Structure from motion

Digital Visual Effects

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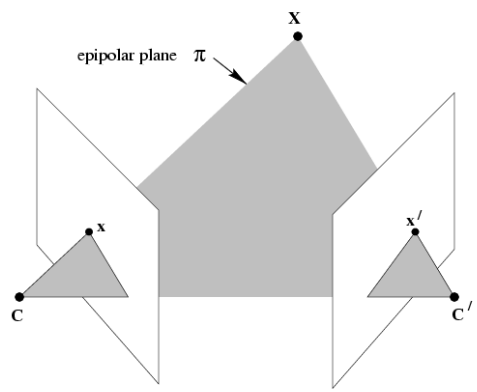
Outline

- Epipolar geometry and fundamental matrix
- Structure from motion
- Factorization method
- Bundle adjustment
- Applications

Epipolar geometry & fundamental matrix

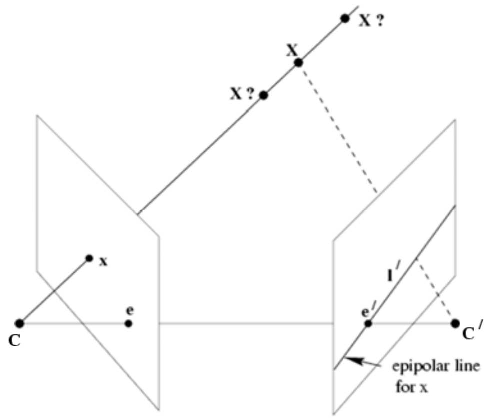
The epipolar geometry

epipolar geometry demo



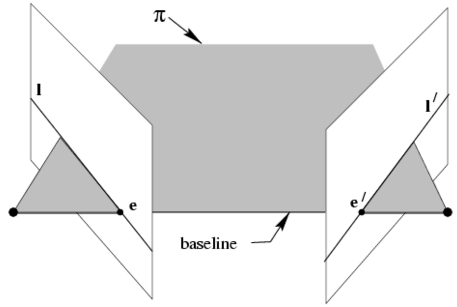
C, C', x, x' and X are coplanar

The epipolar geometry



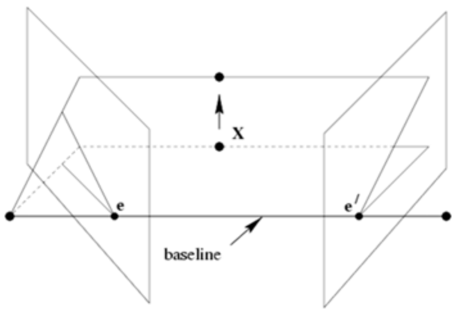
What if only C, C', x are known?

The epipolar geometry



All points on π project on l and l'

The epipolar geometry



Family of planes π and lines l and l' intersect at e and e'

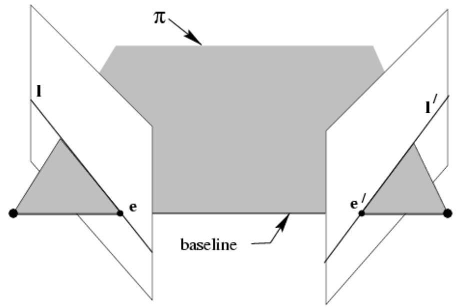
The epipolar geometry

epipolar pole

epipolar geometry demo

= intersection of baseline with image plane

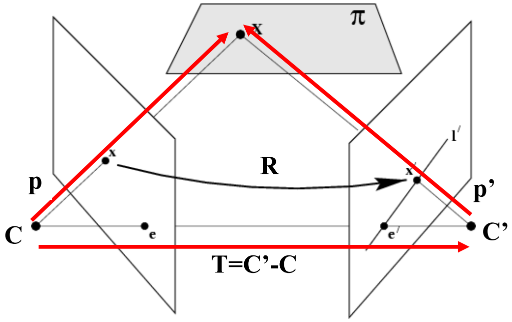
= projection of projection center in the other image



epipolar plane = plane containing baseline

epipolar line = intersection of epipolar plane with image

The fundamental matrix F



Two reference frames are related via the extrinsic parameters

$$\mathbf{p} = \mathbf{R}\mathbf{p}' + \mathbf{T}$$

The fundamental matrix F

$$\mathbf{p} = \mathbf{R}\mathbf{p}' + \mathbf{T}$$

Multiply both sides by $\mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}$

$$[\mathbf{T}]_{\times} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$\mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}\mathbf{p} = \mathbf{p}^{\mathrm{T}}[\mathbf{T}]_{\times}(\mathbf{R}\mathbf{p}' + \mathbf{T})$$

$$\Rightarrow \quad 0 = \mathbf{p}^{\mathrm{T}}\boxed{[\mathbf{T}]_{\times}\mathbf{R}}\mathbf{p}'$$

$$\Rightarrow \quad \mathbf{p}^{\mathrm{T}}\boxed{\mathbf{E}}\mathbf{p}' = 0$$

essential matrix

The fundamental matrix F

$$\mathbf{p}^{\mathrm{T}}\mathbf{E}\mathbf{p}' = 0$$

Let $\mathbf{M}$ and $\mathbf{M}'$ be the intrinsic matrices, then

$$\mathbf{p} = \mathbf{M}^{-1}\mathbf{x} \qquad \mathbf{p}' = \mathbf{M}'^{-1}\mathbf{x}'$$

$$\Rightarrow \quad (\mathbf{M}^{-1}\mathbf{x})^{\mathrm{T}}\mathbf{E}(\mathbf{M}'^{-1}\mathbf{x}') = 0$$

$$\Rightarrow \quad \mathbf{x}^{\mathrm{T}}\boxed{\mathbf{M}^{-\mathrm{T}}\mathbf{E}\mathbf{M}'^{-1}}\mathbf{x}' = 0$$

$$\Rightarrow \quad \mathbf{x}^{\mathrm{T}}\boxed{\mathbf{F}}\mathbf{x}' = 0$$

fundamental matrix

The fundamental matrix F

- The fundamental matrix is the algebraic representation of epipolar geometry
- The fundamental matrix satisfies the condition that for any pair of corresponding points x↔x' in the two images

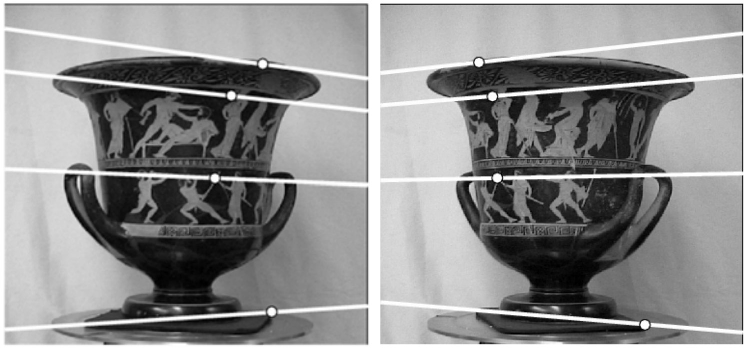
$$\mathrm{x}^{\mathrm{T}}\mathrm{F}\mathrm{x}' = 0 \qquad \left(\mathrm{x}^{\mathrm{T}}\mathrm{l} = 0\right)$$

The fundamental matrix F

F is the unique 3x3 rank 2 matrix that satisfies $x^TFx'=0$ for all $x \leftrightarrow x'$

1. **Transpose:** if F is fundamental matrix for (x,x'), then F^T is fundamental matrix for (x',x)
2. **Epipolar lines:** $l=Fx'$ & $l'=F^Tx$
3. **Epipoles:** on all epipolar lines, thus $e^TFx'=0$, $\forall x'$ $\Rightarrow e^TF=0$, similarly $Fe'=0$
4. **F** has 7 d.o.f. , i.e. 3x3-1(homogeneous)-1(rank2)
5. **F** is a correlation, projective mapping from a point x to a line $l=Fx'$ (not a proper correlation, i.e. not invertible)

The fundamental matrix F



- It can be used for
  - Simplifies matching
  - Allows to detect wrong matches

Estimation of F — 8-point algorithm

- The fundamental matrix F is defined by

$$\mathbf{x}^{\mathrm{T}}\mathbf{F}\mathbf{x}'=0$$

for any pair of matches **x** and **x**' in two images.

- Let **x**=$(u,v,1)^T$ and **x**'=$(u',v',1)^T$, $\mathbf{F}=\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

each match gives a linear equation

$$uu'f_{11}+uv'f_{12}+uf_{13}+vu'f_{21}+vv'f_{22}+vf_{23}+u'f_{31}+v'f_{32}+f_{33}=0$$

8-point algorithm

$$\begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af}=0$, we seek **f** to minimize $\|\mathbf{Af}\|$ subj. $\|\mathbf{f}\|=1$. Find the vector corresponding to the least singular value.

8-point algorithm

- To enforce that F is of rank 2, F is replaced by F' that minimizes $\|\mathbf{F}-\mathbf{F}'\|$ subject to $\det \mathbf{F}'=0$.
- It is achieved by SVD. Let $\mathbf{F}=\mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}$, where

$$\Sigma=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}, \text{ let } \Sigma'=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & 0\end{bmatrix}$$

then $\mathbf{F}'=\mathbf{U}\Sigma'\mathbf{V}^{\mathrm{T}}$ is the solution.

8-point algorithm

```
% Build the constraint matrix
   A = [x2(1,:)'.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
        x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
        x1(1,:)'             x1(2,:)'            ones(npts,1) ];

   [U,D,V] = svd(A);

% Extract fundamental matrix from the column of V
% corresponding to the smallest singular value.
   F = reshape(V(:,9),3,3)';

% Enforce rank2 constraint
   [U,D,V] = svd(F);
   F = U*diag([D(1,1) D(2,2) 0])*V';
```

8-point algorithm

- Pros: it is linear, easy to implement and fast
- Cons: susceptible to noise

Problem with 8-point algorithm

$$\begin{array}{c}\begin{bmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1\\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ u_nu_n' & u_nv_n' & u_n & v_nu_n' & v_nv_n' & v_n & u_n' & v_n' & 1\end{bmatrix} \\ \begin{array}{ccccccccc} \sim10000 & \sim10000 & \sim100 & \sim10000 & \sim10000 & \sim100 & \sim100 & \sim100 & 1\end{array}\end{array} \begin{bmatrix} f_{11}\\ f_{12}\\ f_{13}\\ f_{21}\\ f_{22}\\ f_{23}\\ f_{31}\\ f_{32}\\ f_{33}\end{bmatrix}=0$$

!

Orders of magnitude difference
between column of data matrix
→ least-squares yields poor results

Normalized 8-point algorithm

1. Transform input by $\hat{\mathbf{x}}_i = \mathbf{T}\mathbf{x}_i$, $\hat{\mathbf{x}}'_i = \mathbf{T}'\mathbf{x}'_i$
2. Call 8-point on $\hat{\mathbf{x}}_i, \hat{\mathbf{x}}'_i$ to obtain $\hat{\mathbf{F}}$
3. $\mathbf{F} = \mathbf{T}'^{T}\hat{\mathbf{F}}\mathbf{T}$

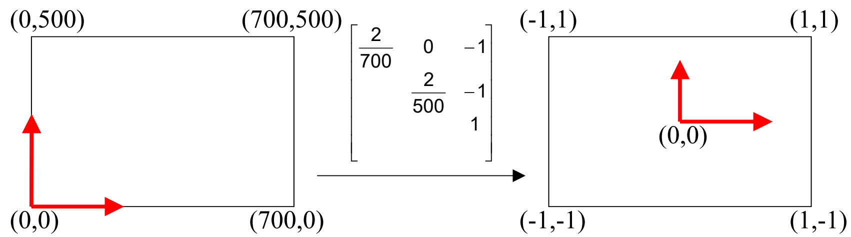
$$\mathbf{x}'^{T}\mathbf{F}\mathbf{x} = 0$$

$$\hat{\mathbf{x}}'^{T}\underbrace{\mathbf{T}'^{-T}\mathbf{F}\mathbf{T}^{-1}}_{\hat{\mathbf{F}}}\hat{\mathbf{x}} = 0$$

Normalized 8-point algorithm

normalized least squares yields good results

Transform image to ~[-1,1]x[-1,1]



Normalized 8-point algorithm

```
[x1, T1] = normalise2dpts(x1);
[x2, T2] = normalise2dpts(x2);
A = [x2(1,:)'.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
     x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
     x1(1,:)'             x1(2,:)'            ones(npts,1) ];

[U,D,V] = svd(A);

F = reshape(V(:,9),3,3)';

[U,D,V] = svd(F);
F = U*diag([D(1,1) D(2,2) 0])*V';

% Denormalise
F = T2'*F*T1;
```

Normalization

```
function [newpts, T] = normalise2dpts(pts)

  c = mean(pts(1:2,:)')';   % Centroid
  newp(1,:) = pts(1,:)-c(1); % Shift origin to centroid.
  newp(2,:) = pts(2,:)-c(2);

  meandist = mean(sqrt(newp(1,:).^2 + newp(2,:).^2));
  scale = sqrt(2)/meandist;

  T = [scale     0    -scale*c(1)
         0     scale  -scale*c(2)
         0       0        1      ];
  newpts = T*pts;
```

RANSAC

repeat

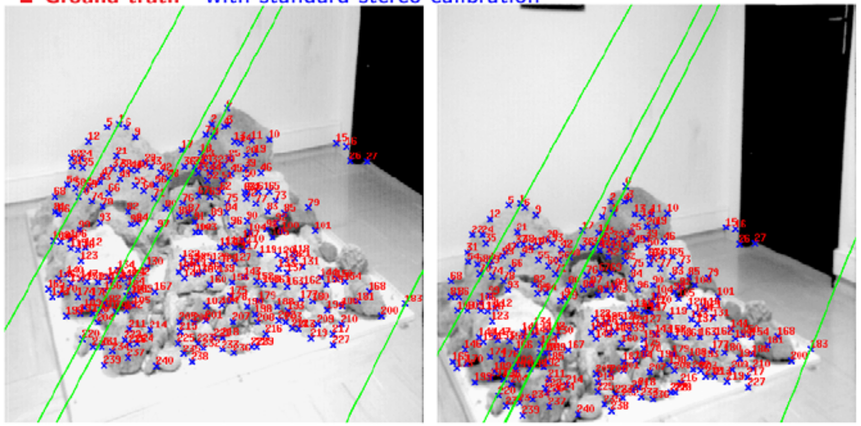
- select minimal sample (8 matches)
- compute solution(s) for F
- determine inliers

until Γ(*#inliers*,*#samples*)>95% or too many times

compute F based on all inliers

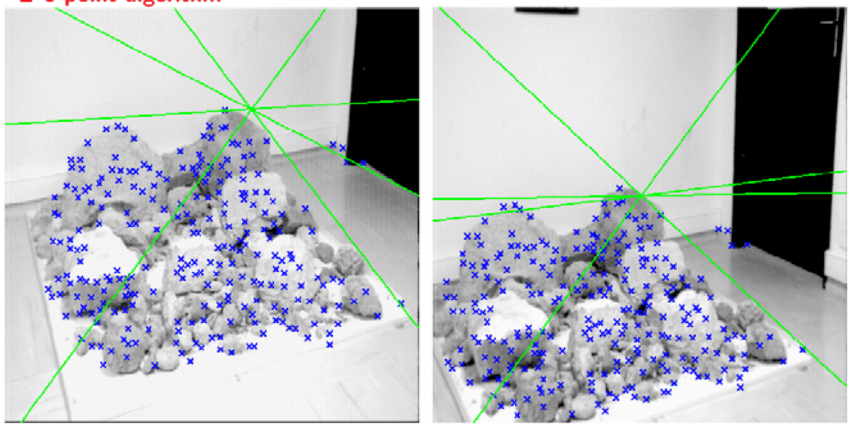
Results (ground truth)

- Ground truth with standard stereo calibration



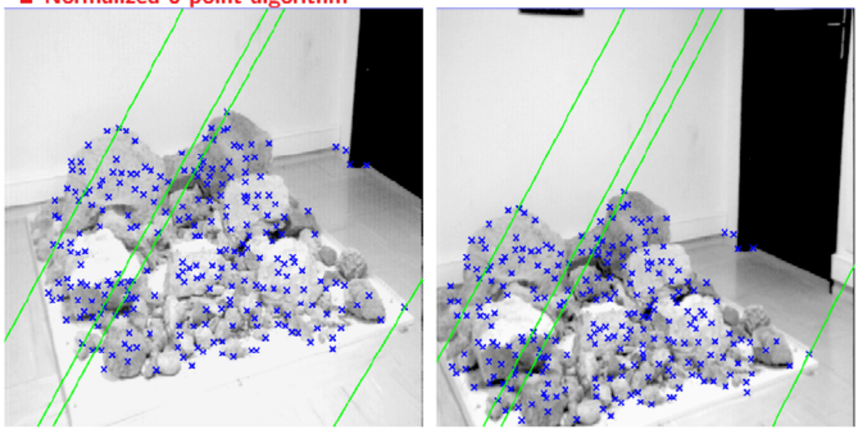
Results (8-point algorithm)

- 8-point algorithm



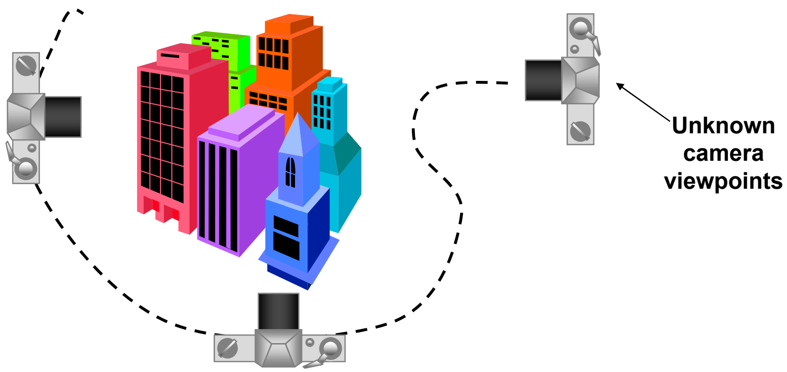
Results (normalized 8-point algorithm)

- Normalized 8-point algorithm



Structure from motion

Structure from motion



structure for motion: automatic recovery of **camera motion** and **scene structure** from two or more images. It is a self calibration technique and called *automatic camera tracking* or *matchmoving*.

Applications

- For computer vision, multiple-view shape reconstruction, novel view synthesis and autonomous vehicle navigation.
- For film production, seamless insertion of CGI into live-action backgrounds

Matchmove

example #1 example #2 example #3

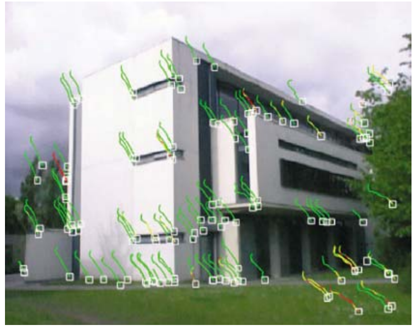
Structure from motion

2D feature tracking → 3D estimation → optimization (bundle adjust) → geometry fitting

SFM pipeline

Structure from motion

- Step 1: Track Features
  - Detect good features, Shi & Tomasi, SIFT
  - Find correspondences between frames
    - Lucas & Kanade-style motion estimation
    - window-based correlation
    - SIFT matching



KLT tracking

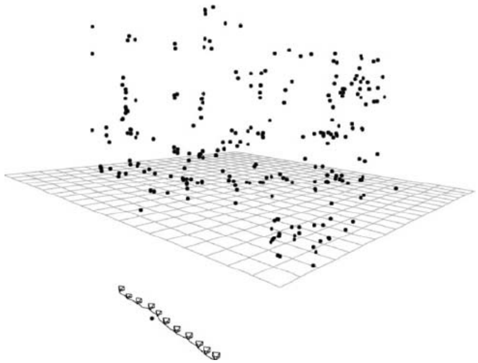
http://www.ces.clemson.edu/~stb/klt/

Structure from Motion

- Step 2: Estimate Motion and Structure
  - Simplified projection model, e.g., **[Tomasi 92]**
  - 2 or 3 views at a time **[Hartley 00]**

Structure from Motion

- Step 3: Refine estimates
  - "Bundle adjustment" in photogrammetry
  - Other iterative methods

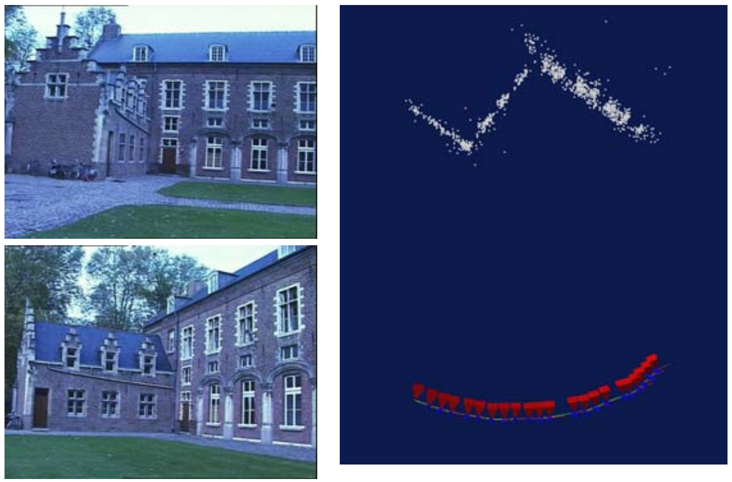


Structure from Motion

- Step 4: Recover surfaces (image-based triangulation, silhouettes, stereo…)

Factorization methods

Problem statement



Notations

- n 3D points are seen in m views
- **q**=*(u,v,1): 2D image point*
- **p**=*(x,y,z,1): 3D scene point*
- Π: projection matrix
- π: projection function
- q_{ij} is the projection of the i-th point on image j
- λ_{ij} projective depth of q_{ij}

$$\mathbf{q}_{ij} = \pi(\Pi_j \mathbf{p}_i) \qquad \pi(x,y,z) = (x/z, y/z)$$

$$\lambda_{ij} = z$$

Structure from motion

- Estimate Π_j and $\mathbf{p}_i$ to minimize

$$\varepsilon(\mathbf{\Pi}_1,\cdots,\mathbf{\Pi}_m,\mathbf{p}_1,\cdots,\mathbf{p}_n) = \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij} \log P(\pi(\mathbf{\Pi}_j \mathbf{p}_i); \mathbf{q}_{ij})$$

$$w_{ij} = \begin{cases} 1 & \text{if } p_i \text{ is visible in view } j \\ 0 & \text{otherwise} \end{cases}$$

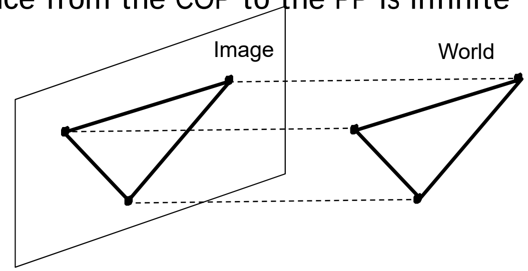
- Assume isotropic Gaussian noise, it is reduced to

$$\varepsilon(\mathbf{\Pi}_1,\cdots,\mathbf{\Pi}_m,\mathbf{p}_1,\cdots,\mathbf{p}_n) = \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ij} \left\| \pi(\mathbf{\Pi}_j \mathbf{p}_i) - \mathbf{q}_{ij} \right\|^2$$

- Start from a simpler projection model

Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

  - Also called "parallel projection": (x, y, z) → (x, y)

SFM under orthographic projection

2D image point | **Orthographic projection incorporating 3D rotation** | **3D scene point** | **image offset**

$$\underset{2\times 1}{\mathbf{q}} = \underset{2\times 3}{\mathbf{\Pi}}\,\underset{3\times 1}{\mathbf{p}} + \underset{2\times 1}{\mathbf{t}}$$

- Trick
  - Choose scene origin to be centroid of 3D points
  - Choose image origins to be centroid of 2D points
  - Allows us to drop the camera translation:

$$\mathbf{q} = \mathbf{\Pi}\mathbf{p}$$

factorization (Tomasi & Kanade)

projection of *n* features in one image:

$$\underbrace{\begin{bmatrix}\mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_n\end{bmatrix}}_{2\times n} = \underbrace{\prod}_{2\times 3} \underbrace{\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n\end{bmatrix}}_{3\times n}$$

projection of *n* features in *m* images

$$\underbrace{\begin{bmatrix}\mathbf{q}_{11} & \mathbf{q}_{12} & \cdots & \mathbf{q}_{1n}\\ \mathbf{q}_{21} & \mathbf{q}_{22} & \cdots & \mathbf{q}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{q}_{m1} & \mathbf{q}_{m2} & \cdots & \mathbf{q}_{mn}\end{bmatrix}}_{2m\times n} = \underbrace{\begin{bmatrix}\Pi_1\\ \Pi_2\\ \vdots\\ \Pi_m\end{bmatrix}}_{2m\times 3} \underbrace{\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n\end{bmatrix}}_{3\times n}$$

W measurement **M** motion **S** shape

Key Observation: *rank*(**W**) <= 3

Factorization

$$\text{known} - \underbrace{\mathbf{W}}_{2m\times n} = \boxed{\underbrace{\mathbf{M}}_{2m\times 3}\ \underbrace{\mathbf{S}}_{3\times n}} - \text{solve for}$$

- Factorization Technique
  - **W** is at most rank 3 (assuming no noise)
  - We can use *singular value decomposition* to factor **W**:

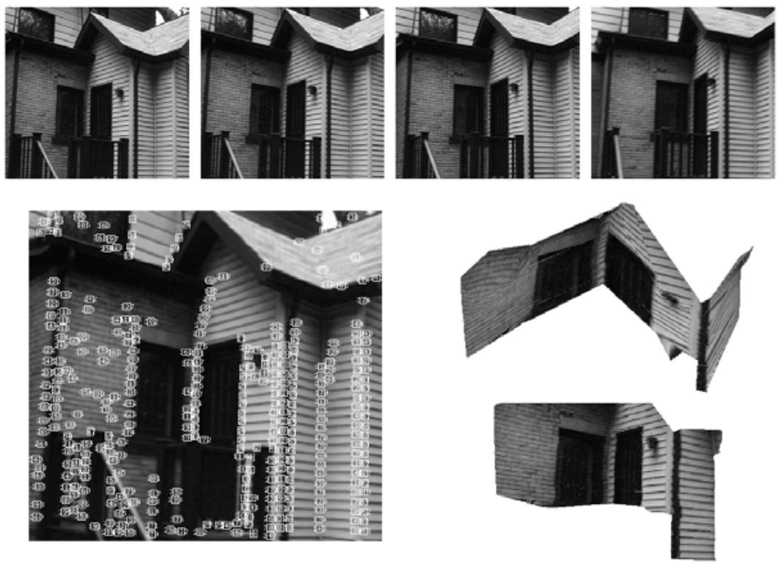
$$\underbrace{\mathbf{W}}_{2m\times n} = \underbrace{\mathbf{M'}}_{2m\times 3}\ \underbrace{\mathbf{S'}}_{3\times n}$$

  - **S'** differs from **S** by a linear transformation *A*:

$$\mathbf{W} = \mathbf{M'S'} = (\mathbf{MA}^{-1})(\mathbf{AS})$$

  - Solve for **A** by enforcing *metric* constraints on **M**

Results



Extensions to factorization methods

- Projective projection
- With missing data
- Projective projection with missing data

Bundle adjustment

Bundle adjustment

- n 3D points are seen in m views
- x_{ij} is the projection of the i-th point on image j
- a_j is the parameters for the j-th camera
- b_i is the parameters for the i-th point
- BA attempts to minimize the projection error

$$\min_{\mathbf{a}_j,\mathbf{b}_i} \sum_{i=1}^{n} \sum_{j=1}^{m} d(\mathbf{Q}(\mathbf{a}_j,\mathbf{b}_i),\ \mathbf{x}_{ij})^2$$

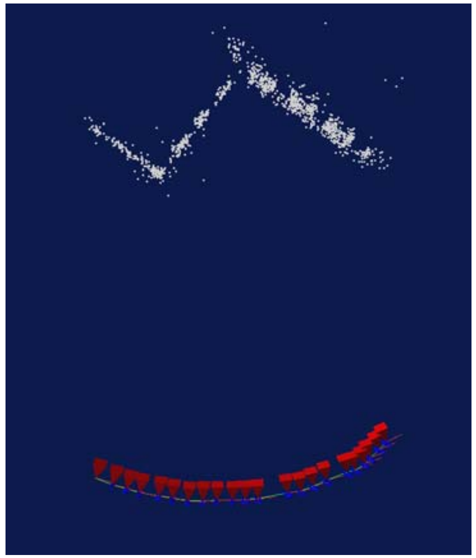
predicted projection

Euclidean distance

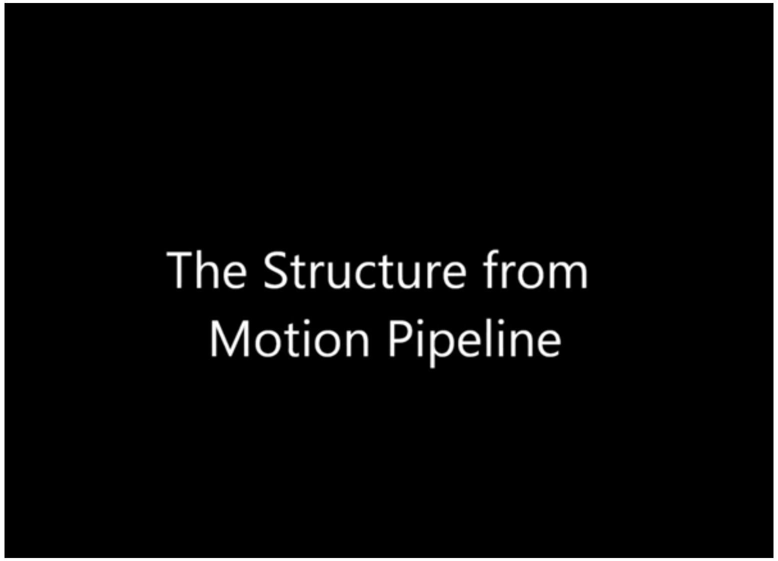
Levenberg-Marquardt method

- LM can be thought of as a combination of steepest descent and the Newton method. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method: slow, but guaranteed to converge. When the current solution is close to the correct solution, it becomes a Newton's method.

Bundle adjustment



MatchMove



Applications of matchmove

Jurassic park

2d3 boujou

Enemy at the Gate, Double Negative

2d3 boujou

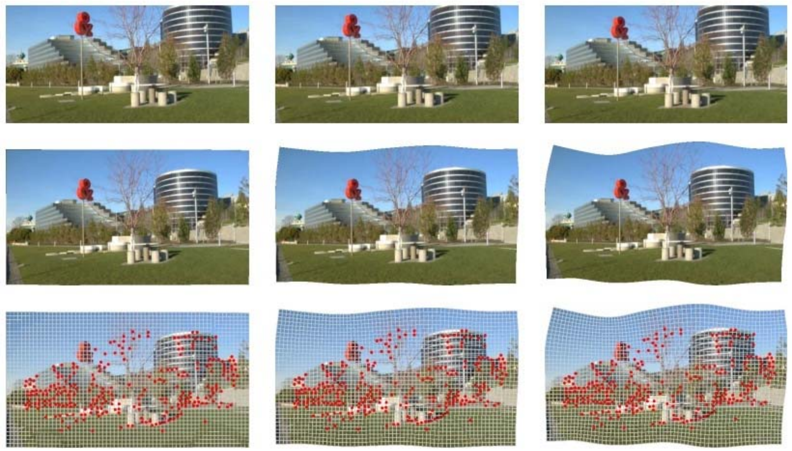
Enemy at the Gate, Double Negative

Photo Tourism

VideoTrace

http://www.acvt.com.au/research/videotrace/

Video stabilization



References

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- Manolis Lourakis and Antonis Argyros, The Design and Implementation of a Generic Sparse Bundle Adjustment Software Package Based on the Levenberg-Marquardt Algorithm, FORTH-ICS/TR-320 2004.
- N. Snavely, S. Seitz, R. Szeliski, Photo Tourism: Exploring Photo Collections in 3D, SIGGRAPH 2006.
- A. Hengel et. al., VideoTrace: Rapid Interactive Scene Modelling from Video, SIGGRAPH 2007.

Project #3 MatchMove

- It is more about using tools in this project
- You can choose either calibration or structure from motion to achieve the goal
- Calibration
- Voodoo/Icarus

- Examples from previous classes, #1, #2
- https://www.youtube.com/user/theActionMovieKid/videos