Bilateral Filters

Digital Visual Effects

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with slides by Fredo Durand, Ramesh Raskar, Sylvain Paris, Soonmin Bae

Image Denoising

noisy image | naïve denoising Gaussian blur | better denoising edge-preserving filter

Smoothing an image without blurring its edges.

A Wide Range of Options

- Diffusion, Bayesian, Wavelets…
  - All have their pros and cons.
- Bilateral filter
  - not always the best result [Buades 05] but often good
  - easy to understand, adapt and set up

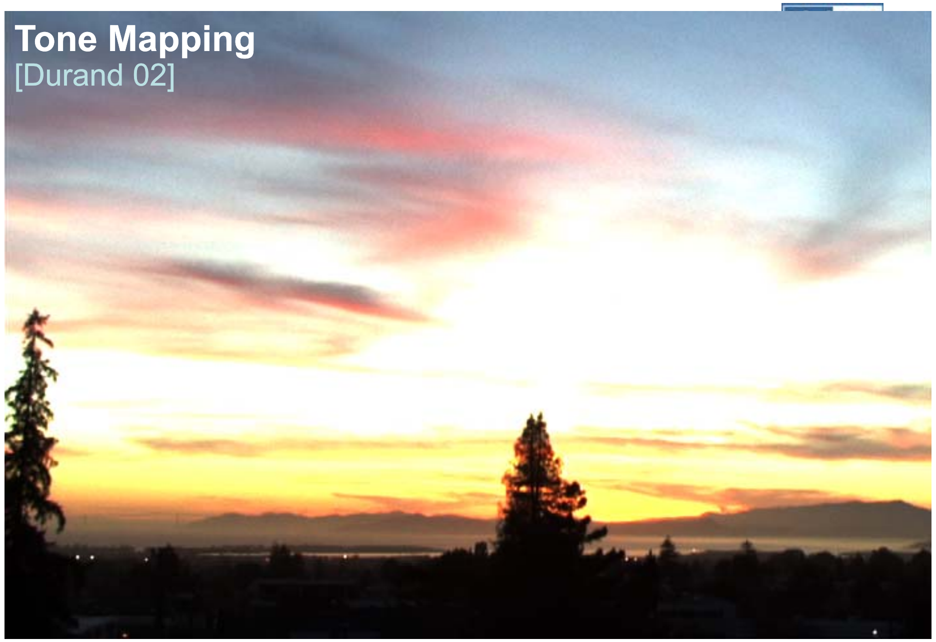
Basic denoising

Noisy input | Median 5x5

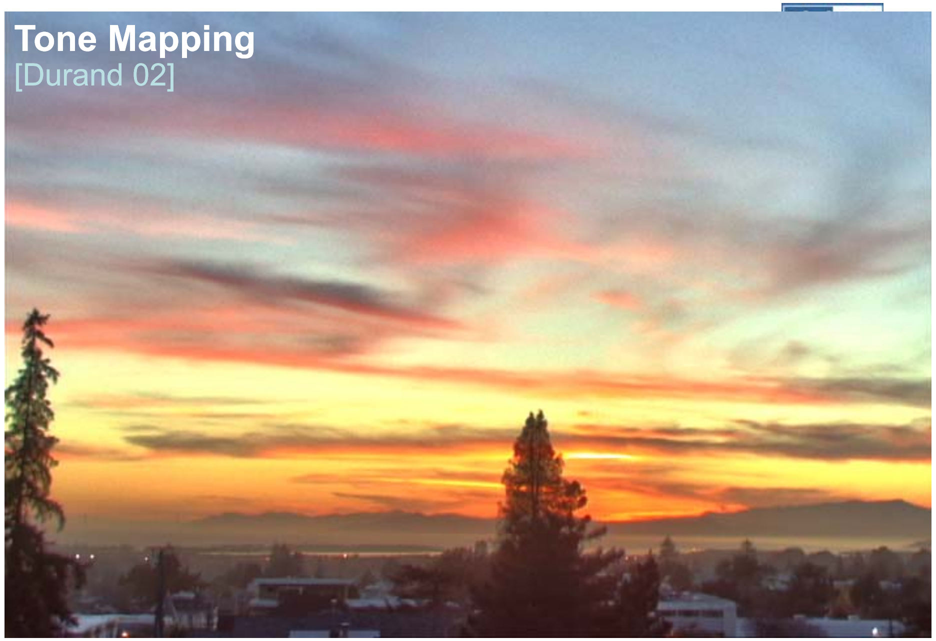
Basic denoising

Noisy input

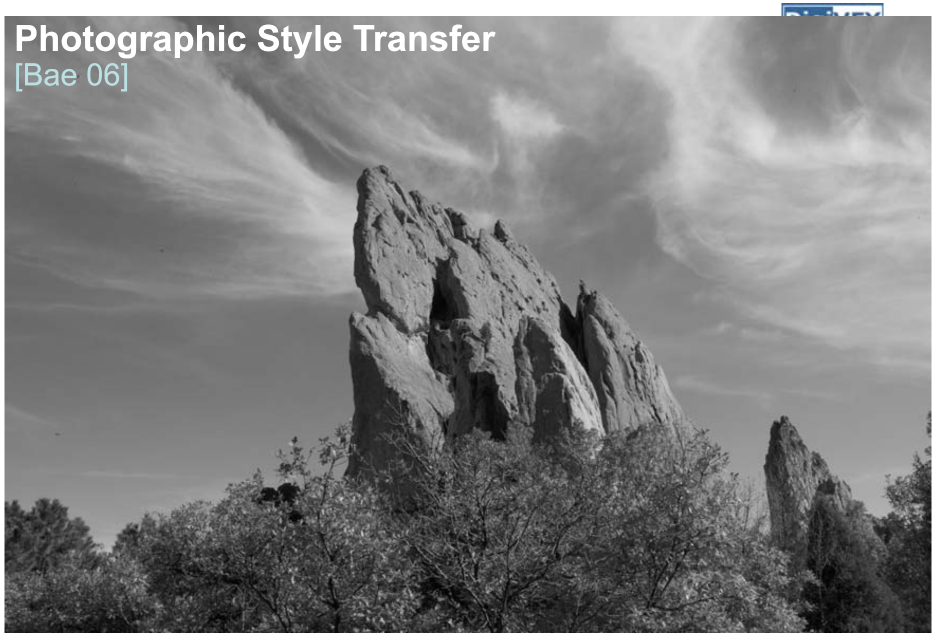
Bilateral filter 7x7 window



HDR input



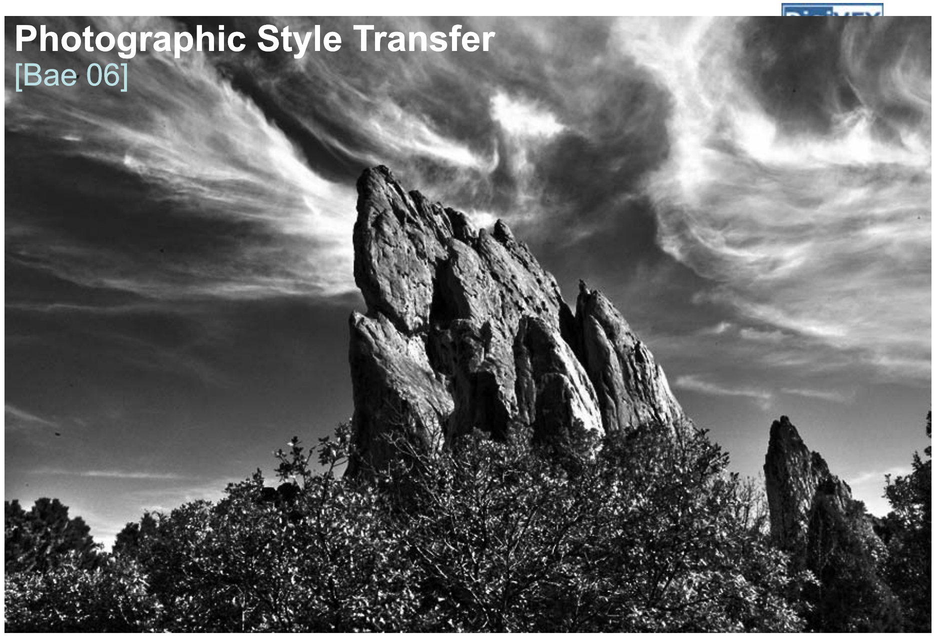
output



input

Photographic Style Transfer

[Bae 06]



output

Cartoon Rendition

[Winnemöller 06]

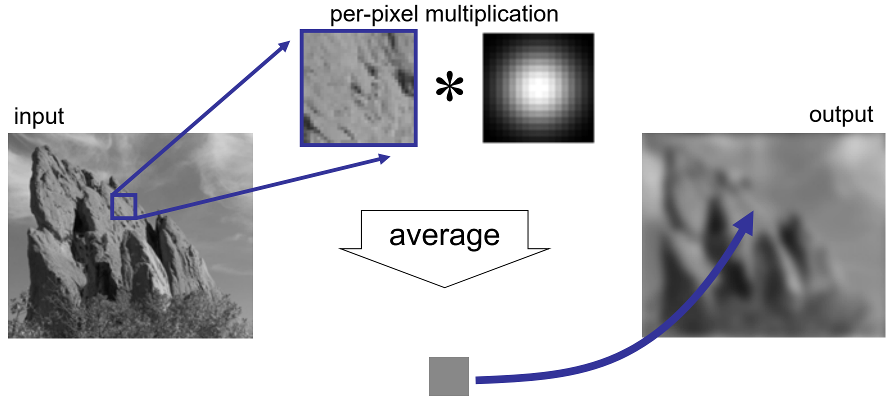
input

Cartoon Rendition

[Winnemöller 06]

output

Gaussian Blur

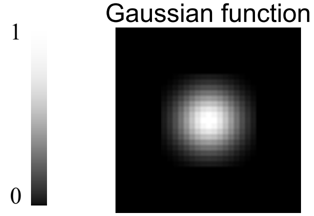


Equation of Gaussian Blur

Same idea: **weighted average of pixels.**

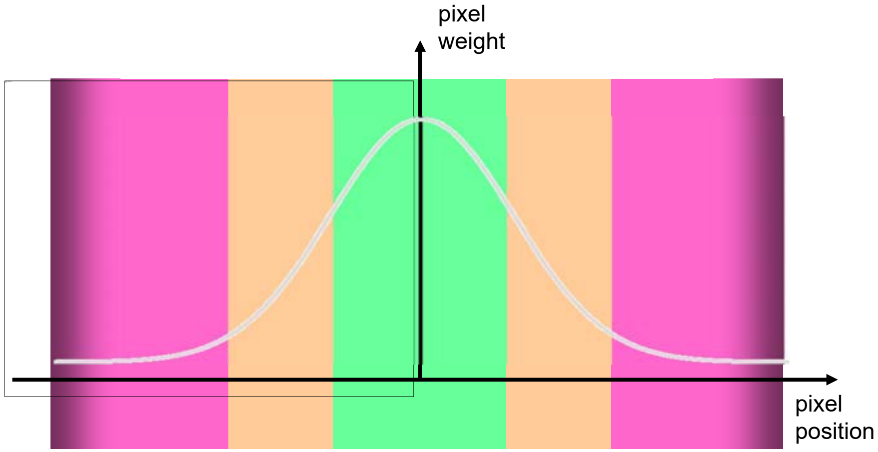
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|)\, I_{\mathbf{q}}$$

normalized Gaussian function



Gaussian Profile

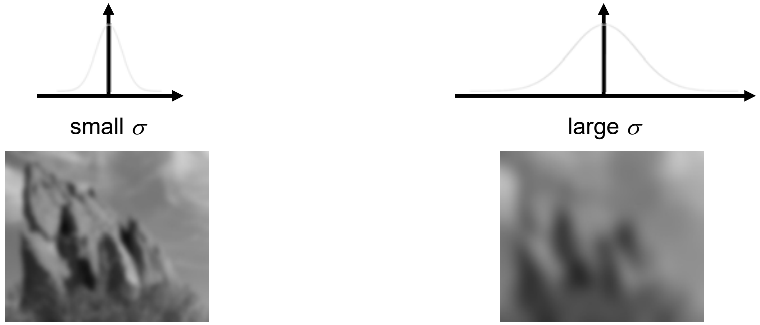
$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



Spatial Parameter

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_\sigma(\| \mathbf{p} - \mathbf{q} \|)\, I_{\mathbf{q}}$$

size of the window

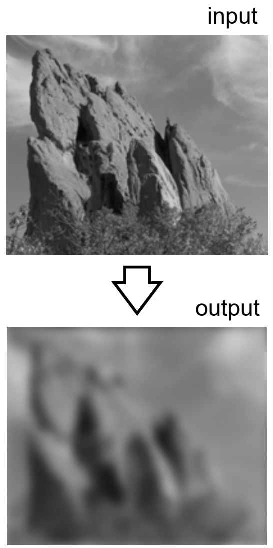


Properties of Gaussian Blur

- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT…)

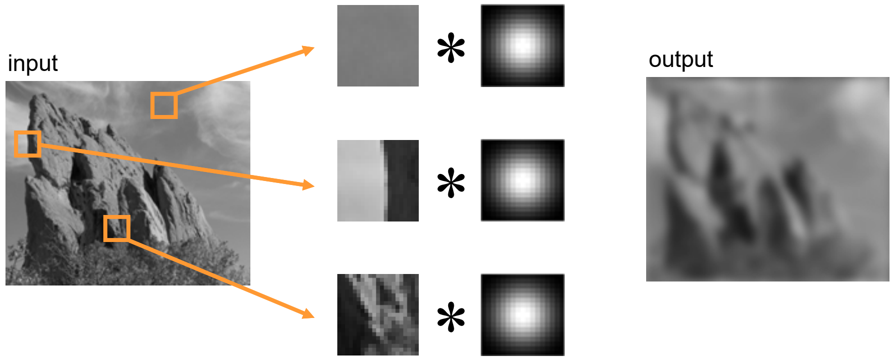
Properties of Gaussian Blur

- Does smooth images
- But smoothes too much: **edges are blurred.**
  - Only spatial distance matters
  - No edge term



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} \underbrace{G_\sigma(\| \mathbf{p} - \mathbf{q} \|)}_{\text{space}}\, I_{\mathbf{q}}$$

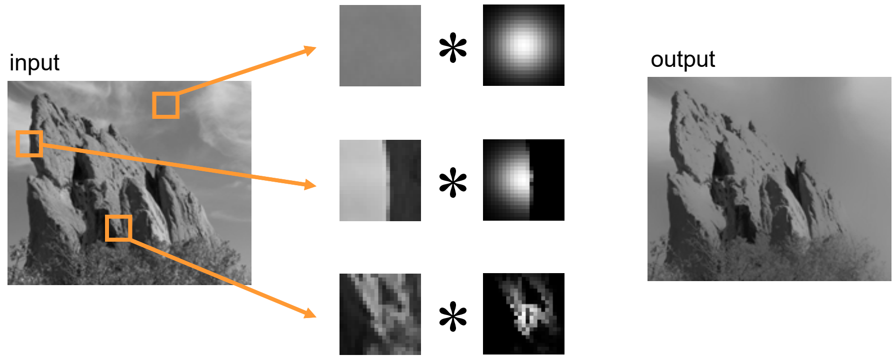
Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

Bilateral Filter No Averaging across Edges

[Aurich 95, Smith 97, Tomasi 98]



The kernel shape depends on the image content.

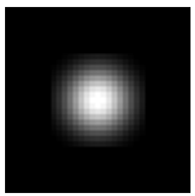
Bilateral Filter Definition

Same idea: **weighted average of pixels.**

new — not new — new

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

normalization factor — ***space*** weight — ***range*** weight



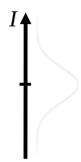
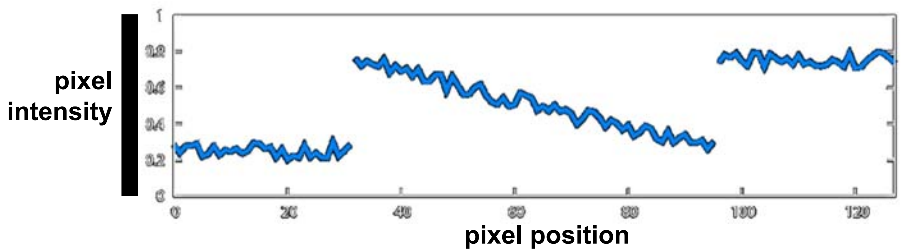


Illustration a 1D Image

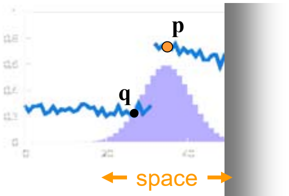
- 1D image = line of pixels

- Better visualized as a plot



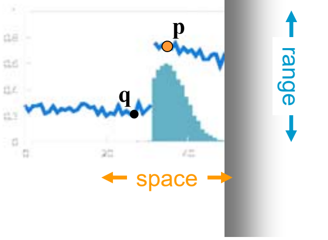
Gaussian Blur and Bilateral Filter

Gaussian blur



Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\| \mathbf{p} - \mathbf{q} \|) I_{\mathbf{q}}$$

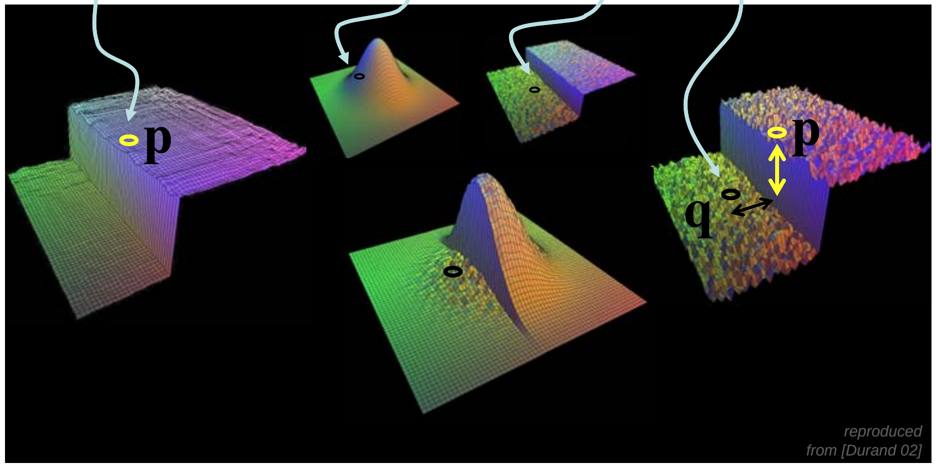
space

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) I_{\mathbf{q}}$$

normalization, space, range

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \; G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) \; I_{\mathbf{q}}$$



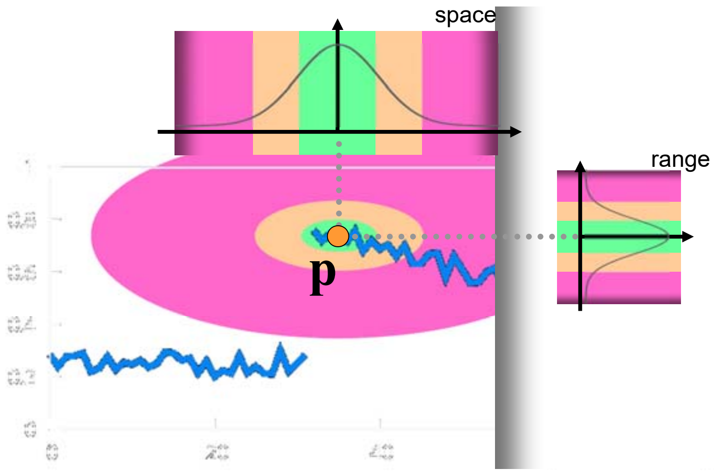
Space and Range Parameters

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) I_{\mathbf{q}}$$

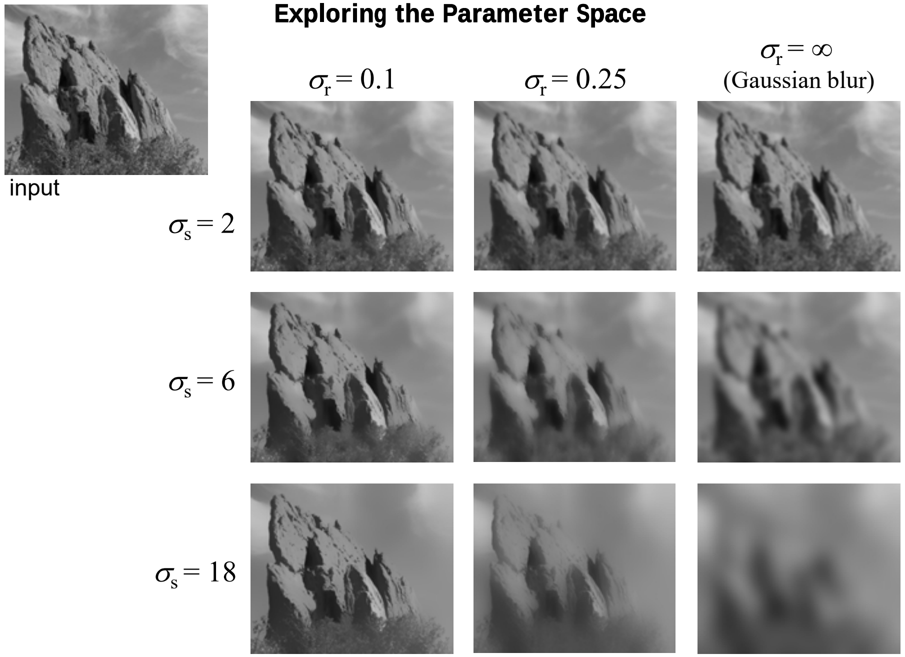
- space σ_s : spatial extent of the kernel, size of the considered neighborhood.
- range σ_r : “minimum” amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



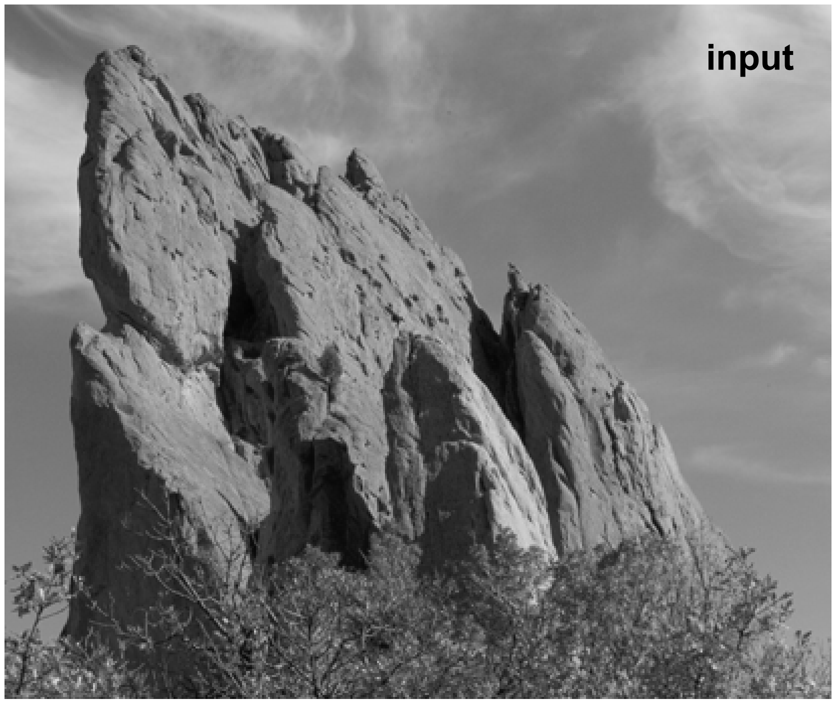
Exploring the Parameter Space

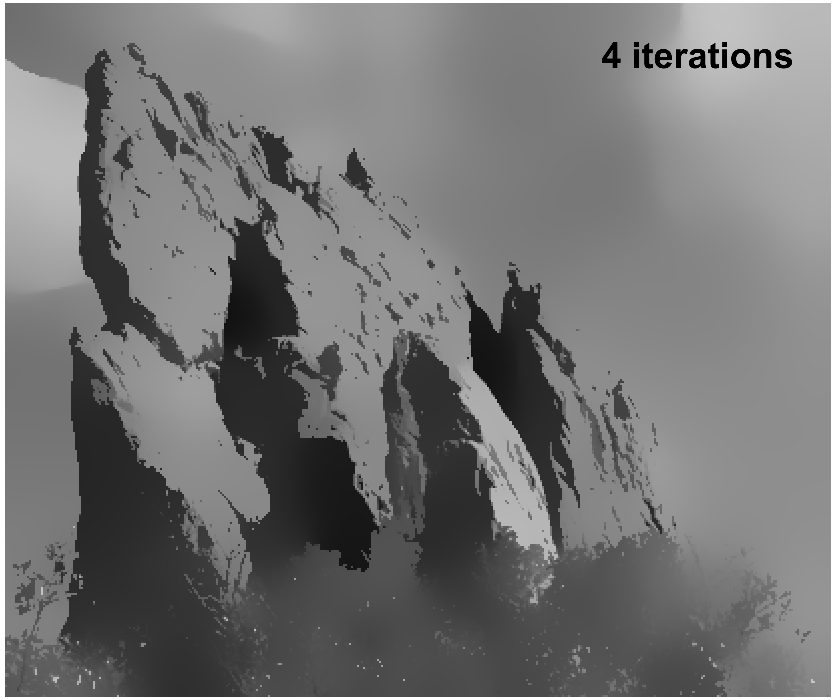


Iterating the Bilateral Filter

$$I_{(n+1)} = BF[I_{(n)}]$$

- Generate more piecewise-flat images
- Often not needed in computational photo, but could be useful for applications such as NPR.





Advantages of Bilateral Filter

- Easy to understand
  - Weighted mean of nearby pixels
- Easy to adapt
  - Distance between pixel values
- Easy to set up
  - Non-iterative

Hard to Compute

- Nonlinear $BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$
- Complex, spatially varying kernels
  - Cannot be precomputed, no FFT…

- Brute-force implementation is slow > 10min

But Bilateral Filter is Nonlinear

- Slow but some accelerations exist:
  - [Elad 02]: Gauss-Seidel iterations
    - **Only for many iterations**
  - [Durand 02, Weiss 06]: fast approximation
    - **No formal understanding** of accuracy versus speed
    - [Weiss 06]: Only **box function** as spatial kernel

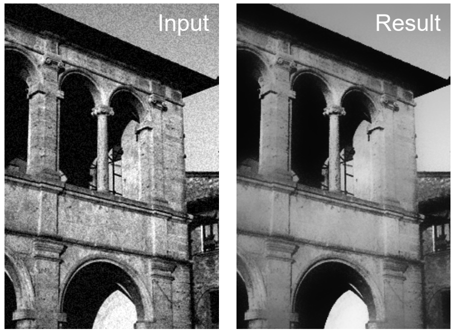
A Fast Approximation of the Bilateral Filter using a Signal Processing Approach

Sylvain Paris and Frédo Durand

Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology

Definition of Bilateral Filter

- [Smith 97, Tomasi 98]
- Smoothes an image and preserves edges
- Weighted average of neighbors
- Weights
  - Gaussian on *space* distance
  - Gaussian on *range* distance
  - sum to 1

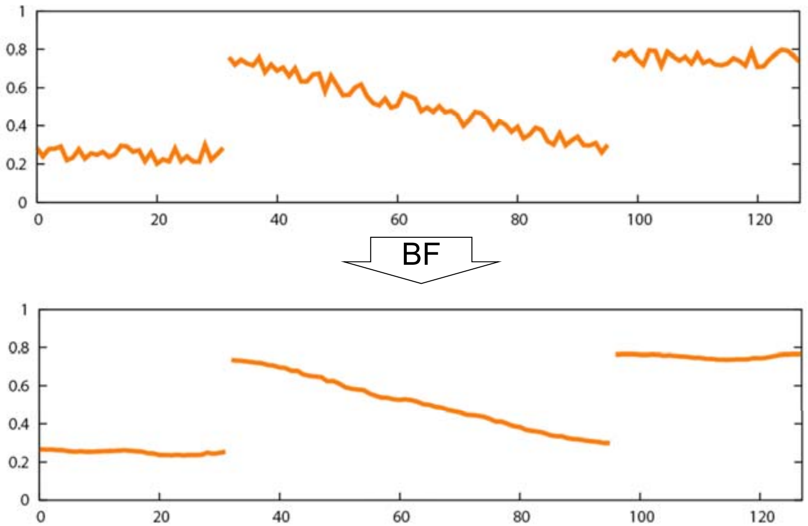


$$I_{\mathbf{p}}^{\mathrm{bf}} = \frac{1}{W_{\mathbf{p}}^{\mathrm{bf}}} \sum_{\mathbf{q}\in\mathcal{S}} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} I_{\mathbf{q}}$$

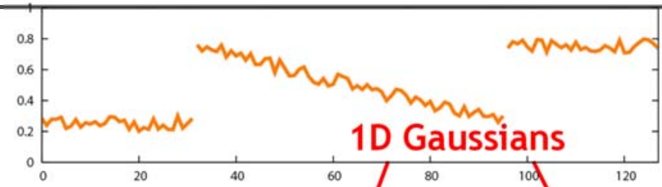
Contributions

- Link with **linear filtering**
- **Fast** and **accurate** approximation

Intuition on 1D Signal



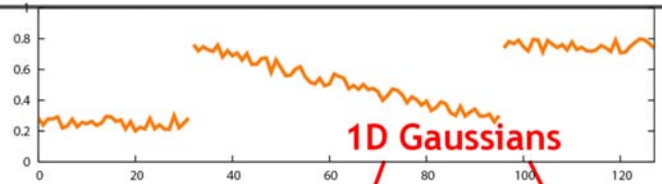
Basic idea



1D Gaussians

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G(\mathbf{q};\mathbf{p},\sigma_s)\, G(I_{\mathbf{q}};I_{\mathbf{p}},\sigma_r)\, I_{\mathbf{q}}$$

Basic idea



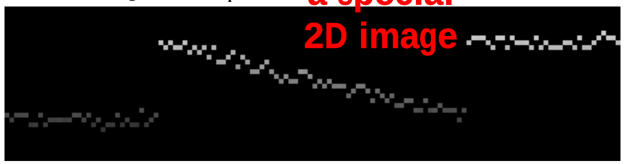
1D Gaussians

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G(\mathbf{q};\mathbf{p},\sigma_s)\, G(I_{\mathbf{q}};I_{\mathbf{p}},\sigma_r)\, I_{\mathbf{q}}$$

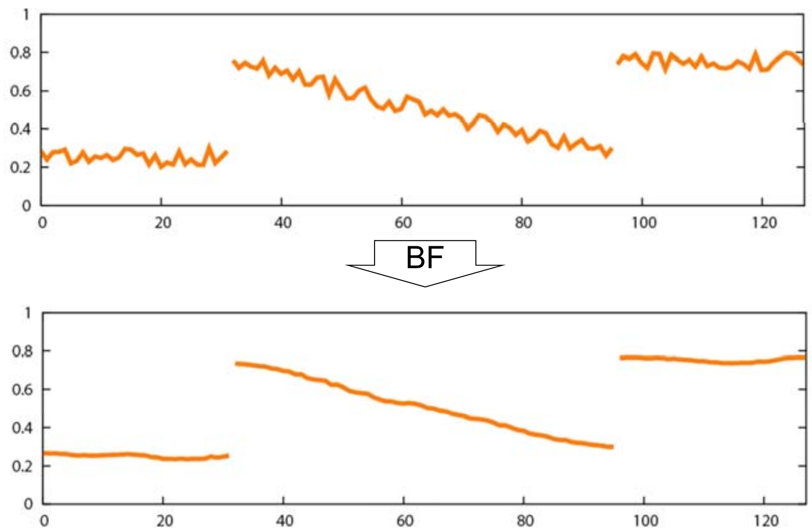
2D Gaussians

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{<\mathbf{q},I'_{\mathbf{q}}>\in S'} G(\mathbf{q},I_{\mathbf{q}};\mathbf{p},I_{\mathbf{p}},\sigma_s,\sigma_r)\, I_{<\mathbf{q},I'_{\mathbf{q}}>}$$

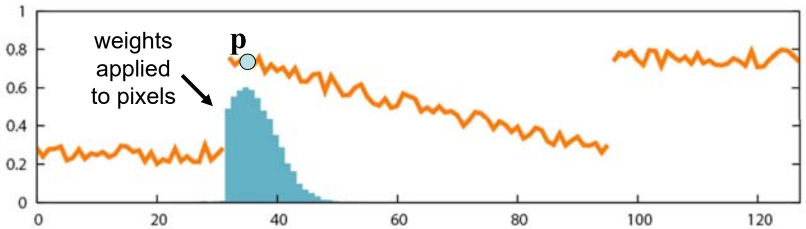
a special 2D image



Intuition on 1D Signal

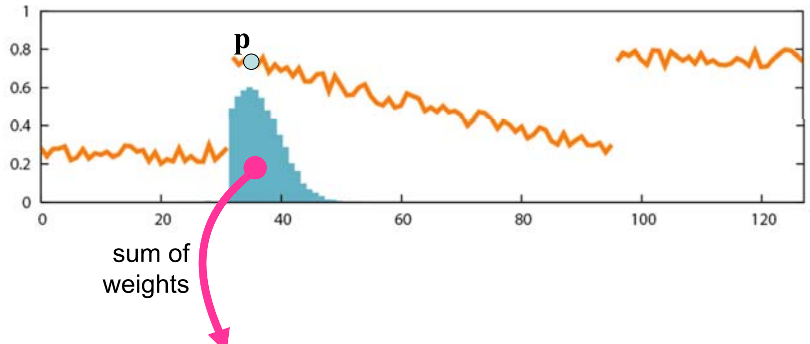


Intuition on 1D Signal
Weighted Average of Neighbors



- Near and similar pixels have influence.
- Far pixels have no influence.
- Pixels with different value have no influence.

Link with Linear Filtering
1. Handling the Division



$$I_{\mathbf{p}}^{\mathrm{bf}} = \frac{1}{W_{\mathbf{p}}^{\mathrm{bf}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

Handling the division with a **projective space**.

Formalization: Handling the Division

$$I_{\mathbf{p}}^{\mathrm{bf}} = \frac{1}{W_{\mathbf{p}}^{\mathrm{bf}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

$$W_{\mathbf{p}}^{\mathrm{bf}} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)$$

- Normalizing factor as homogeneous coordinate
  - Multiply both sides by $W_{\mathbf{p}}^{\mathrm{bf}}$

$$\begin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} I_{\mathbf{p}}^{\mathrm{bf}} \\ W_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} I_{\mathbf{q}} \\ 1 \end{pmatrix}$$

Formalization: Handling the Division

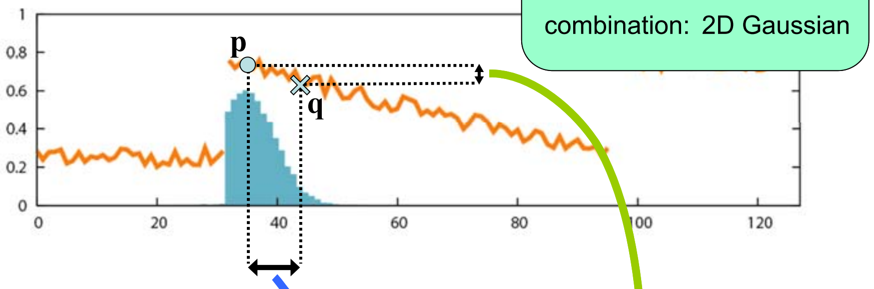
$$\begin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} I_{\mathbf{p}}^{\mathrm{bf}} \\ W_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix} \quad \text{with } W_{\mathbf{q}}=1$$

- Similar to homogeneous coordinates in projective space
- Division delayed until the end
- Next step: Adding a dimension to make a convolution appear

Link with Linear Filtering
2. Introducing a Convolution

space: 1D Gaussian
× range: 1D Gaussian

combination: 2D Gaussian

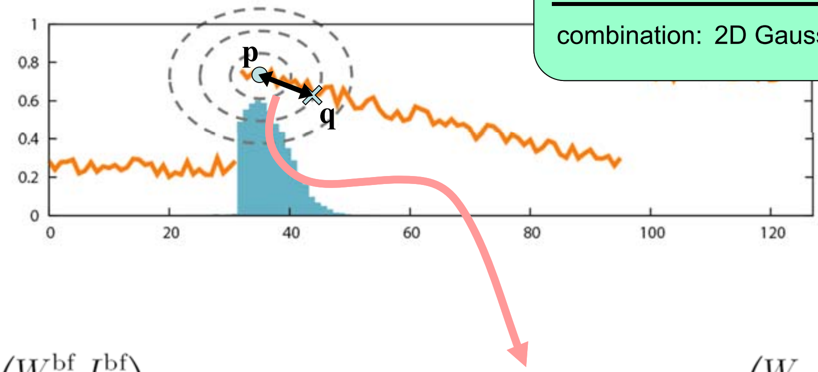


$$\begin{pmatrix} W^{\mathrm{bf}}_{\mathbf{p}} I^{\mathrm{bf}}_{\mathbf{p}} \\ W^{\mathrm{bf}}_{\mathbf{p}} \end{pmatrix} = \sum_{\mathbf{q}\in\mathcal{S}} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

Link with Linear Filtering
2. Introducing a Convolution

space: 1D Gaussian
× range: 1D Gaussian

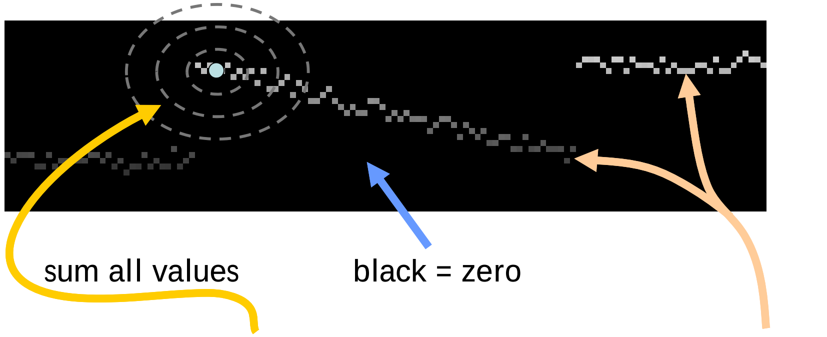
combination: 2D Gaussian



$$\begin{pmatrix} W^{\mathrm{bf}}_{\mathbf{p}} I^{\mathrm{bf}}_{\mathbf{p}} \\ W^{\mathrm{bf}}_{\mathbf{p}} \end{pmatrix} = \sum_{\mathbf{q}\in\mathcal{S}} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{space x range}} \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

Corresponds to a 3D Gaussian on a 2D image.

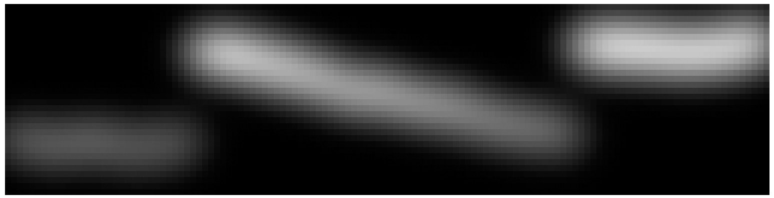
Link with Linear Filtering
2. Introducing a Convolution



$$\begin{pmatrix} W^{\mathrm{bf}}_{\mathbf{p}} I^{\mathrm{bf}}_{\mathbf{p}} \\ W^{\mathrm{bf}}_{\mathbf{p}} \end{pmatrix} = \sum_{(\mathbf{q},\zeta)\in\mathcal{S}\times\mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

sum all values multiplied by kernel ⇨ convolution

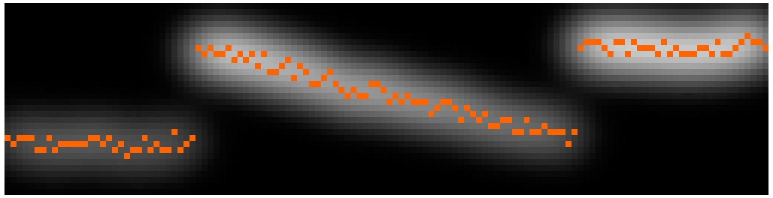
Link with Linear Filtering
2. Introducing a Convolution



result of the convolution

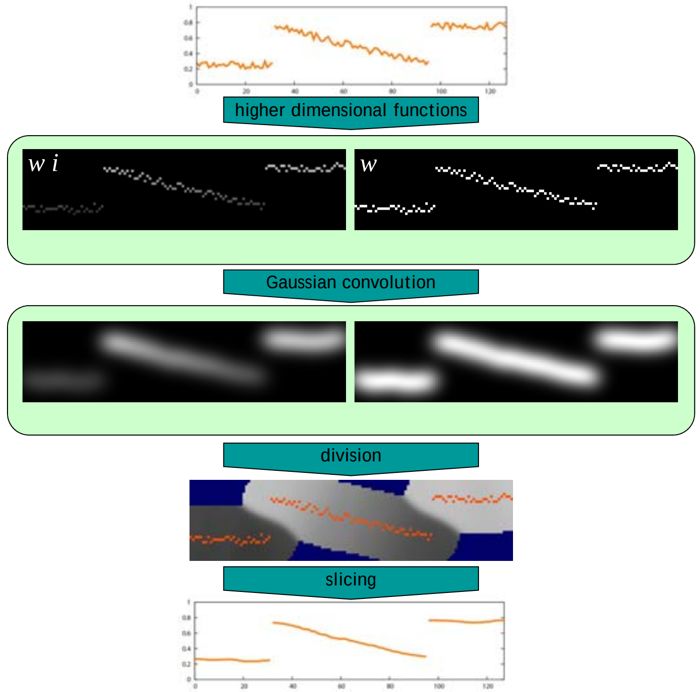
$$\begin{pmatrix} W^{\mathrm{bf}}_{\mathbf{p}} I^{\mathrm{bf}}_{\mathbf{p}} \\ W^{\mathrm{bf}}_{\mathbf{p}} \end{pmatrix} = \sum_{(\mathbf{q},\zeta)\in\mathcal{S}\times\mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

Link with Linear Filtering
2. Introducing a Convolution



result of the convolution

$$\begin{pmatrix} W^{\text{bf}}_{\mathbf{p}}\, I^{\text{bf}}_{\mathbf{p}} \\ W^{\text{bf}}_{\mathbf{p}} \end{pmatrix} = \sum_{(\mathbf{q},\zeta)\in\mathcal{S}\times\mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_{\mathbf{q}}\, I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

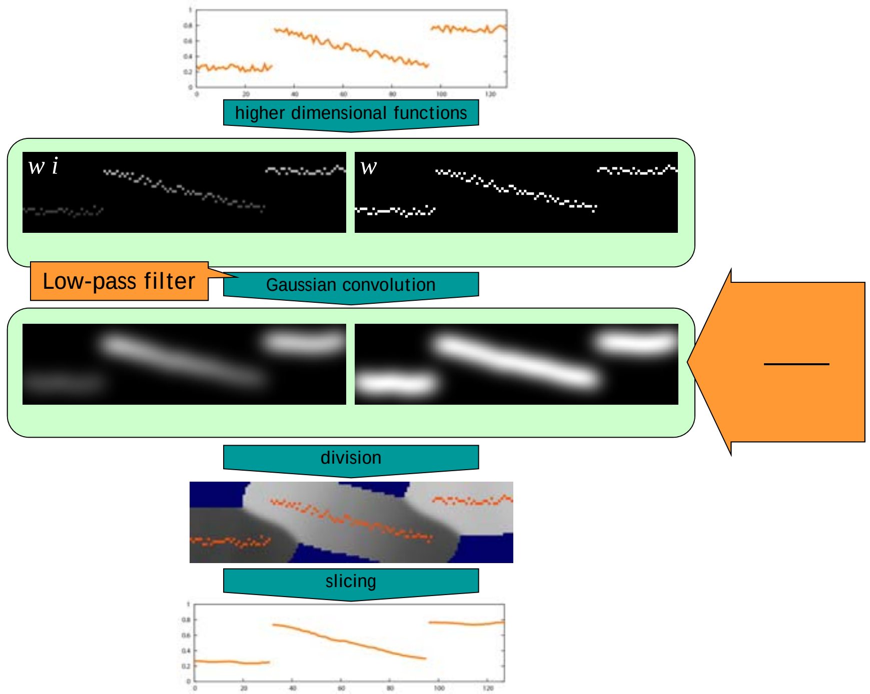


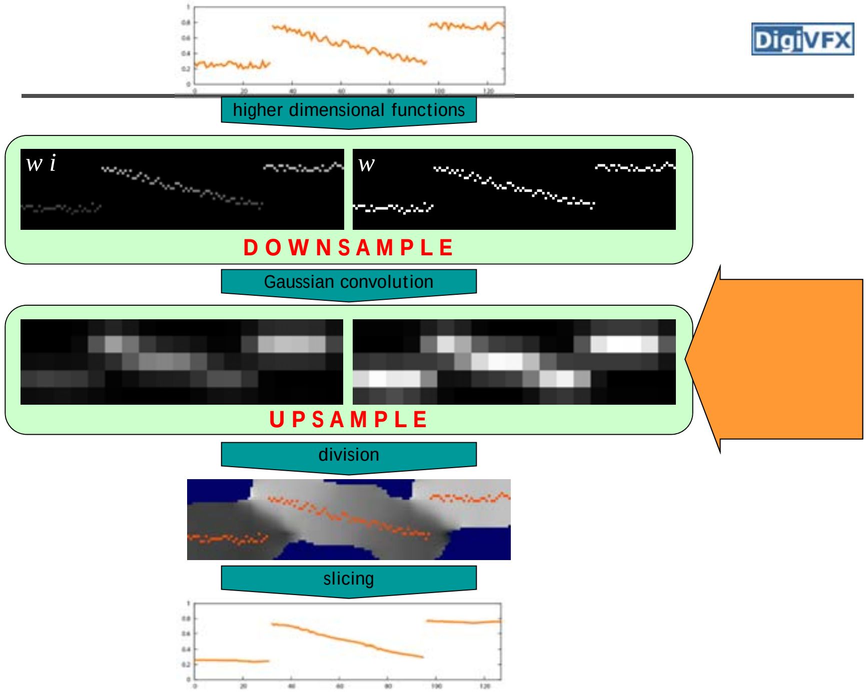
Reformulation: Summary

linear:	$(w^{\text{bf}}\, i^{\text{bf}}, w^{\text{bf}})$	$=$	$g_{\sigma_s,\sigma_r} \otimes (wi, w)$
nonlinear:	$I^{\text{bf}}_{\mathbf{p}}$	$=$	$\dfrac{w^{\text{bf}}(\mathbf{p}, I_{\mathbf{p}})\; i^{\text{bf}}(\mathbf{p}, I_{\mathbf{p}})}{w^{\text{bf}}(\mathbf{p}, I_{\mathbf{p}})}$

1. Convolution in higher dimension
   - expensive but well understood (linear, FFT, etc)
2. Division and slicing
   - nonlinear but simple and pixel-wise

Exact reformulation



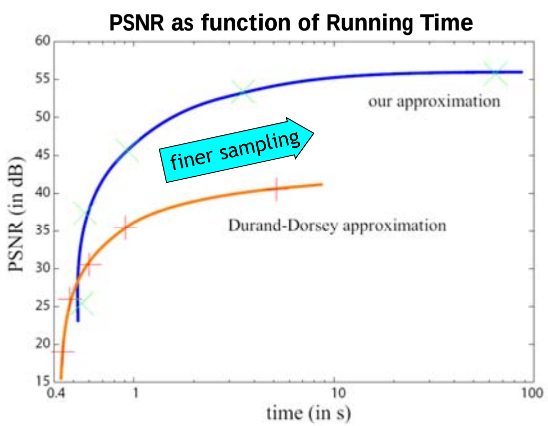


Fast Convolution by Downsampling

- Downsampling cuts frequencies above Nyquist limit
  - Less data to process
  - But induces error
- Evaluation of the approximation
  - Precision versus running time
  - Visual accuracy

Accuracy versus Running Time

- Finer sampling increases accuracy.
- More precise than previous work.

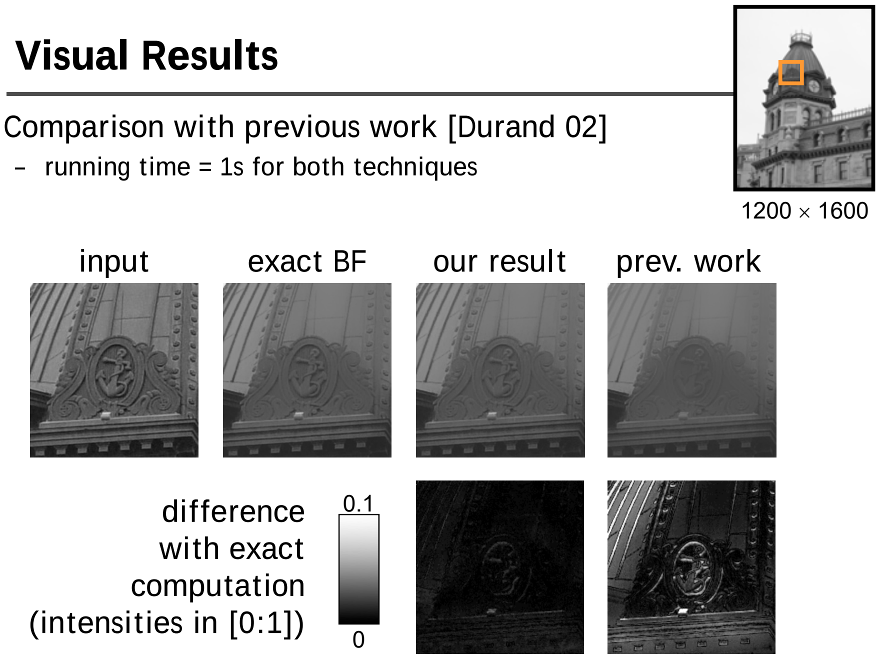


Digital photograph
1200 × 1600

Straightforward implementation is over 10 minutes.

Visual Results

- Comparison with previous work [Durand 02]
  - running time = 1s for both techniques



Two-scale Tone Management for Photographic Look

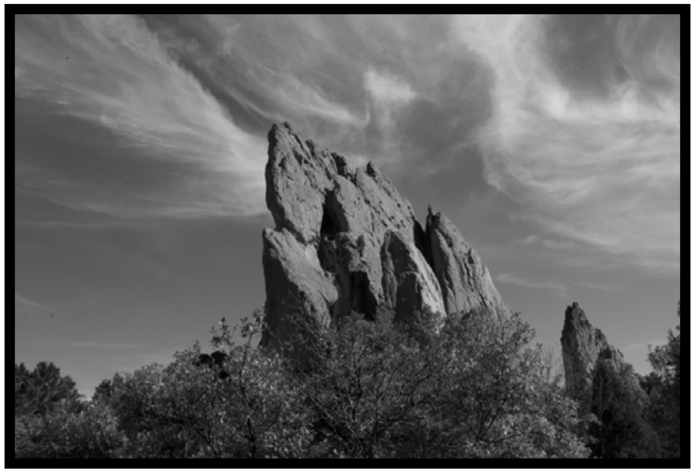
Soonmin Bae, Sylvain Paris, and Frédo Durand
MIT CSAIL

SIGGRAPH2006

Ansel Adams

Ansel Adams, *Clearing Winter Storm*

An Amateur Photographer



A Variety of Looks

Goals

- Control over photographic look
- Transfer "look" from a model photo

For example,

we want  with the look of 

Aspects of Photographic Look

- Subject choice
- Framing and composition

➔ Specified by input photos

Input

- Tone distribution and contrast

➔ Modified based on model photos

Model

Tonal Aspects of Look

Ansel Adams

Kenro Izu

Tonal aspects of Look - Global Contrast

Ansel Adams

Kenro Izu

High Global Contrast **Low Global Contrast**

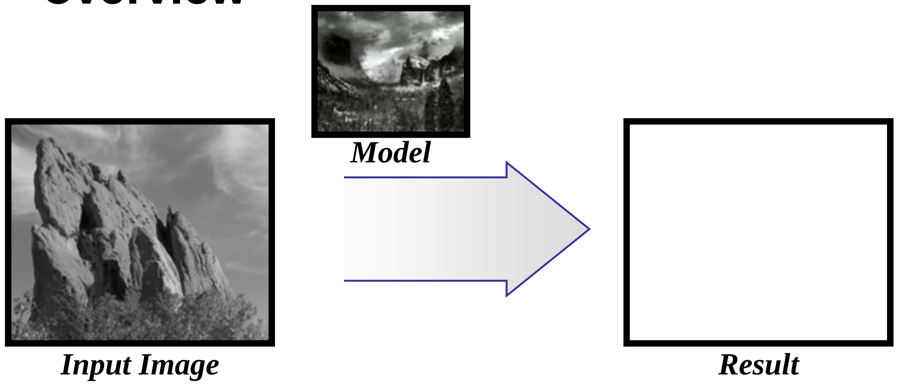
Tonal aspects of Look - Local Contrast

Ansel Adams

Kenro Izu

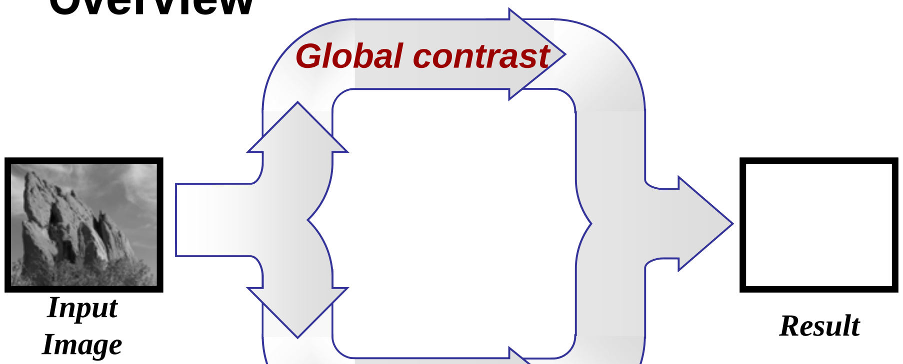
Variable amount of texture **Texture everywhere**

Overview



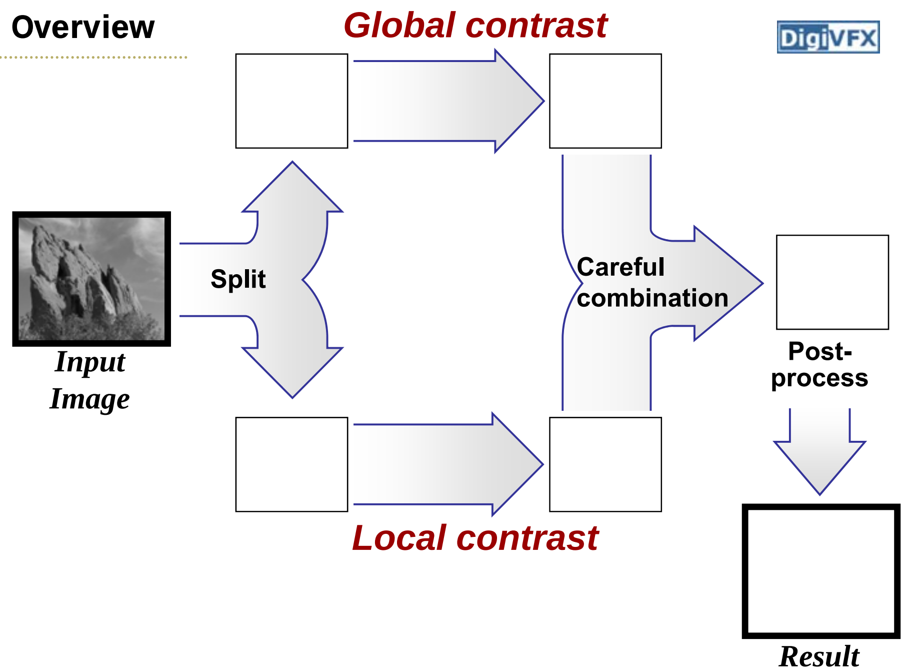
- Transfer look between photographs
  - Tonal aspects

Overview

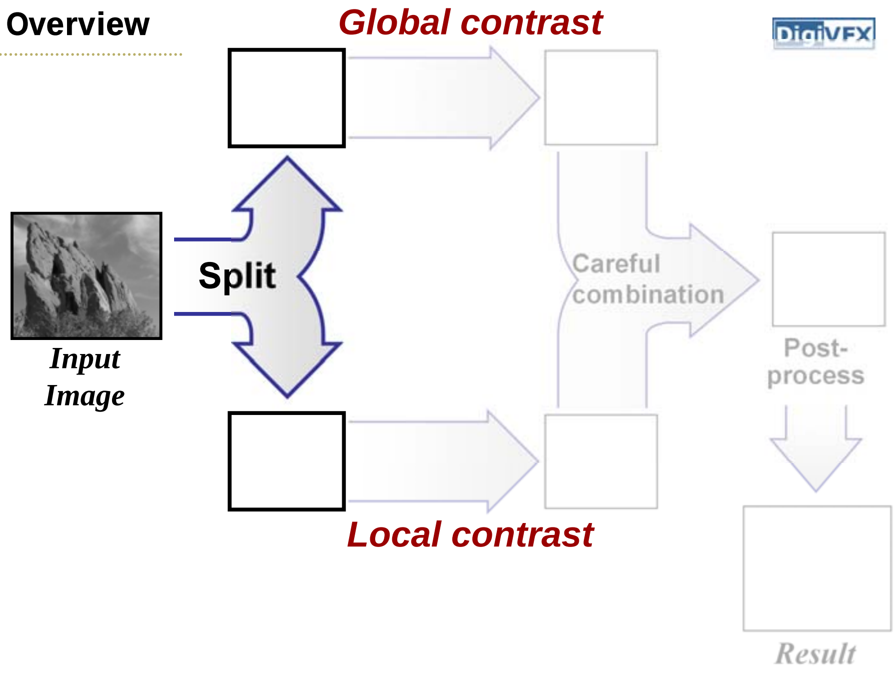


- Separate global and local contrast

Overview



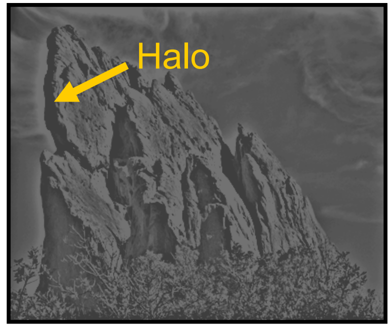
Overview



Split Global vs. Local Contrast

- Naïve decomposition: low vs. high frequency
  - Problem: introduce blur & halos

Low frequency
Global contrast



High frequency
Local contrast

Bilateral Filter

- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]

After bilateral filtering
Global contrast

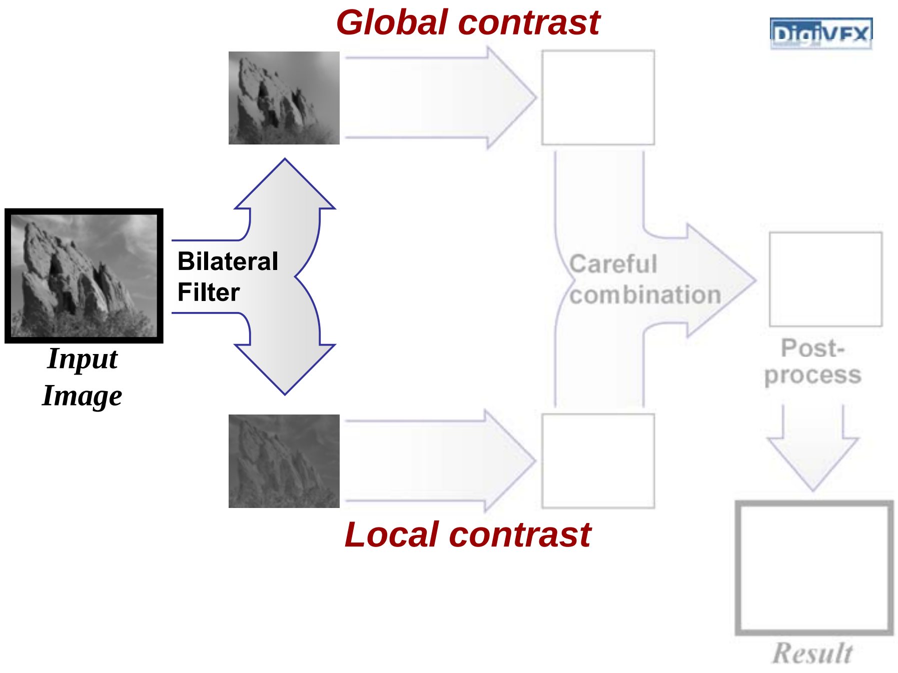
Residual after filtering
Local contrast

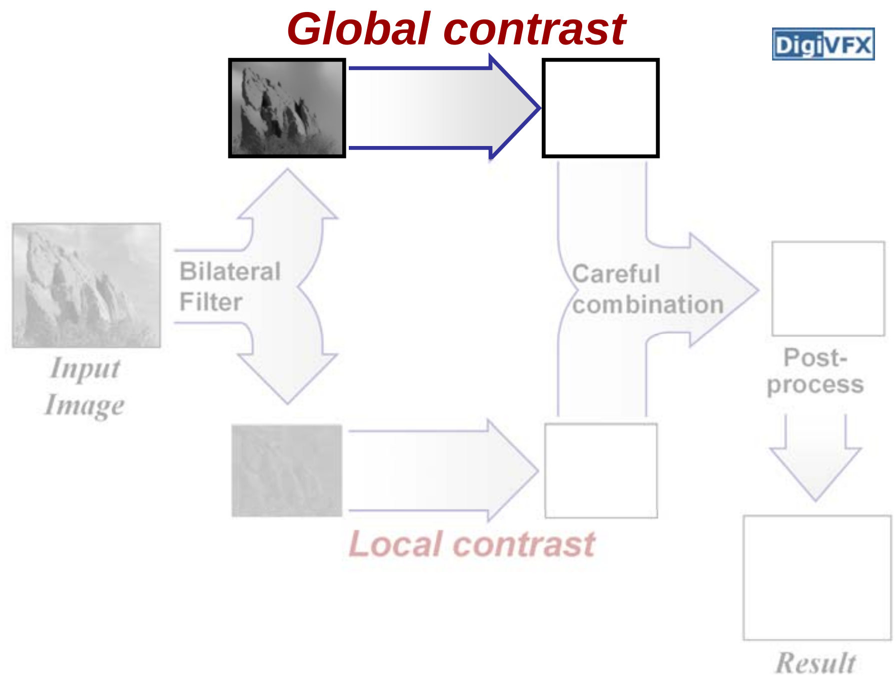
Bilateral Filter

- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]

After bilateral filtering
Global contrast

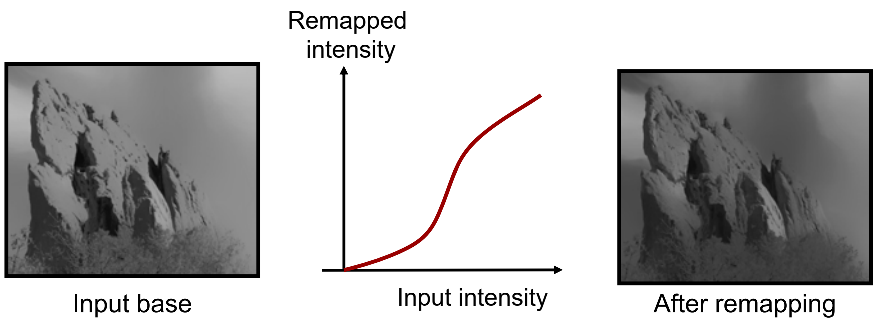
Residual after filtering
Local contrast





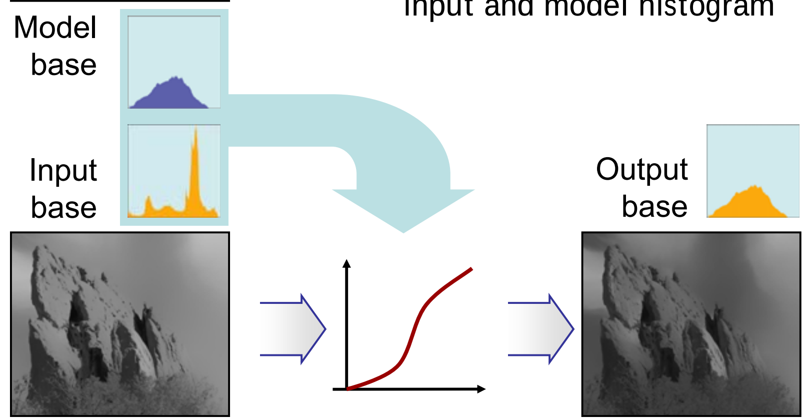
Global Contrast

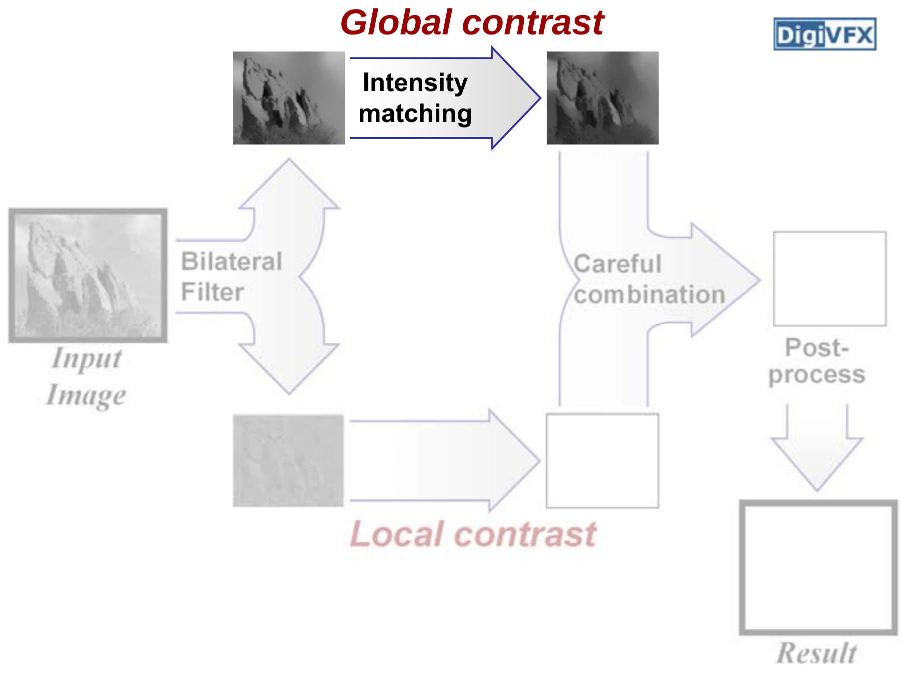
- Intensity remapping of base layer

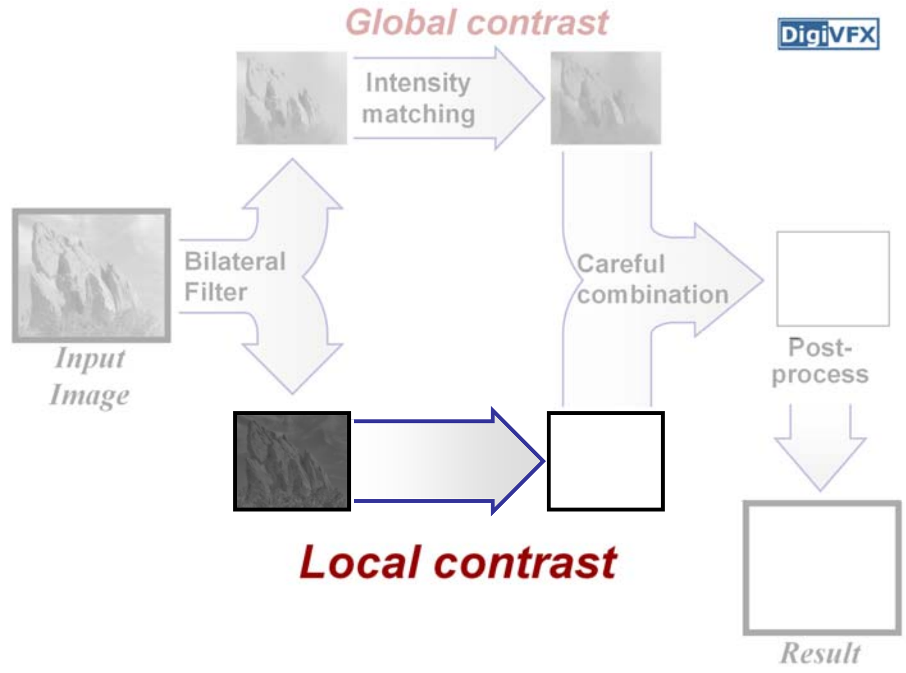


Global Contrast (Model Transfer)

- Histogram matching
  - Remapping function given input and model histogram

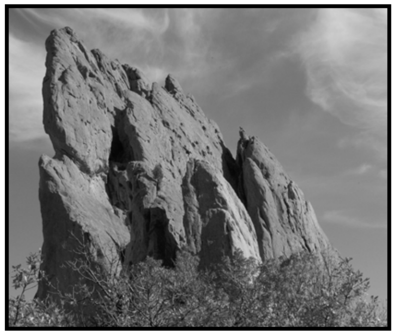




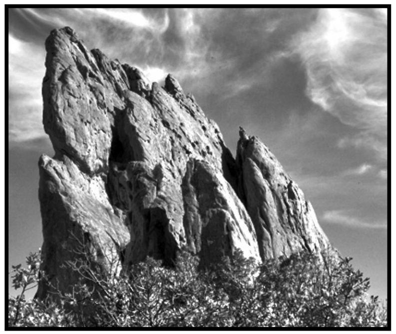


Local Contrast: Detail Layer

- Uniform control:
  - Multiply all values in the detail layer

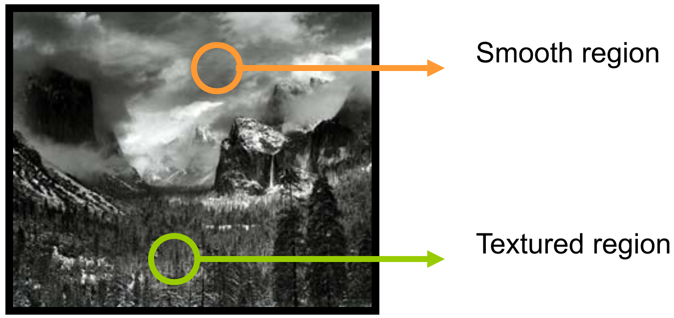


Input



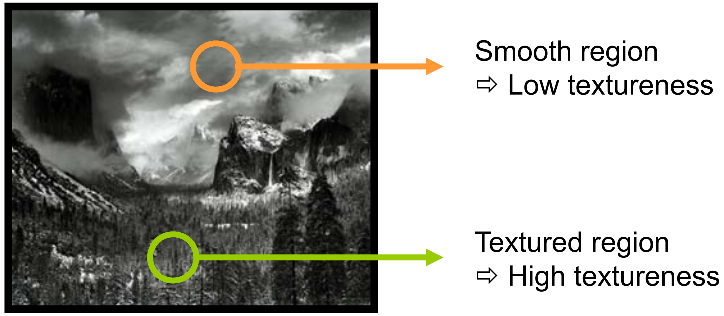
Base + 3 × Detail

The amount of local contrast is not uniform

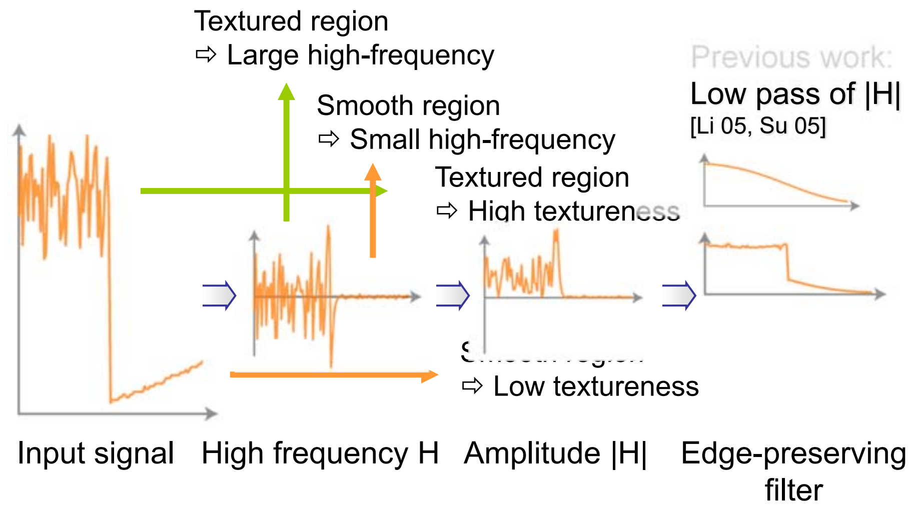


Local Contrast Variation

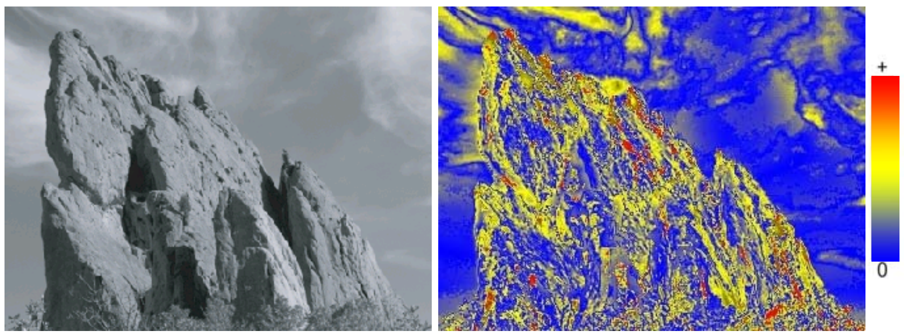
- We define "textureness": amount of local contrast
  - at each pixel based on surrounding region



"Textureness": 1D Example

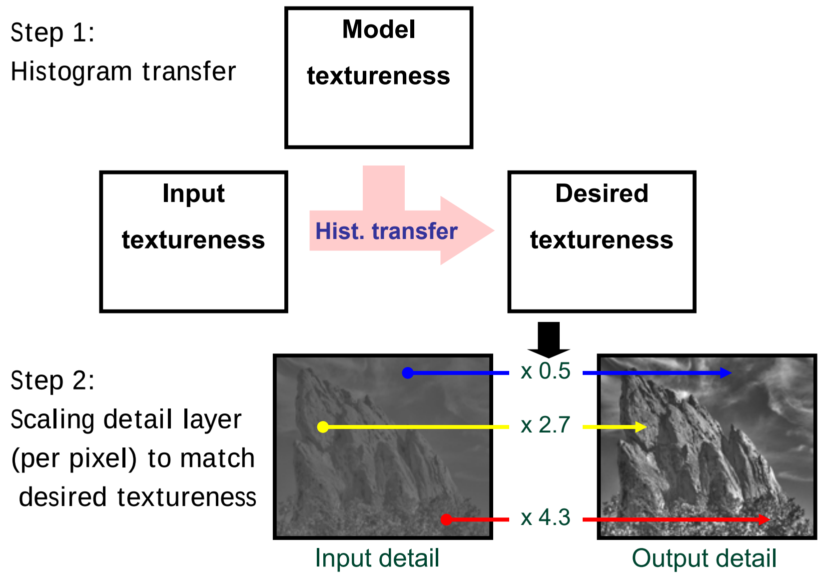


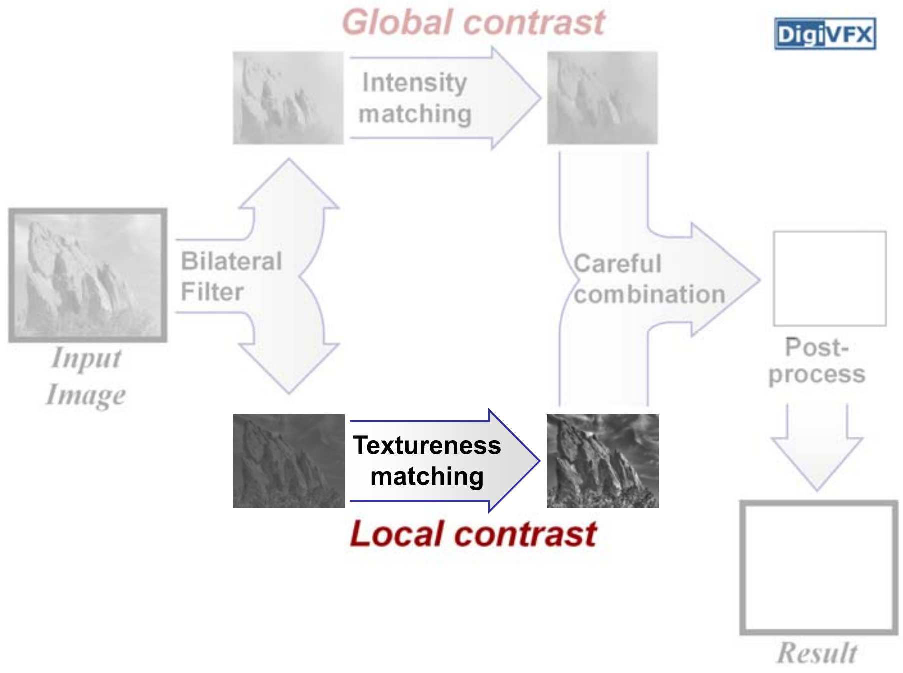
Textureness

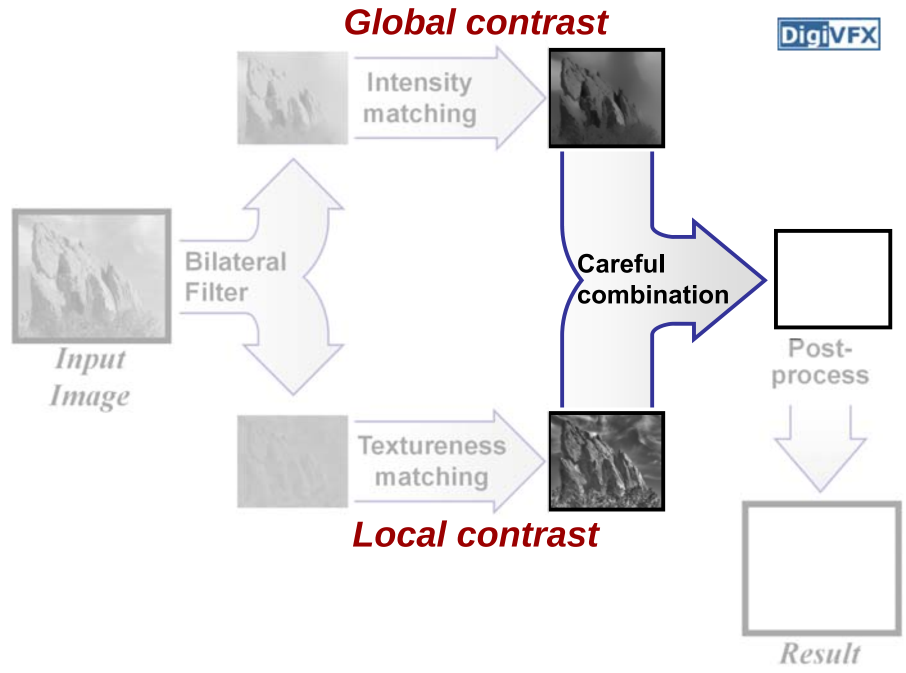


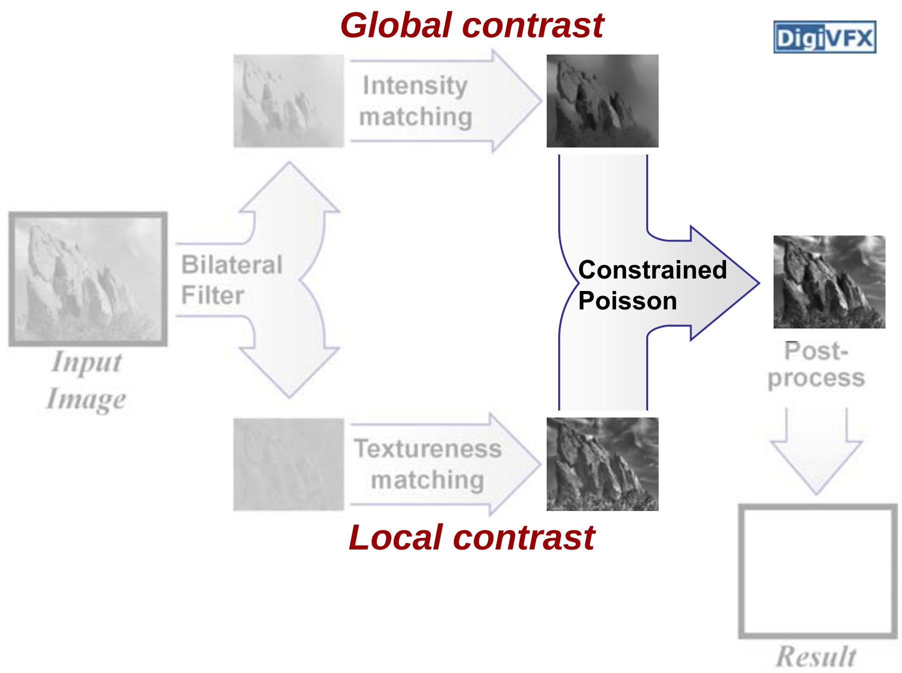
Input Textureness

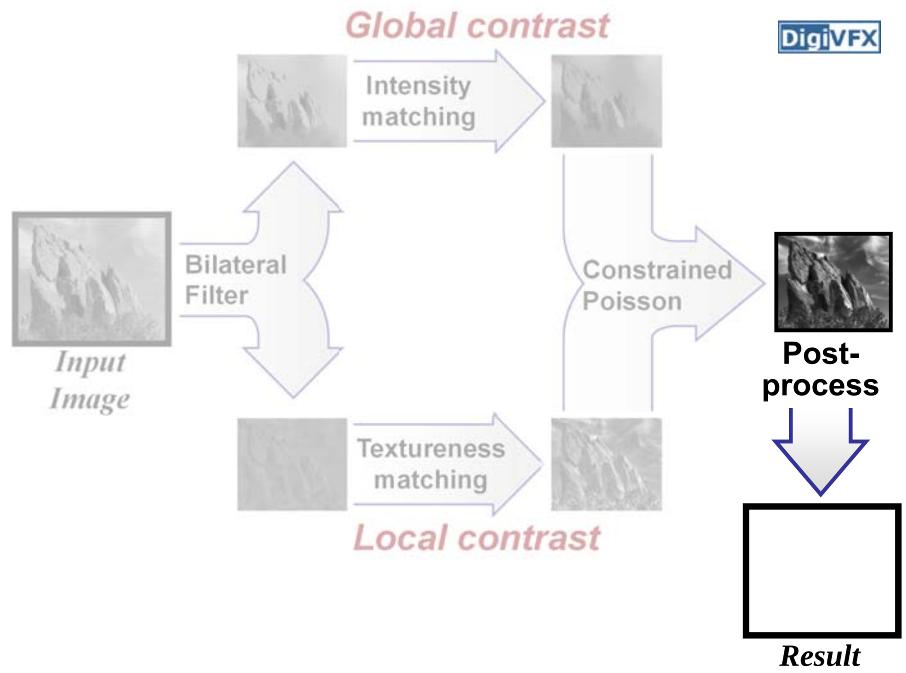
Textureness Transfer




Global contrast
Intensity matching
Bilateral Filter
Input Image
Careful combination
Post-process
Textureness matching
Local contrast
Result


Global contrast
Intensity matching
Bilateral Filter
Input Image
Careful combination
Post-process
Textureness matching
Local contrast
Result


Global contrast
Intensity matching
Bilateral Filter
Input Image
Constrained Poisson
Post-process
Textureness matching
Local contrast
Result


Global contrast
Intensity matching
Bilateral Filter
Input Image
Constrained Poisson
Post-process
Textureness matching
Local contrast
Result

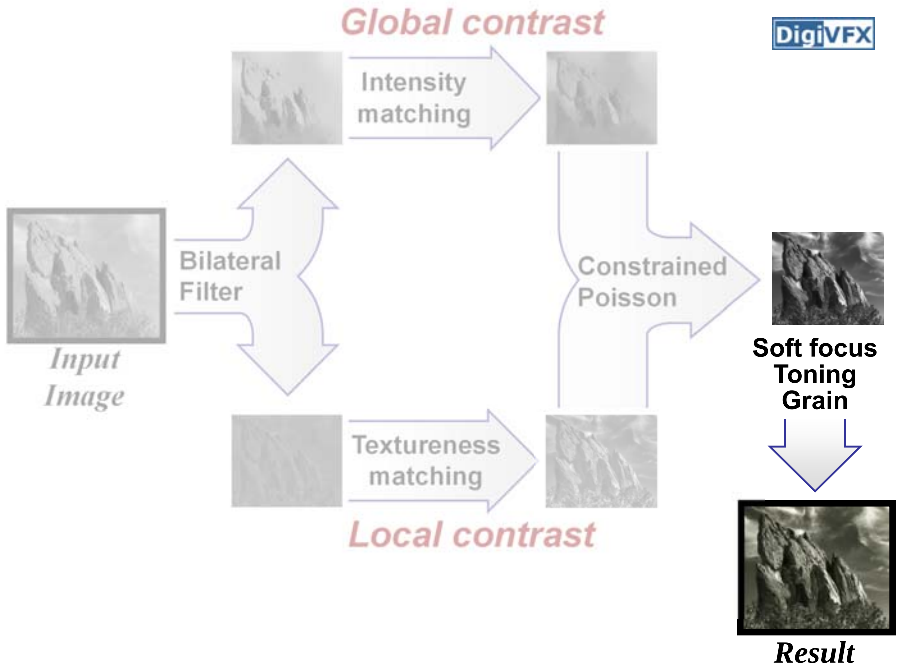
Additional Effects

- **Soft focus** (high frequency manipulation)
- **Film grain** (texture synthesis [Heeger 95])
- **Color toning** (chrominance = *f* (luminance))

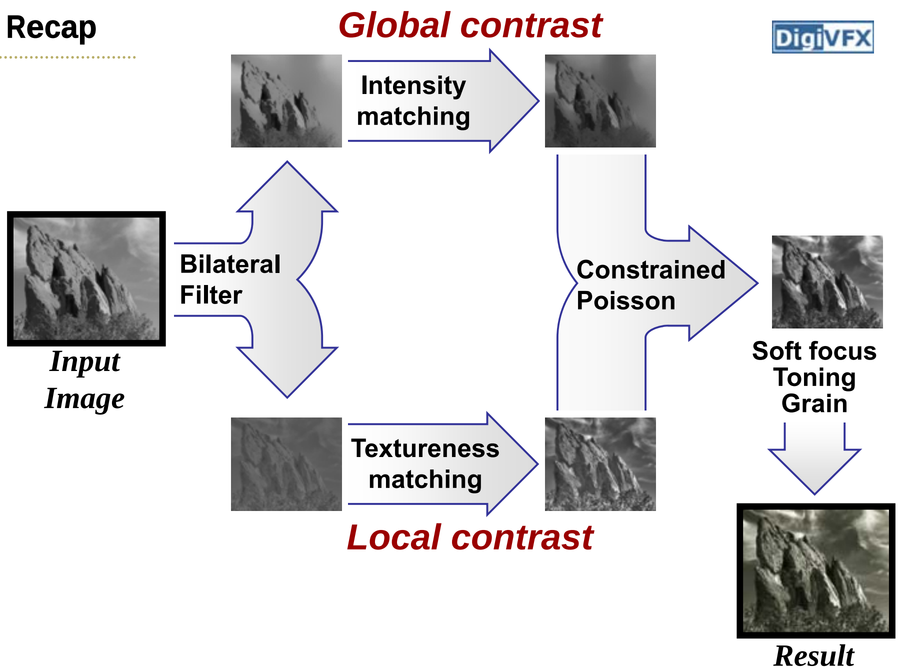
model

before effects

after effects



Recap



Results

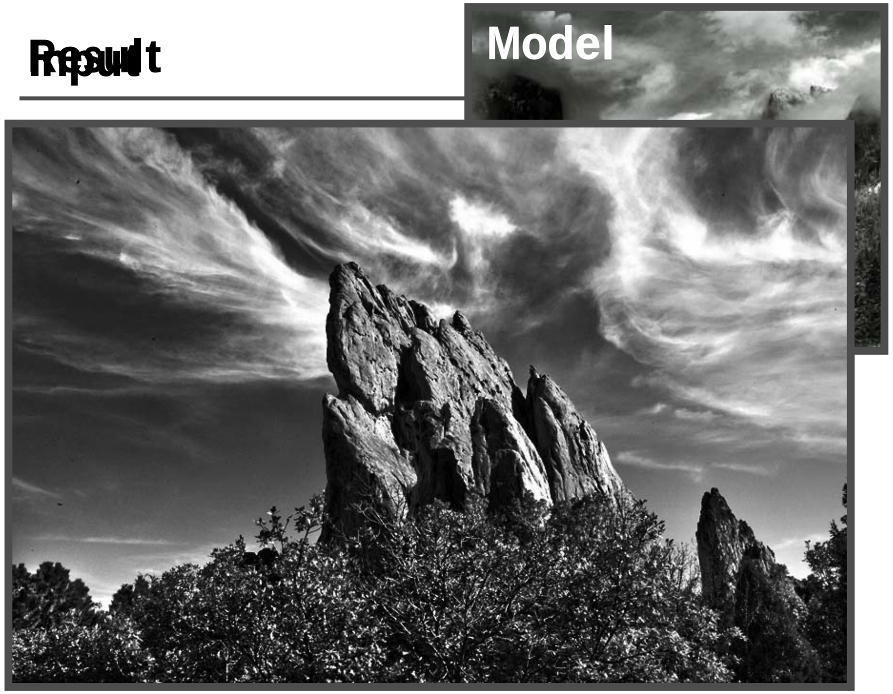
User provides input and model photographs.

➔ Our system **automatically** produces the result.

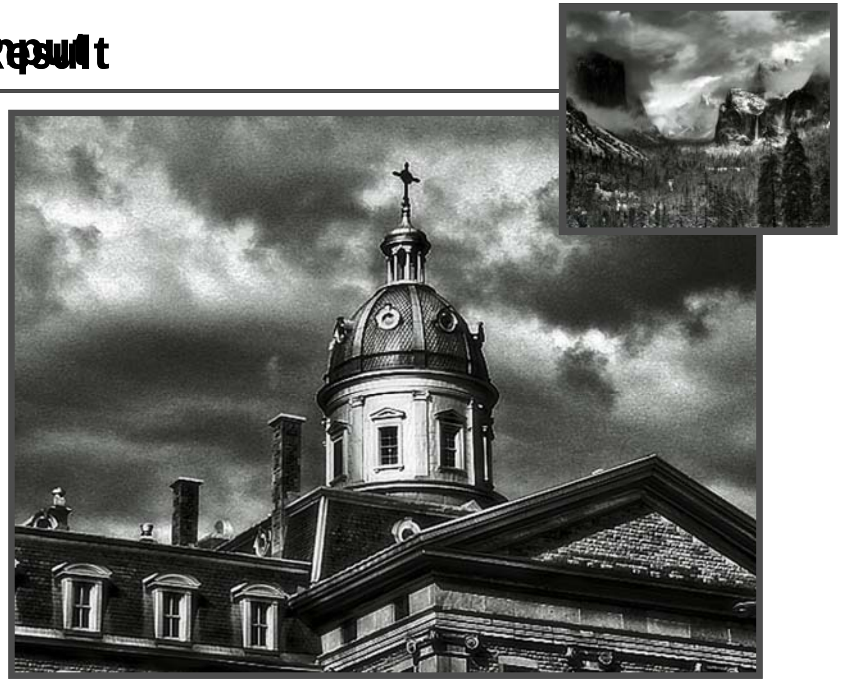
Running times:

- 6 seconds for 1 MPixel or less
- 23 seconds for 4 MPixels
- multi-grid Poisson solver and fast bilateral filter [Paris 06]

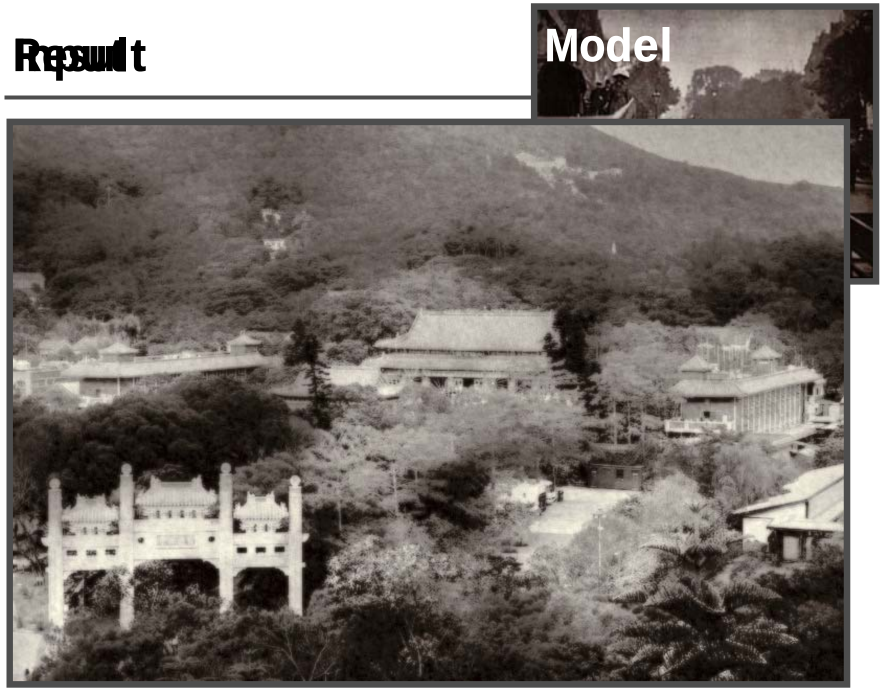
Input / Result



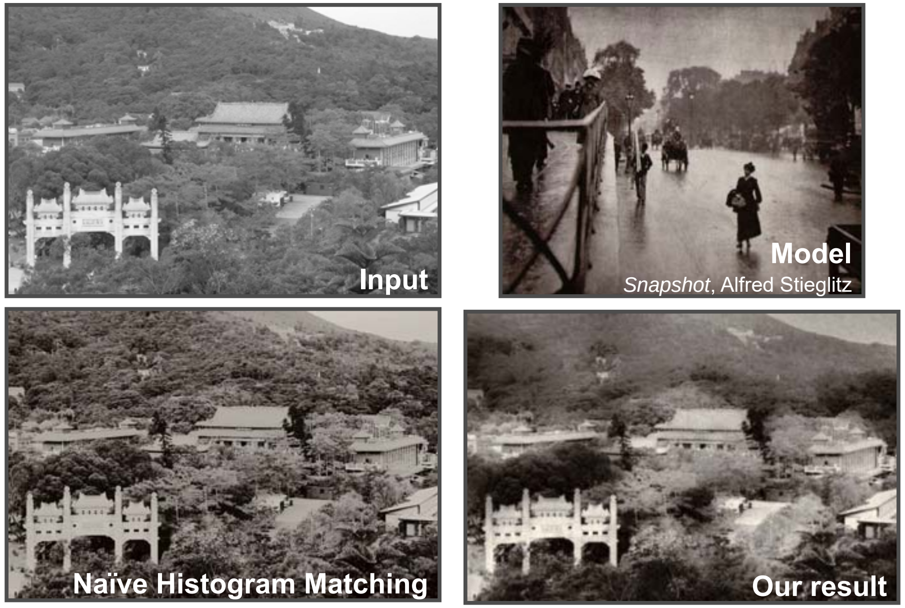
Input / Result



Input / Result



Comparison with Naïve Histogram Matching



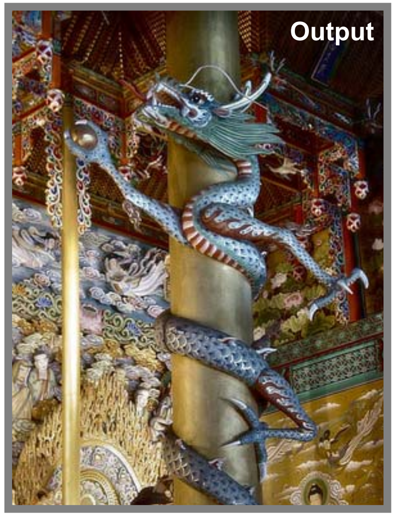
Local contrast, sharpness unfaithful

Comparison with Naïve Histogram Matching

Local contrast too low

Color Images

- **Lab color space: modify only luminance**



Limitations

- Noise and JPEG artifacts
  - amplified defects

- Can lead to unexpected results if the image content is too different from the model
  - Portraits, in particular, can suffer

Conclusions

- Transfer "look" from a model photo
- Two-scale tone management
  - Global and local contrast
  - New edge-preserving textureness
  - Constrained Poisson reconstruction
  - Additional effects

Joint bilateral filtering

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q \; f(||p-q||) \; g(||I_p - I_q||)$$

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q \; f(||p-q||) \; g(||\tilde{I}_p - \tilde{I}_q||)$$

Flash / No-Flash Photo Improvement (Petschnigg04) (Eisemann04)

Merge best features: warm, cozy candle light (no-flash)
low-noise, detailed flash image

Overview

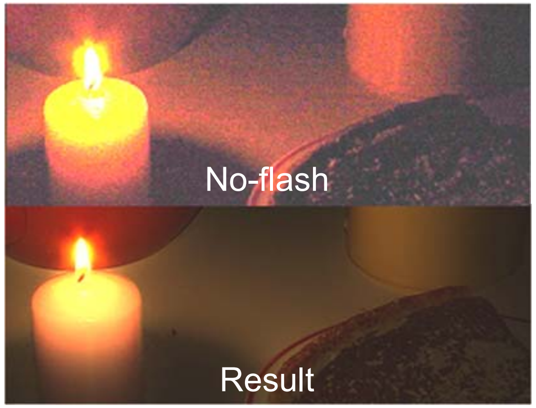
Basic approach of both flash/noflash papers

Remove noise + details from image A,

Keep as image A Lighting

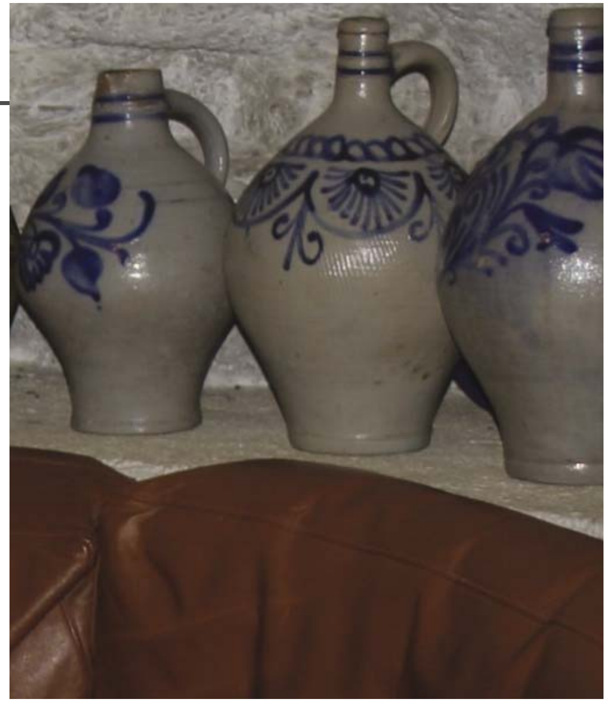
Obtain noise-free details from image B,

Discard Image B Lighting



Petschnigg:

- Flash

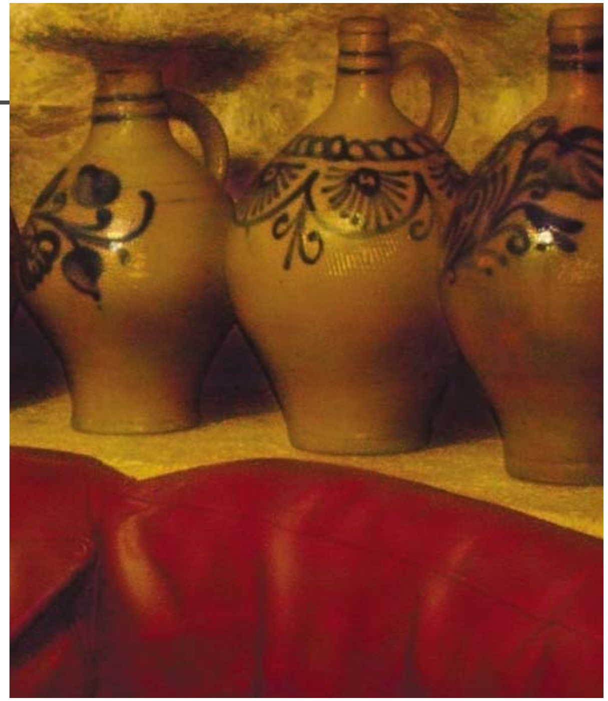


Petschnigg:

- No Flash,

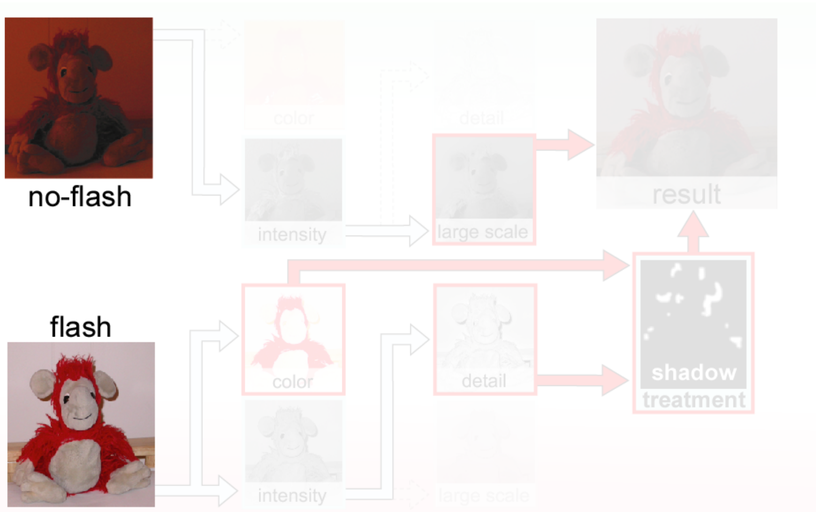
Petschnigg:

- Result



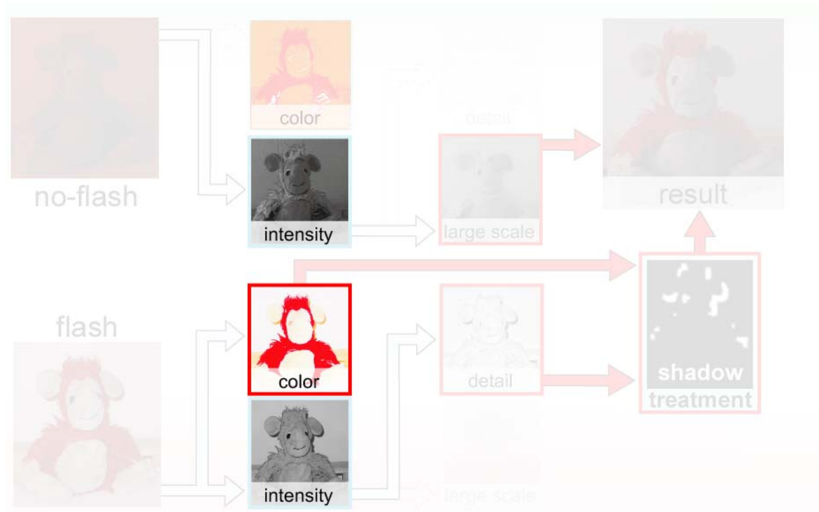
Our Approach

Registration



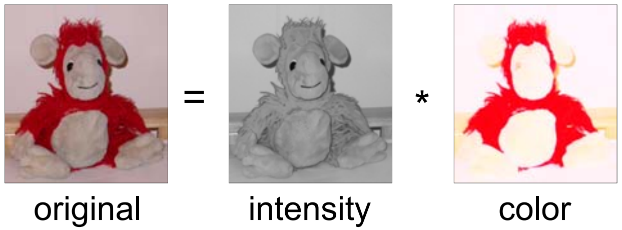
Our Approach

Decomposition



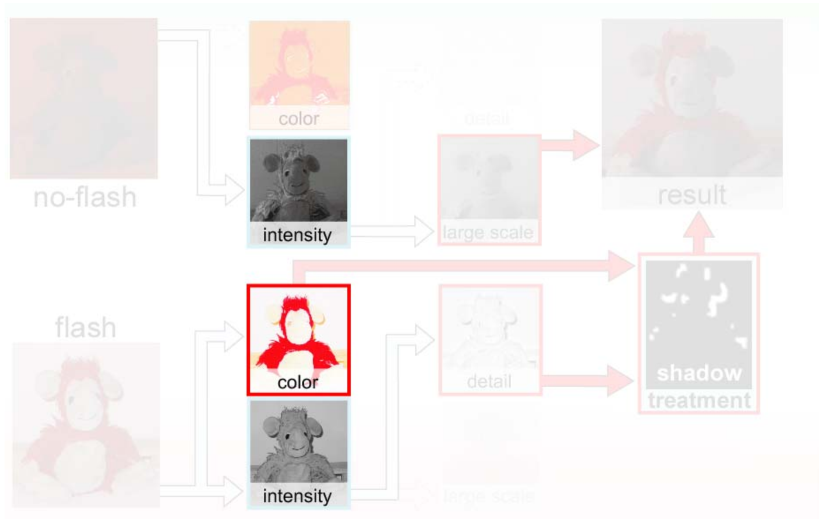
Decomposition

Color / Intensity:



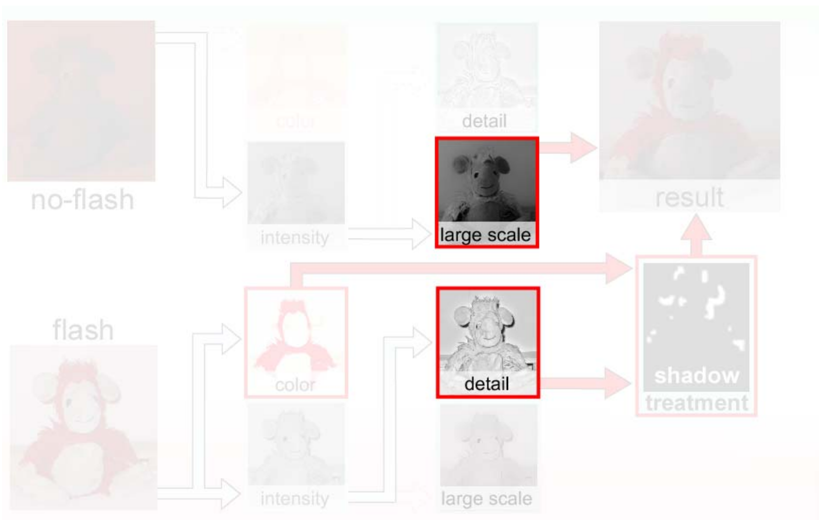
Our Approach

Decomposition



Our Approach

Decoupling



Decoupling

- Lighting : Large-scale variation
- Texture : Small-scale variation

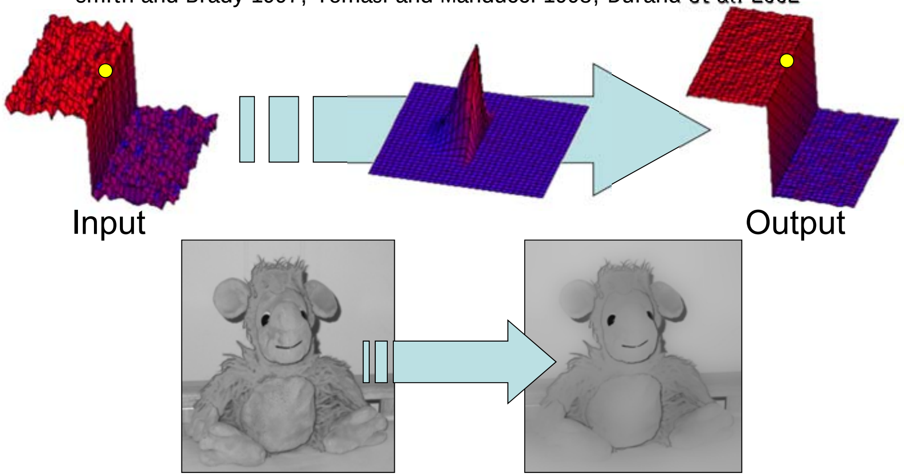
Lighting

Texture

Large-scale Layer

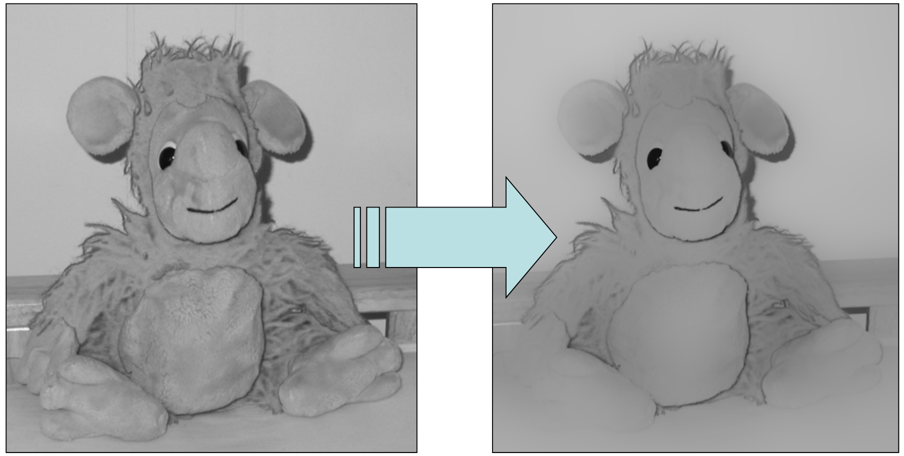
- Bilateral filter – edge preserving filter

Smith and Brady 1997; Tomasi and Manducci 1998; Durand et al. 2002



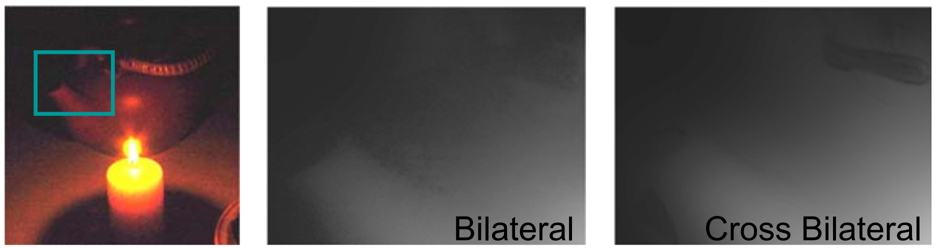
Large-scale Layer

- Bilateral filter

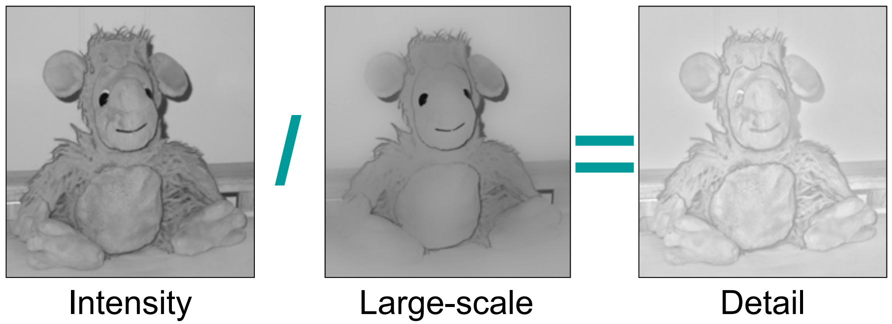


Cross Bilateral Filter

- Similar to joint bilateral filter by Petschnigg et al.
- When no-flash image is too noisy
- Borrow similarity from flash image
  - ➢ edge stopping from flash image



Detail Layer



Recombination: Large scale * Detail = Intensity

Recombination

Large-scale
No-flash

Detail
Flash

Intensity
Result

Recombination: Large scale * Detail = Intensity

Recombination

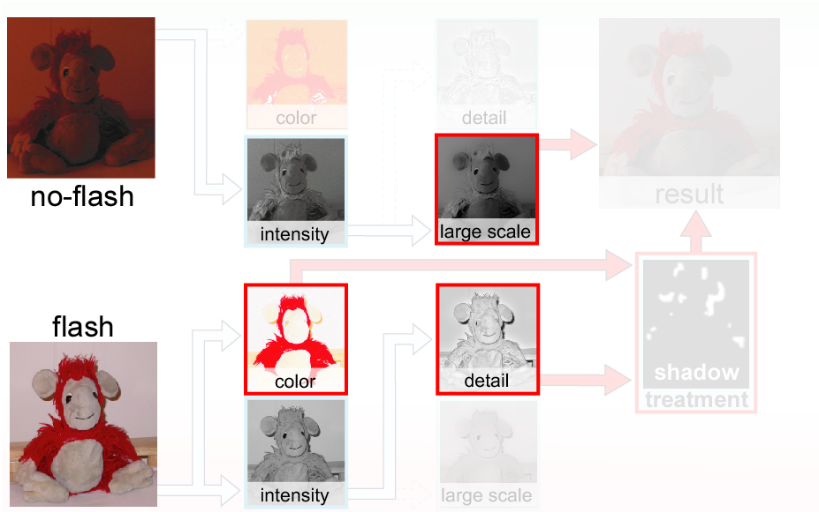
Intensity
Result

Color
Flash

Result

Recombination: Intensity * Color = Original

Our Approach



Results

No-flash

Flash