M) = \emptyset$ if there does not exist any node structure equivalent to $v$ given $M$.

An example is shown in Figure 2. In Figure 2(a), suppose $M = \{m_1\}$, then $SE(v, M) = \{u_1, u_2\}$. Note that $u_4$ is not included in $SE(v, M)$ because it is not the same type as $v$. Figure 2(b), assuming an additional vertex $m_2$ is added into $M$ (note that the nodes in $M$ do not need to be of the same type), we can find that the set $SE(v, M)$ becomes smaller since $u_2 \notin N(m_2)$ and has to be removed from $SE(v, M)$. This example shows that adding nodes into $M$ may decrease the number of nodes in $SE(v, M)$. In Figure 2(c), when $M = \{m_1, m_2, m_3\}$, no vertex is connected to every element in $M$. Therefore, $SE(v, M) = \phi$.

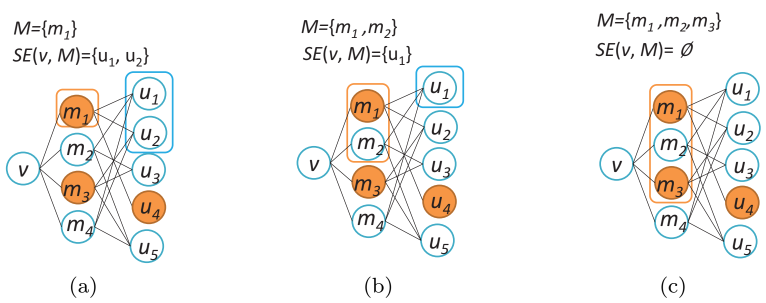

**Fig. 2** Three examples to show $SE(v, M)$. (a) $u_1$ and $u_2$ are structure equivalent(SE) to $v$ given $M$ =$\{m_1\}$. (b) By adding $m_2$ into $M$, $u_2$ becomes not SE to $v$ since $u_2$ has no link with $m_2$. (c) Similarly by adding $m_3$ into M, SE(v, M) becomes empty set. Note that here we assume two types of nodes: ones with filled color, one without.

**Definition 2** Given a vertex $v$ and a non-empty set $M \subseteq N(v)$, we define that $v$'s *uniqueness can be identified* by the 2-tuples set $[M, SE(v, M)]$. This set is called a UID of $v$.

Note that $SE(v, M)$ can be empty as it is possible to interpret such situation as "$v$ is unique because there is no other vertex that connects to $M$ as $v$ does". When $SE(v, M)$ is non-empty, then the uniqueness of $v$ can be interpreted as "$v$ is unique because, besides the vertices in $SE(v, M)$, the only vertex that connects to $M$ is $v$". Next we introduce an interesting and useful property of UIDs.

*Property 1* Given a query vertex $v$, its UIDs can always be found within two-hops of $v$.

*Proof* Since $M \subseteq N(v)$, according to Definition 1, $\forall u \in SE(v, M)$, $M \subseteq N(u)$. Therefore, $SE(v, M) \subseteq N_2(v)$. That is, any UID, $[M, SE(v, M)]$, is within two-hops of $v$. □

By Definition 2, there are many possible UIDs given a query vertex $v$. Therefore, we define a function to compare the quality of UIDs.

**Definition 3** Given two UIDs, $D_1 = [M_1, SE(v, M_1)]$ and $D_2 = [M_2, SE(v, M_2)]$, we define a comparison function $Q(v, M_1, M_2)$ as follows.

(a) $Q(v, D_1, D_2) = 1$, i.e., the quality of $D_1$ is better than that of $D_2$, if (i) $|SE(v, M_1)| < |SE(v, M_2)|$ or (ii) $|SE(v, M_1)| = |SE(v, M_2)|$ and $|M_1| < |M_2|$.
(b) $Q(v, D_1, D_2) = 0$, i.e., the quality of $D_1$ equals to that of $D_2$, if $|SE(v, M_1)| = |SE(v, M_2)|$ and $|M_1| = |M_2|$.
(c) Otherwise, $Q(v, D_1, D_2) = -1$, i.e., the quality of $D_1$ is worse than that of $D_2$

Note that $Q(v, D_1, D_2) = -Q(v, D_2, D_1)$. Furthermore, $Q$ is transitive as when $Q(v, D1, D2) = 1$ and $Q(v, D2, D3) = 1$, $Q(v, D1, D3) = 1$.

In Definition 3, the UIDs with a smaller $|SE(v, M)|$ is preferred. This reflects the intuition that having less structurally equivalent nodes in the UID indicates a more unique vertex $v$. Therefore our primary goal is to minimize $|SE(v, M)|$ of the identified UIDs. We treat $|M|$ as a secondary criterion because using smaller $M$ to obtain the same $SE(v, M)$ implies that the query vertex can be identified with fewer critical neighbors.

In the three UIDs shown in Figure 2, the quality of the UID in Figure 2(c) is the best since there is no SE node for the query vertex. The UID of the query vertex in Figure 2(a) is the least unique of the three.

We then further introduce a problem of finding mutual identification group (MUID) given a query vertex. An MUID is a set of vertices where each vertex can be uniquely identified by the subset of the remaining vertices in the set.

**Definition 4** In a graph $G(V, E)$, given a vertex $v \in V$, a set of vertices $X \subseteq V$ is a *mutual identification group* (MUID) of $v$ if the following two conditions hold.

(a) $v \in X$.
(b) $\forall u \in X, \exists D \subseteq X$, $D$ is a UID of $v$

The quality measure of MUIDs is conceptually similar to UIDs. For each MUID, the primary goal is to minimize $|SE(v, M)|$ and secondary goal is to minimize $|M|$ of the UIDs.

For the reason that each vertex does have a UID, it is less straightforward to define the quality of MUID. Again, we still prefer a smaller $SE(v, M)$ size than the $M$ size. To compare the $|SE(v, M)|$ of MUIDs generated from different models, we propose two different metrics. The first metric uses the union of the SE set of each node in the MUID, while the second metric evaluates the summation of the size of the SE set of each node in the MUID. The same rule applies to $M$ as the secondary criteria. The union measure evaluates how the vertices in the MUID exploit other members to identify themselves. If the nodes tend to include the same set of vertices to identify themselves, the union size would be smaller, creating better MUID. The sum of size of SE tells us whether we only need a small set of vertices to uniquely identify each individual member, regardless whether those vertices are overlapped or not.

Then, we can formally define the metric for MUID: Given an MUID $X$, for all $v \in X$, the UID $D_v = [M_v, SE(v, M_v)]$. For the metric of union size, we define size of union of SE set (USE) and size union of $M$ set (UM):

$$USE = |\bigcup_{v \in X} SE(v, M_v)|, and$$

$$UM = |\bigcup_{v \in X} M_v|.$$

We are now able to compare the quality of two MUIDs by replacing $|SE(v, M)|$ and $|M|$ in Definition 3 with $USE$ and $UM$. Similarly, for the metric of sum of size, we define total size of SE set (TSE) and total size of $M$ set (TM):

$$TSE = \sum_{v \in X} |SE(v, M_v)|, and$$

$$TM = \sum_{v \in X} |M_v|.$$

We have considered to use other quality measure of UID such as comparing $|M| + |SE(v, M)|$, i.e. do not differentiate the importance of $|M|$ and $|SE(v, M)|$ but just compare the UID size. Under this measure, the three examples in Figure 2 has the same quality which is against intuition because case (c) clearly identify v from others. Minimizing the above criteria while failed to minimize $|SE(v, M)|$ (i.e. due to a minimum $|M|$), and we will leave the ego with a set of structure equivalent nodes, which does not make it “unique”. For the same reason, another alternative measure to put more emphasis on minimizing size of $|M|$ than $|SE(v, M)|$ cannot highlight the uniqueness of vertices. The concept of priority of SE set and M set can be also applied to MUID. We have also considered other measures such as measuring the number of type included in UID or MUID found. For example, when forming a team, including more types means including more experts that know different skills. One of our future work is to modify the current proposed models to optimize such objective function.

## 3.2 Complexity Analysis

In this section we prove that finding the optimal UID with a minimal $M$ under the condition of a minimal $SE(v, M)$ is NP-hard. We first prove that increasing $M$ can only decrease the size of $SE(v, M)$ in Theorem 1. Based on this theorem, we can reduce the *Set Covering Optimization Problem*, which is known as an NP-hard problem, to our problem of finding the optimal UID.

We first observe an interesting property that adding more nodes into an existing $M$ set will cause some nodes in $SE(v, M)$ being removed from the set. Therefore, $|SE(v, M)|$ can only remain the same or become smaller, it can never become larger when more nodes are added into $M$.

**Theorem 1** *Given a query vertex $v$ and $M' \supseteq M$, then $SE(v, M') \subseteq SE(v, M)$.*

*Proof By Definition 1, since for all $u' \in SE(v, M')$, we have:*

$$\begin{aligned} u' &\in \bigcap_{m \in M'} N(m) \\ &= [(\bigcap_{m \in M} N(m)) \cap (\bigcap_{m \in M' - M} N(m))] \\ &\subseteq (\bigcap_{m \in M} N(m)) \\ &= SE(v, M). \end{aligned}$$

*Therefore, for all* $u' \in SE(v, M')$*,* $u' \in SE(v, M)$ *is proved, i.e.,* $SE(v, M') \subseteq SE(v, M)$*.* □

This theorem implies that if $M = N(v)$, $SE(v, M)$ would be the minimal SE set since any other $|M'| < |M|$ can only produce $SE(v, M') \supseteq SE(v, M)$. Therefore, we can conclude that $SE(v, M_{max} = N(v))$ is the minimal SE set because $N(v)$ is the largest possible set for $M$. Next, we want to show that the *Set Covering Optimization Problem*, which is known as an NP-hard problem, can be reduced to the problem of finding optimal UID with the minimal size of $M$ to obtain the optimal $SE(v, M)$.

**Theorem 2** *Given a query vertex* $v$*, with the neighbor set* $N(v) = m_1, m_2, ..., m_y$*. The optimization of UID first tries to find a minimum SE set,* $SE_{min}$*, then given the condition tries to find a minimum* $M$ *set such that* $M \subseteq N(v)$ *and* $SE(v, M) = SE_{min}$*. This problem is NP-hard.*

*Proof The Set Covering Optimization Problem is given a universe set* $U = u_1, u_2, ..., u_N$ *and a set of subsets of* $U$*,* $S = S_1, S_2, ..., S_k$ *where* $s_i \subseteq U, i \in [1, k]$*, to find a subset* $S_{opt} \subseteq S$ *that* $\bigcup_{s \in S_{opt}} s = U$*.*

*For the problem of finding optimal UID, Theorem 1 tells us that each time when we add a node into* $M$*, some nodes in* $T = N_2(v) - SE_{min}$ *can be removed from the eventual SE set where* $SE_{min} = SE(v, N(v))$ *and the elements in it are not removable. Therefore, for each* $m = m_i$*, we can identify a subset* $T_i = T - SE(v, m_i)$*. We can say that for each* $m_i$*, there is a corresponding set* $T_i$ *representing nodes that can be excluded from the potential SE set,* $T$*, when* $m_i$ *is added into* $M$*. Note that in our problem the goal is to find a minimum set* $M = M_{min}$ *such that* $SE(v, M_{min}) = SE_{min}$*. In other words, we want to find a minimum* $M$ *set which can remove all nodes in* $T$ *to find the minimal SE set.*

*To reduce the Set Covering Optimization to finding optimal UID, we first create a ego vertex* $v$*, creating a neighbor* $m_i$ *to* $v$ *for each* $S_i \subseteq S$*, and then create a vertex* $t_j$ *for each* $u_j \in U$*. Finally for each* $m_i$*, we link* $m_i$ *with* $t_j$ *if* $u_j \in U - S_i$*. That is, the set* $U$ *to be covered in the Set Covering Optimization Problem can be mapped to the removable SE set,* $T$*, in the finding optimal UID problem and the covering subsets,* $S_i$*, can be mapped to the identifying set of neighbors,* $T_i$*. Since by generating graph and finding the optimal UID in this way, the Set Covering Optimization Problem, which known as an NP-hard problem, can be also solved, the problem of finding optimal UID is proven to be NP-hard.* □

Here we suspect the MUID problem is also NP-hard because in MUID all vertices need to be uniquely identified by the rest. However, we will leave the detailed theoretical analysis of it to the future work.

## 4 Finding Optimal UIDs

In this section, we propose an exact and several greedy methods to find the uniquely identify set that satisfies a desired criterion. Since finding an optimal

**Algorithm 1:** Exhaustive Search

```
   Input: A Graph $G = (V, E)$, a query vertex $v$
   Output: UID set of $v$
1  Get $N(v) = \{m_1, m_2, ..., m_d\}$;
2  $d \longleftarrow |N(v)|$
3  $M \longleftarrow 2^d$ combination set of $N(v)$ //$M(i)$ is the $i^{th}$ element $M$;
4  $optM \longleftarrow M(0)$
5  for $i = 1$ to $2^d - 1$ do
6      if $Q(v, M(i), optM) = 1$ then
7          $optM \longleftarrow M(i)$
8      end
9  end
10 $optUID \longleftarrow [optM, SE(optM, v)]$
11 Output $optUID$;
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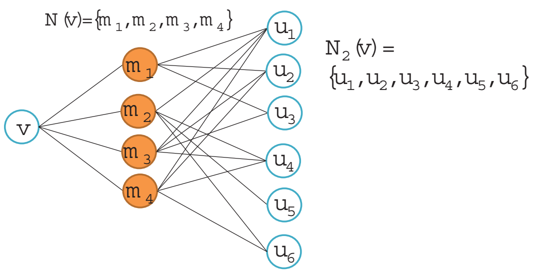


**Fig. 3** The example graph to find UID given $v$.

UID is an NP-hard problem, here we propose an exhaustive search method to find the exact outcome and some heuristic-based methods to identify the sub-optimal result for more efficiency. In Algorithm 1, an exhaustive search algorithm that requires $O(2^d)$ time complexity is provided which identifies the optimal UID, where $d$ is the degree of the query node $v$. It basically tries all the combination of $M$. Note that this method is not very efficient for nodes with high degree. For example, in our real world datasets there are a few nodes with hundreds or even higher degree, which makes running the exhaustive algorithm impractical.

To improve the efficiency, we propose the following three greedy methods, *1-Hop+*, *One-Neighbor*, and *Multiple-Neighbor*, to find sub-optimal UID sets faster. *1-Hop+* and *One-Neighbor* are naïve baselines to be compared with *Multiple-Neighbor*. We will use the example graph of query $v$ in Figure 3 for demonstration.

In the *1-Hop+* method, the algorithm first includes all neighbors of $v$ as $M$, and then adds $SE(v, M)$ into the UID set. This method extracts all the one-degree neighbors of $v$ with their SE nodes. Figure 4 shows the UID $m_1, m_2, m_3, m_4, u_1$ with $u_1$ being the only node that connects to all the nodes in $M$. Note that this method guarantees to find the minimal SE set; however, it may produce the largest $M$ set.

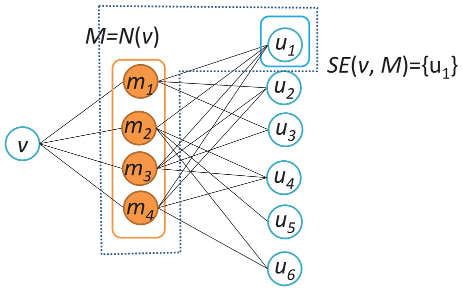


**Fig. 4** The UID of $v$ found by the *1-Hop+* method.

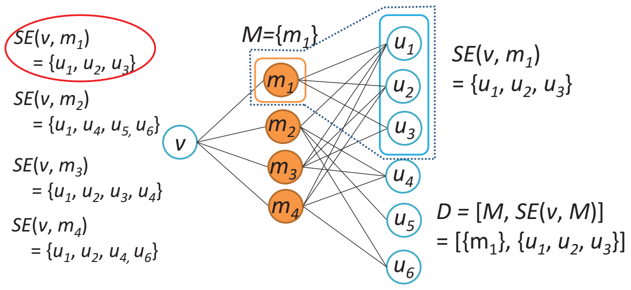

SE(v, m1) = {u1, u2, u3}
SE(v, m2) = {u1, u4, u5, u6}
SE(v, m3) = {u1, u2, u3, u4}
SE(v, m4) = {u1, u2, u4, u6}
M={m1}
v
m1
m2
m3
m4
u1
u2
u3
u4
u5
u6
SE(v, m1) = {u1, u2, u3}
D = [M, SE(v, M)] = [{m1}, {u1, u2, u3}]

**Fig. 5** The UID of $v$ found by the *One-Neighbor* method.

In the *One-Neighbor* method, given a vertex $v$, we choose a neighbor $m \in N(v)$ and then obtain $SE(v, M = \{m\})$ to be included in the UID. Since the goal is to minimize the SE size, we observe the size of SE set produced by different $m$ and choose the minimal one. In Figure 5, $m_1$ is chosen and it produces $SE(v, m_1) = \{u_1, u_2, u_3\}$

The two algorithms have low time complexity of $O(d)$ where $d$ is the degree of query node $v$. The *1-Hop+* method produces the minimal SE set but the maximal $M$. The *One-Neighbor* method keeps the minimal size of $M$, but in general does not minimize the size of the SE set. To address the above concerns, Algorithm 2 introduces the *Multiple-Neighbor* method whose goal is to identify a minimal $M$ to minimize SE set. In this method, we greedily add neighbor nodes of $v$ into $M$, guaranteeing the optimal SE set and hoping to obtain a minimal $M$.

With the greedy heuristic to pick neighbors of $v$ into $M$, *SEgain* is defined to represent "the vertices to be removed from $SE(v, M)$ after adding a node into $M$".

**Definition 5** Given UID of a query vertex $v$ as $D = [M, SE(v, M)]$, and for $M' = M \cup \{n\}$, where $n \in N(v) - M$, then $SEgain(v, n, M) = SE(v, M) - SE(v, M')$.

In Theorem 3 we prove that the set size of $SEgain(v, n, M)$ is monotonic with the size of $M$. In this property, $\forall u \in SE(v, N(v))$, $u$ cannot be in $SEgain(v, n, M)$ for any $v, n, M$ since there is no larger subset $M \subseteq N(v)$ than $N(v)$ itself.

**Theorem 3** *Given $SE(v, M), SE(v, M'), n \in N(v)$, $n \notin M$ and $n \notin M'$, if $M' \supseteq M$ then $SEgain(v, n, M') \subseteq SEgain(v, n, M)$.*

*Proof By Theorem 1, we have the following derivation.*

*(a) $\because M' \supseteq M$, $\therefore SE(v, M') \subseteq SE(v, M)$, and*
*(b) $\because M' \cup \{n\} \supseteq M \cup \{n\}$, $\therefore SE(v, M' \cup \{n\}) \subseteq SE(v, M \cup \{n\})$.*

*From (a) and (b), we have*

$$\begin{aligned} SEgain(v, n, M') &= SE(v, M') - SE(v, M' \cup \{n\}) \\ &\subseteq SE(v, M) - SE(v, M' \cup \{n\}) \\ &\subseteq SE(v, M) - SE(v, M \cup \{n\}) \\ &= SEgain(v, n, M). \end{aligned}$$

□

The *Multiple-Neighbor* method starts from the state of $M = \phi$ and $SE(v, M) = N_2(v)$ given the query vertex $v$. It iteratively calculates the $SEgain(v, n, M)$ of each neighbor $n$ that is not included in $M$, adding the neighbor with the largest $|SEgain(v, n, M)|$ into $M$ and removing the vertices in $SEgain(v, n, M)$ from the current $SE(v, M)$. It ends when there is no neighbor vertex with $|SEgain(v, n, M)| > 0$. This method guarantees that the result UID has a minimal SE set.

**Theorem 4** *The UID of query vertex $v$ found by the* Multiple-Neighbor *method is guaranteed with minimal SE set, i.e., if there is no neighbor $n \in N(v) - M$ satisfying $|SEgain(v, n, M)| > 0$, then $SE(v, M)$ is minimal.*

*Proof Let $SE_{min} = SE(v, N(v))$ be the minimal SE set and let the current state of the* Multiple-Neighbor *method be $SE(v, M_v)$. Suppose that $SE(v, M_v)$ is not minimal, then the following two conditions must both hold.*

*(a) $\exists u$ such that $u \notin SE_{min}$ and $u \in SE(v, M_v)$, and*
*(b) $\neg\exists n \in N(v) - M_v$ such that $SEgain(v, n, M_v) > 0$.*

*Since $u \notin SE_{min}$, there exits $n \in N(v) - M$ such that $n \notin N(u)$. If we add $n$ into $M_v$, then $u \notin SE(v, M_v \cup n)$. Because $u \in SE(v, M_v)$ and $u \notin SE(v, M_v \cup n)$, $u \in SE(v, M_v)_v - SE(v, M_v \cup n) = SEgain(v, n, M_v)$.* □

Figure 6 shows how the *Multiple-Neighbor* method is applied to the same example graph. In Figure 6(a), $SEgain(v, m_x, M)$ is generated for $m_1$, $m_2$, $m_3$ and $m_4$, while $m_1$ is selected to be added into $M$ because it has the largest *SEgain*. $SEgain(v, m_1, M)$ =$\{u_4, u_5, u_6\}$ is then removed from $SE(v, M)$. In Figure 6(b), *SEgain* of the rest of the neighbors are updated and $m_2$ is then added. After $m_2$ is added, it leaves only $u_1$ in $SE(v, M)$ and the algorithm halts because the condition $|SEgain(v, x, M)| > 0$ cannot be satisfied by adding any neighbor.

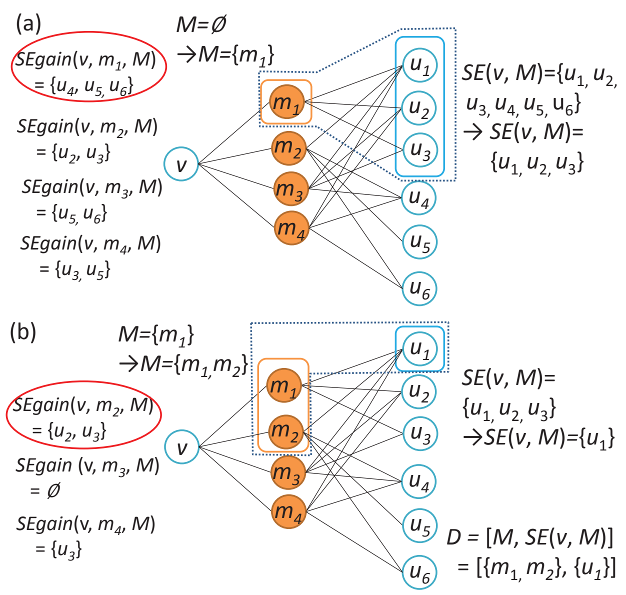

(a)
M=∅
→M={m1}
SEgain(v, m1, M) = {u4, u5, u6}
SEgain(v, m2, M) = {u2, u3}
SEgain(v, m3, M) = {u5, u6}
SEgain(v, m4, M) = {u3, u5}
SE(v, M)={u1, u2, u3, u4, u5, u6}
→ SE(v, M)= {u1, u2, u3}
(b)
M={m1}
→M={m1,m2}
SEgain(v, m2, M) = {u2, u3}
SEgain (v, m3, M) = ∅
SEgain(v, m4, M) = {u3}
SE(v, M)= {u1, u2, u3}
→SE(v, M)={u1}
D = [M, SE(v, M)] = [{m1, m2}, {u1}]

**Fig. 6** The UID of $v$ found by the *Multiple-Neighbor* method (a) $m_1$ is chosen as the first vertex to be added into $M$ (b) $m_2$ is chosen as the second vertex to be added into $M$.

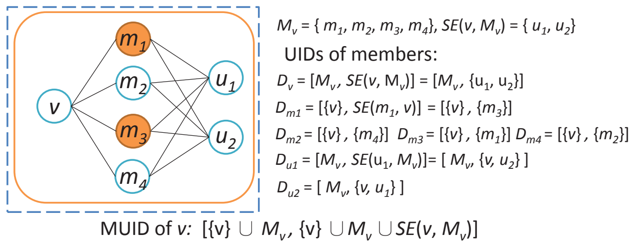
**Algorithm 2:** THE MULTIPLE-NEIGHBOR METHOD

```
   Input: A Graph $G = (V, E)$, a query vertex $v$
   Output: UID $D$ of $v$
1  $SE \longleftarrow N_2(v)$
2  $M \longleftarrow \phi$
3  while $|SE| > 0$ do
4      $m_{max} \longleftarrow argmax_m |SEgain(v, m, M)|, m \in N(v)$
5      if $m_{max} \leq 0$ and $|M| \geq 1$ then
6          break;
7      end
8      $M \longleftarrow M \cup m_{max}$
9      $SE \longleftarrow SE - N(m_{max})$
10 end
11 $D \longleftarrow [M, SE]$
12 output $D$;
```

## 5 Finding MUIDs

In Section 4, we introduced four methods to find the UID given a vertex $v$. We then extend the three algorithms proposed in Section 4, except exhaustive search, to identify the MUID given a vertex $v$.

Note that in MUID, not only the uniqueness of the query nodes but also that of the rest of the nodes in the set has to be uniquely identified. That is to say, after adding nodes into the set to uniquely identify the query vertex, one would then need to make sure the introduced nodes can be uniquely identified



m1
m2
m3
m4
v
u1
u2
$M_v = \{m_1, m_2, m_3, m_4\}, SE(v, M_v) = \{u_1, u_2\}$
UIDs of members:
$D_v = [M_v, SE(v, M_v)] = [M_v, \{u_1, u_2\}]$
$D_{m1} = [\{v\}, SE(m_1, v)] = [\{v\}, \{m_3\}]$
$D_{m2} = [\{v\}, \{m_4\}]$ $D_{m3} = [\{v\}, \{m_1\}]$ $D_{m4} = [\{v\}, \{m_2\}]$
$D_{u1} = [M_v, SE(u_1, M_v)] = [M_v, \{v, u_2\}]$
$D_{u2} = [M_v, \{v, u_1\}]$
MUID of $v$: $[\{v\} \cup M_v, \{v\} \cup M_v \cup SE(v, M_v)]$

**Fig. 7** The MUID of $v$ found by the *1-Hop+* by redefining M and SE set

by adding more nodes. In the worst case, one would have to include every node in the graph into the MUID. The quality of MUID is measured by the average quality of UID of each vertex in the MUID. Note that the quality of UID is the same as the one defined in Definition 3.

For the *1-Hop+* method, we first prove that the UID found given a vertex $v$ can be an MUID for $v$ by redefining $M$ and SE set.

**Theorem 5** *Given a vertex $v$, and the UID identified by the* 1-Hop+ *method $D = [M_v, SE(v, M_v)]$, $\forall u \in D$ we can find a UID $D_u = [M_u, SE(u, M_u)]$ such that $M_u \subseteq \{v\} \cup M_v$ and $SE(u, M_u) \subseteq \{v\} \cup M_v \cup SE(v, M_v)$.*

*Proof* *We have to show that for each $m \in M_v$ we can find UID $D_m = [M_m, SE(m, M_m)]$ such that $M_m \subseteq \{v\} \cup M_v$ and $SE(m, M_m) \subseteq \{v\} \cup M_v$, and the same for $u \in SE(v, M_v)$. This can be proved by the following two statements.*

*(a) For $m \in M_v$, the UID of $m$ can be found as $M_m = \{v\}$ and $SE(m, M_m) = \{x|x \in M_v - \{m\}, type(x) = type(m)\}$.*
*(b) For $u \in SE(v, M_v)$, the UID of $u$ can be found as $M_u = M_v$ and $SE(u, M_u) = SE(v, M_v) \cup \{v\} - \{u\}$.*

*From (a) and (b), we have shown that $\forall u \in D$, it is possible to identify $u$ with UID, $D_u = [M_u, SE(u, M_u)]$, such that $M_u \subseteq \{v\} \cup M_v$ and $SE(u, M_u) \subseteq \{v\} \cup M_v \cup SE(v, M_v)$. Therefore the UID of $v$ identified by the* 1-Hop+ *method has been proven to be an MUID of $v$ by redefining $M$ and SE set.* □

In Figure 7 an example of redefining UID by *1-Hop+* method is shown. The problem for the MUID obtained this way is that for each query node, all the neighbor nodes are included in the MUID. For nodes with large degrees, the MUID generated will be large in size, which is not preferable. Also for the neighbor nodes of $v$, they are SE to other neighbors with the same type, leaving large SE set in their own UIDs.

In the *One-Neighbor* method, given a vertex $v$, we obtain a UID, $D_v = [\{m\}, SE(v, m)]$. To extend the *One-Neighbor* method for MUID, we propose *One-Neighbor-MUID* method to further uniquely identify the newly introduced nodes. The proposal is to obtain the UID for $m$, $D_m$, in a similar manner. With $D_m = [\{v\}, SE(m, v)]$ as shown in Figure 8, $D_v \cup D_m =$

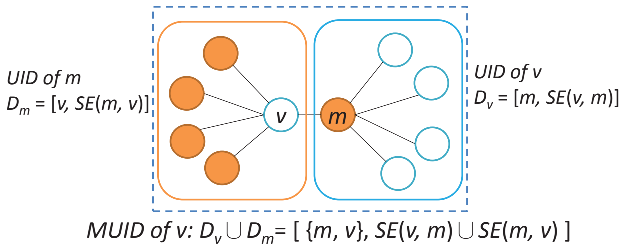


**Fig. 8** The MUID of $v$ found by the *One-Neighbor-MUID*, obtained by combining UIDs of $v$ and $m$

$[\{m\} \cup \{v\}, SE(v,m) \cup SE(m,v)]$ becomes the MUID of $v$. The problem for the *One-Neighbor-MUID* is that for each node, there could be many SE nodes, which is not desirable in general.

**Theorem 6** *Given a vertex $v$ and one of its neighbors $m$ and the UID, $D_v = [\{m\}, SE(v,m)]$, $D_m = [\{v\}, SE(m,v)]$, can be identified by the* One-Neighbor *method. The union of UIDs, $D_v \cup D_m$, is an MUID of $v$.*

*Proof* *Given $v$ and $m$, which are uniquely identified by UID, $D_v$ and $D_m$, respectively, we have to show that for each element $u \in SE(v,m)$ and $u \in SE(m,v)$ we can find UID, $D_u = [M_u, SE(u, M_u)]$.*

*(a) For each $u \in SE(v,m)$, its UID can be found as $M_u = \{m\}$ and $SE(u, M_u) = SE(v,m) \cup \{v\} - \{u\}$.*
*(b) For each $u \in SE(m,v)$, its UID can be found as $M_u = \{v\}$ and $SE(u, M_u) = SE(m,v) \cup \{m\} - \{u\}$.*

*From (a) and (b), we can conclude that every vertex $u \in D_v \cup D_m$ can be identified by a UID $D_u$, where $D_u \subseteq D_v \cup D_m$. We prove that the result of* One-Neighbor-MUID*, $D_v \cup D_m$, satisfies the requirement as an MUID of $v$.* □

We have showed that both *1-Hop+* and *One-Neighbor-MUID* can be extended to obtain MUID given a query vertex $v$. However, the quality of MUID produced by these methods is usually not optimal. To overcome such deficiency, we try to extend the *Multiple-Neighbor* method for the MUID problem. As proposed previously, the spirit of the *Multiple-Neighbor* method is to choose a small neighborhood subset $M$ of a given query $v$, leading to a smallest $SE(v, M)$ to uniquely identify $v$. To extend the method for finding MUID as *Multiple-Neighbor-MUID*, given the query vertex $v$ and its UID $D = M_v \cap SE(v, M_v)$, we have to make sure that all newly included $m \in M_v$ and $u \in SE(v, M_v)$ need to be uniquely identified, the process continues. It is not hard to show that each $u$ belonging to $SE(v, M_v)$ obtained from our *Multiple-Neighbor* method has been uniquely identified. Unfortunately, their corresponding UIDs might not be optimal because there could exist another set $M$ identifies better SE sets. Thus, we need to include more vertices into the MUID to identify the best result set.

We have proposed to use a heuristic $SEgain(v, n, M) = SE(v, M) - N(n)$ as a criterion to choose the next neighbor to be added in the *Multiple-Neighbor* method. This criterion calculates the number of nodes that can be removed from the UID if $n \in N(v)$ is added. However, when it comes to *Multiple-Neighbor-MUID*, all the newly added nodes in $M$ and $SE(v, M)$ also have to be uniquely identified. As shown in Figure 9, to estimate how many additional nodes would be introduced after adding $m_1$, we need to estimate how many vertices are required to identify $M_{m1}$, and to identify the further introduced vertices.

Similar to the *Multiple-Neighbor* method for the UID problem, here we define a heuristic function to estimate the quality of neighbors as candidates to be added into $M$. The idea is to execute the UID *Multiple-Neighbor* method for one pass on all nodes and record the $M$ and $SE(v, M)$ sets of each node $v$ as $UID_{SE}(v)$ and $UID_M(v)$. When choosing the neighbors of $v$ into $M$ in *Multiple-Neighbor-MUID*, we give higher priority for neighbor $n$ with smaller $|SE(v, M)|$ in UID, or $|UID_{SE}(n)|$. Given equal-sized $|UID_{SE}(n)|$, we then define the secondary criteria as minimizing $|UID_M(n) - M|$. This is because that exploiting neighbors already in $M$ could potentially introduce fewer new vertices. Given equal-sized $|UID_{SE}(n)|$ and $|UID_M(n)-M|$ the tertiary criteria is larger $SEgain(v, m, M)$. Note that for $n$ to be chosen into $M$, $|SEgain(v, n, M)|$ must be larger than 0. Algorithm 3 describes the algorithm in details.

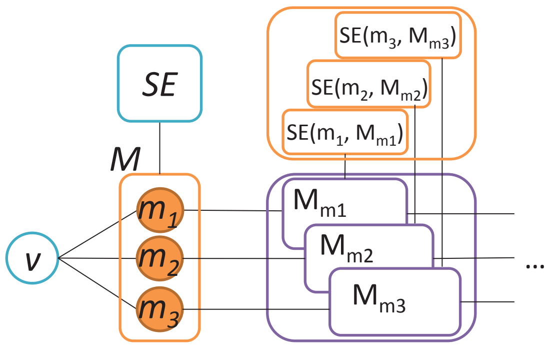


**Fig. 9** Example of *Multiple-Neighbor-MUID*.

## 6 Experimental Results

We introduce 3 real and 3 synthetic datasets for evaluation in Section 6.1. Then, we display the experimental results of finding UID and MUID, respectively. Finally, we report some case studies on MUID examples.

**Algorithm 3:** *Multiple-Neighbor-MUID* Method

```
Input: A Graph $G = (V, E)$, a query vertex $v$, precomputed $SE(v, M)$ and $M$ in
       UID
Output: MUID set $X$ of $v$
1  Get $N(v) = \{m_1, m_2, ..., m_d\}$;
2  $M_x \longleftarrow \emptyset$
3  $SE_x \longleftarrow \emptyset$
4  $UID_{SE}(v) \longleftarrow$ Read $SE(v, M)$ of $v$'s UID
5  $UID_M(v) \longleftarrow$ Read $M$ of $v$'s UID
6  $W$ is a stack and initially empty.
7  $//W$ stores not yet uniquely identified vertices
8  push $v$ into $W$
9  while $|W| > 0$ do
10     $w \longleftarrow$ pop first element from $W$
11     $SE_w \longleftarrow SE(w, M_x)$ $M_w \longleftarrow \emptyset$
12     $N_x \longleftarrow N(v) - X$
13     while $|SE_w| > 0$ do
14         for $n \in N_x$ do
15             if $|SEgain(w, n, M_w)| = 0$ then
16                 $N_x \longleftarrow N_x - \{n\}$
17             end
18         end
19         if $N_x = \emptyset$ then
20             break;
21         end
22         $minUID_{SE} \longleftarrow$ max integer
23         $minUID_M \longleftarrow$ max integer
24         $maxSEG \longleftarrow$ min integer
25         $n_{opt} \longleftarrow null$
26         for $n \in N_x$ do
27             if $(UID_{SE}(n) < minUID_{SE})$ or
28             $(UID_{SE}(n) = minUID_{SE}$ and $UID_M(n) < minUID_M)$ or
29             $(UID_{SE}(n) = minUID_{SE}$ and $UID_M(n) = minUID_M$ and
               $SEgain(w, n, M_w) > maxSEG)$ then
30                 $minUID_{SE} \longleftarrow UID_{SE}(n)$
31                 $minUID_M \longleftarrow UID_M(n)$
32                 $maxSEG \longleftarrow SEgain(w, n, M_w)$
33                 $n_{opt} \longleftarrow n$
34             end
35         end
36         $M_w \longleftarrow M_w \cup \{n_{opt}\}$
37         $SE_w \longleftarrow SE_w - N(n_{opt})$
38         push $n_{opt}$ into $W$
39     end
40     for $u \in SE_w$ do
41         if $N(u) \supset N(w)$ then
42             p
43         end
44         ush $u$ into $W$ $//\exists M_u$ that $w \notin SE(u, M_u)$
45     end
46     $M_x \longleftarrow M_x \cup M_w$
47     $SE_x \longleftarrow SE_x \cup SE_w$
48 end
49 Output $X = [M_x, SE_x]$;
```

**Table 2** Data sets statement.

| Name | —V— | —E— | Number of types |
|---|---|---|---|
| KDD Movie Dataset | 35311 | 168868 | 20 |
| TW Academic Network | 63122 | 770155 | 6 |
| HepTh Citation Network | 41840 | 933149 | 4 |
| Erdős-Rényi Model | 50000 | 249128 | 3 |
| Barabási-Albert Model | 50000 | 249179 | 3 |
| Watts-Strogatz Model | 50000 | 300000 | 3 |

## 6.1 Experiment Settings

We exploit three real datasets and three synthetic datasets to evaluate our model. The three real datasets include KDD movie data set, HepTh citation network, and academic co-author network. The KDD movie data set is a movie-centric heterogeneous information network. There are 35,311 nodes belonging to 20 different types. The node types include movie, actor, director, place, producer, . . . etc. There are a total of 168,868 edges in this network connecting different entities. The second real data set encodes paper-author relationship among numerous scholars and published research papers. This network consists of 63,122 nodes and 770,155 edges. There are six types of nodes - student, professor, department, college, and Chinese and English keywords. For the third dataset, we choose a commonly used citation graph, Arxiv HEPTH (High Energy Physics - Theory), provided in KDD Cup 2003. There are 93,319 edges connecting 41,840 nodes with four types, author, paper, journal, and email domain. For each dataset, a giant connected component (GCC) is obtained for our experiments. We also create three synthetic datasets of 50,000 nodes each, based on three different social network generation models. We apply Erdős-Rényi model [2] to produce a random graph; the Barabási-Albert model [1] to produce scale-free networks; and the Watts-Strogatz model [10] to produce graphs that can mimic the small world phenomena. For the Erdős-Rényi Random Graph (ER) model , we set the connection probability to be 0.0002. For the Barabási-Albert (BA) model, we set the number of initial nodes to 5. For the Watts-Strogatz (WS) model , we set the mean degree to 6, and the edge rewiring probability to 0.18. We assume there are three types of vertices in the network and randomly assign a type to each node.

In Table 3 we compare different models by averaging their rankings (1 to 3). Note that the ranking is generated using all vertices in the graph based on Definition 3 (i.e., we compare $|SE(v, M)|$ first, and the second criteria is $|M|$ given identical $|SE(v, M)|$). Finally, the methods with lower average rank are considered having better performance, namely obtaining better quality of UIDs.

For MUID, we compare the performance of the three greedy methods only because the exhaustive search method becomes intractable. Similar to our experiment on UID, Table 4 displays the comparison between the 3 greedy-based algorithms on MUID using the metrics described in Section 3.1. We return the average rank (calculated over all vertices) of each model.

**Table 3** The UID experimental results of the datasets. In the table, "MN", "ON", "OH", "EX" stand for the *Multi-Neighbor* method, the *One-Neighbor* method, the *1-Hop+* method, and exhaustive search, respectively. The last row records the execution time on Taiwan Academic network.

| **Dataset** | **MN** | **ON** | **OH** | **EX** |
|---|---|---|---|---|
| KDD Movie | 1.00 | 1.80 | 2.21 | 1.00 |
| TW Academic | 1.01 | 2.65 | 3.63 | 1.00 |
| HepTh | 1.00 | 2.50 | 3.39 | 1.00 |
| ER | 1.00 | 2.70 | 3.70 | 1.00 |
| BA | 1.00 | 2.62 | 3.62 | 1.00 |
| WS | 1.00 | 2.30 | 3.29 | 1.00 |
| Execution time of TW Academic (sec.) | 383.06 | 271.84 | 256.22 | 3876.46 |

## 6.2 Data Analysis

We first discuss the results for UID in Table 3. In all datasets, the *Multiple-Neighbor* method obtains as good results as the exhaustive search method does. The *One-Neighbor* method, although generally cannot achieve the optimal solution, usually does better than the *1-Hop+* method. The *1-Hop+* method guarantees the minimal SE set at the cost of including all neighbors into M. The *One-Neighbor* method includes only one neighbor into $M$ and therefore cannot guarantee minimizing the size of SE set. Therefore, it is not hard to show that for datasets where the optimal UID can be obtained with only one neighbor, the *One-Neighbor* method would outperform the *1-Hop+* method. The last row of the table records the execution time to generate the UIDs for all vertices on the Academic network dataset. It shows that the *Multiple-Neighbor* method, although requires slightly more time than the other two greedy methods, performs significantly more efficient than the exhaustive search does.

In Table 4 we discuss the quality of results of the MUID identified by each of the three methods. In both metrics, *Multiple-Neighbor-MUID* obtained the best result and it generally does a better job minimizing the $TSE$ than $USE$. Table 5 shows the mean of union size as well as the $TSE$ and $TM$ on the Taiwan academic network dataset. The *Multiple-Neighbor-MUID* obtains the minimal $TSE$ and $USE$. The *One-Neighbor-MUID* method always obtains the minimal $UM$ as 2 (the query and its neighbor). Although the *1-Hop+* method guarantees the minimal SE set for the UID of the ego vertex, it includes full neighborhood of the ego vertex. The neighbors of the same type would be SE to each other, therefore it introduces not only a large set of $M$ but also a large SE set. For the KDD Movie dataset, the *One-Neighbor-MUID* and the *1-Hop+* methods seem to perform closer to *Multiple-Neighbor-MUID* than with other datasets. It is because in this dataset more than 40% vertices have very low degrees. Also, we have observed that in the dataset with more node types, the performance of the *One-Neighbor-MUID* method improves because such data provides more chances for *One-Neighbor* method to uniquely identify the query node using only one neighbor node.

**Table 4** The MUID experimental results in the datasets. “Union” and “Sum” represent the two metric ranks described in Section 3.1

| **Dataset** | **Metrics** | **MN** | **ON** | **OH** |
|---|---|---|---|---|
| KDD Movie | Union | 1.33 | 1.61 | 2.07 |
| | Sum | 1.01 | 1.81 | 2.19 |
| TW Academic | Union | 1.20 | 2.43 | 2.32 |
| | Sum | 1.00 | 2.55 | 2.40 |
| HepTh | Union | 1.04 | 1.92 | 2.79 |
| | Sum | 1.02 | 1.95 | 2.78 |
| ER | Union | 1.00 | 2.02 | 2.96 |
| | Sum | 1.00 | 2.14 | 2.84 |
| BA | Union | 1.00 | 2.00 | 2.98 |
| | Sum | 1.00 | 2.04 | 2.95 |
| WS | Union | 1.05 | 1.76 | 2.99 |
| | Sum | 1.05 | 1.78 | 2.97 |

**Table 5** The MUID experimental results in mean of union size and mean of sum of size on Taiwan Academic network. $USE$, $UM$, $TSE$, $TM$ are defined in Section 3.1

| **Dataset** | **Metrics** | **MN** | **ON** | **OH** |
|---|---|---|---|---|
| TW Academic | USE | 3.00 | 10.45 | 12.58 |
| | UM | 10.19 | 2.00 | 13.20 |
| | TSE | 80.04 | 425.27 | 1563.26 |
| | TM | 29.49 | 12.16 | 32.58 |

**Table 6** The comparison between heuristic sets in *Multiple-Neighbor-MUID*. *Full* stands for the heuristic setting including the three heuristics. $H_A$ excludes $UID_{SE}$, $H_B$ excludes $UID_M$, and $H_C$ excludes both of $UID_{SE}$ and $UID_M$(i.e., $H_C$ includes only $SEgain$).

| **Dataset** | **Metrics** | *Full* | $H_A$ | $H_B$ | $H_C$ |
|---|---|---|---|---|---|
| KDD Movie | Union | 1.07 | 2.00 | 1.32 | 1.63 |
| | Sum | 1.07 | 2.01 | 1.32 | 1.64 |
| TW Academic | Union | 1.31 | 2.32 | 1.45 | 1.83 |
| | Sum | 1.36 | 2.34 | 1.45 | 1.83 |
| HepTh | Union | 1.35 | 1.58 | 1.78 | 1.82 |
| | Sum | 1.38 | 1.60 | 1.79 | 1.83 |

Furthermore, we compare the effectiveness of the heuristics introduced in Section 5: $UID_{SE}$, $UID_M$, $SEgain$. In Table 6 the *Full* setting utilizes all three heuristics and performs the best. The setting, $H_A(UID_{SE}$ is excluded), generally performs the worst. It is because that using $UID_M$ without $UID_{SE}$ may include neighbors with many SE nodes. This also explains why on the HepTh dataset the $H_A$looks fine because there are fewer SE nodes. Also, it seems that $UID_M$ is the least effective heuristic among the three.

## 6.3 Case Study

Here we conduct a case study to demonstrate real-world results for MUID using the KDD Movie dataset. Figure 10 shows an example of three different MUIDs discovered by the three proposed algorithms. The *1-hop+* method

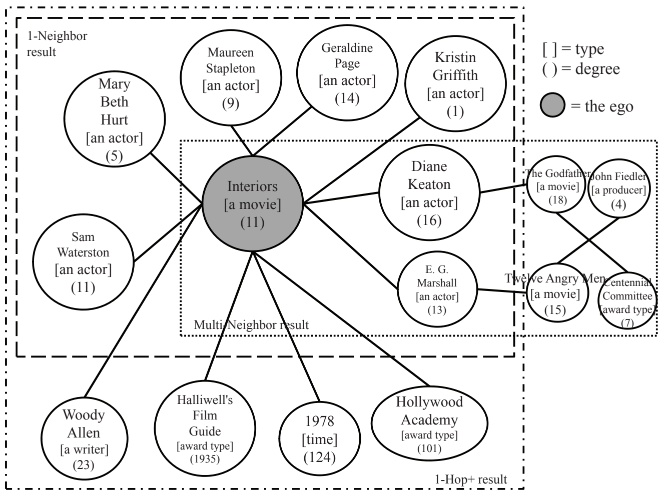

1-Neighbor result
Maureen Stapleton [an actor] (9)
Geraldine Page [an actor] (14)
Kristin Griffith [an actor] (1)
[ ] = type
( ) = degree
= the ego
Mary Beth Hurt [an actor] (5)
Interiors [a movie] (11)
Diane Keaton [an actor] (16)
The Godfather [a movie] (18)
John Fiedler [a producer] (4)
Sam Waterston [an actor] (11)
E. G. Marshall [an actor] (13)
Twelve Angry Men [a movie] (15)
Centennial Committee [award type] (7)
Multi-Neighbor result
Woody Allen [a writer] (23)
Halliwell's Film Guide [award type] (1935)
1978 [time] (124)
Hollywood Academy [award type] (101)
1-Hop+ result

**Fig. 10** Three Different Groups. A case that MN wins in all evaluation metrics.

simply obtains the neighbors of the ego vertex (i.e., movie Interiors marked in grey). *One-Neighbor* adds a single node, actor Kristin Griffith, first because this vertex has degree 1 and will not further produce SE nodes that need to be further identified. The *Multiple-Neighbor* method first selects two neighbor nodes since they are less likely to introduce too many nodes that require a large amount of vertices to identify.

Table 7 shows the size of MUID sets produced by different models. Note that for the *Multiple-Neighbor* method, from $TSE$ we can realize each node in MUID can be uniquely identified by its M set without the requirement of an SE set, which implies the vertices in this set are more unique than those in other MUID sets.

After examining the MUID results in KDD Movie dataset, we have identified three interesting patterns: Outliers, high-degree nodes and movie series. We will discuss each pattern in detail below.

**Outliers.** Figure 11 displays a frequent pattern that contains some inactive individuals in the network. Firstly, a vertex with small degree (e.g., an actor named David Tom) is chosen as the ego. In this case, 1-Hop+ and One-Neighbor MUID methods return the same set of nodes. It is because that this ego has only one single neighbor (movie *Stepfather 3*), which has to be included by both methods. Then, it is necessary to include SE('David Tom', 'Stepfather 3') so that Tom can be unique. Then one can observe that the mission has been accomplished because all the 7 nodes in the set can be uniquely identified by the remaining nodes in the set. Generally speaking, in the movie dataset, we found plenty of MUID of such flavor: an outlier ego, with its neighbor and some SE nodes. The *Multiple-Neighbor* method first includes two more nodes to identify Season Hubley and Priscilla Barnes so their own SE set can be minimized. Then, a movie *Island in The Stream* has

**Table 7** The MUID experimental results in union size and sum of size for the cases in this section.

| **Queried vertex** | **Metrics** | **MN** | **ON** | **OH** |
|---|---|---|---|---|
| Interiors | USE | 0 | 7 | 9 |
| | UM | 7 | 2 | 12 |
| | TSE | 0 | 42 | 44 |
| | TM | 12 | 8 | 22 |
| David Tom | USE | 6 | 6 | 6 |
| | UM | 5 | 2 | 2 |
| | TSE | 20 | 30 | 30 |
| | TM | 13 | 7 | 7 |
| Saving Private Ryan | USE | 0 | 2 | 8 |
| | UM | 6 | 2 | 13 |
| | TSE | 0 | 2 | 16 |
| | TM | 10 | 3 | 24 |
| Blondie on a Budget | USE | 5 | 17 | 6 |
| | UM | 9 | 2 | 5 |
| | TSE | 9 | 272 | 30 |
| | TM | 49 | 18 | 28 |

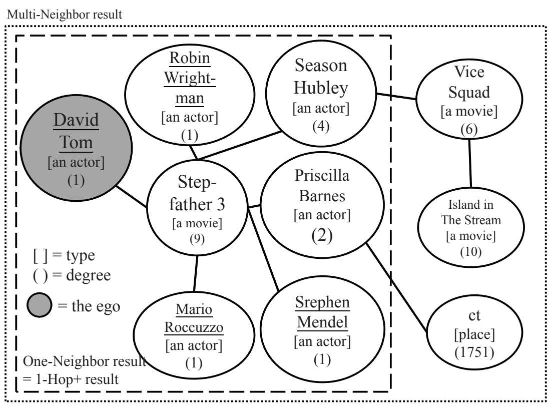

Multi-Neighbor result
Robin Wright-man [an actor] (1)
Season Hubley [an actor] (4)
Vice Squad [a movie] (6)
David Tom [an actor] (1)
Step-father 3 [a movie] (9)
Priscilla Barnes [an actor] (2)
Island in The Stream [a movie] (10)
[ ] = type
( ) = degree
= the ego
Mario Roccuzzo [an actor] (1)
Srephen Mendel [an actor] (1)
ct [place] (1751)
One-Neighbor result = 1-Hop+ result

**Fig. 11** Outliers. The MN performed best in TSE.

to be includes to uniquely identify *Vice Squad*. Note that although the value $|SE(v, M)|+|M|$ of the *Multiple-Neighbor* method is not as small as the other methods in these examples, it generally produces the better union and sum of SE sets, which implies that some additional vertices might be added to reduce the size of the SE set.

**High-degree Nodes.** An example of high degree nodes is illustrated in Figure 12. By taking *Saving Private Ryan* as the ego, the Multiple-Neighbor select DreamWorks and Jeremy Davies to uniquely identify the ego node, indicating that this is the only movie performed by J. Davis and published by DreamWorks. To identifying both neighbors, we include 3 more nodes and finally produce a MUID with an empty SE set, which means all nodes in this set are unique given certain neighbors. The One-Neighbor algorithm chooses Omaha Beach to uniquely identify the ego. Then, it continues to find Dream-

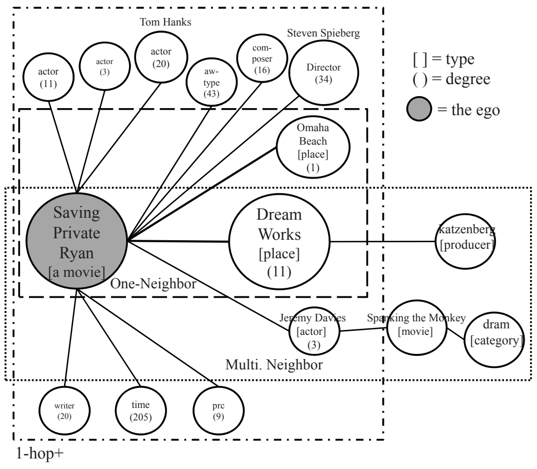


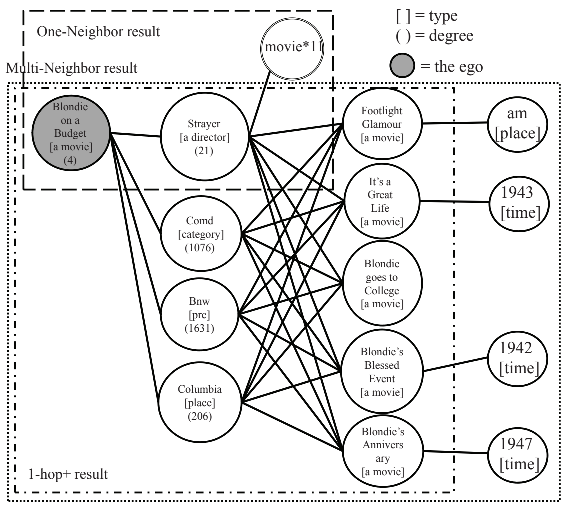
**Fig. 12** High-degree Nodes. The $MN$ outperformed others in both SE sizes.

Works (place) to uniquely identify Omaha Beach. Although the total size of graph is small, it does not produce empty SE sets as both places have each other as the SE node. The 1-Hop+ method included all 12 neighbor nodes. In general, nodes with higher degree have higher chance to connect to vertices with rare type and fewer connections (e.g., Omaha beach).

**Series of movies.** The next example describes the cases where there exist many SE nodes. Figure 13 uses *Blondie on a Budget* as the ego, and shows that there are 5 nodes that are SE to this ego. It is inevitable for any algorithm to include these 5 nodes in order to distinguish the movie from others. The Multiple-Neighbor produces the set that includes some relevant information (e.g., director, category, etc.) about the movie, and those 5 SE nodes. For recommendation purpose, we may find those movies are suitable candidates to replace each other. Furthermore, additional information about the 5 added movies (e.g., place, time) is provided, so they can be identified with minimal SE sets.

## 7 Conclusions

This paper shows that the uniqueness of a node can be captured in many different ways: using all the neighbors plus some similar nodes (1-Hop+), using a combination of 1st degree and 2nd degree neighbors (One-Neighbor method), greedily adding vertices that are likely to be unique themselves (Multiple-Neighbor method). This paper proposes and proves some valid methods to identify a closure set of mutually identifiable nodes, and focus on finding one with small SE set size in UID or smaller union or summation SE set size in MUID. The future work lies in the investigation of some other metrics such as diversity of types, coverage, etc, and comes up with efficient and effective models to identify such sets and their complexity analysis accordingly.



[ ] = type
( ) = degree
= the ego
One-Neighbor result
Multi-Neighbor result
1-hop+ result
movie*11
Blondie on a Budget [a movie] (4)
Strayer [a director] (21)
Comd [category] (1076)
Bnw [prc] (1631)
Columbia [place] (206)
Footlight Glamour [a movie]
It's a Great Life [a movie]
Blondie goes to College [a movie]
Blondie's Blessed Event [a movie]
Blondie's Anniversary [a movie]
am [place]
1943 [time]
1942 [time]
1947 [time]

**Fig. 13** Series of movies. A case that MN performed best; while ON performed worst.

## References

1. R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. *Reviews of Modern Physics*, 74:47–97, 2002.
2. P. Erdős and A. Rényi. On the evolution of random graphs. *Bull. Inst. Int. Stat.*, 38:343, 1961.
3. S. Fortunato. Community detection in graphs. *Physics Reports*, 486(3-5):75–174, 2010.
4. T. Lappas, K. Liu, and E. Terzi. Finding a team of experts in social networks. In *Proc. of ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining*, 2009.
5. C.-T. Li and S.-D. Lin. Egocentric Information Abstraction for Heterogeneous Social Networks. In *ASONAM*, 2009.
6. C.-T. Li and M.-K. Shan. Team Formation for Generalized Tasks in Expertise Social Networks. In *IEEE SocialCom*, 2010.
7. M. Newman. Detecting community structure in networks. *The European Physical Journal B - Condensed Matter and Complex Systems*, 38(2):321–330, 2004.
8. M. Sozio and A. Gionis. The community-search problem and how to plan a successful cocktail party. In *Proc. of ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining*, 2010.
9. J. Tang, J. Zhang, L. Yao, J. Li, L. Zhang, and Z. Su. ArnetMiner: Extraction and Mining of Academic Social Networks. In *Proc. of ACM SIGKDD Int. Conf. on Knowledge Discovery and Data Mining*, 2008.
10. D. Watts and S. Strogatz. Collective dynamics of small-world networks. *Nature*, 363:202–204, 1998.
11. D.-N. Yang, Y.-L. Chen, W.-C. Lee, and M.-S. Chen. Social-Temporal Group Query with Acquaintance Constraint. In *Proc. of Int. Conf. on Very Large Data Bases*, 2011.
12. B. Zhou and J. Pei. Preserving Privacy in Social Networks against Neighborhood Attacks. In *Proc. of IEEE Int. Conf. on Data Engineering*, 2008.
13. J. Zhu, Z. Nie, X. Liu, B. Zhang, and J.-R. Wen. StatSnowball: a Statistical Approach to Extracting Entity Relationships. In *Proc. of Int. World Wide Web Conf.*, 2009.