Gradient domain operations

Digital Visual Effects

Yung-Yu Chuang

with slides by Fredo Durand, Ramesh Raskar, Amit Agrawal

Gradient domain operators

sources/destinations cloning seamless cloning

Gradient Domain Manipulations

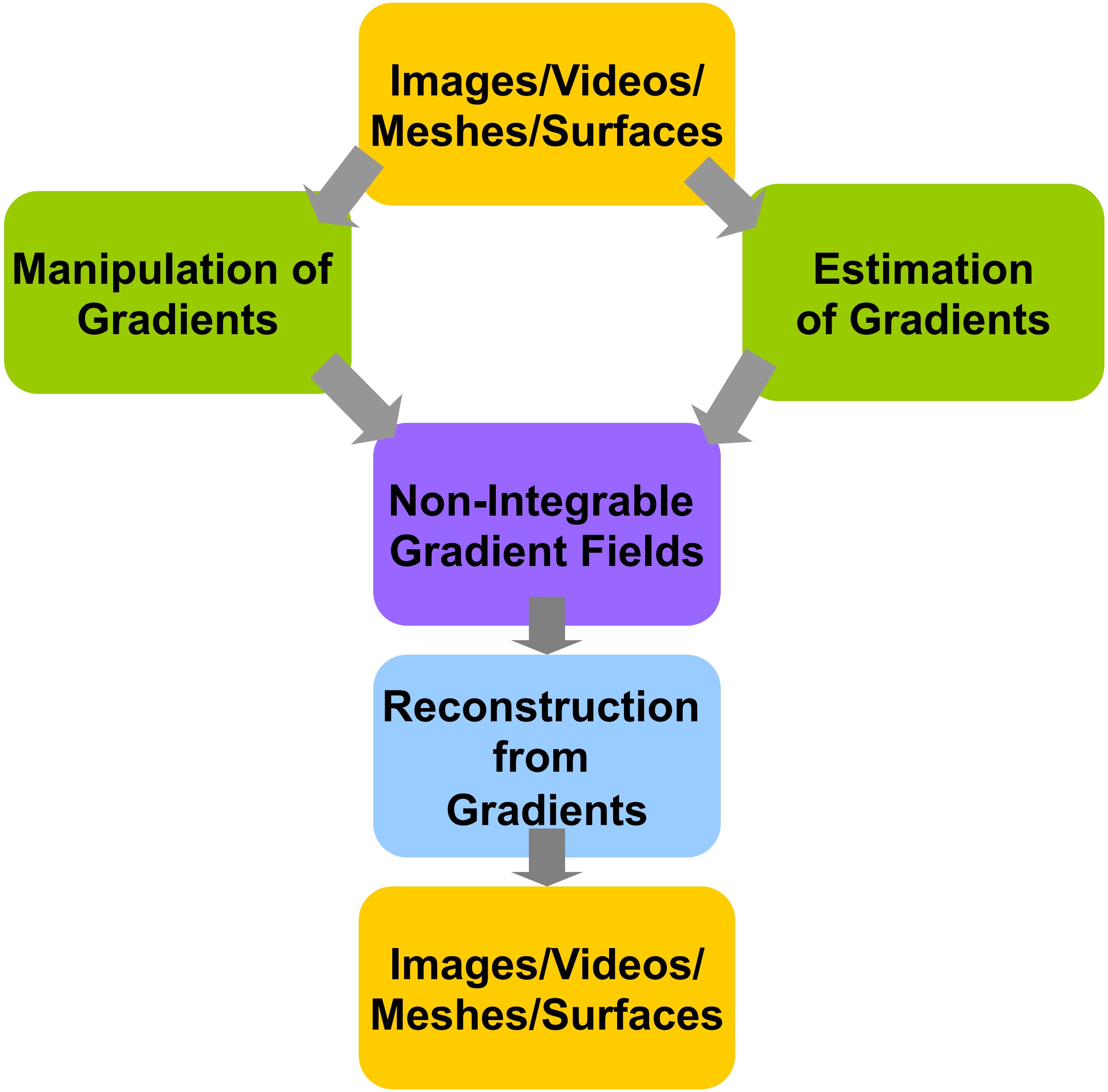
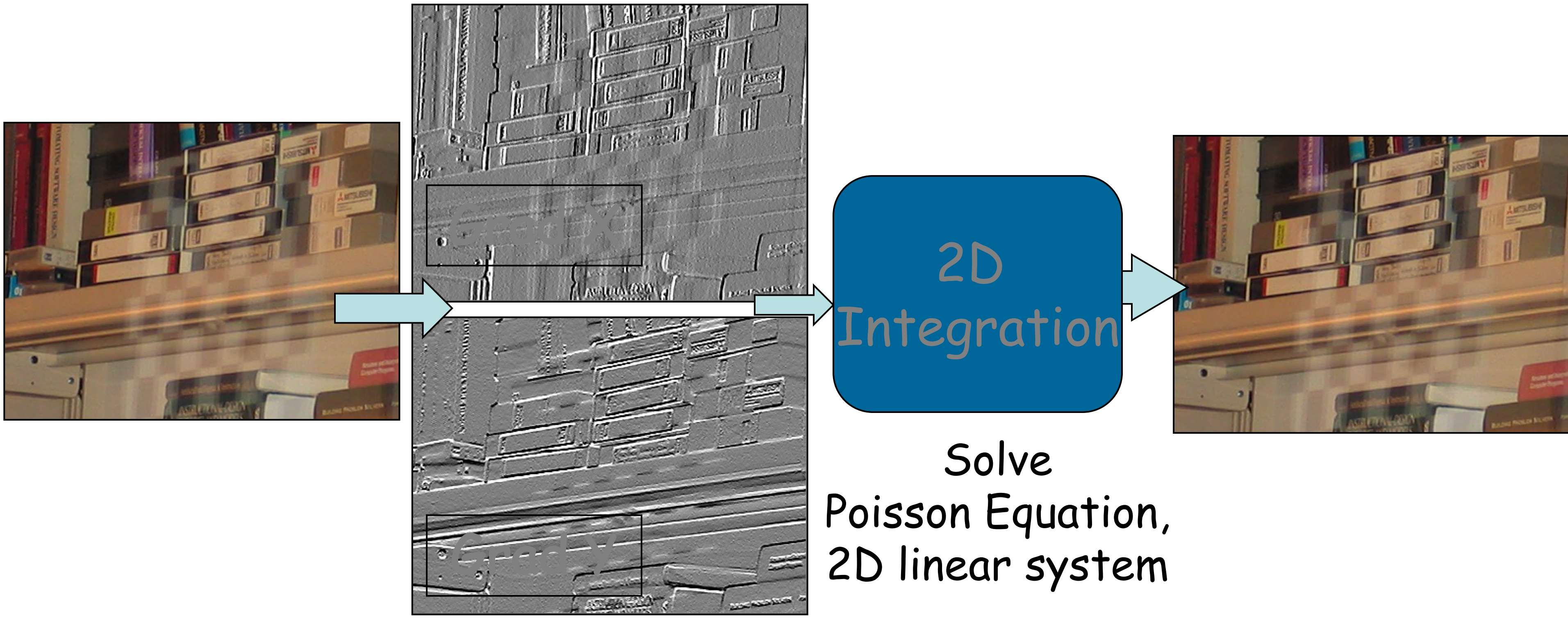
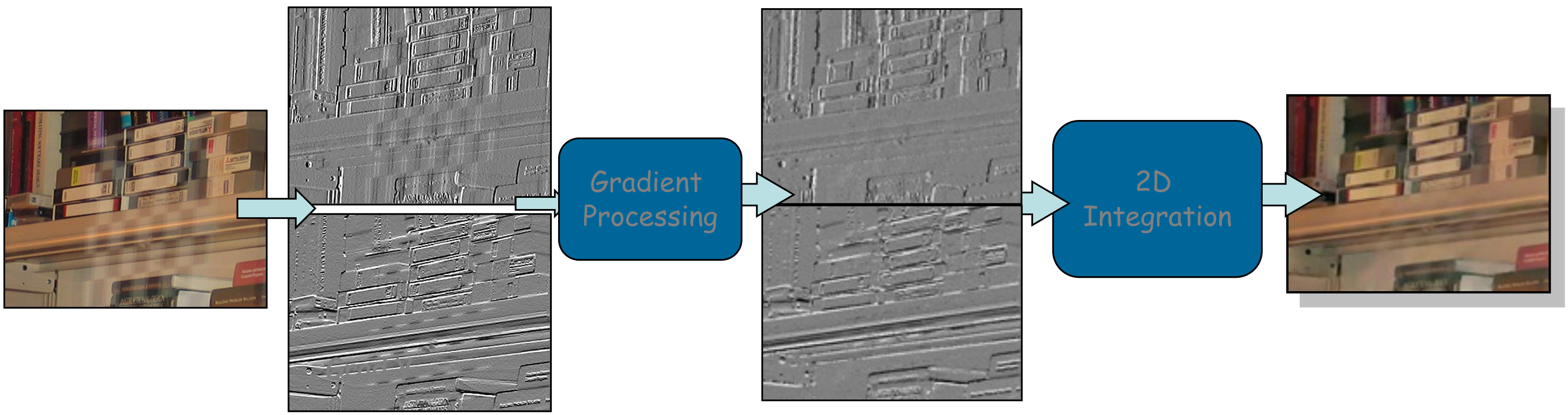


Image Intensity Gradients in 2D



Intensity Gradient Manipulation

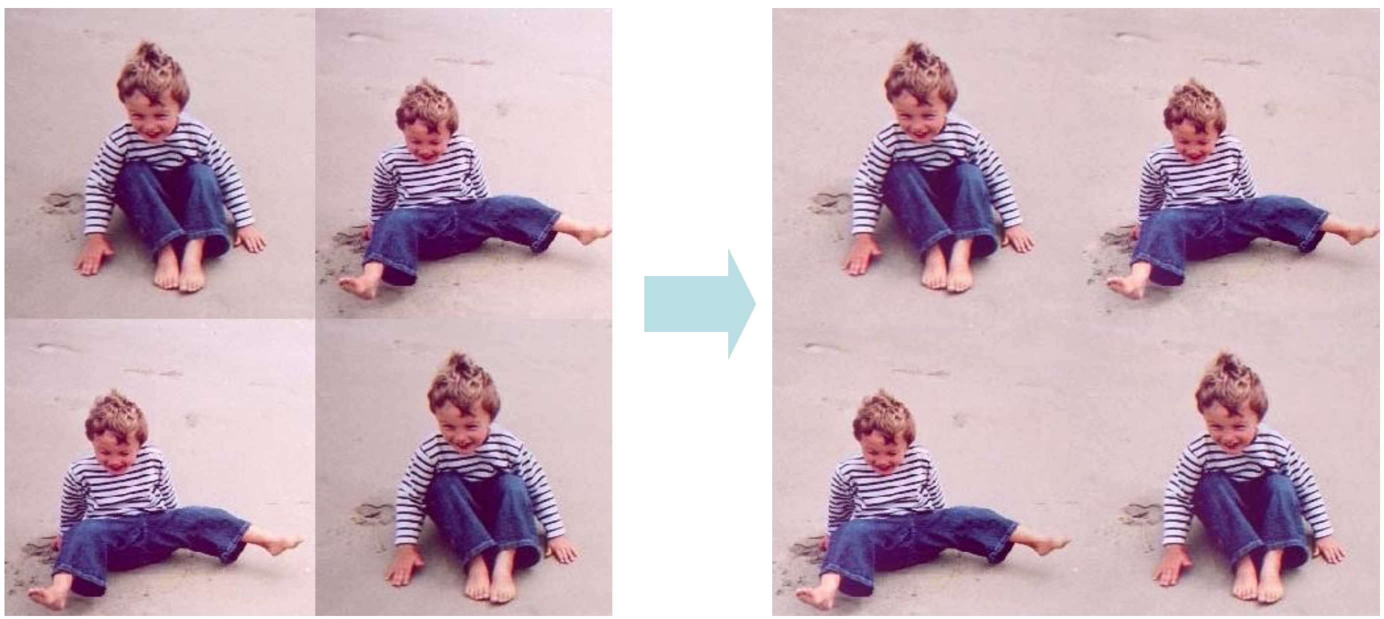
A Common Pipeline



1. Gradient manipulation
2. Reconstruction from gradients

Example Applications

Removing Glass Reflections



Seamless Image Stitching

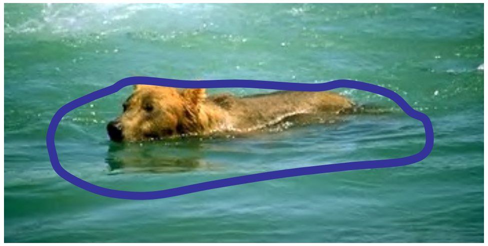
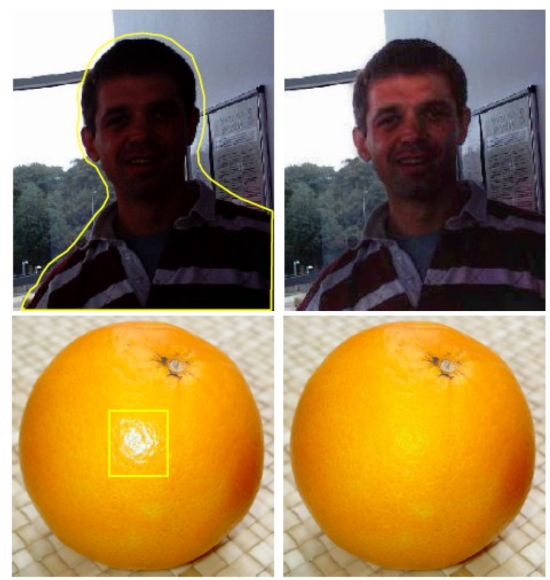


Image Editing

Changing Local Illumination



Original PhotoshopGrey Color2Gray

Color to Gray Conversion

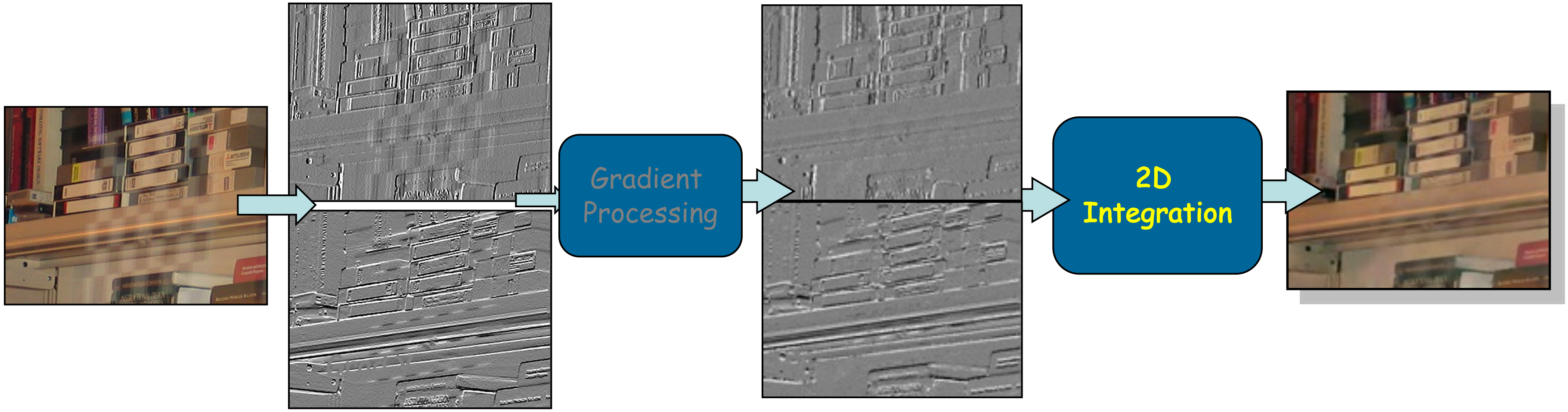
High Dynamic Range Compression

Edge Suppression under Significant Illumination Variations

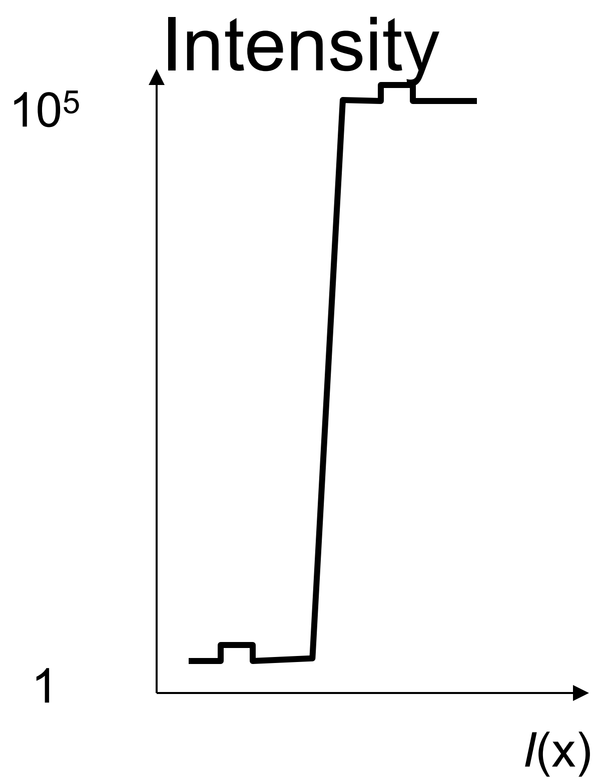
Fusion of day and night images

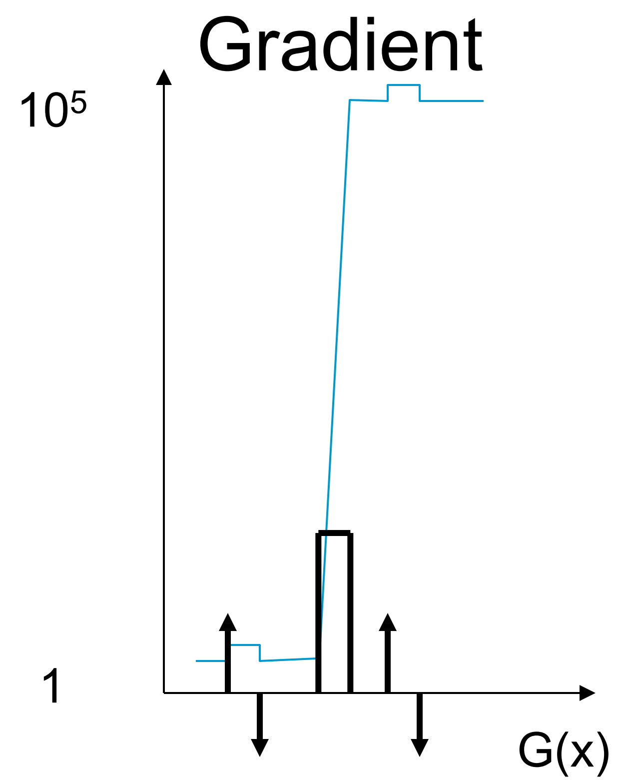
Intensity Gradient Manipulation

A Common Pipeline



Intensity Gradient in 1D



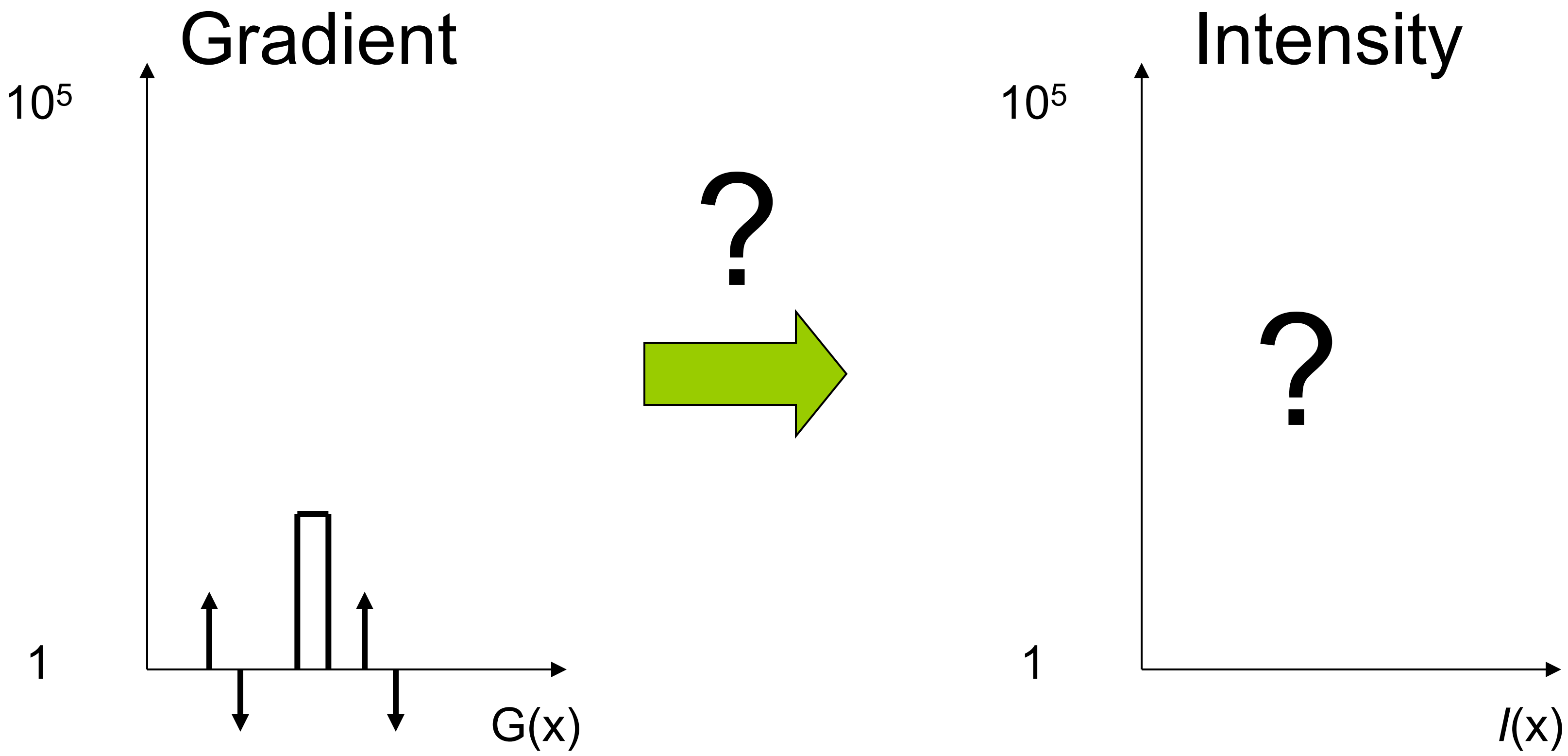


Gradient at x,

$$G(x) \quad = \quad I(x+1) - I(x)$$

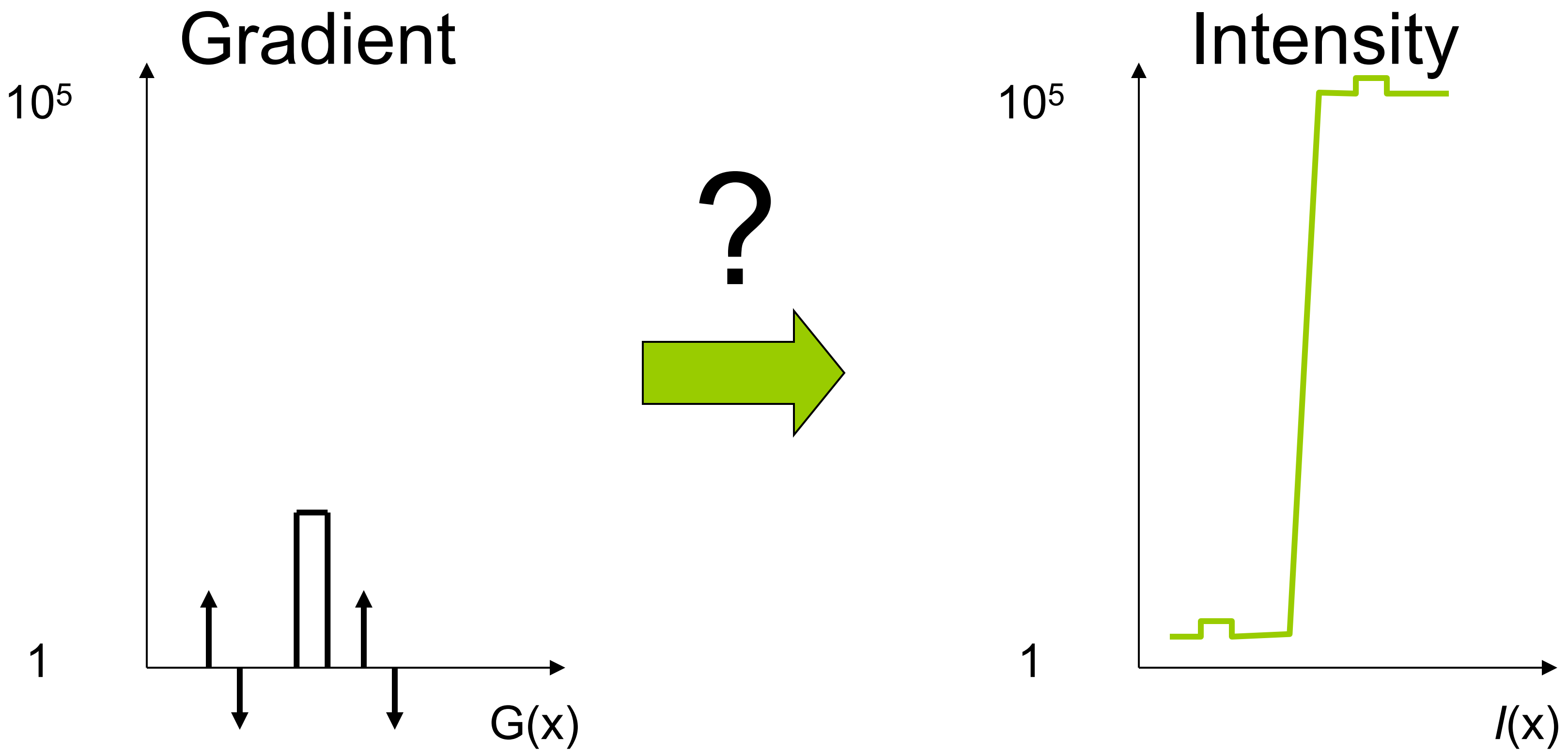
Forward Difference

Reconstruction from Gradients



For *n* intensity values, about *n* gradients

Reconstruction from Gradients



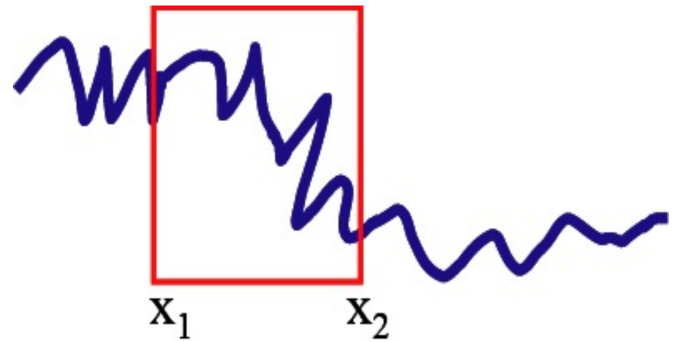
1D Integration

$$I(x) = I(x-1) + G(x)$$

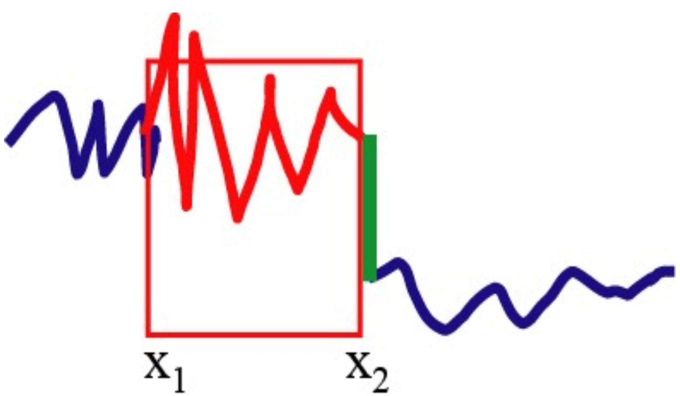
Cumulative sum

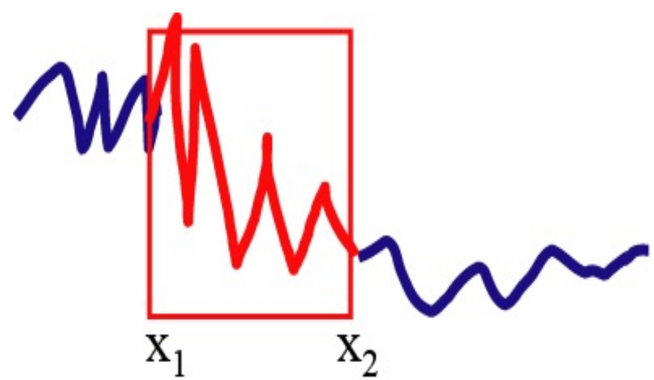
1D case with constraints

DigiVFX

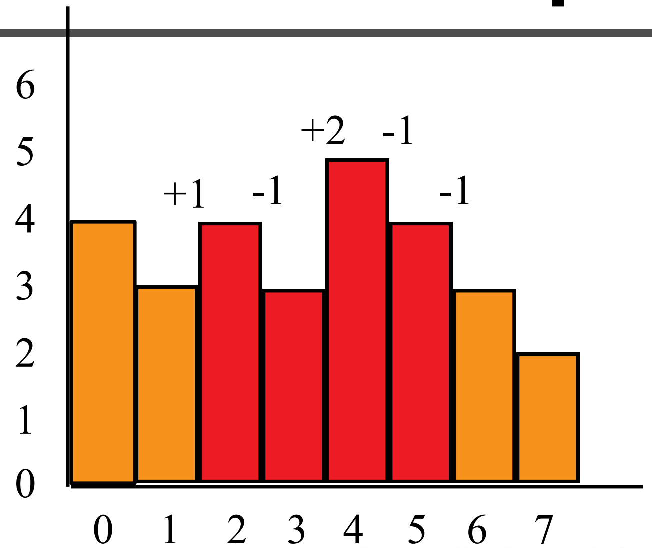
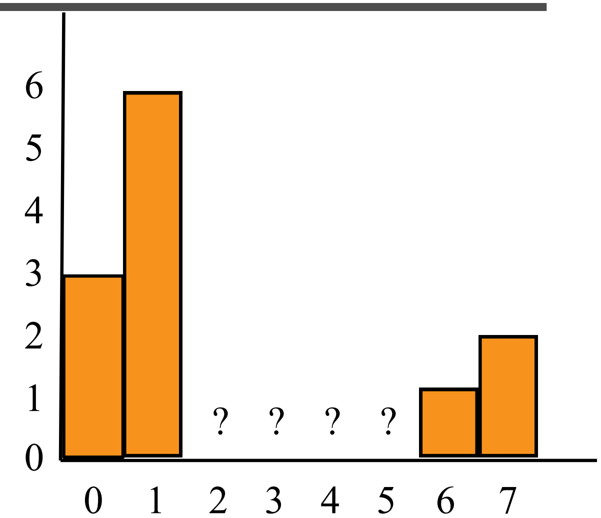
Seamlessly paste  onto 

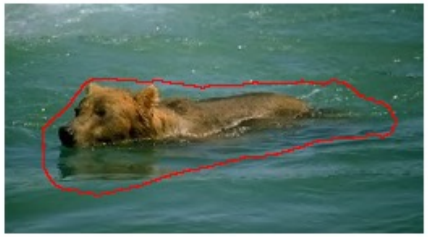
Just add a linear function so that the boundary condition is respected

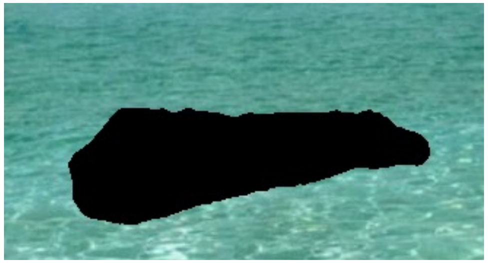




Discrete 1D example: minimization

- Copy  to 

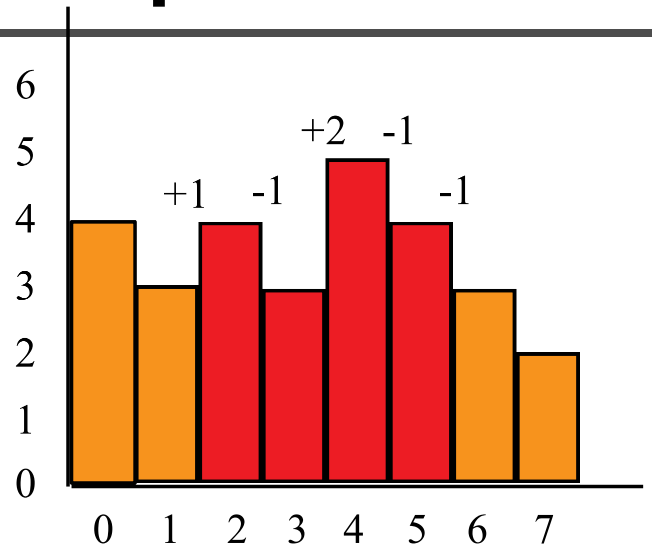
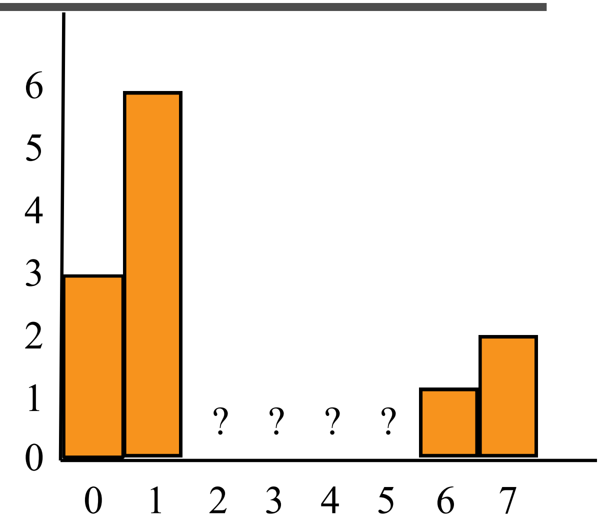




- Min $((f_2-f_1)-1)^2$
- Min $((f_3-f_2)-(-1))^2$
- Min $((f_4-f_3)-2)^2$
- Min $((f_5-f_4)-(-1))^2$
- Min $((f_6-f_5)-(-1))^2$

With
$f_1=6$
$f_6=1$

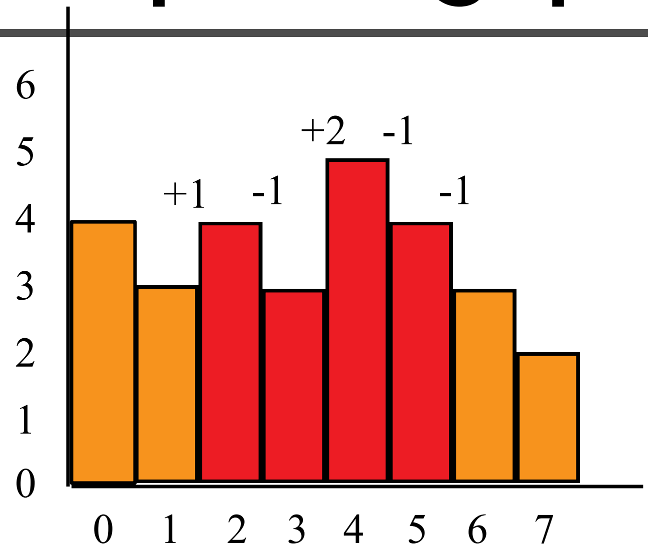
1D example: minimization

- Copy  to 

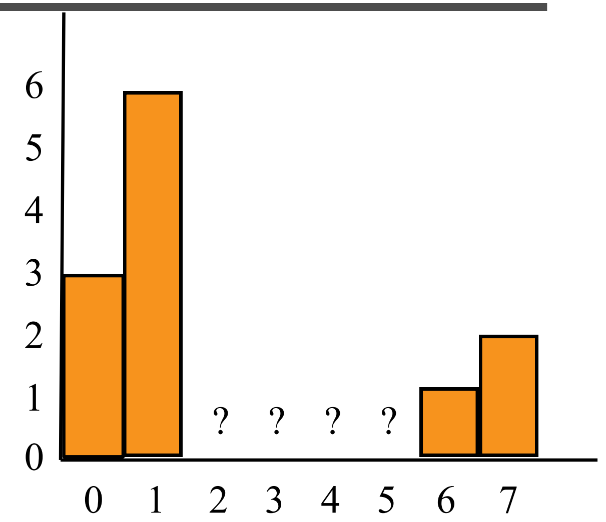
- Min $((f_2-6)-1)^2$ ==> $f_2^2+49-14f_2$
- Min $((f_3-f_2)-(-1))^2$ ==> $f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$
- Min $((f_4-f_3)-2)^2$ ==> $f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$
- Min $((f_5-f_4)-(-1))^2$ ==> $f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$
- Min $((1-f_5)-(-1))^2$ ==> $f_5^2+4-4f_5$

1D example: big quadratic

- Copy

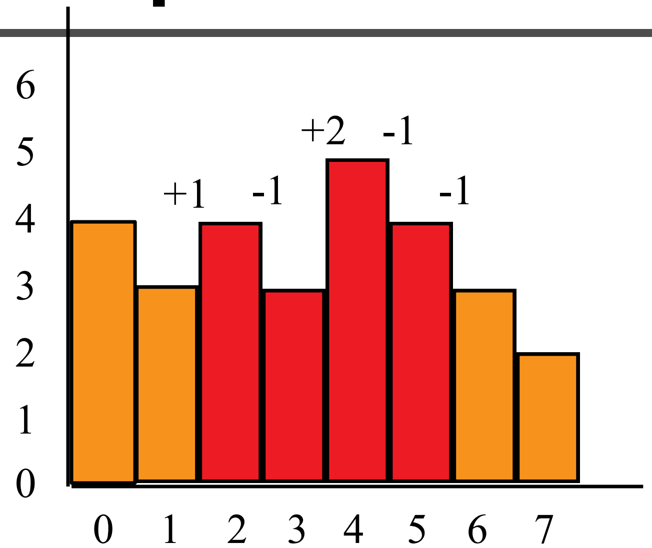
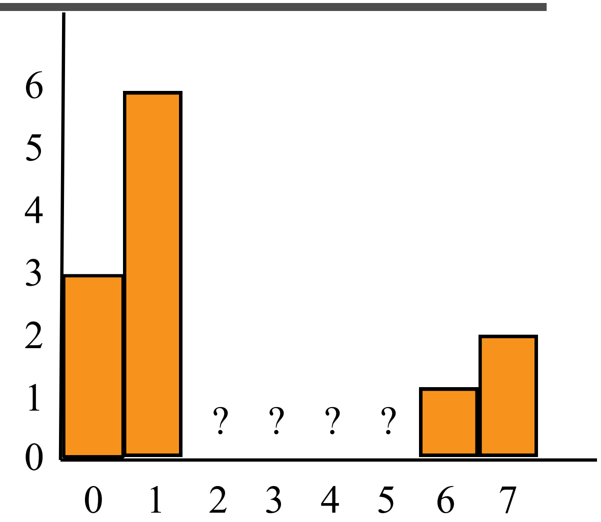


to



- Min $(f_2^2+49-14f_2$
  $+ f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$
  $+ f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$
  $+ f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$
  $+ f_5^2+4-4f_5)$
  Denote it Q

1D example: derivatives

- Copy  to 

Min $(f_2^2+49-14f_2$
$+ f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$
$+ f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$
$+ f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$
$+ f_5^2+4-4f_5)$

Denote it Q

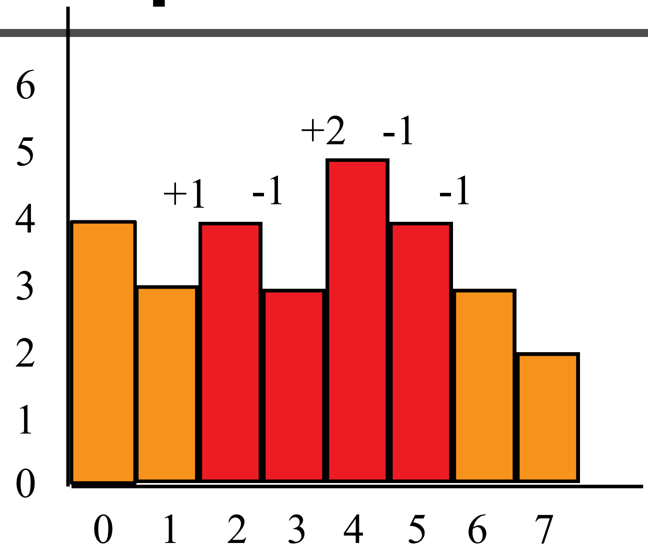
$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$

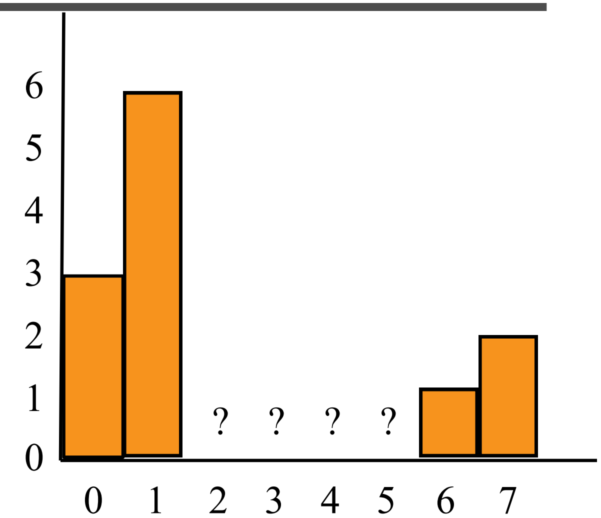
$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$

$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$

$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

1D example: set derivatives to zero

- Copy 

to 

$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$

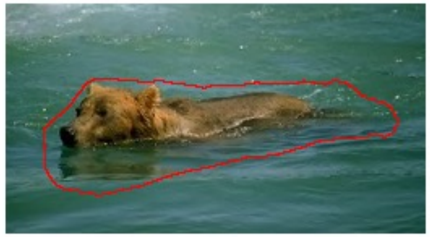
$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$

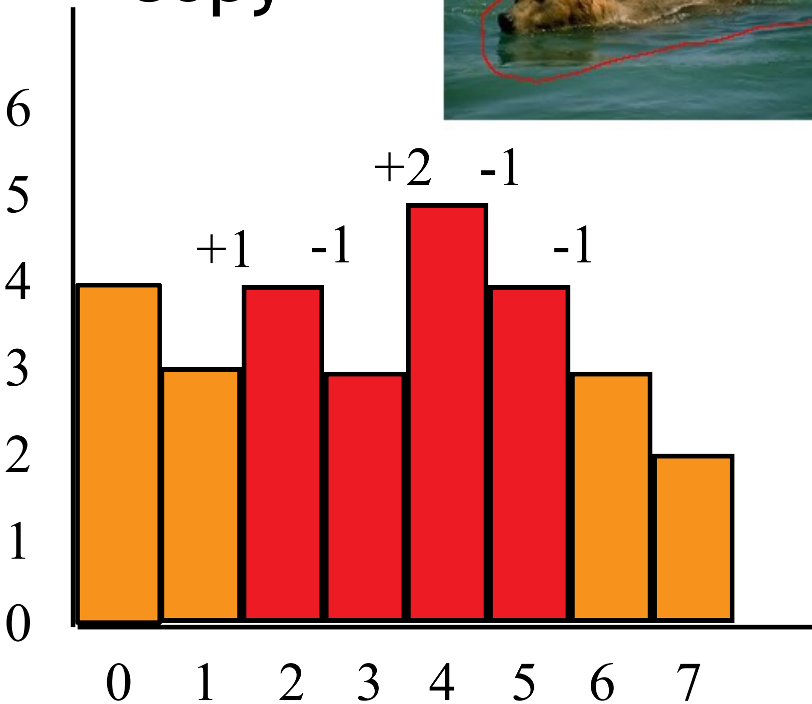
$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$

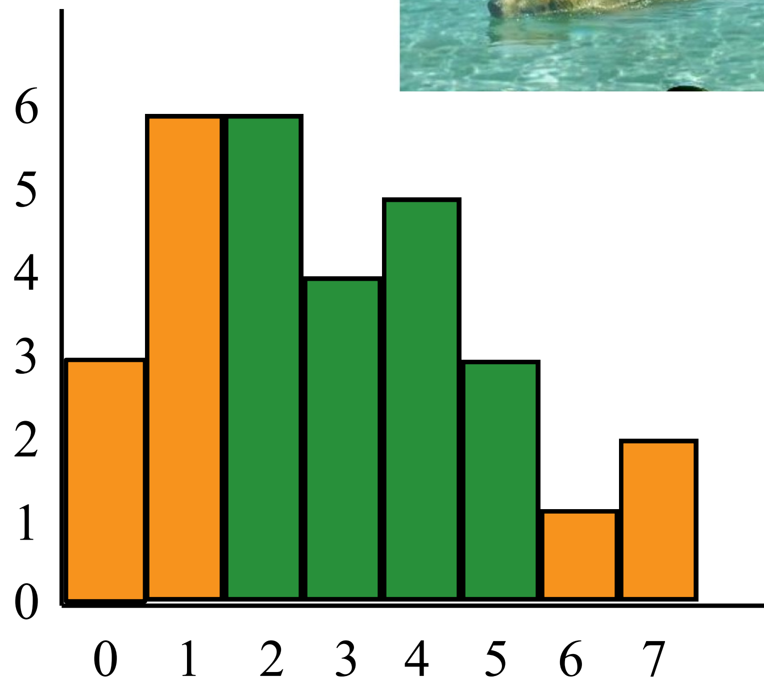
$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

==> $$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

1D example

- Copy  to 

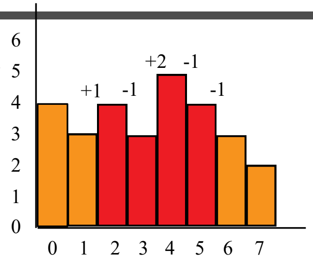


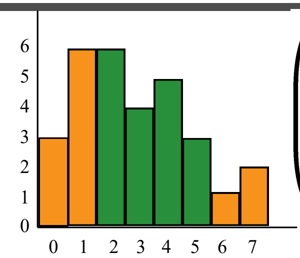


$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \\ 3 \end{pmatrix}$$

1D example: remarks

- Copy 

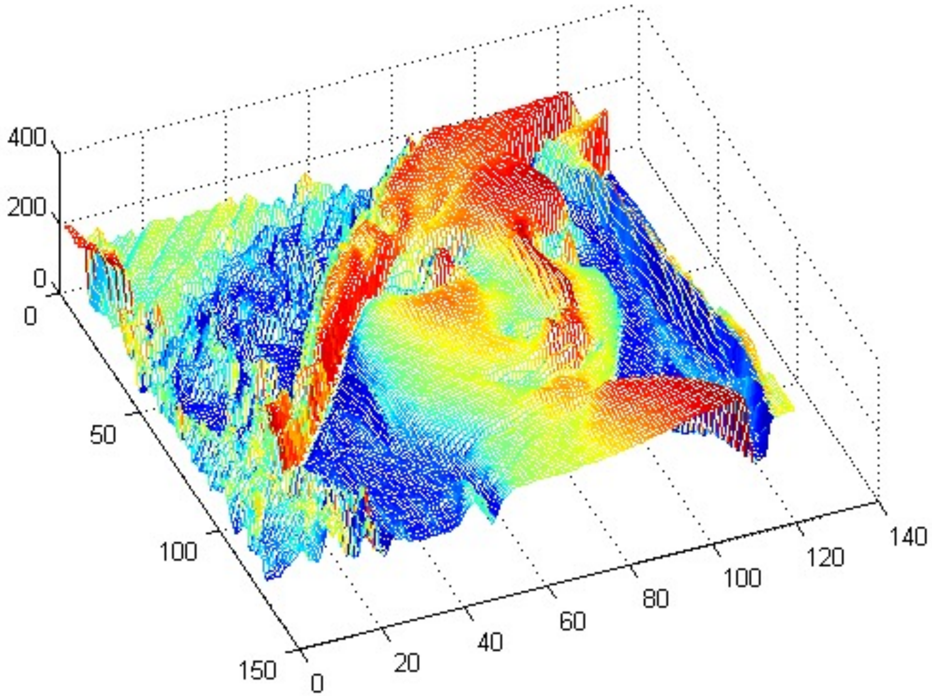
to 

$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

- Matrix is sparse
- Matrix is symmetric
- Everything is a multiple of 2
  - because square and derivative of square
- Matrix is a convolution (kernel -2 4 -2)
- Matrix is independent of gradient field. Only RHS is
- Matrix is a second derivative

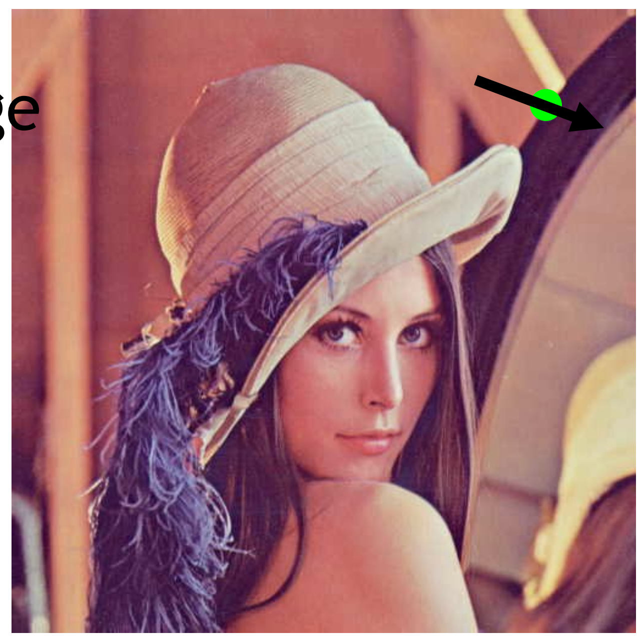
2D example: images

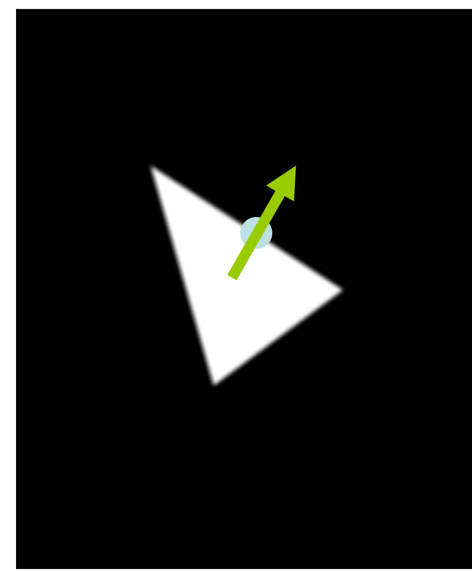
- Images as scalar fields
  - R^2 -> R



Gradients

- Vector field (gradient field)
  - Derivative of a scalar field
- Direction
  - Maximum rate of change of scalar field
- Magnitude
  - Rate of change





Example

Image
I(x,y)

I_x

I_y

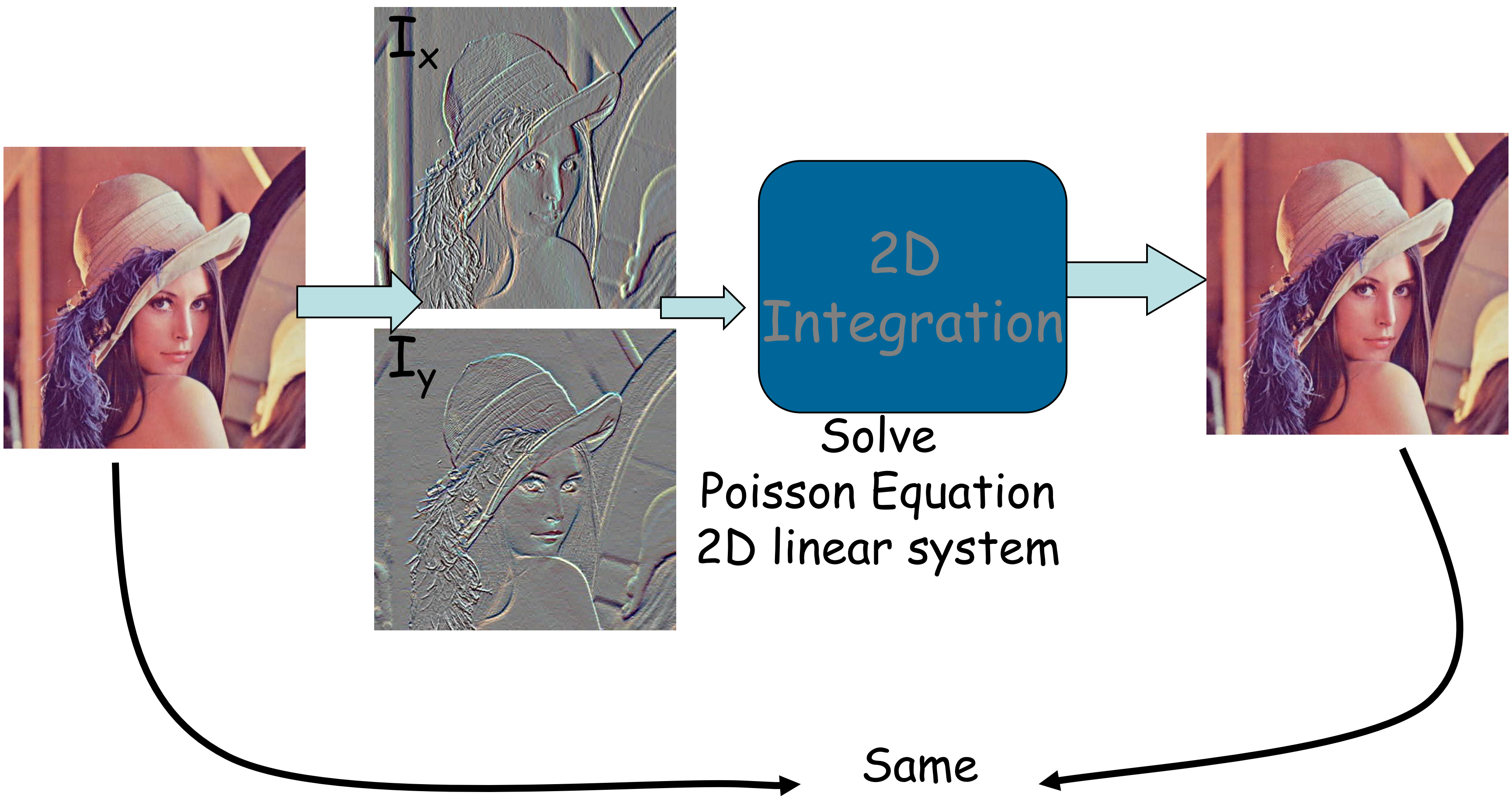
Gradient at x,y as Forward Differences

$$G_x(x,y) = I(x+1, y) - I(x,y)$$
$$G_y(x,y) = I(x, y+1) - I(x,y)$$

$$G(x,y) = (G_x, G_y)$$

Reconstruction from Gradients

Sanity Check:
Recovering Original Image

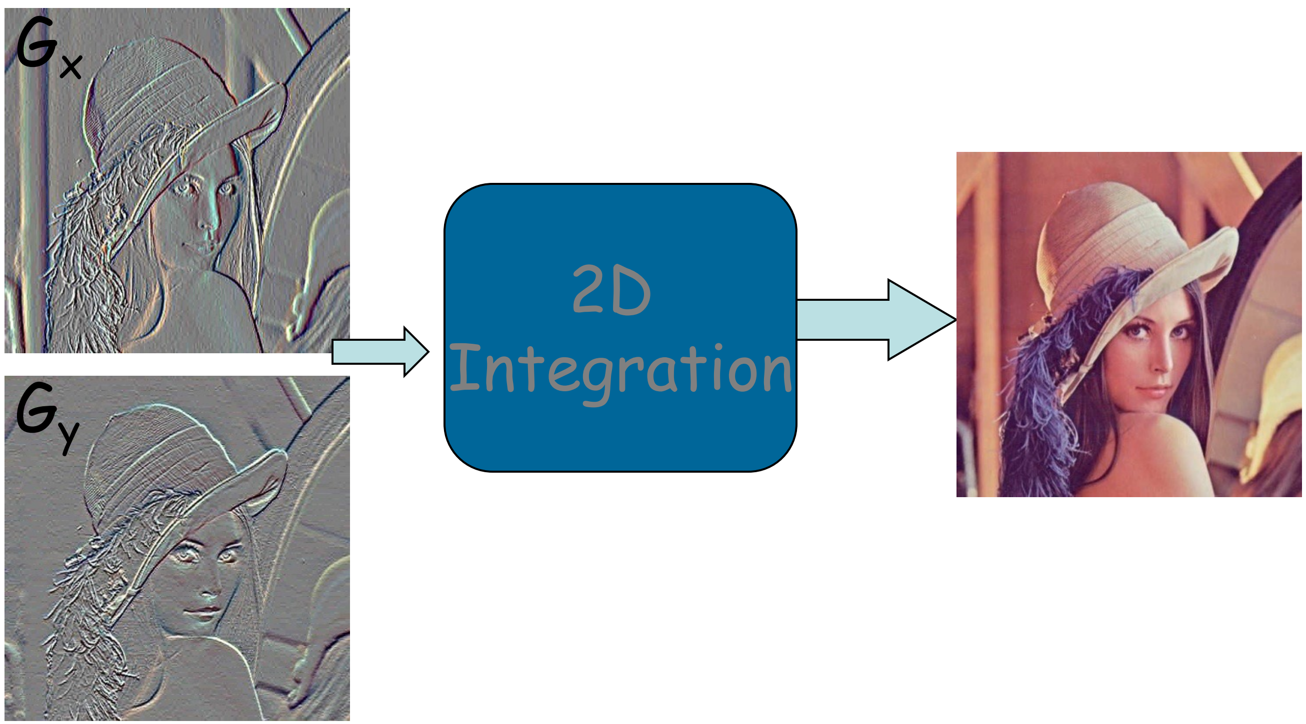


Reconstruction from Gradients

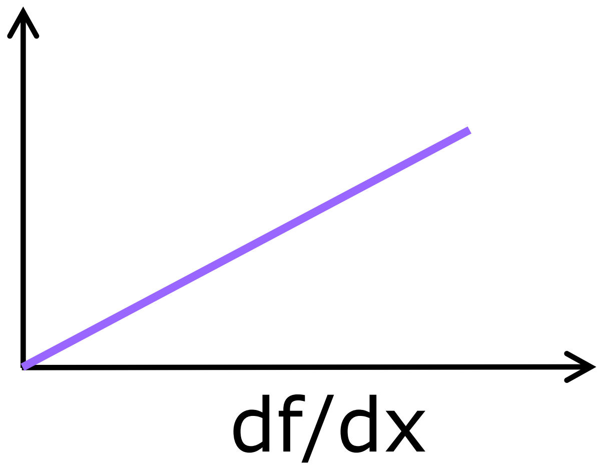
Given $G(x,y) = (G_x, G_y)$

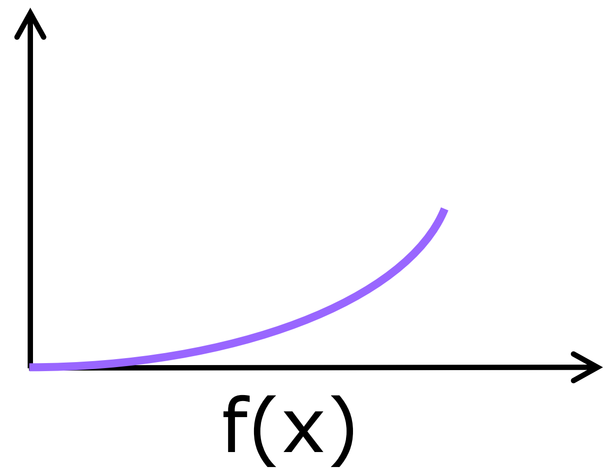
How to compute $I(x,y)$ for the image ?

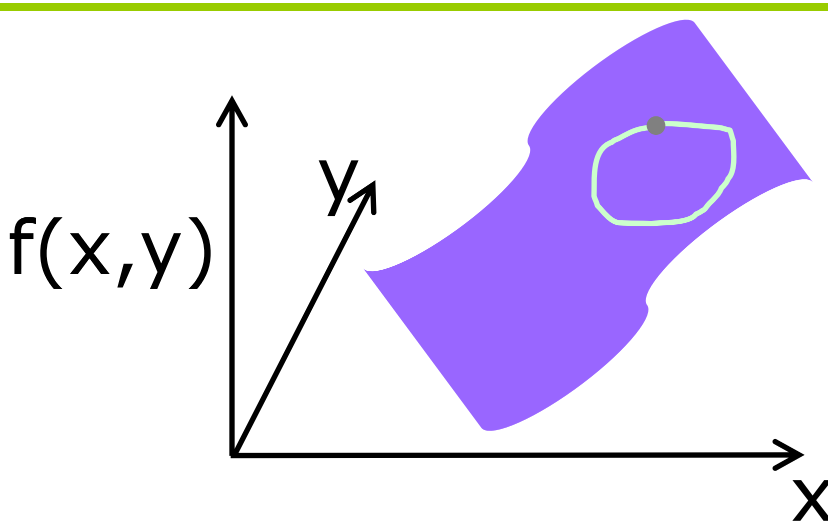
For n^2 image pixels, $2n^2$ gradients !



2D Integration is non-trivial







Reconstruction depends on chosen path

Reconstruction from Gradient Field G

- Look for image ***I*** with gradient closest to ***G*** in the least squares sense.
- ***I*** minimizes the integral: $\iint F(\nabla I, G)\,dx\,dy$

$$F(\nabla I, G) = \|\nabla I - G\|^2 = \left(\frac{\partial I}{\partial x} - G_x\right)^2 + \left(\frac{\partial I}{\partial y} - G_y\right)^2$$

$$\Rightarrow \quad \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

Solve $$\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

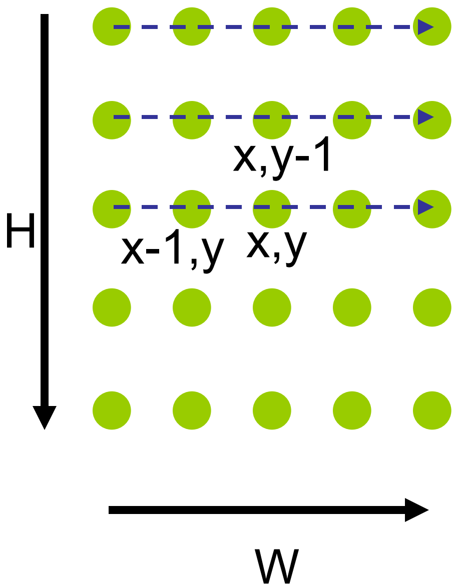
$$G_x(x,y) - G_x(x-1,y) + G_y(x,y) - G_y(x,y-1)$$

$$I(x+1,y) + I(x-1,y) + I(x,y+1) + I(x,y-1) - 4I(x,y)$$

$$\begin{bmatrix} \\ ..\ 1\ \ldots\ \ 1\ -4\ 1\ \ldots\ \ 1\ .. \\ \\ \end{bmatrix} \begin{bmatrix} \\ \mathbf{I} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Linear System

$$-4I(x,y)+I(x,y+1)+I(x,y-1)+I(x+1,y)+I(x-1,y)=u(x,y)$$



$$\underbrace{\left[. \quad 1 \quad . \quad . \quad . \quad 1 \quad -4 \quad 1 \quad . \quad . \quad . \quad 1 \quad .\right]}_{A}
\begin{bmatrix} . \\ . \\ I(x-1,y) \\ . \\ . \\ . \\ I(x,y-1) \\ I(x,y) \\ I(x,y+1) \\ . \\ . \\ . \\ I(x+1,y) \\ . \\ . \end{bmatrix}_{x}
=
\begin{bmatrix} . \\ . \\ . \\ . \\ . \\ . \\ . \\ u(x,y) \\ . \\ . \\ . \\ . \\ . \\ . \\ . \end{bmatrix}_{b}$$

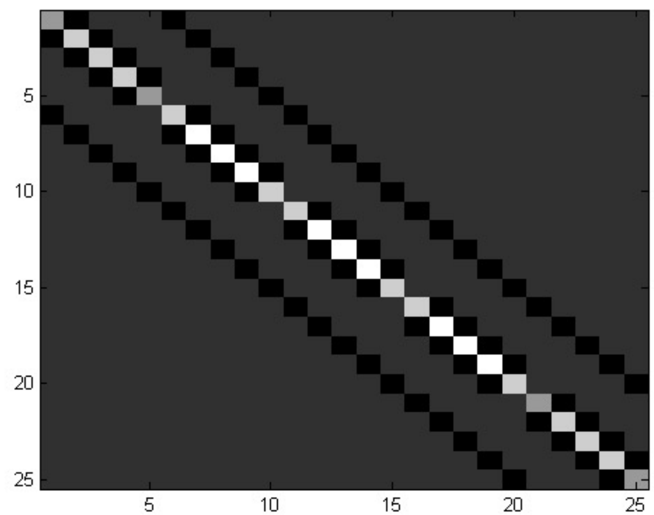
H (span between the 1 entries of A, and between $I(x-1,y)$ and $I(x,y)$ in x)

A x b

Sparse Linear system

$$
\begin{bmatrix}
 & & 1 & -4 & 1 & & & & 1 & & & & \\
 & & & 1 & -4 & 1 & & & & 1 & & & \\
1 & & & & 1 & -4 & 1 & & & & 1 & & \\
 & 1 & & & & 1 & -4 & 1 & & & & 1 & \\
 & & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & & 1 & & & & 1 & -4 & 1 & & & \\
 & & & & 1 & & & & 1 & -4 & 1 & &
\end{bmatrix}
$$

A matrix

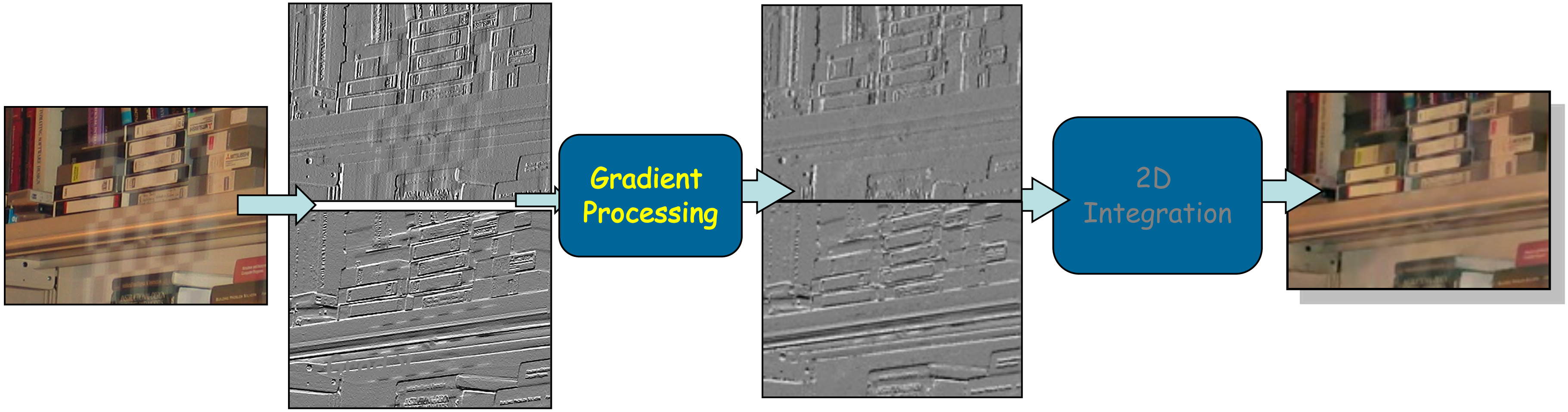


Solving Linear System

- Image size N*N
- Size of A ~ N^2 by N^2
- Impractical to form and store A
- Direct Solvers
- Basis Functions
- Multigrid
- Conjugate Gradients

Intensity Gradient Manipulation

A Common Pipeline



Gradient Domain Manipulations: Overview

(A) Per pixel

(B) Corresponding gradients in two images

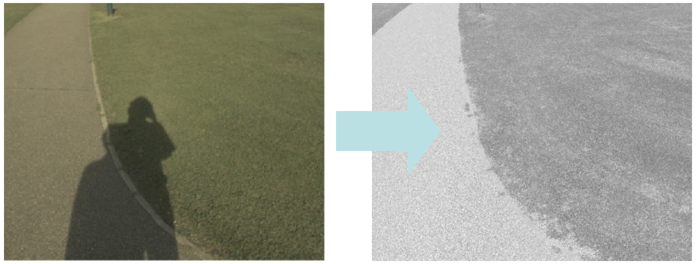
(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

A. Per Pixel Manipulations

- Non-linear operations
  - HDR compression, local illumination change

- Set to zero
  - Shadow removal, intrinsic images, texture de-emphasis



High Dynamic Range Imaging

DigiVFX

Images from Raanan Fattal

Gradient Domain Compression

HDR
Image L

Log L

Gradients
L_x, L_y

Multiply

2D
Integration

Gradient Attenuation
Function G

Illumination Invariant Image

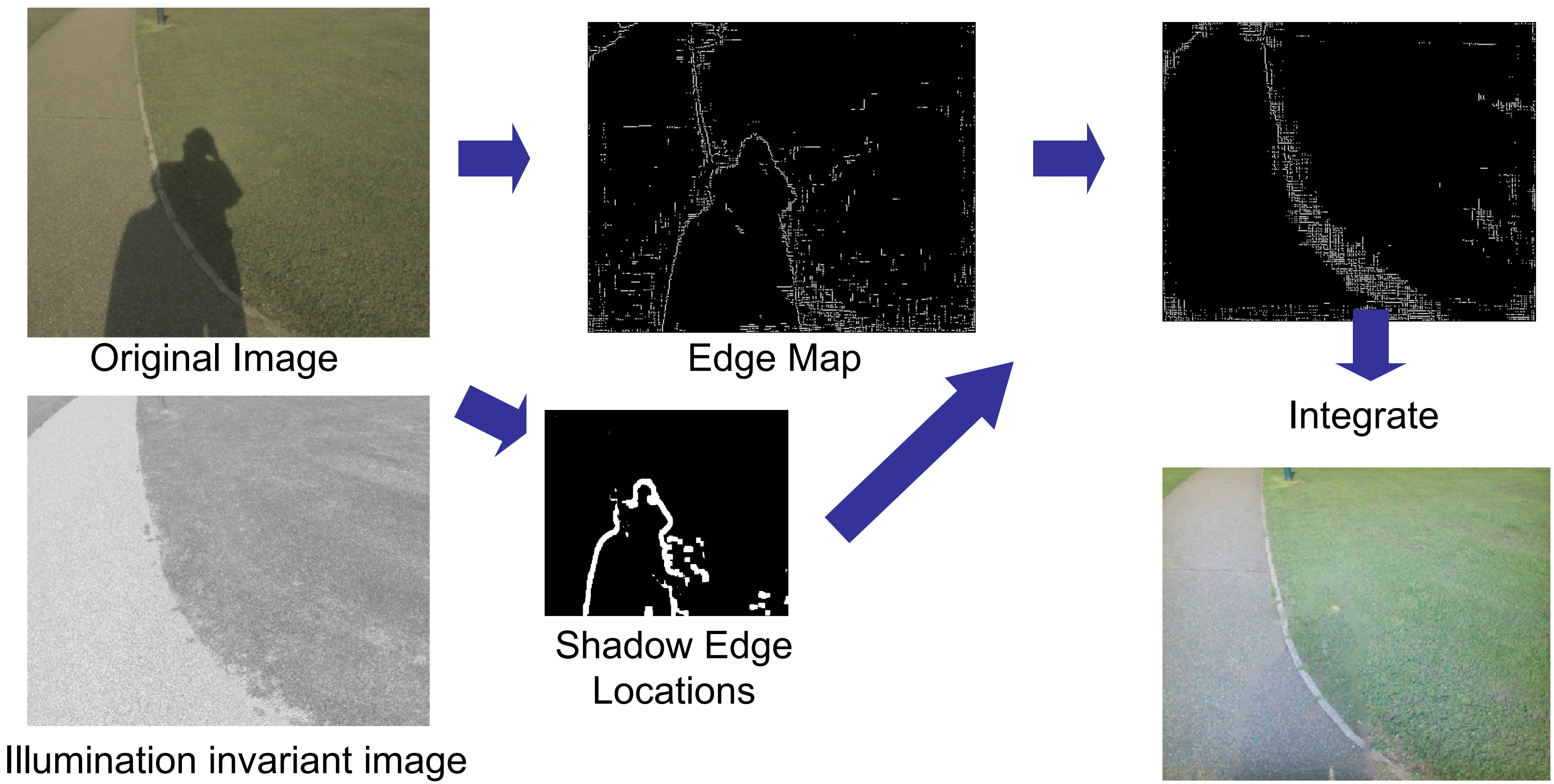
Original Image

Illumination invariant image

- Assumptions
  - Sensor response = delta functions R, G, B in wavelength spectrum
  - Illumination restricted to Outdoor Illumination

G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

Shadow Removal Using Illumination Invariant Image



G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

Gradient Domain Manipulations: Overview

(A) Per pixel

(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Photography Artifacts: Flash Hotspot

Ambient

Flash

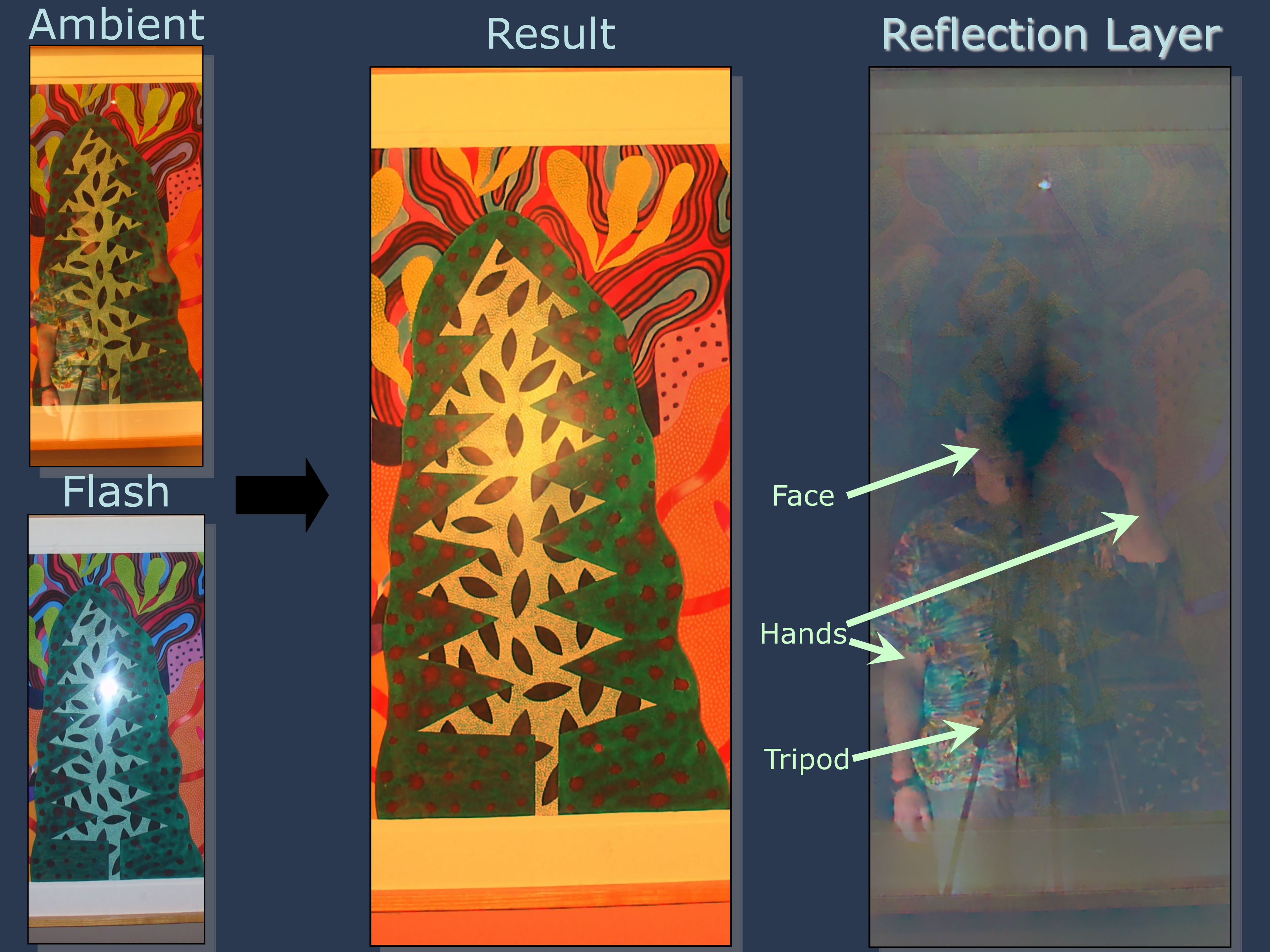
Flash Hotspot

Reflections due to Flash

Underexposed

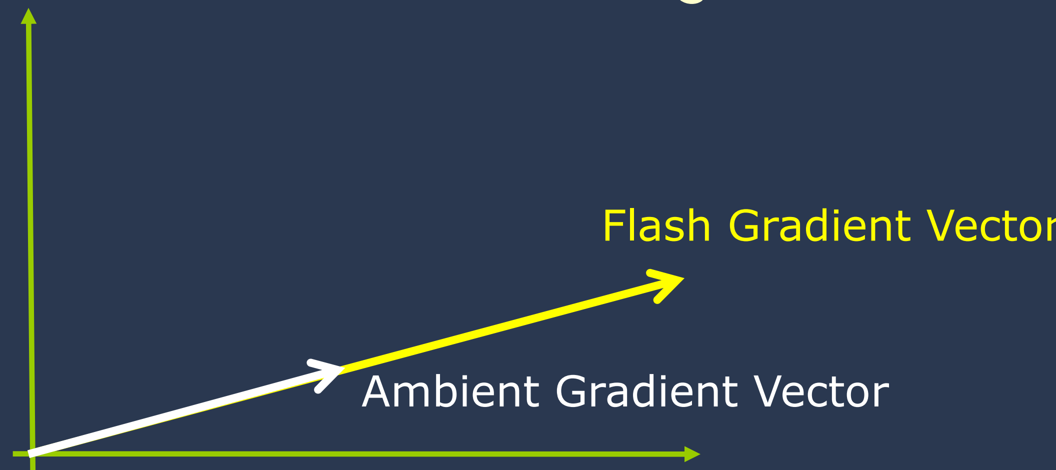
Reflections

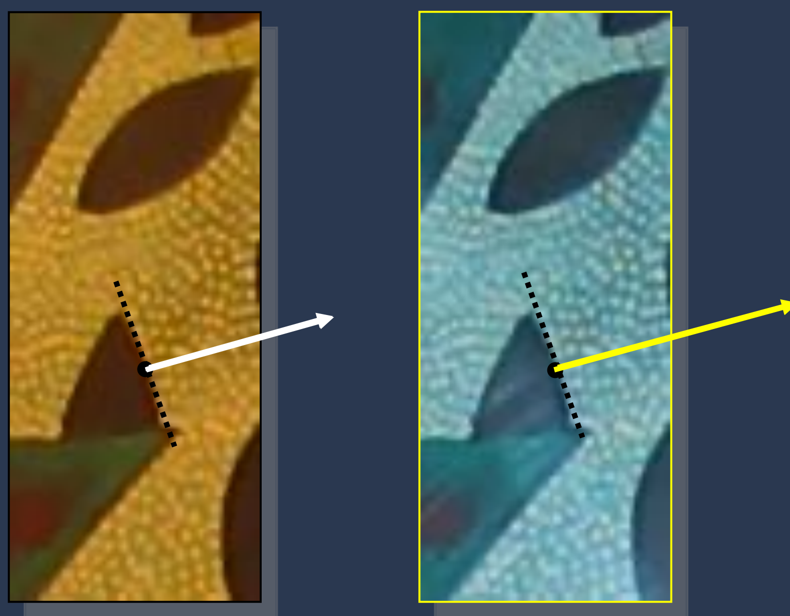
Self-Reflections and Flash Hotspot


Ambient
Flash
Result
Reflection Layer
Face
Hands
Tripod

Intensity Gradient Vectors in Flash and Ambient Images

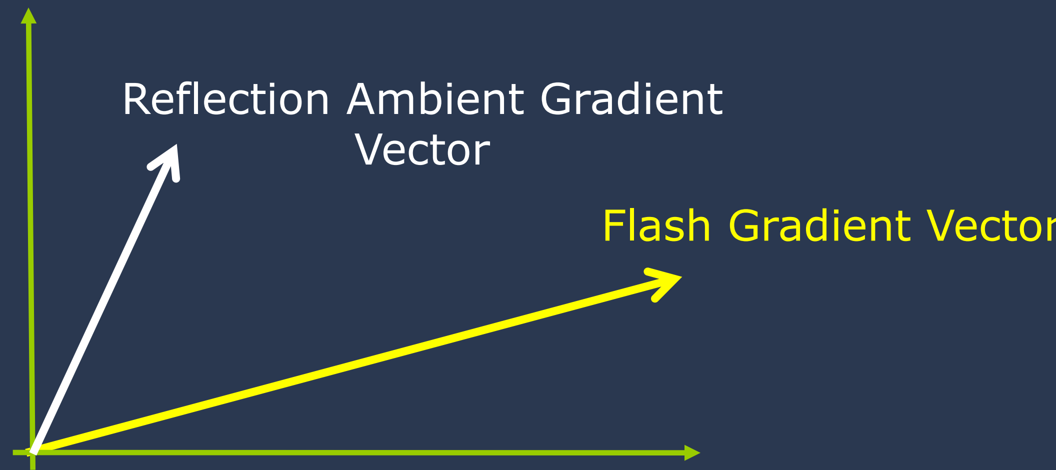
Same gradient vector direction





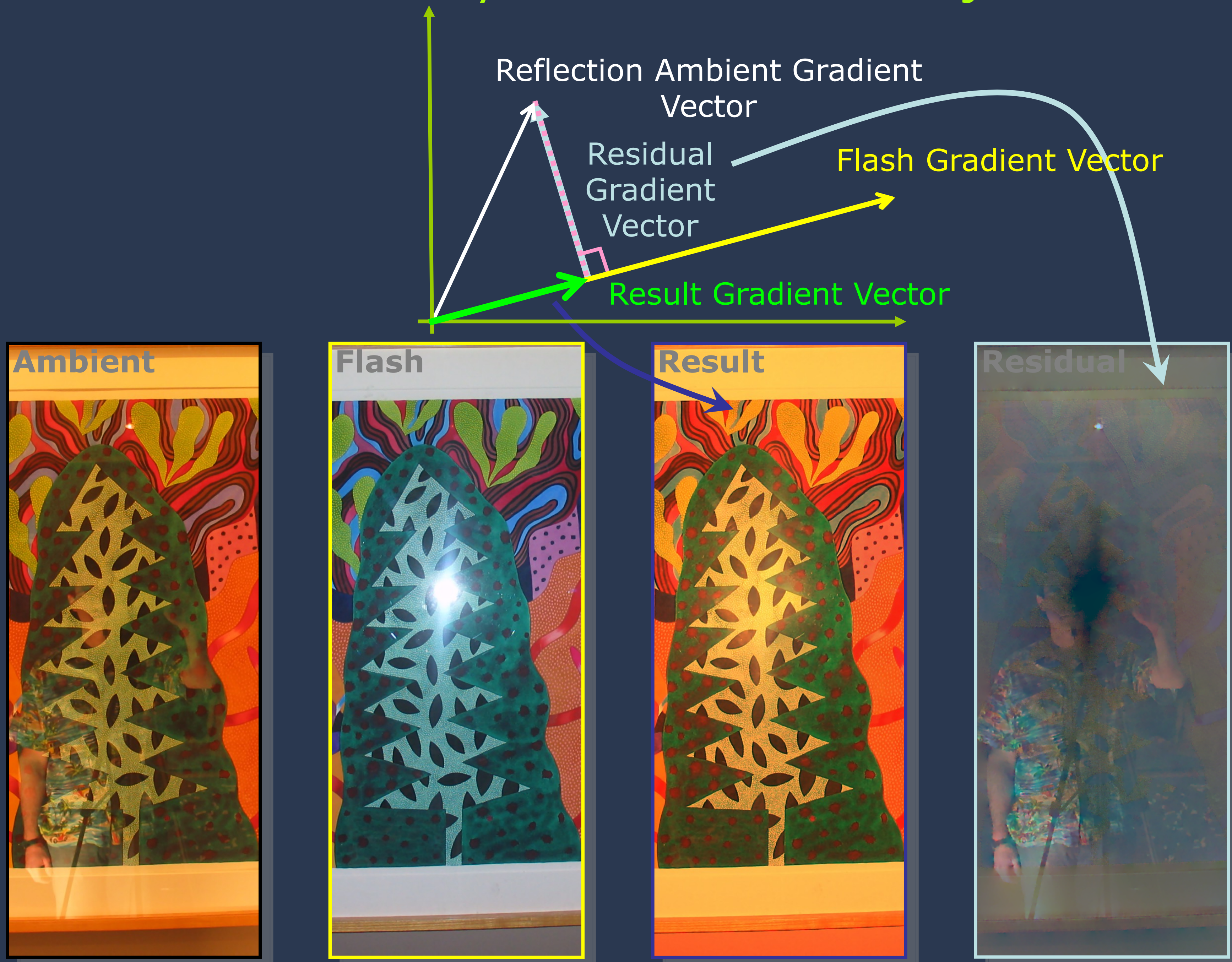
No reflections

Different gradient vector directions



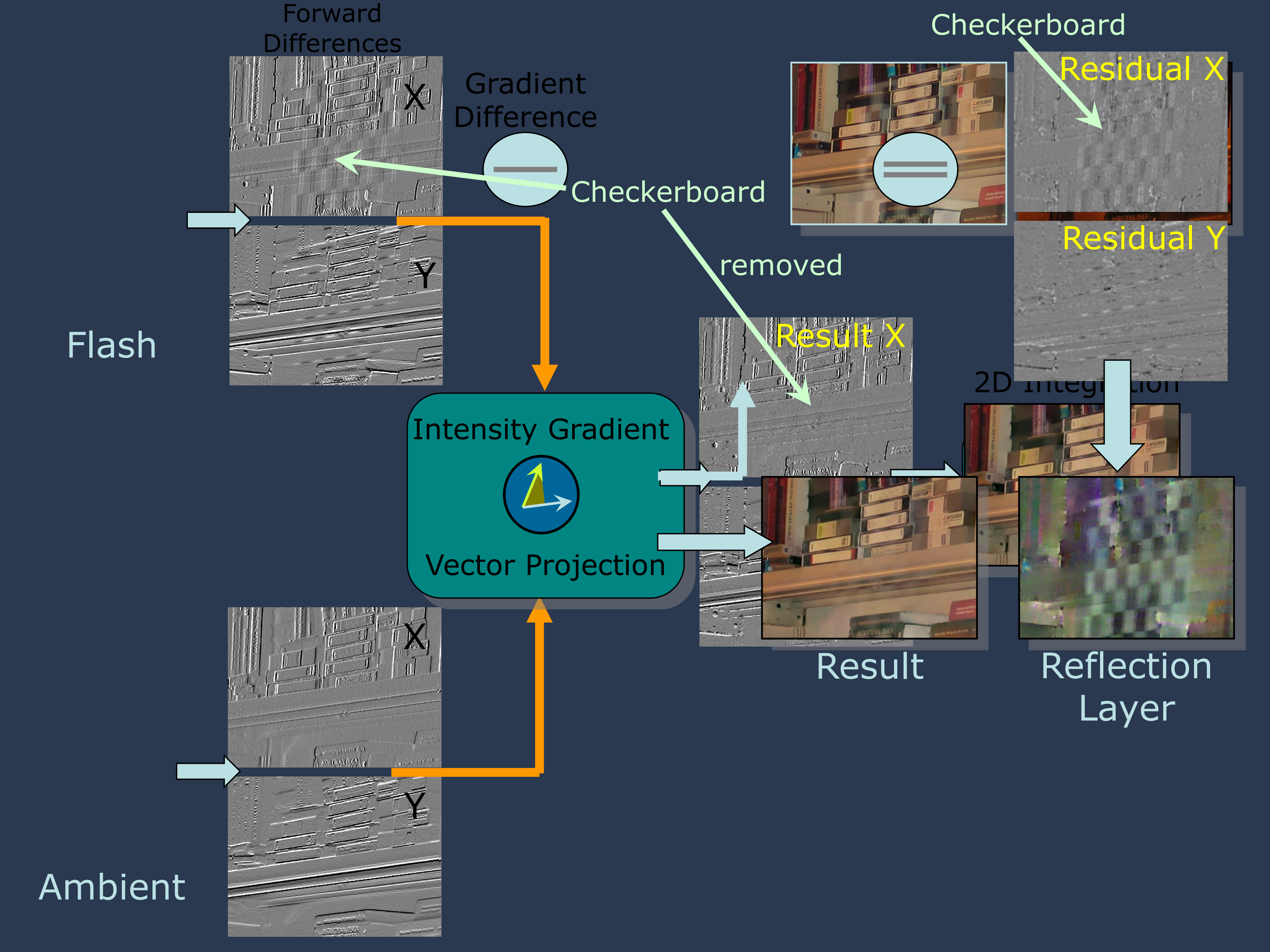
With reflections

Intensity Gradient Vector Projection




Ambient
Flash
Projection =
Result
Residual =
Reflection Layer


Flash
Ambient
Checkerboard
outside glass window
Reflections on
glass window



Forward Differences
X
Y
Gradient Difference
Checkerboard
removed
Checkerboard
Residual X
Residual Y
Flash
Result X
Intensity Gradient
Vector Projection
Result
Reflection Layer
X
Y
Ambient

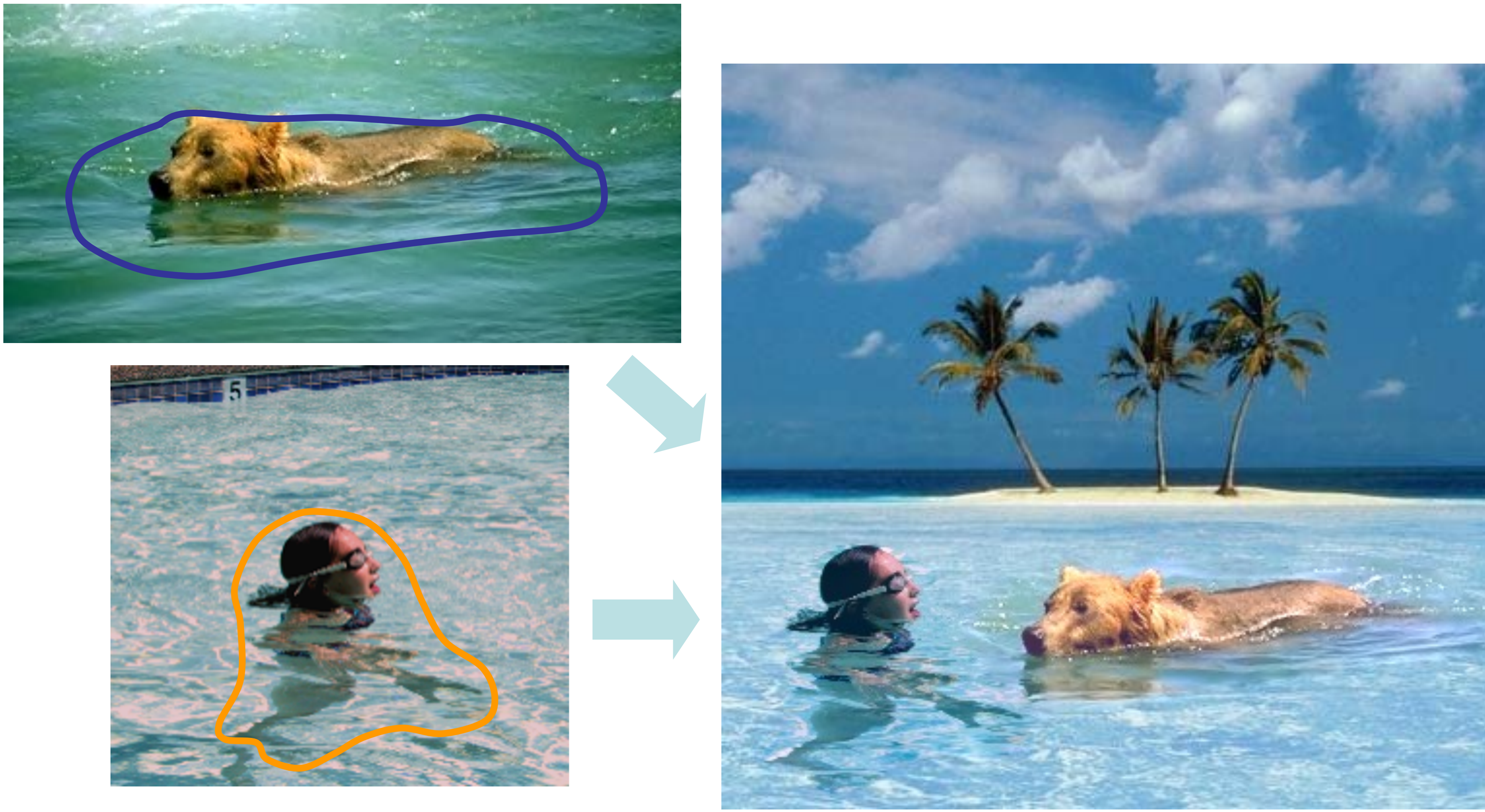
Poisson Image Editing

- Precise selection: tedious and unsatisfactory
- Alpha-Matting: powerful but involved
- Seamless cloning: loose selection but no seams?

Conceal

Copy Background gradients (user strokes)

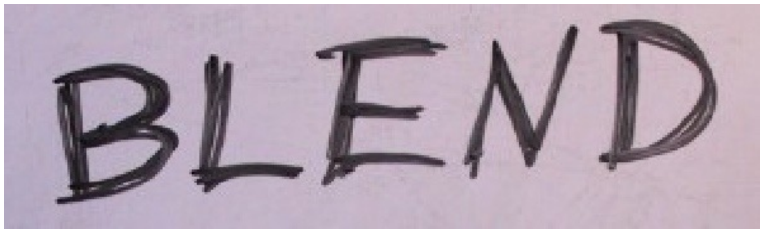
Compose



Source Images

Target Image

Transparent Cloning



$$\mathbf{v} = \frac{\nabla f^* + \nabla g}{2}$$

Largest variation from source and destination at each point

Gradient Domain Manipulations: Overview

(A) Per pixel

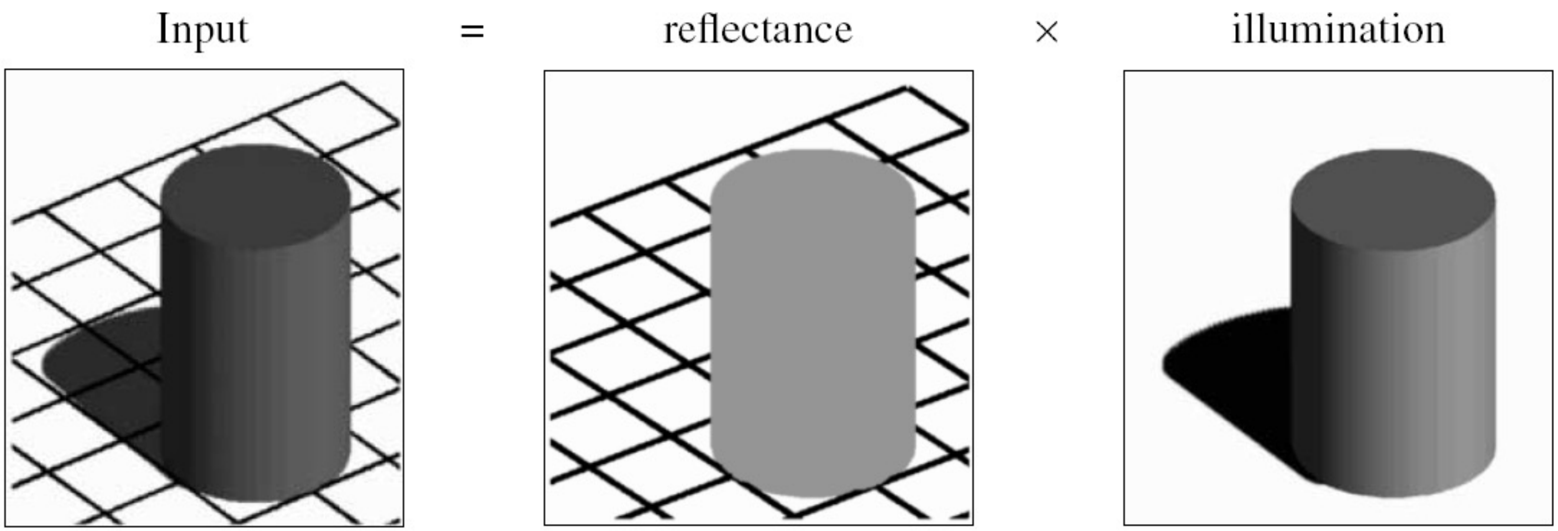
(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Intrinsic images

- I = L * R
- L = illumination image
- R = reflectance image



Intrinsic images

- Use multiple images under different illumination
- Assumption
  - Illumination image gradients = Laplacian PDF
  - Under Laplacian PDF, Median = ML estimator

- At each pixel, take Median of gradients across images
- Integrate to remove shadows

Yair Weiss, “Deriving intrinsic images from image sequences”, ICCV 2001

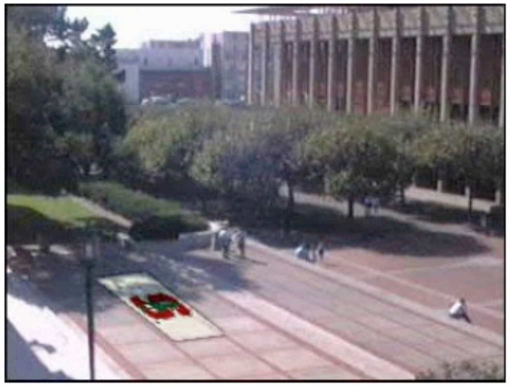
frame 1

frame 11

ML reflectance

Shadow free

Intrinsic Image



Result = Illumination Image * (Label in Intrinsic Image)

Gradient Domain Manipulations: Overview

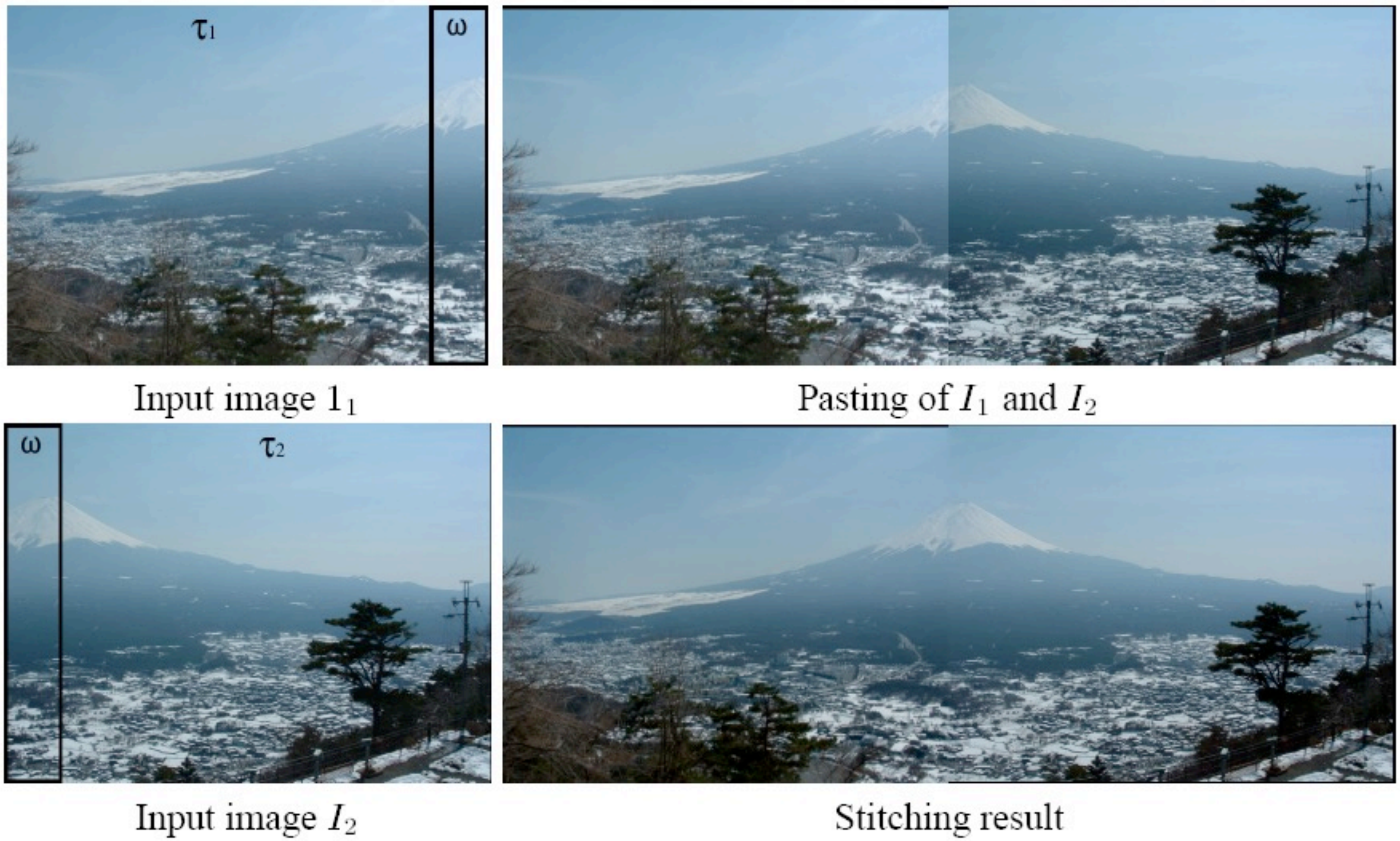
(A) Per pixel

(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Seamless Image Stitching



Anat Levin, Assaf Zomet, Shmuel Peleg and Yair Weiss, "Seamless Image Stitching in the Gradient Domain", ECCV 2004