Gradient domain operations

Digital Visual Effects

Yung-Yu Chuang

with slides by Fredo Durand, Ramesh Raskar, Amit Agrawal

Gradient domain operators

sources/destinations — cloning — seamless cloning

Gradient Domain Manipulations

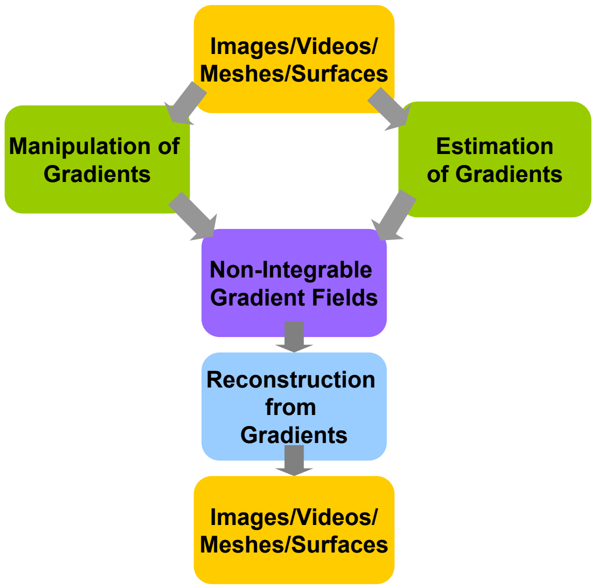
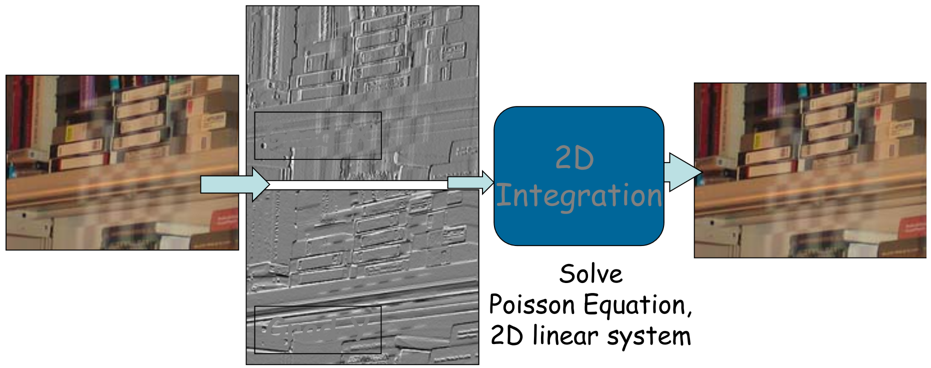
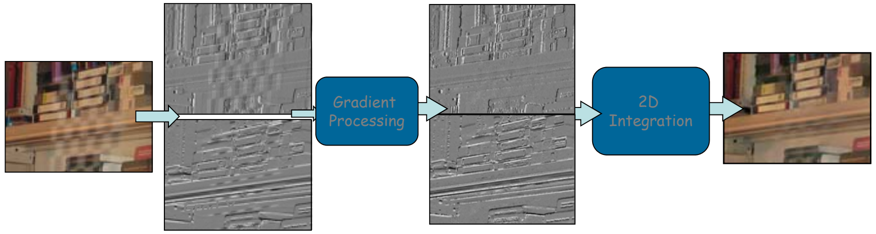


Image Intensity Gradients in 2D



Intensity Gradient Manipulation

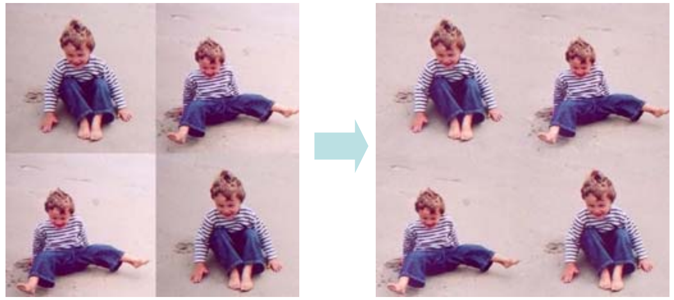
A Common Pipeline



1. Gradient manipulation
2. Reconstruction from gradients

Example Applications

Removing Glass Reflections



Seamless Image Stitching

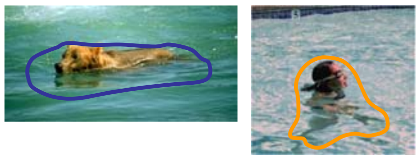
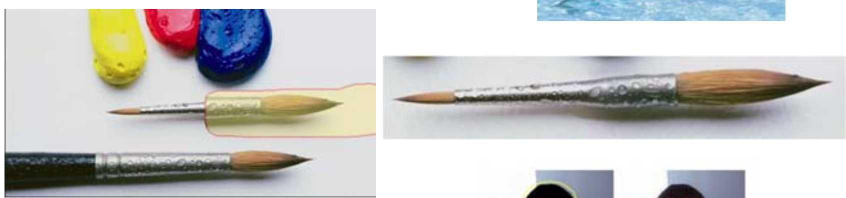
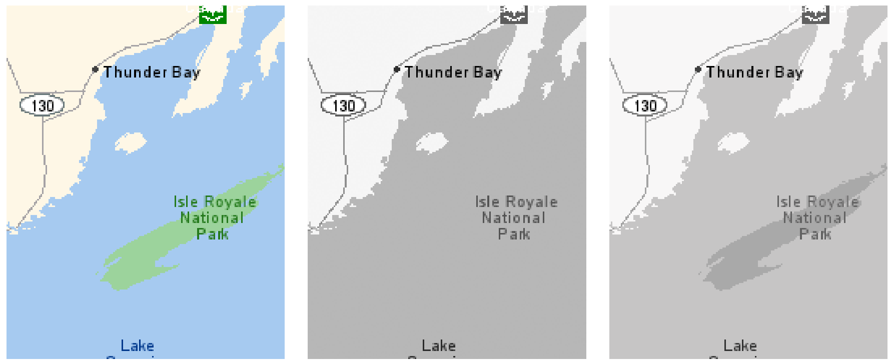


Image Editing



Changing Local Illumination



Original PhotoshopGrey Color2Gray

Color to Gray Conversion

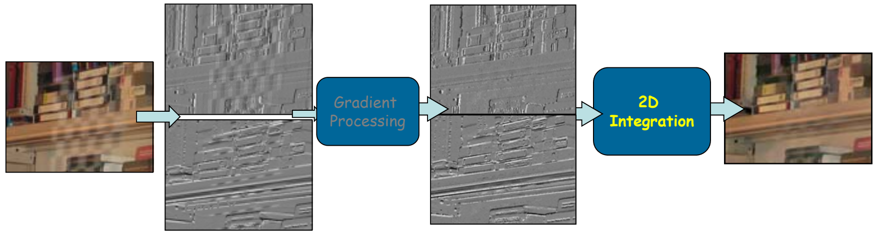
High Dynamic Range Compression

Edge Suppression under Significant Illumination Variations

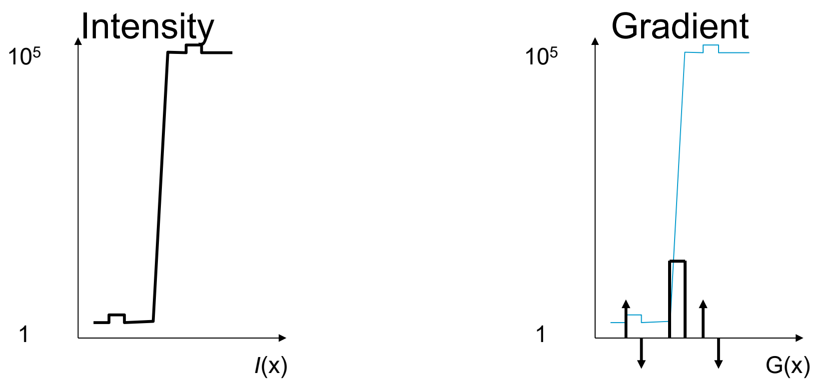
Fusion of day and night images

Intensity Gradient Manipulation

A Common Pipeline



Intensity Gradient in 1D

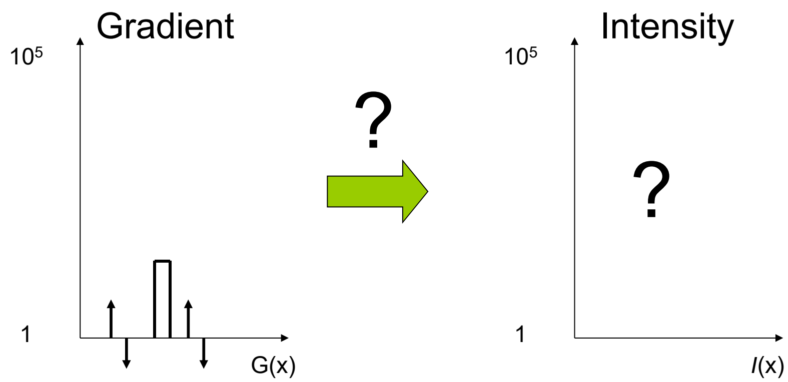


Gradient at x,

$G(x) = I(x+1) - I(x)$

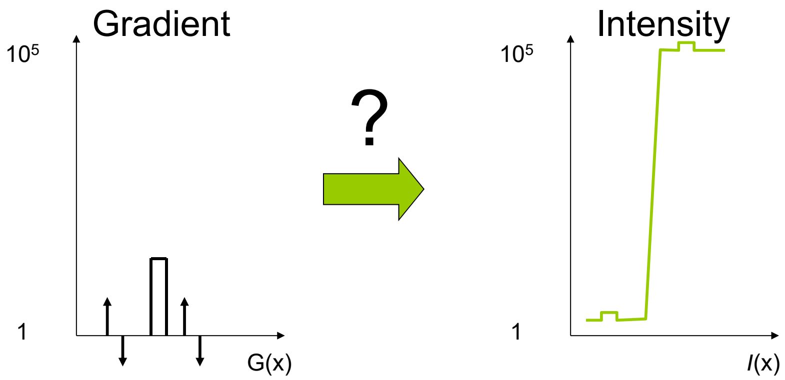
Forward Difference

Reconstruction from Gradients



For *n* intensity values, about *n* gradients

Reconstruction from Gradients



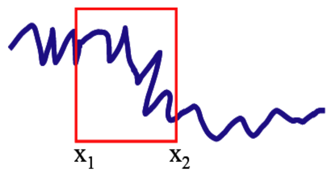
1D Integration

$$I(x) = I(x-1) + G(x)$$

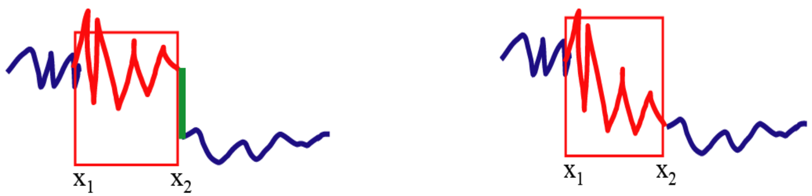
Cumulative sum

DigiVFX

1D case with constraints

Seamlessly paste  onto 

Just add a linear function so that the boundary condition is respected



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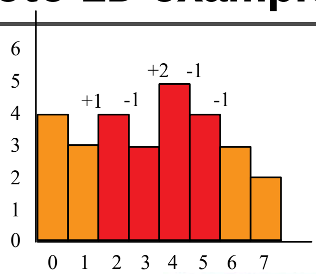
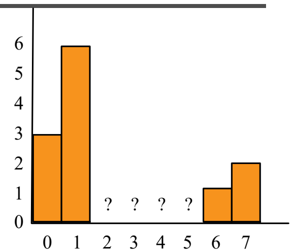
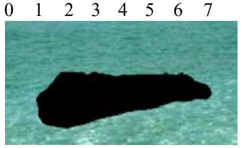
Discrete 1D example: minimization

- Copy



to





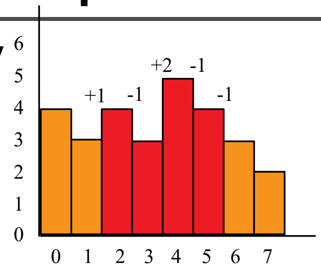
- Min $((f_2-f_1)-1)^2$
- Min $((f_3-f_2)-(-1))^2$
- Min $((f_4-f_3)-2)^2$
- Min $((f_5-f_4)-(-1))^2$
- Min $((f_6-f_5)-(-1))^2$

With
$f_1=6$
$f_6=1$

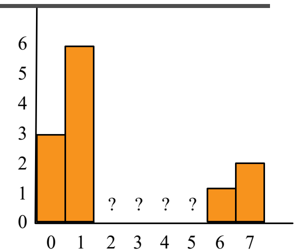
DigiVFX

1D example: minimization

- Copy



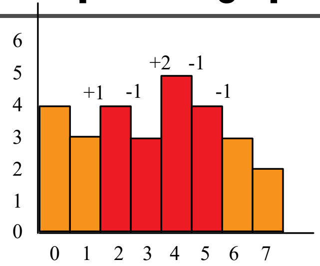
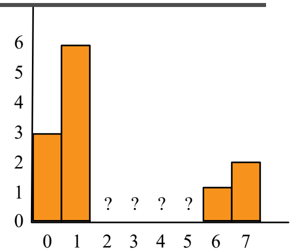
to



- Min $((f_2-6)-1)^2$ ==> $f_2^2+49-14f_2$
- Min $((f_3-f_2)-(-1))^2$ ==> $f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$
- Min $((f_4-f_3)-2)^2$ ==> $f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$
- Min $((f_5-f_4)-(-1))^2$ ==> $f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$
- Min $((1-f_5)-(-1))^2$ ==> $f_5^2+4-4f_5$

1D example: big quadratic

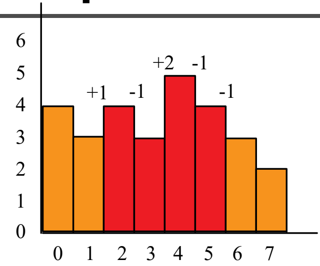
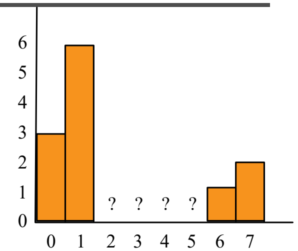
DigiVFX

- Copy  to 

- Min (f_2^2+49-14f_2
  + f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2
  + f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3
  + f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4
  + f_5^2+4-4f_5)
  Denote it Q

1D example: derivatives

DigiVFX

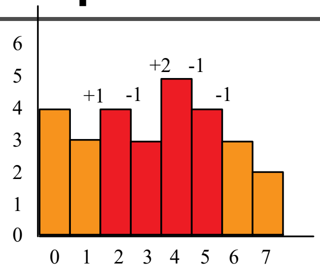
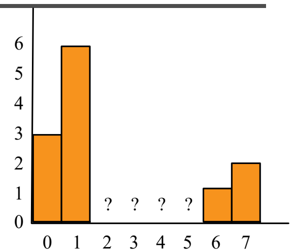
- Copy  to 

Min (f_2^2+49-14f_2
+ f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2
+ f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3
+ f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4
+ f_5^2+4-4f_5)
Denote it Q

$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$
$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$
$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$
$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

1D example: set derivatives to zero

DigiVFX

- Copy  to 

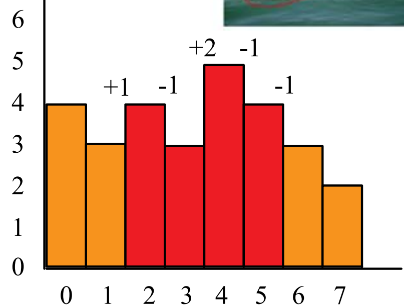
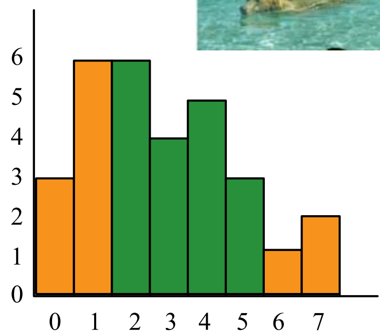
$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$
$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$
$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$
$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

==>
$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

1D example

DigiVFX

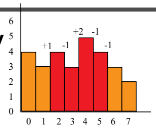
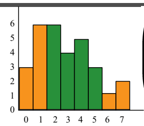
- Copy  to 

$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \\ 3 \end{pmatrix}$$

1D example: remarks

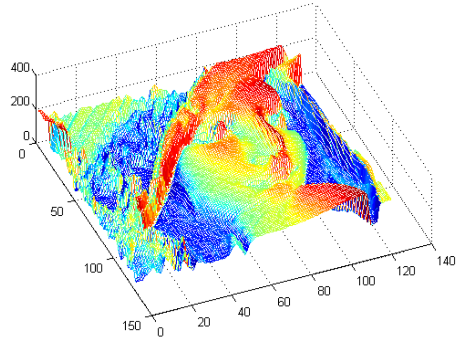
- Copy  to 

$$\begin{pmatrix}4&-2&0&0\\-2&4&-2&0\\0&-2&4&-2\\0&0&-2&4\end{pmatrix}\begin{pmatrix}f_2\\f_3\\f_4\\f_5\end{pmatrix}=\begin{pmatrix}16\\-6\\6\\2\end{pmatrix}$$

- Matrix is sparse
- Matrix is symmetric
- Everything is a multiple of 2
  - because square and derivative of square
- Matrix is a convolution (kernel -2 4 -2)
- Matrix is independent of gradient field. Only RHS is
- Matrix is a second derivative

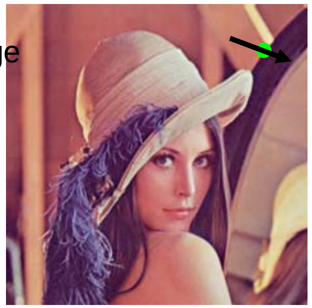
2D example: images

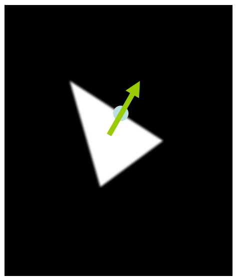
- Images as scalar fields
  - R^2 -> R



Gradients

- Vector field (gradient field)
  - Derivative of a scalar field
- Direction
  - Maximum rate of change of scalar field
- Magnitude
  - Rate of change





Example

Image
I(x,y)

I_x

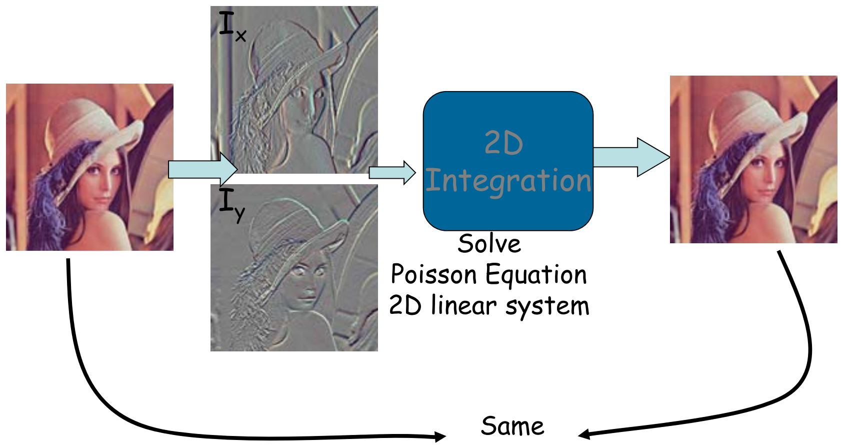
I_y

Gradient at x,y as Forward Differences

$$G_x(x,y) = I(x+1, y) - I(x,y)$$
$$G_y(x,y) = I(x, y+1) - I(x,y)$$
$$G(x,y) = (G_x, G_y)$$

Reconstruction from Gradients

Sanity Check:
Recovering Original Image

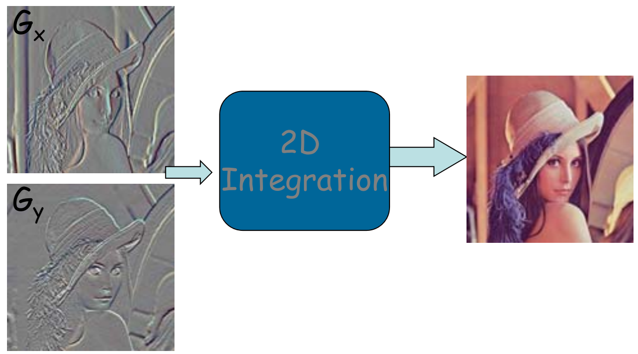


Reconstruction from Gradients

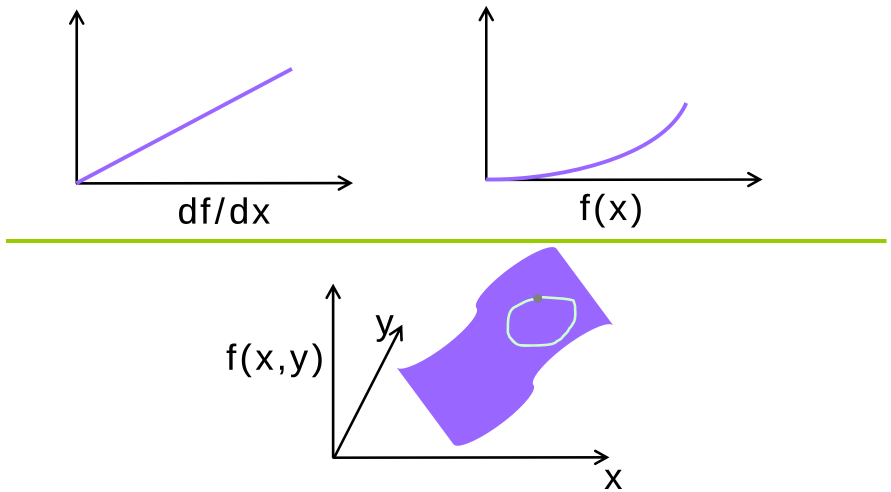
Given $G(x,y) = (G_x, G_y)$

How to compute $I(x,y)$ for the image ?

For n^2 image pixels, $2n^2$ gradients !



2D Integration is non-trivial



Reconstruction depends on chosen path

Reconstruction from Gradient Field G

- Look for image ***I*** with gradient closest to ***G*** in the least squares sense.
- ***I*** minimizes the integral: $\iint F(\nabla I, G)\,dxdy$

$$F(\nabla I, G) = \|\nabla I - G\|^2 = \left(\frac{\partial I}{\partial x} - G_x\right)^2 + \left(\frac{\partial I}{\partial y} - G_y\right)^2$$

$$\Rightarrow \quad \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

Solve $\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$

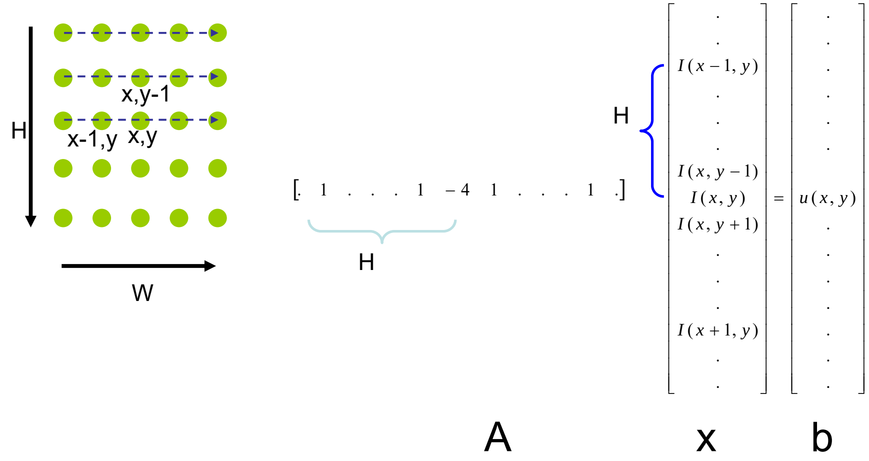
$$G_x(x,y) - G_x(x-1,y) + G_y(x,y) - G_y(x,y-1)$$

$$I(x+1,y) + I(x-1,y) + I(x,y+1) + I(x,y-1) - 4I(x,y)$$

$$\begin{bmatrix} \\ ..\,1\,\ldots\quad 1\,{-4}\,1\,\ldots\quad 1\,.. \\ \\ \end{bmatrix} \begin{bmatrix} \\ \mathbf{I} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Linear System

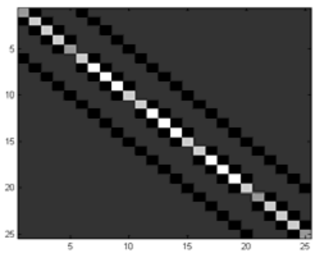
$$-4I(x,y) + I(x,y+1) + I(x,y-1) + I(x+1,y) + I(x-1,y) = u(x,y)$$



Sparse Linear system

$$\begin{bmatrix} & & 1 & -4 & 1 & & & & 1 & & & & \\ & & & 1 & -4 & 1 & & & & 1 & & & \\ 1 & & & & 1 & -4 & 1 & & & & 1 & & \\ & 1 & & & & 1 & -4 & 1 & & & & 1 & \\ & & 1 & & & & 1 & -4 & 1 & & & & 1 \\ & & & 1 & & & & 1 & -4 & 1 & & & \\ & & & & 1 & & & & 1 & -4 & 1 & & \end{bmatrix}$$

A matrix



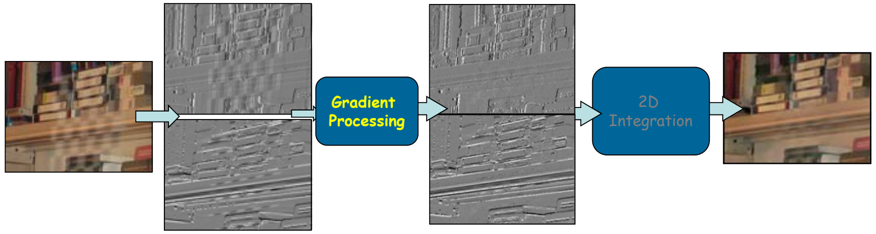
Solving Linear System

- Image size N*N
- Size of A ~ N^2 by N^2
- Impractical to form and store A

- Direct Solvers
- Basis Functions
- Multigrid
- Conjugate Gradients

Intensity Gradient Manipulation

A Common Pipeline



Gradient Domain Manipulations: Overview

(A) Per pixel

(B) Corresponding gradients in <u>two images</u>

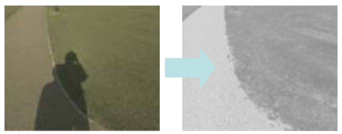
(C) Corresponding gradients in <u>multiple images</u>

(D) Combining gradients <u>along seams</u>

A. Per Pixel Manipulations

- Non-linear operations
  - HDR compression, local illumination change

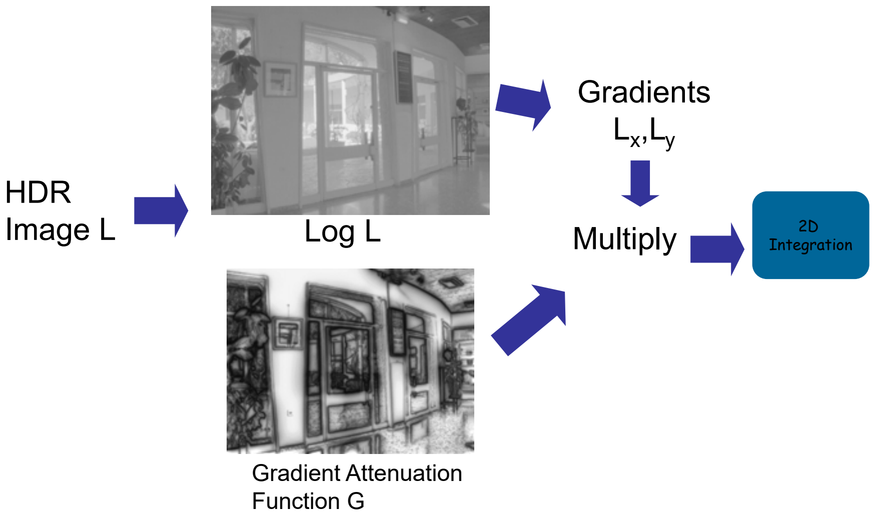
- Set to zero
  - Shadow removal, intrinsic images, texture de-emphasis



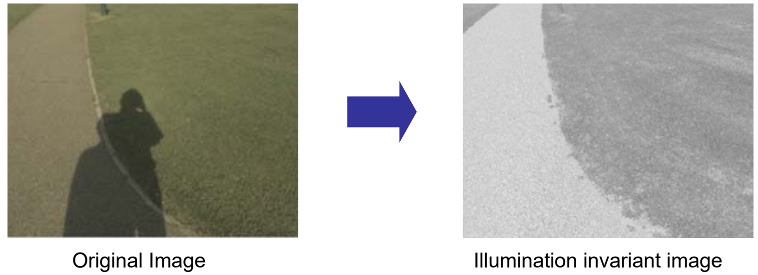
High Dynamic Range Imaging

Images from Raanan Fattal

Gradient Domain Compression



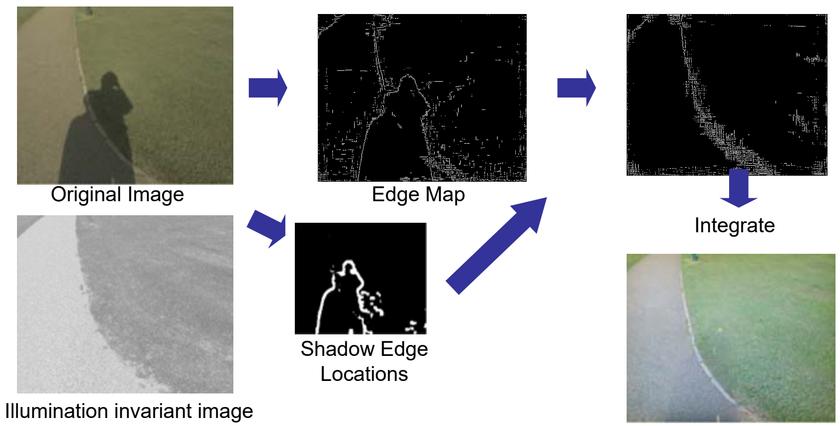
Illumination Invariant Image



- Assumptions
  - Sensor response = delta functions R, G, B in wavelength spectrum
  - Illumination restricted to Outdoor Illumination

G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

Shadow Removal Using Illumination Invariant Image



G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

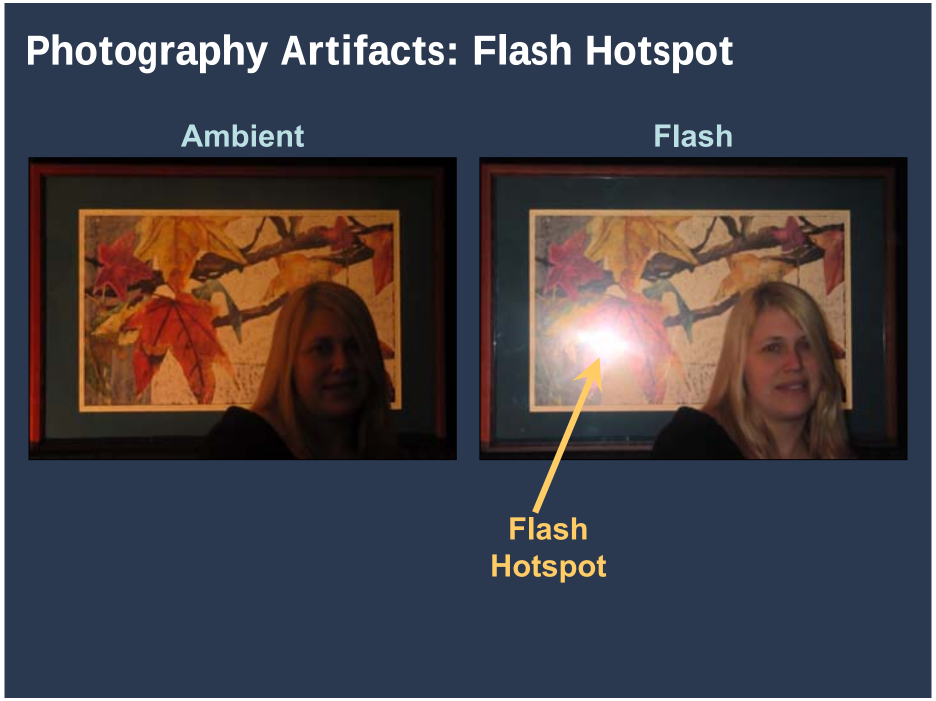
Gradient Domain Manipulations: Overview

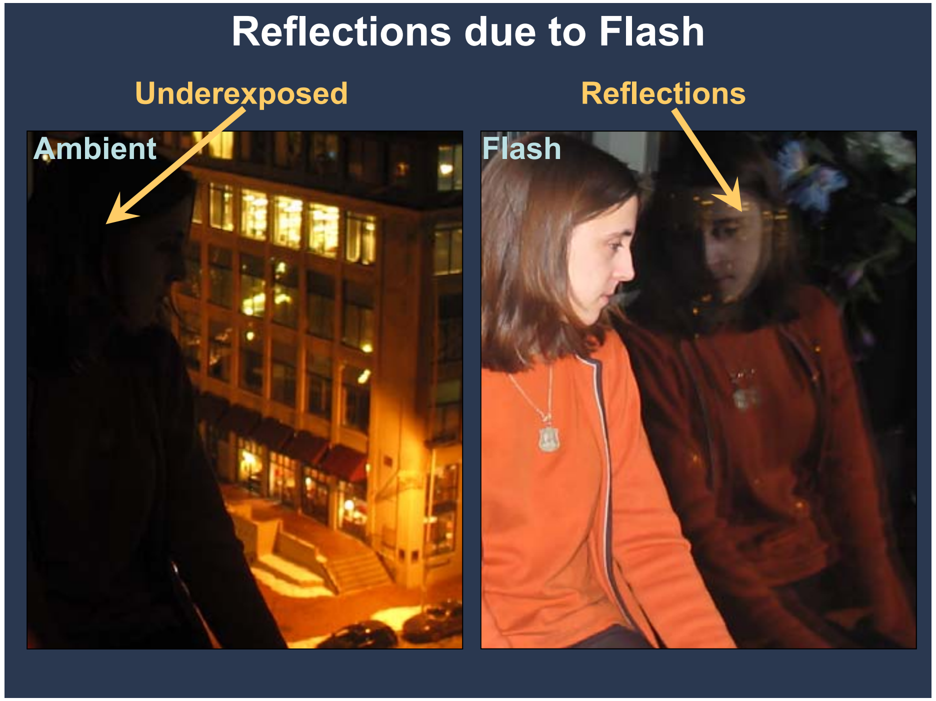
(A) Per pixel

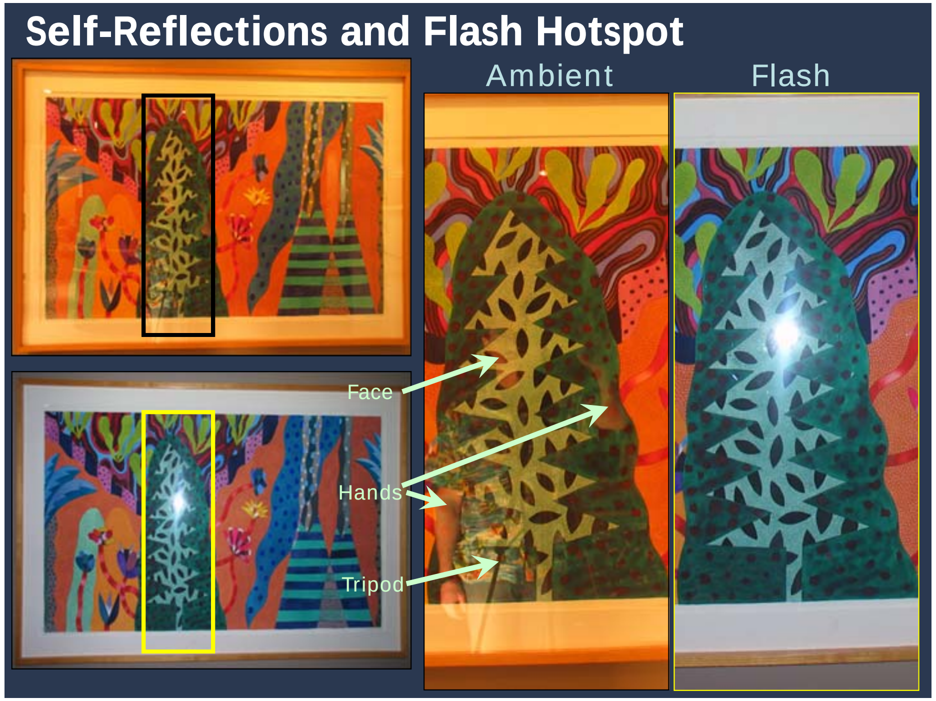
(B) Corresponding gradients in two images

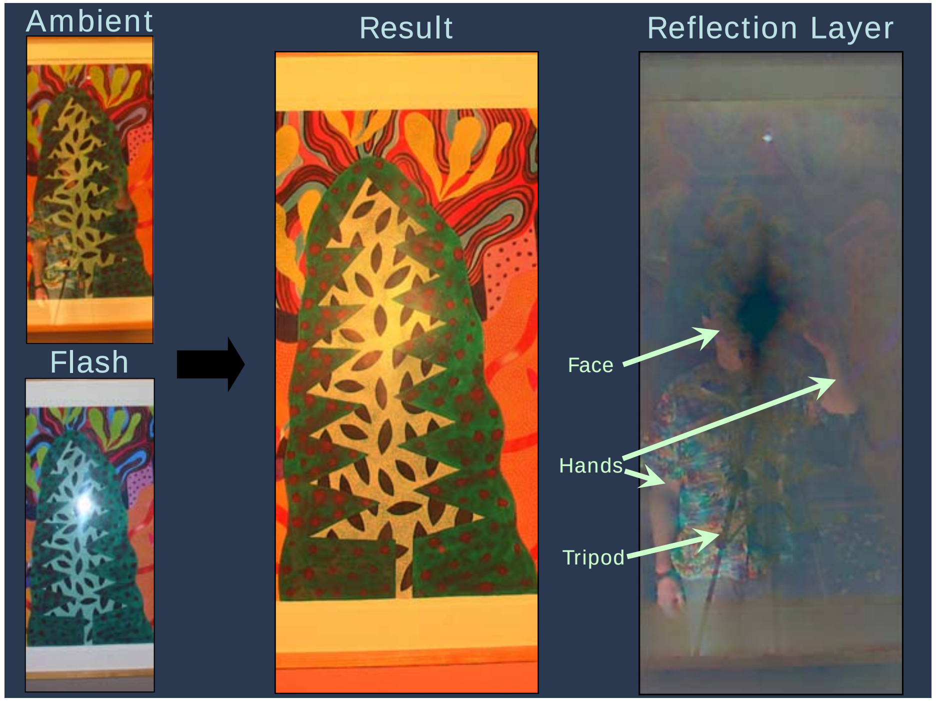
(C) Corresponding gradients in multiple images

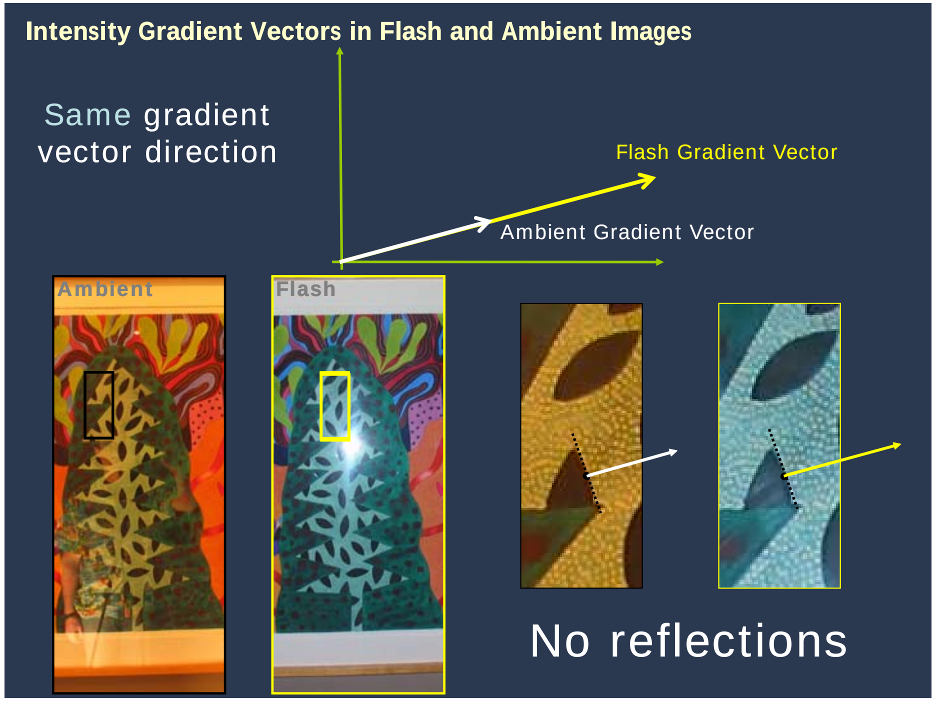
(D) Combining gradients along seams


Photography Artifacts: Flash Hotspot
Ambient
Flash
Flash Hotspot

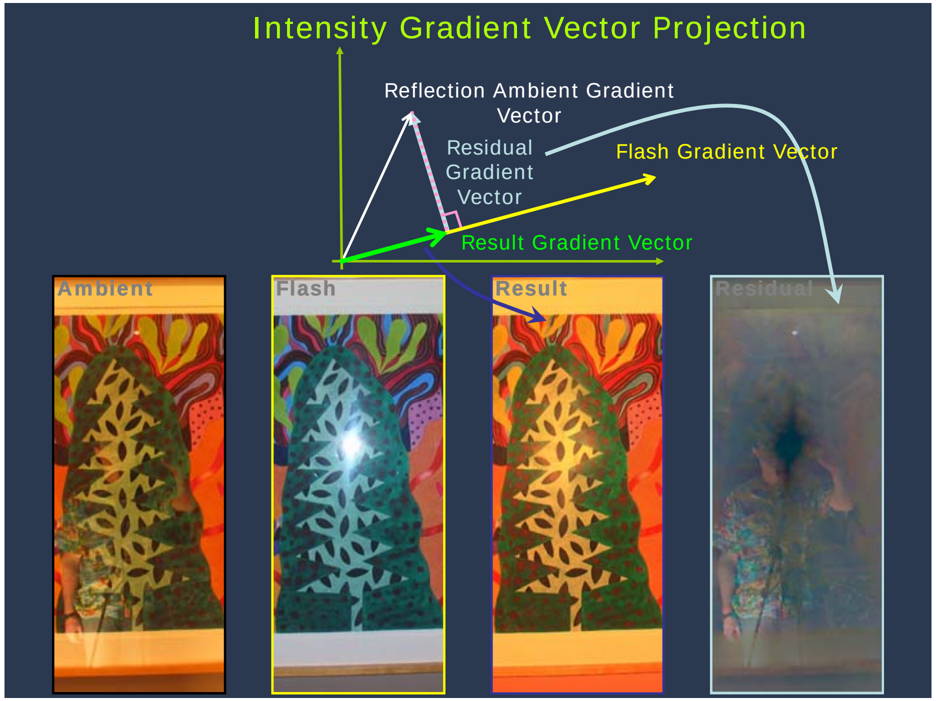

Reflections due to Flash
Underexposed
Reflections
Ambient
Flash

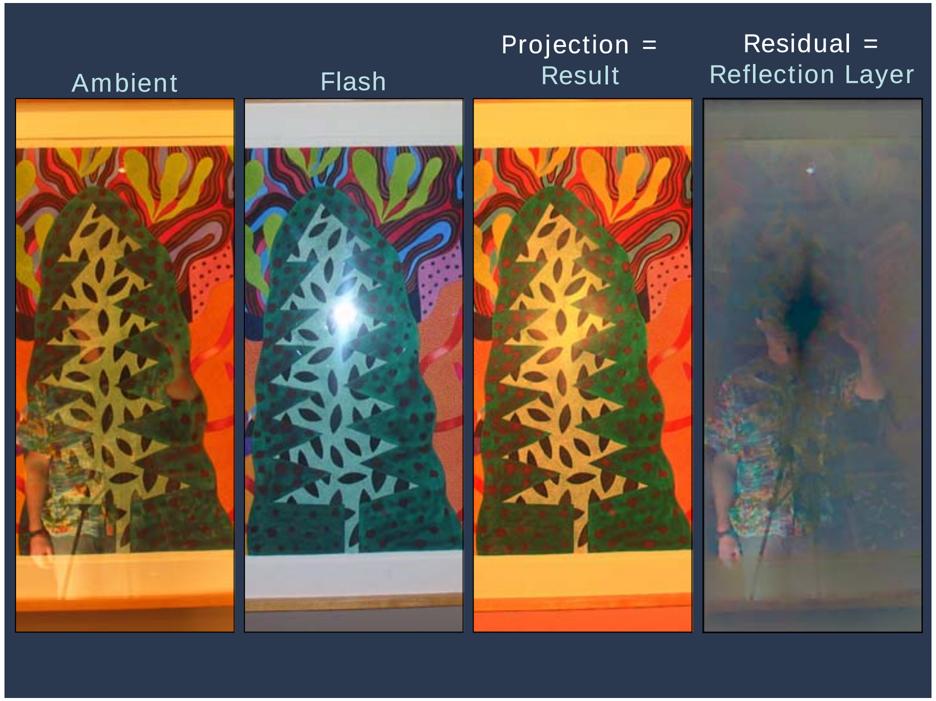

Self-Reflections and Flash Hotspot
Ambient
Flash
Face
Hands
Tripod

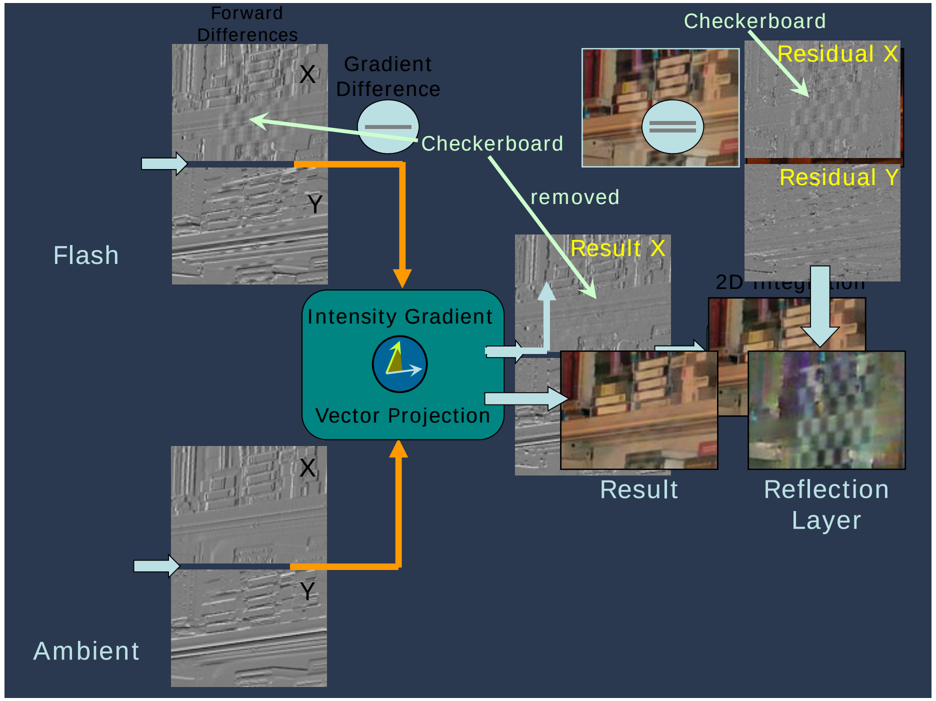

Ambient
Result
Reflection Layer
Flash
Face
Hands
Tripod


Intensity Gradient Vectors in Flash and Ambient Images
Same gradient vector direction
Flash Gradient Vector
Ambient Gradient Vector
Ambient
Flash
No reflections


Reflection Ambient Gradient Vector
Different gradient vector directions
Flash Gradient Vector
Ambient
Flash
With reflections


Intensity Gradient Vector Projection
Reflection Ambient Gradient Vector
Residual Gradient Vector
Flash Gradient Vector
Result Gradient Vector
Ambient
Flash
Result
Residual


Ambient
Flash
Projection = Result
Residual = Reflection Layer



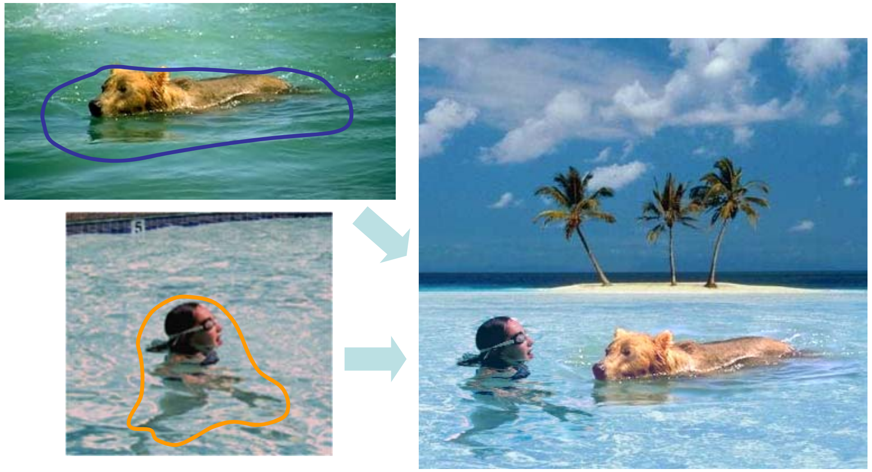
Poisson Image Editing

- Precise selection: tedious and unsatisfactory
- Alpha-Matting: powerful but involved
- Seamless cloning: loose selection but no seams?

Conceal

Copy Background gradients (user strokes)

Compose



Source Images Target Image

Transparent Cloning

$$\mathbf{v} = \frac{\nabla f^* + \nabla g}{2}$$

Largest variation from source and destination at each point

Gradient Domain Manipulations: Overview

(A) Per pixel

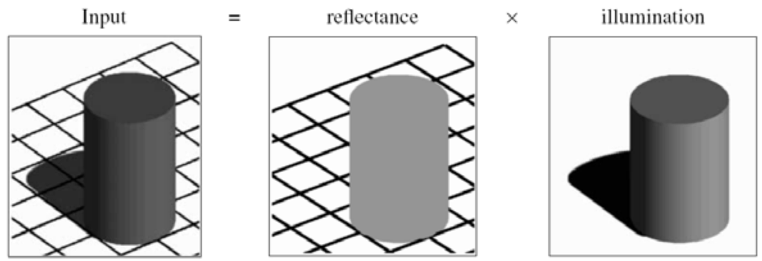
(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Intrinsic images

- I = L * R
- L = illumination image
- R = reflectance image



Intrinsic images

- Use multiple images under different illumination
- Assumption
  - Illumination image gradients = Laplacian PDF
  - Under Laplacian PDF, Median = ML estimator
- At each pixel, take Median of gradients across images
- Integrate to remove shadows

Yair Weiss, "Deriving intrinsic images from image sequences", ICCV 2001

frame 1 | frame 11 | ML reflectance

Shadow free
Intrinsic Image

Result = Illumination Image * (Label in Intrinsic Image)

Gradient Domain Manipulations: Overview

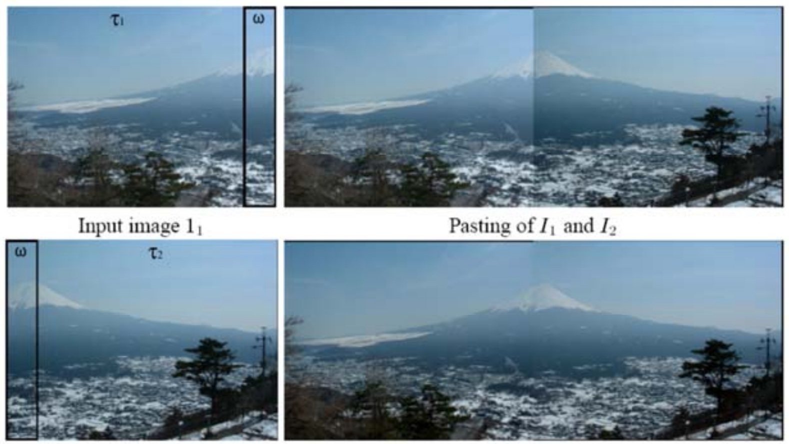
(A) Per pixel

(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Seamless Image Stitching



Input image I_1 | Pasting of I_1 and I_2

Input image I_2 | Stitching result

Anat Levin, Assaf Zomet, Shmuel Peleg and Yair Weiss, "Seamless Image Stitching in the Gradient Domain", ECCV 2004