# Bilateral Filters

Digital Visual Effects

Yung-Yu Chuang

### **Image Denoising**





noisy image



naïve denoising Gaussian blur



better denoising edge-preserving filter

Smoothing an image without blurring its edges.



### A Wide Range of Options

- Diffusion, Bayesian, Wavelets...
  - All have their pros and cons.

#### Bilateral filter

- not always the best result [Buades 05] but often good
- easy to understand, adapt and set up

# Basic denoising



Noisy input \_\_\_\_ Median 5x5



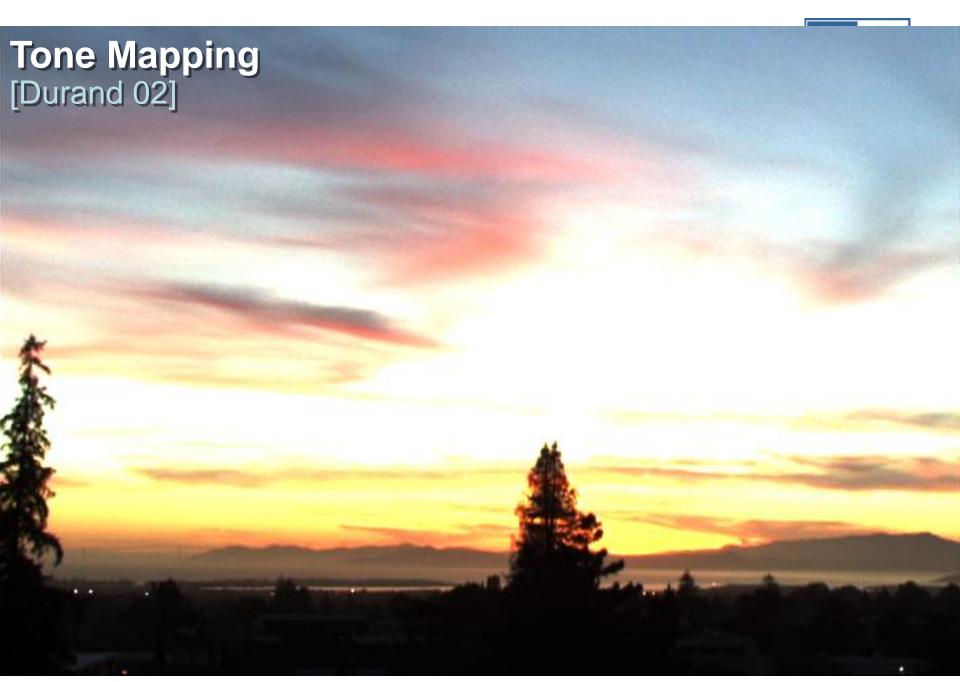
# Basic denoising

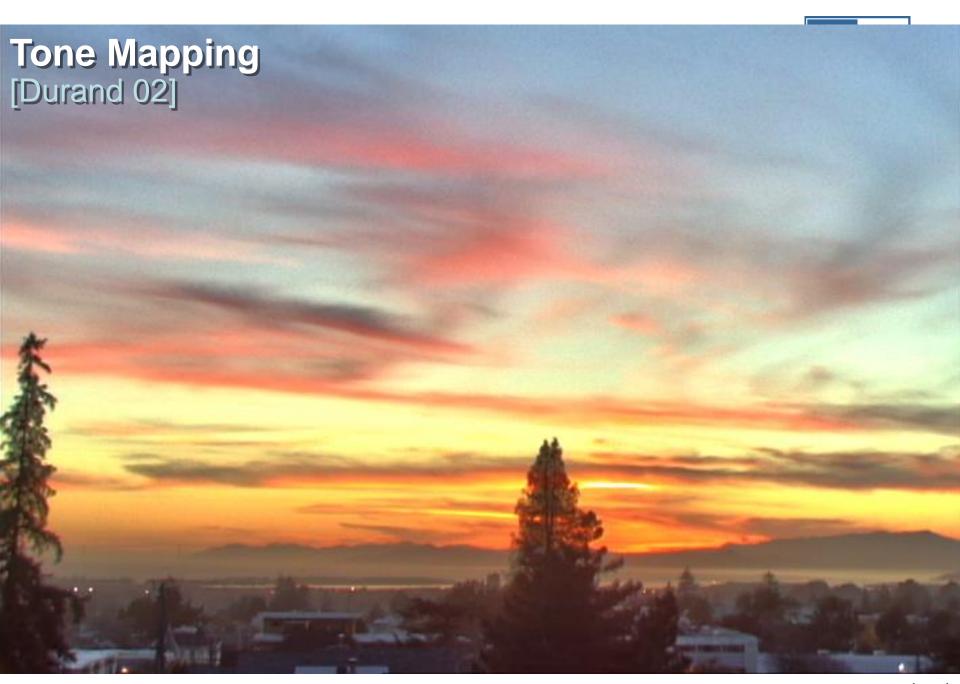


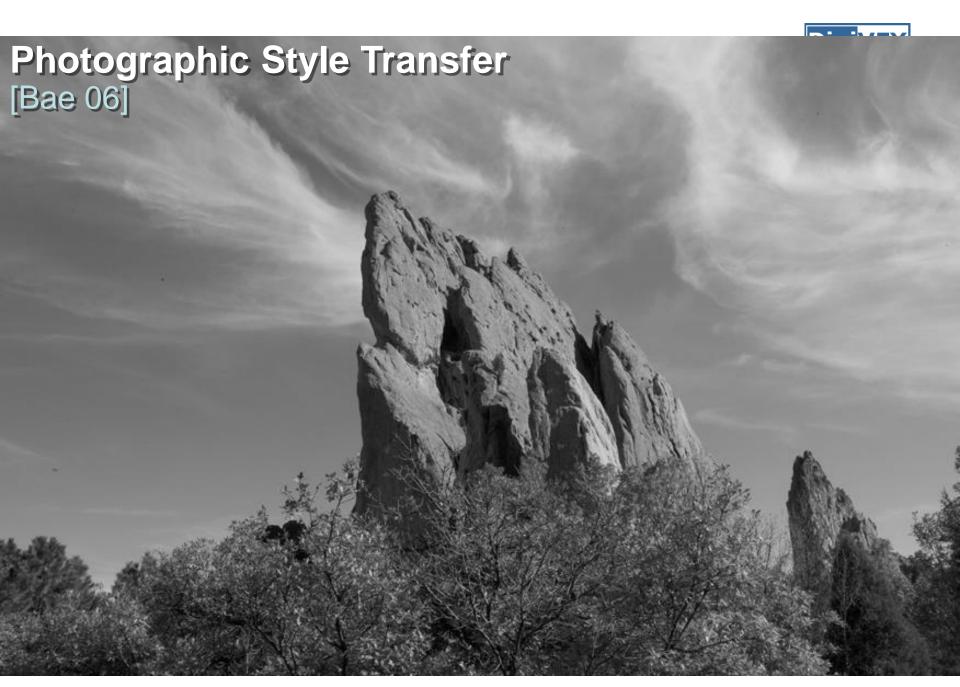
Noisy input

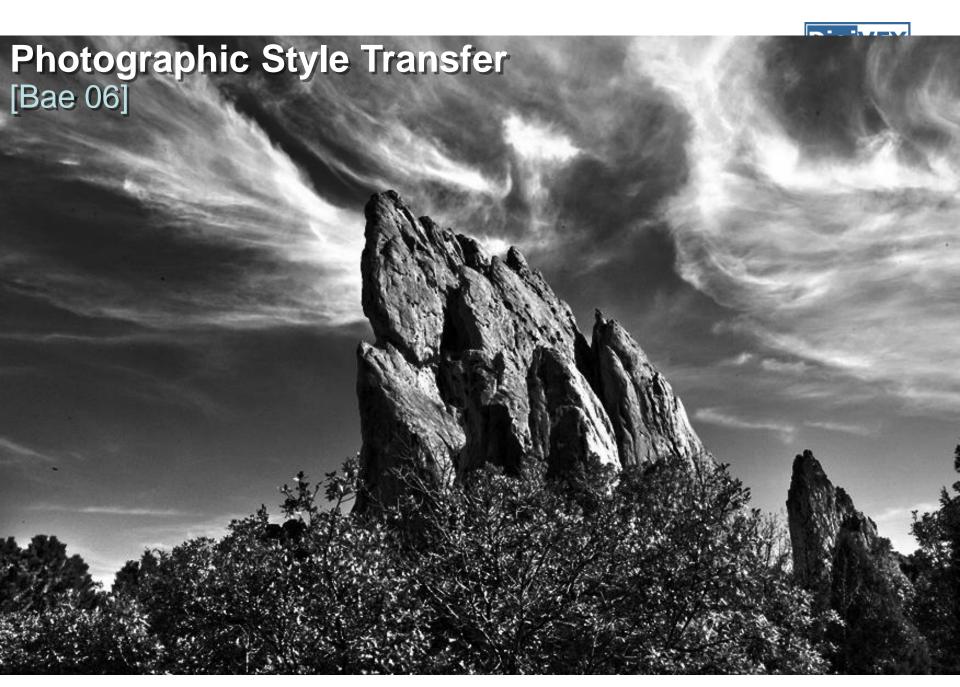
Bilateral filter 7x7 window



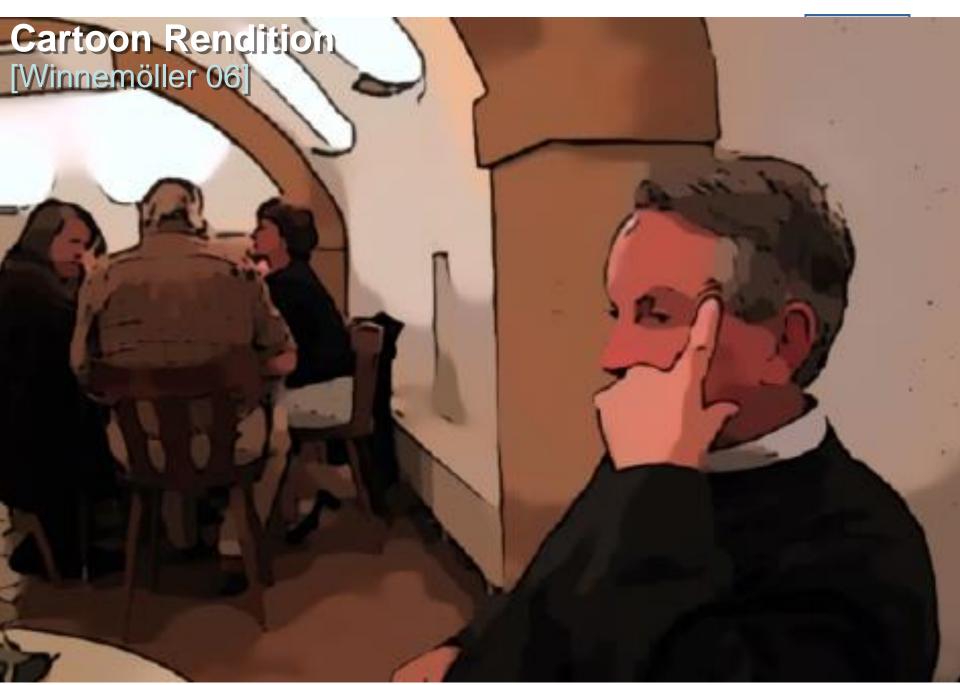






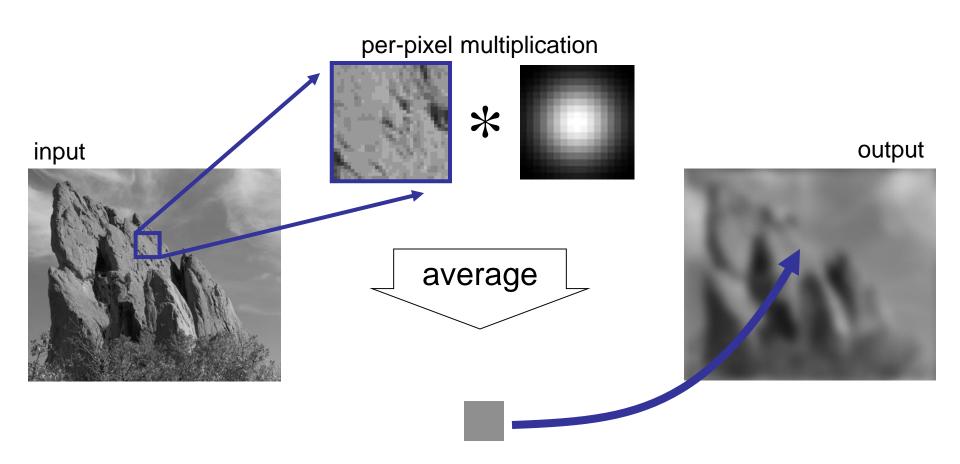






## Gaussian Blur









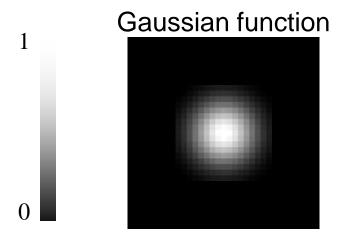




# Equation of Gaussian Blur

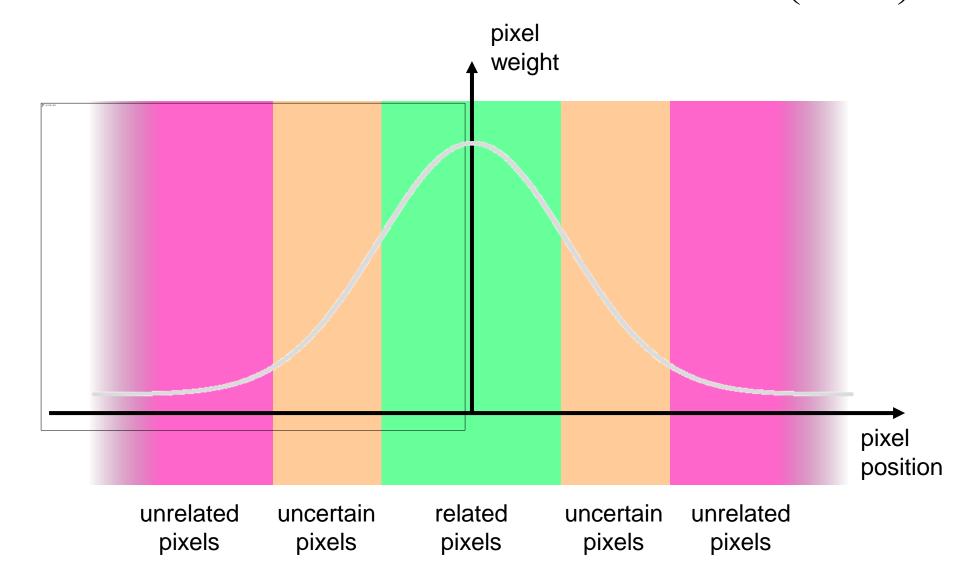
Same idea: weighted average of pixels.

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\|\mathbf{p} - \mathbf{q}\|) I_{\mathbf{q}}$$



### Gaussian Profile

$$G_{\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



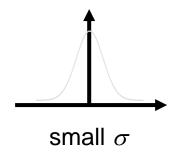
# **Spatial Parameter**



input

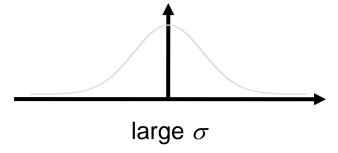
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\mathbf{q}}(||\mathbf{p} - \mathbf{q}||) I_{\mathbf{q}}$$

size of the window





limited smoothing





strong smoothing

#### **Digi**VFX

# Properties of Gaussian Blur

- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT...)





- Does smooth images
- But smoothes too much: edges are blurred.
  - Only spatial distance matters
  - No edge term

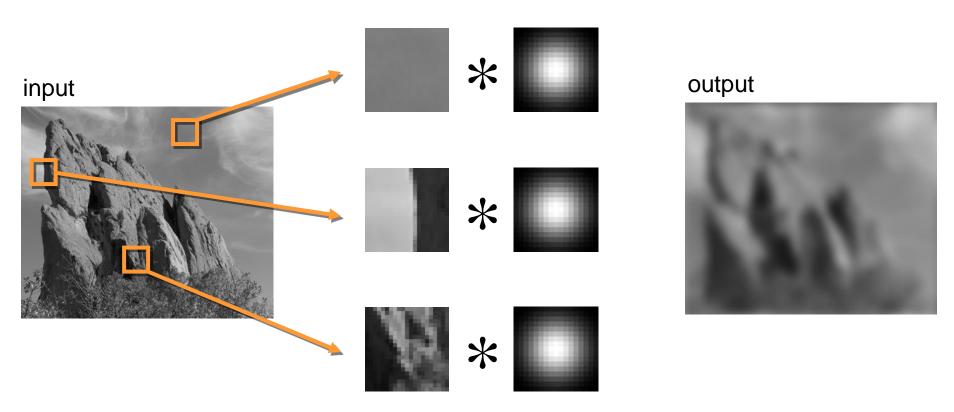


output

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(||\mathbf{p} - \mathbf{q}||) I_{\mathbf{q}}$$
space

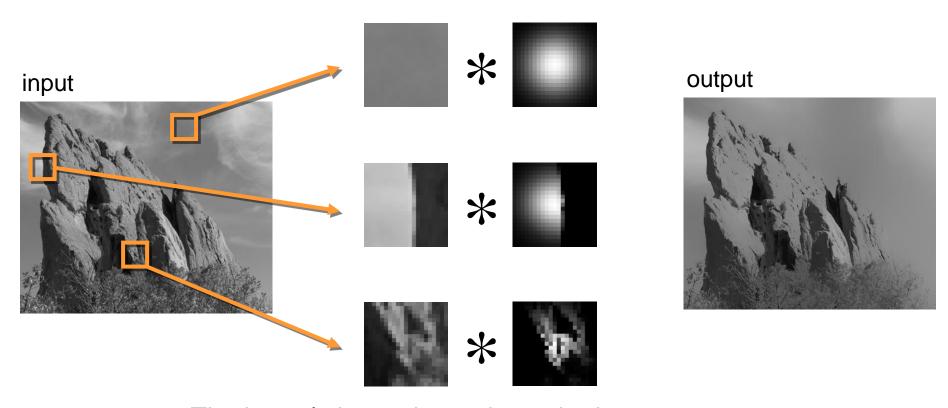


# Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

# Bilateral Filter No Averaging across Edges

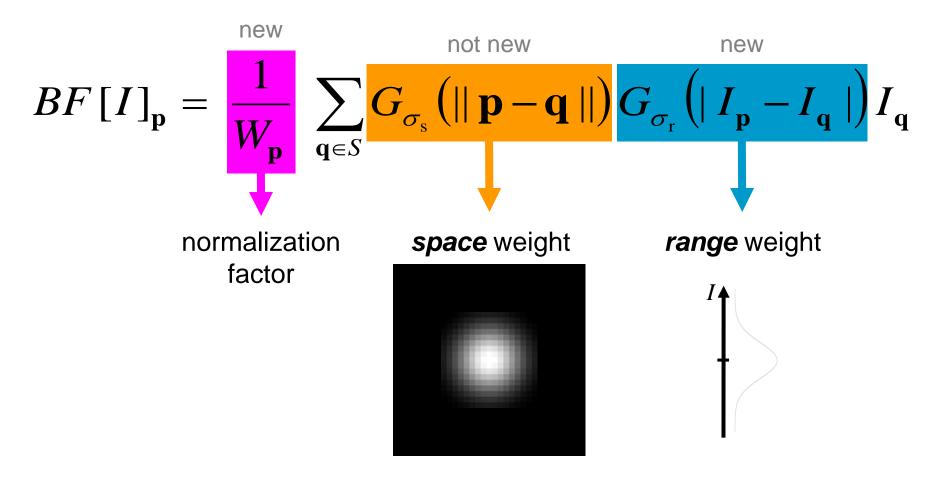


The kernel shape depends on the image content.



#### **Bilateral Filter Definition**

Same idea: weighted average of pixels.



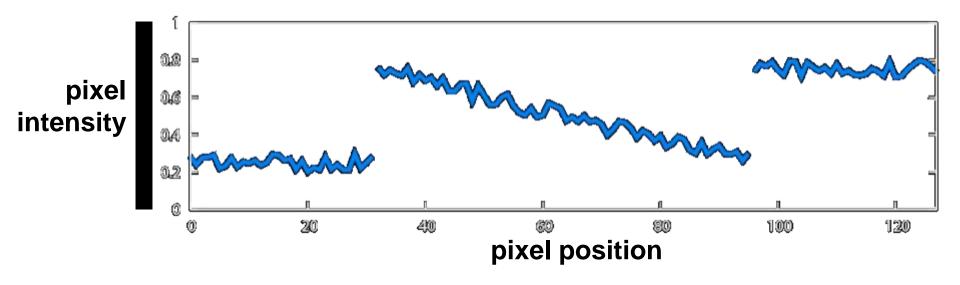
# Illustration a 1D Image



• 1D image = line of pixels



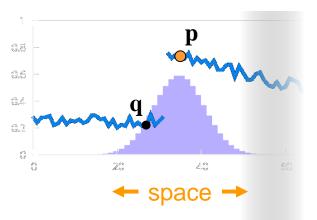
Better visualized as a plot



#### Gaussian Blur and Bilateral Filter Digivex

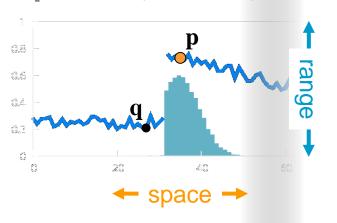


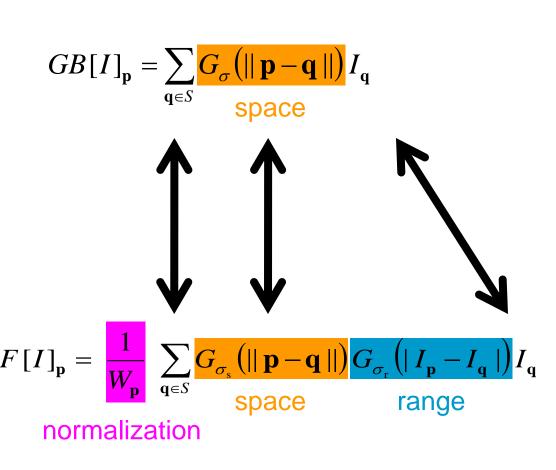
#### Gaussian blur



#### Bilateral filter

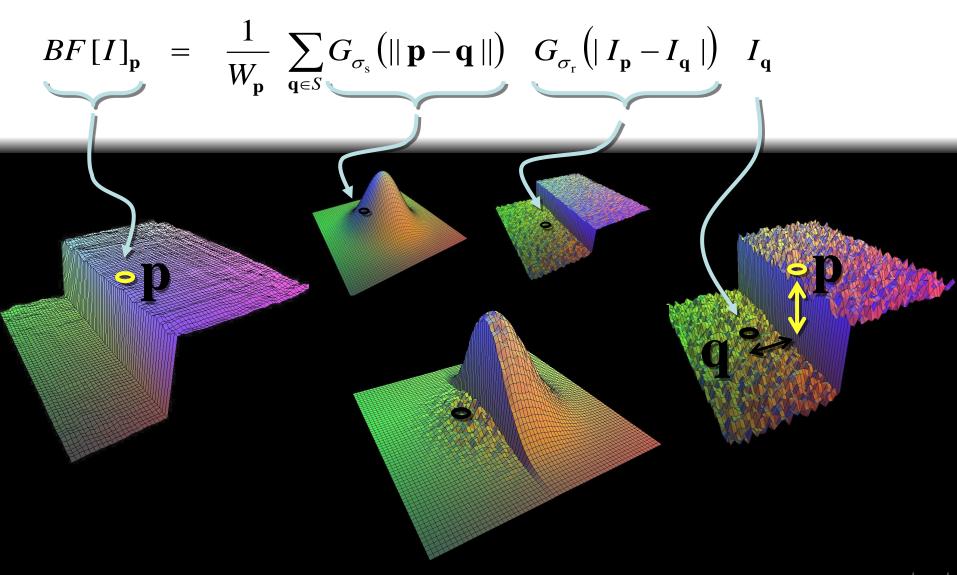
[Aurich 95, Smith 97, Tomasi 98]





# Bilateral Filter on a Height Field







# Space and Range Parameters

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_{s}} (||\mathbf{p} - \mathbf{q}||) G_{\sigma_{r}} (|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

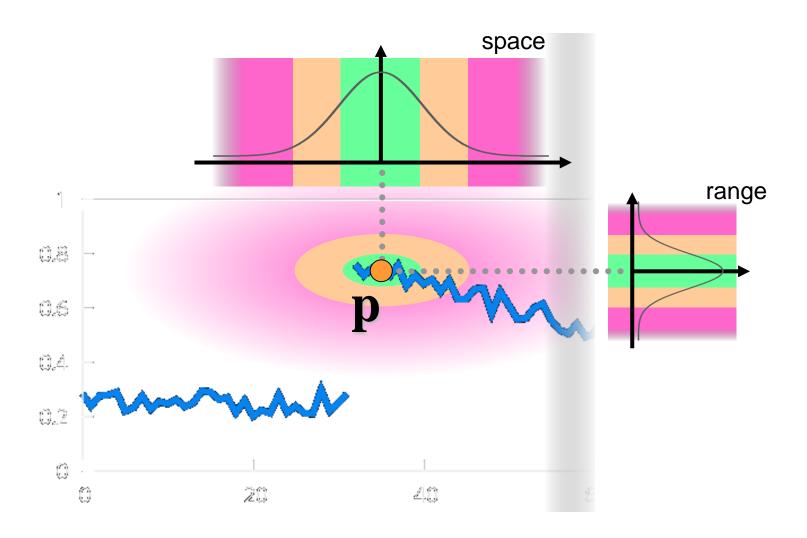
• space  $\sigma_s$ : spatial extent of the kernel, size of the considered neighborhood.

• range  $\sigma_{\rm r}$  : "minimum" amplitude of an edge

#### Influence of Pixels



Only pixels close in space and in range are considered.



input

#### **Exploring the Parameter Space**

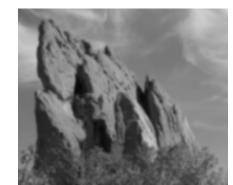
$$\sigma_{\rm r} = 0.1$$



 $\sigma_{\rm r} = 0.25$ 



 $\sigma_{\rm r} = \infty$  (Gaussian blur)





 $\sigma_{s} = 2$ 















# Iterating the Bilateral Filter

$$I_{(n+1)} = BF\left[I_{(n)}\right]$$

- Generate more piecewise-flat images
- Often not needed in computational photo, but could be useful for applications such as NPR.









#### Advantages of Bilateral Filter



- Easy to understand
  - Weighted mean of nearby pixels
- Easy to adapt
  - Distance between pixel values
- Easy to set up
  - Non-iterative

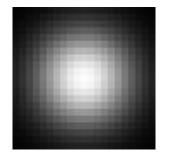
# Hard to Compute

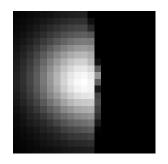


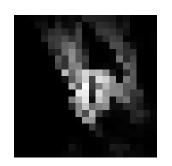
Nonlinear

$$BF\left[I\right]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_{s}} \left( ||\mathbf{p} - \mathbf{q}|| \right) G_{\sigma_{r}} \left( |I_{\mathbf{p}} - I_{\mathbf{q}}| \right) I_{\mathbf{q}}$$

- Complex, spatially varying kernels
  - Cannot be precomputed, no FFT...









• Brute-force implementation is slow > 10min



#### But Bilateral Filter is Nonlinear

- Slow but some accelerations exist:
  - [Elad 02]: Gauss-Seidel iterations
    - Only for many iterations

- [Durand 02, Weiss 06]: fast approximation
  - No formal understanding of accuracy versus speed
  - [Weiss 06]: Only **box function** as spatial kernel

# A Fast Approximation of the Bilateral Filter using a Signal Processing Approach

Sylvain Paris and Frédo Durand

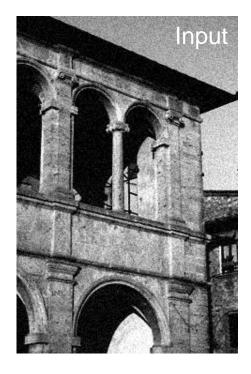
Computer Science and Artificial Intelligence Laboratory Massachusetts Institute of Technology





#### **Definition of Bilateral Filter**

- [Smith 97, Tomasi 98]
- Smoothes an image and preserves edges
- Weighted average of neighbors
- Weights
  - Gaussian on space distance
  - Gaussian on range distance
  - sum to 1





$$I_{\mathbf{p}}^{\mathrm{bf}} = \frac{1}{W_{\mathbf{p}}^{\mathrm{bf}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_{\mathrm{s}}}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_{\mathrm{r}}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

#### **Contributions**

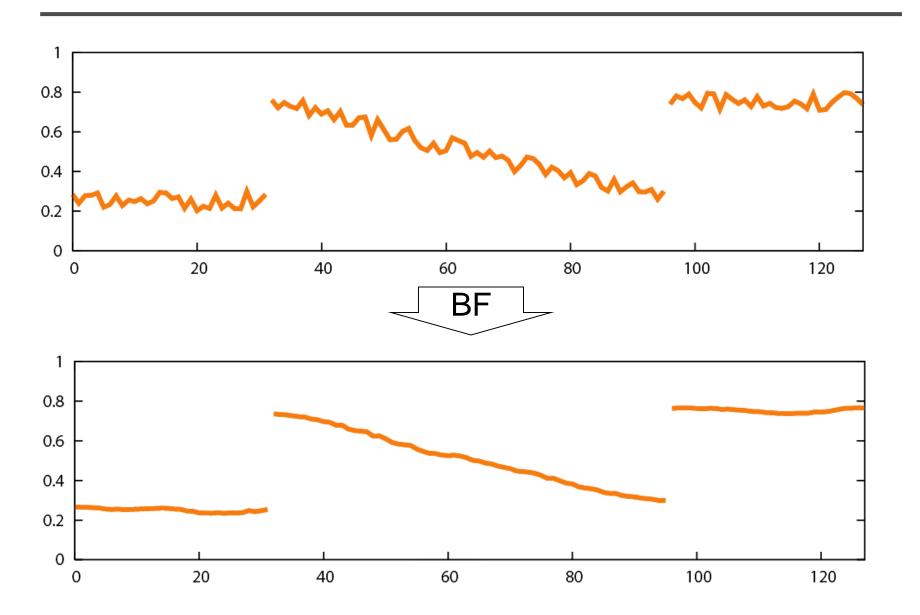


Link with linear filtering

• Fast and accurate approximation

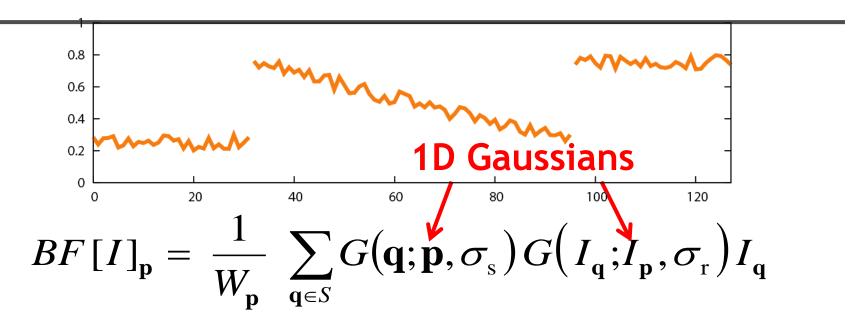
#### Intuition on 1D Signal





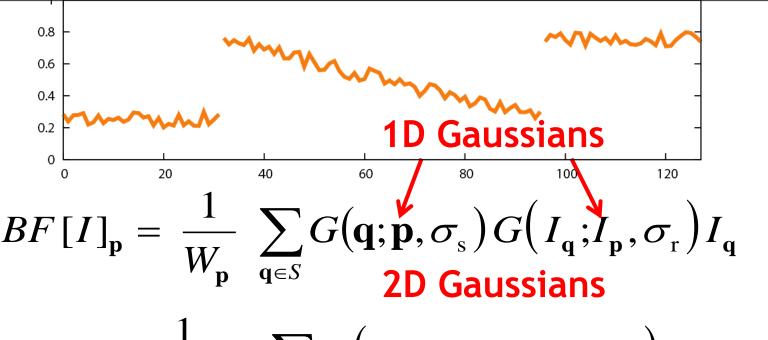
#### Basic idea





#### Basic idea

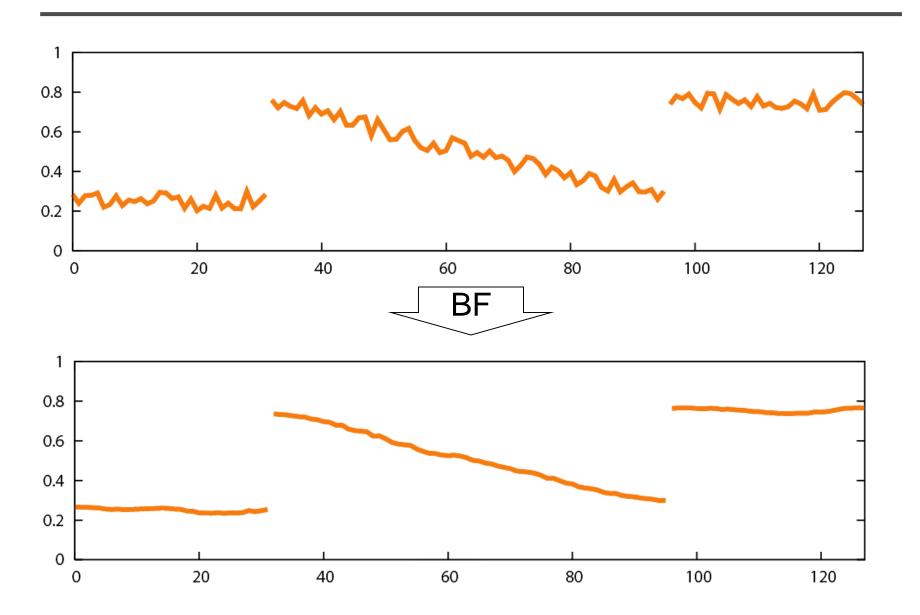




$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\langle \mathbf{q}, I'_{\mathbf{q}} \rangle \in S'} \mathbf{G}(\mathbf{q}, I_{\mathbf{q}}; \mathbf{p}, I_{\mathbf{p}}, \sigma_{s}, \sigma_{r}) I_{\langle \mathbf{q}, I'_{\mathbf{q}} \rangle}$$

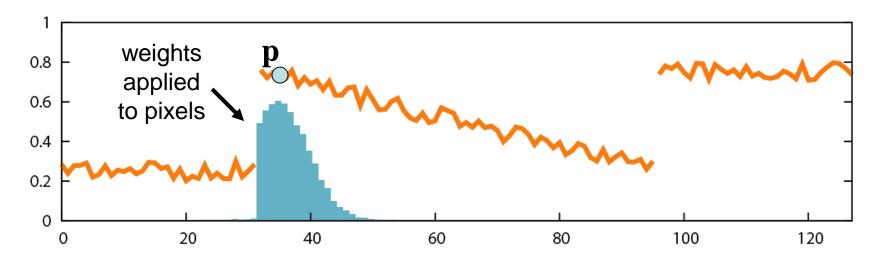
#### Intuition on 1D Signal





# Intuition on 1D Signal Weighted Average of Neighbors

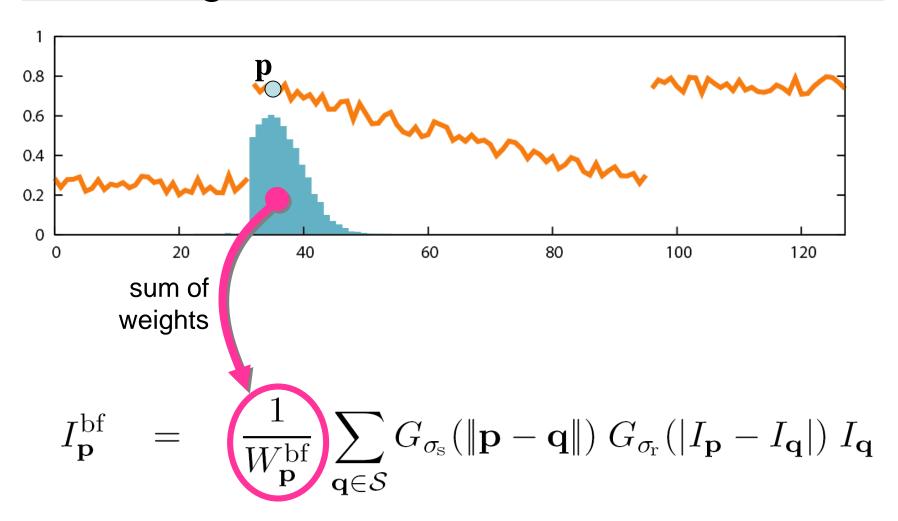




- Near and similar pixels have influence.
- Far pixels have no influence.
- Pixels with different value have no influence.

#### **Digi**VFX

#### 1. Handling the Division



Handling the division with a **projective space**.



#### Formalization: Handling the Division

$$I_{\mathbf{p}}^{\mathrm{bf}} = \frac{1}{W_{\mathbf{p}}^{\mathrm{bf}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_{\mathrm{s}}}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_{\mathrm{r}}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

$$W_{\mathbf{p}}^{\mathrm{bf}} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_{\mathrm{s}}}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_{\mathrm{r}}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)$$

- Normalizing factor as homogeneous coordinate
  - ullet Multiply both sides by  $W_{f p}^{
    m bf}$

$$\begin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} I_{\mathbf{p}}^{\mathrm{bf}} \\ W_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_{s}}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_{r}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} I_{\mathbf{q}} \\ 1 \end{pmatrix}$$

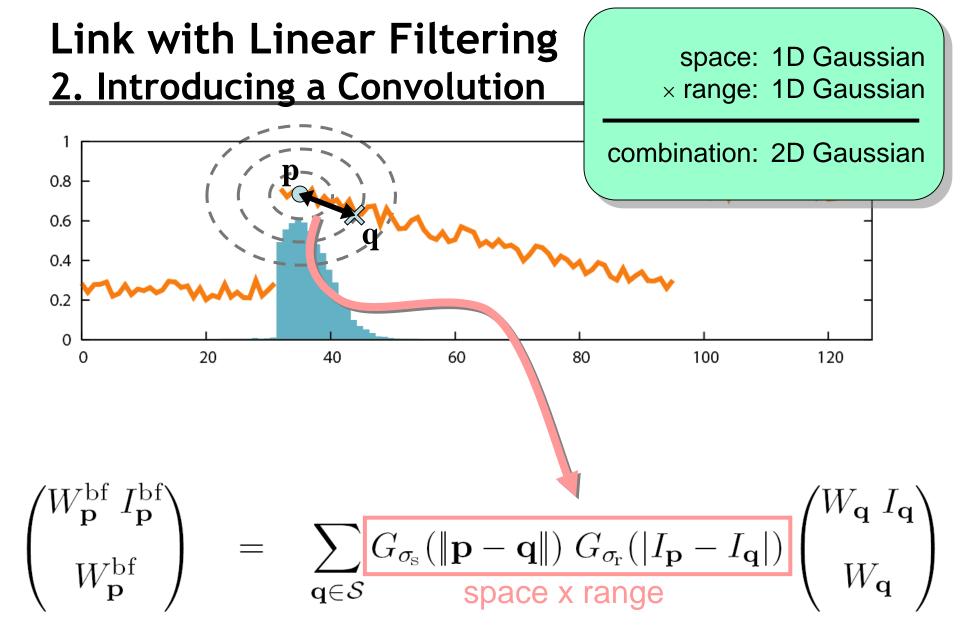


#### Formalization: Handling the Division

$$\begin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} I_{\mathbf{p}}^{\mathrm{bf}} \\ W_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_{s}}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_{r}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} W_{\mathbf{q}} I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix} \text{ with } W_{\mathbf{q}} = 1$$

- Similar to homogeneous coordinates in projective space
- Division delayed until the end
- Next step: Adding a dimension to make a convolution appear

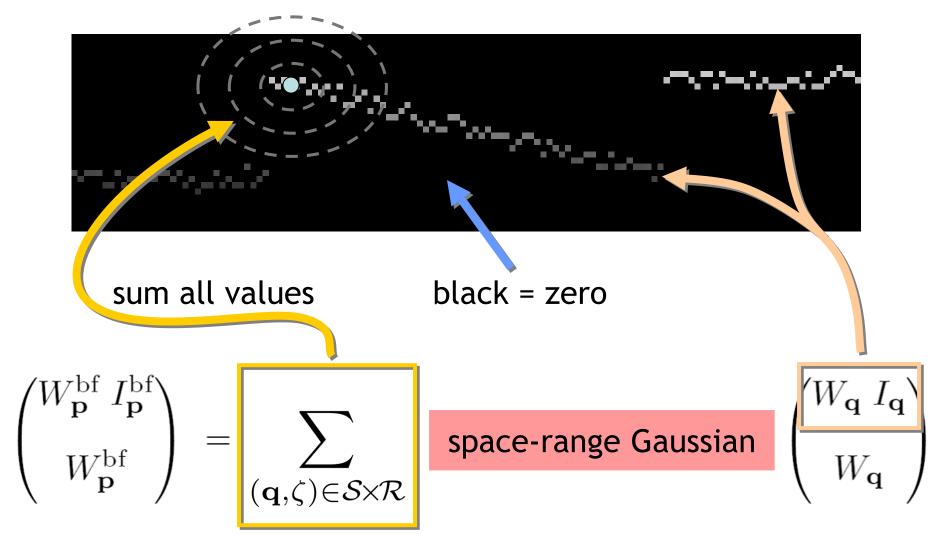
#### Link with Linear Filtering space: 1D Gaussian 2. Introducing a Convolution × range: 1D Gaussian combination: 2D Gaussian 0.8 0.6 0.4 0.2 0 20 40 60 80 00 120 space



Corresponds to a 3D Gaussian on a 2D image.



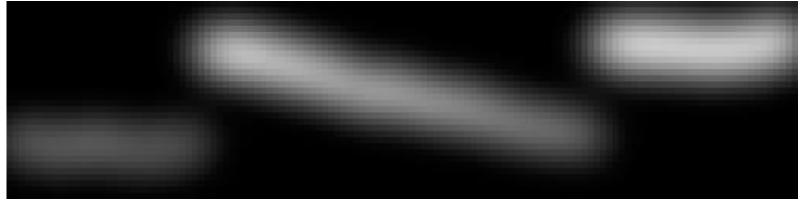
#### 2. Introducing a Convolution



sum all values multiplied by kernel ⇒ convolution



#### 2. Introducing a Convolution

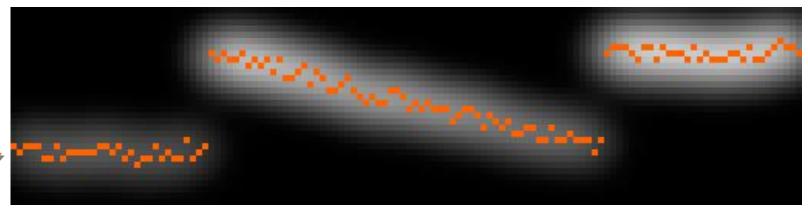


result of the convolution

$$\begin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} \ I_{\mathbf{p}}^{\mathrm{bf}} \\ W_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} \ = \ \sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}} \quad \text{space-range Gaussian} \quad \begin{pmatrix} W_{\mathbf{q}} \ I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

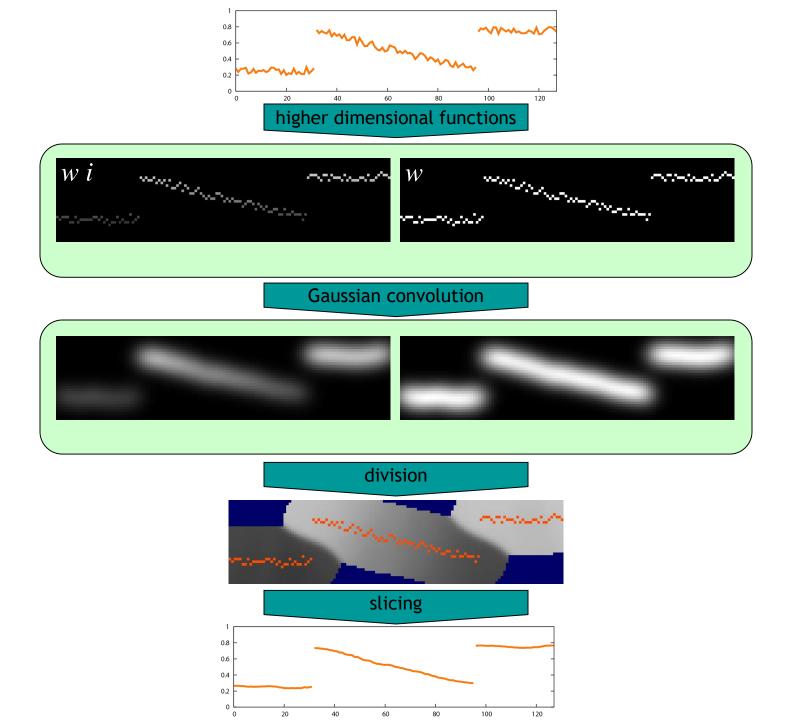


#### 2. Introducing a Convolution



result of the convolution

$$egin{pmatrix} W_{\mathbf{p}}^{\mathrm{bf}} \ I_{\mathbf{p}}^{\mathrm{bf}} \end{pmatrix} = \sum_{(\mathbf{q},\zeta) \in \mathcal{S} imes \mathcal{R}} \quad ext{space-range Gaussian} \ \begin{pmatrix} W_{\mathbf{q}} \ I_{\mathbf{q}} \end{pmatrix}$$







linear: 
$$(w^{\mathrm{bf}}\ i^{\mathrm{bf}}, w^{\mathrm{bf}}) = g_{\sigma_{\!\!\mathbf{s}}, \sigma_{\!\!\mathbf{r}}} \otimes (wi, w)$$
nonlinear:  $I^{\mathrm{bf}}_{\mathbf{p}} = \frac{w^{\mathrm{bf}}(\mathbf{p}, I_{\mathbf{p}})\ i^{\mathrm{bf}}(\mathbf{p}, I_{\mathbf{p}})}{w^{\mathrm{bf}}(\mathbf{p}, I_{\mathbf{p}})}$ 

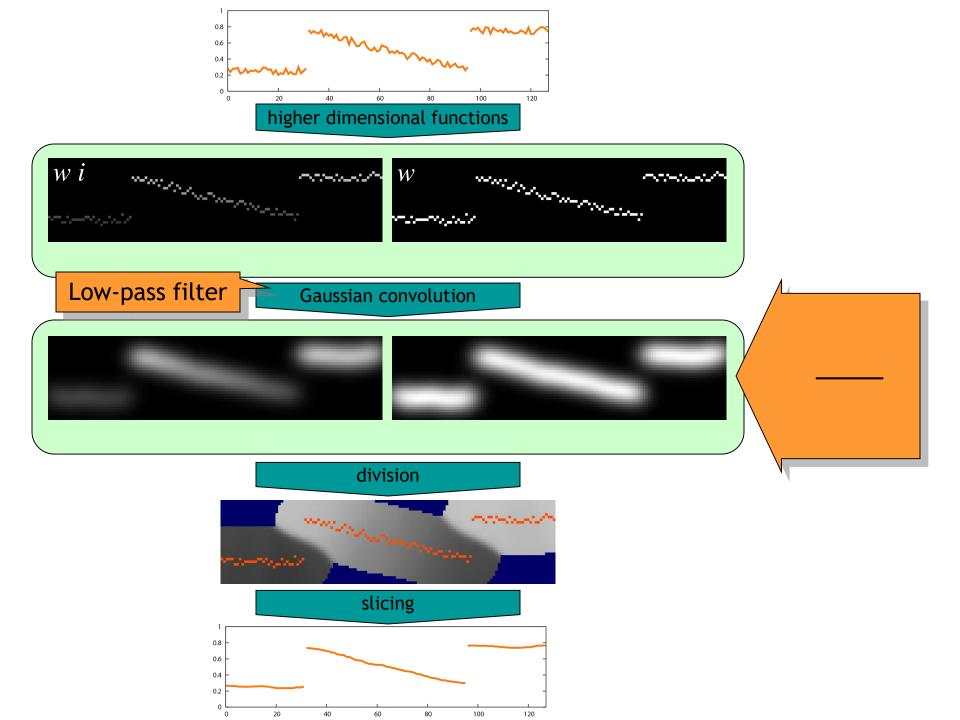
#### 1. Convolution in higher dimension

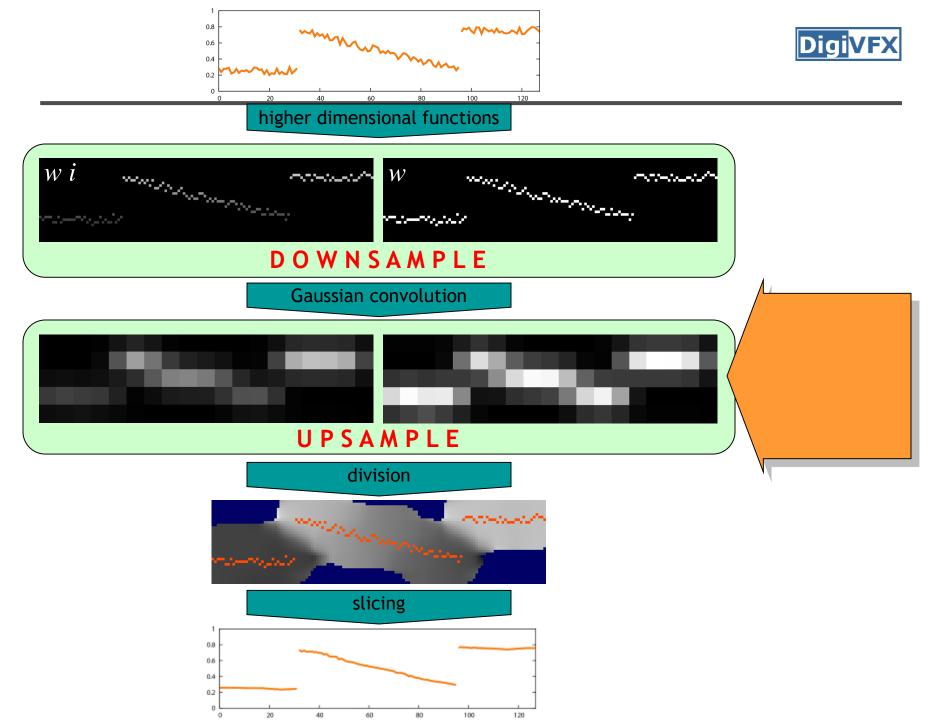
expensive but well understood (linear, FFT, etc)

#### 2. Division and slicing

nonlinear but simple and pixel-wise

#### **Exact reformulation**







#### Fast Convolution by Downsampling

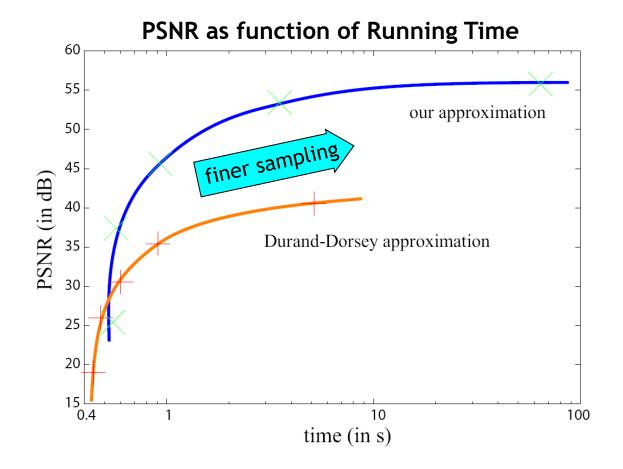
- Downsampling cuts frequencies above Nyquist limit
  - Less data to process
  - But induces error

- Evaluation of the approximation
  - Precision versus running time
  - Visual accuracy

#### **Accuracy versus Running Time**



- Finer sampling increases accuracy.
- More precise than previous work.



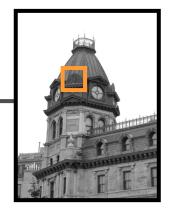


Digital photograph 1200 × 1600

Straightforward implementation is over 10 minutes.

#### **Visual Results**

- Comparison with previous work [Durand 02]
  - running time = 1s for both techniques



 $1200 \times 1600$ 

input







prev. work

difference with exact computation (intensities in [0:1])









# Two-scale Tone Management for Photographic Look

Soonmin Bae, Sylvain Paris, and Frédo Durand MIT CSAIL

# SIGGRAPH2006

#### **Ansel Adams**





Ansel Adams, Clearing Winter Storm

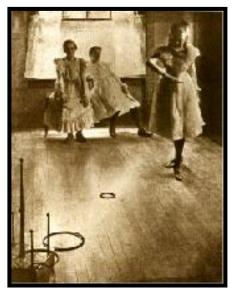


#### An Amateur Photographer



#### A Variety of Looks











#### Goals



- Control over photographic look
- Transfer "look" from a model photo

#### For example,

we want



with the look of



#### (

#### Aspects of Photographic Look

- Subject choice
- Framing and composition
- Specified by input photos
- Tone distribution and contrast
- → Modified based on model photos



Input



Model

#### **Tonal Aspects of Look**

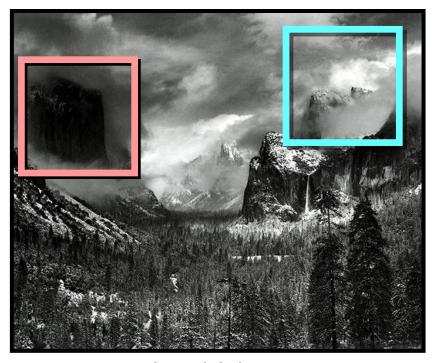






Ansel Adams Kenro Izu

## Tonal aspects of Look - Global Contrast





**Ansel Adams** 

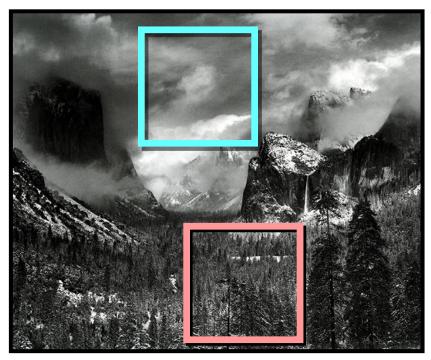
Kenro Izu

**High Global Contrast** 

**Low Global Contrast** 

# Tonal aspects of Look - Local Contrast







**Ansel Adams** 

Kenro Izu

Variable amount of texture

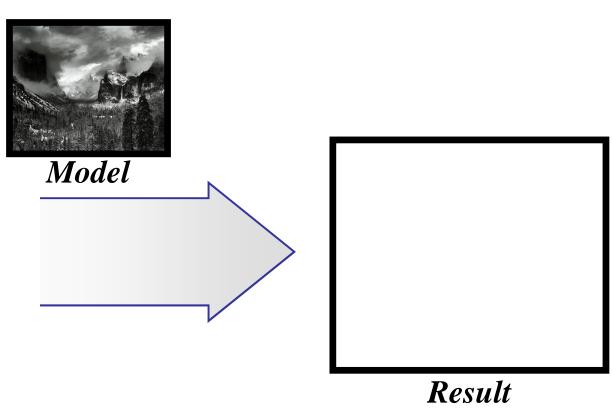
**Texture everywhere** 

#### Overview

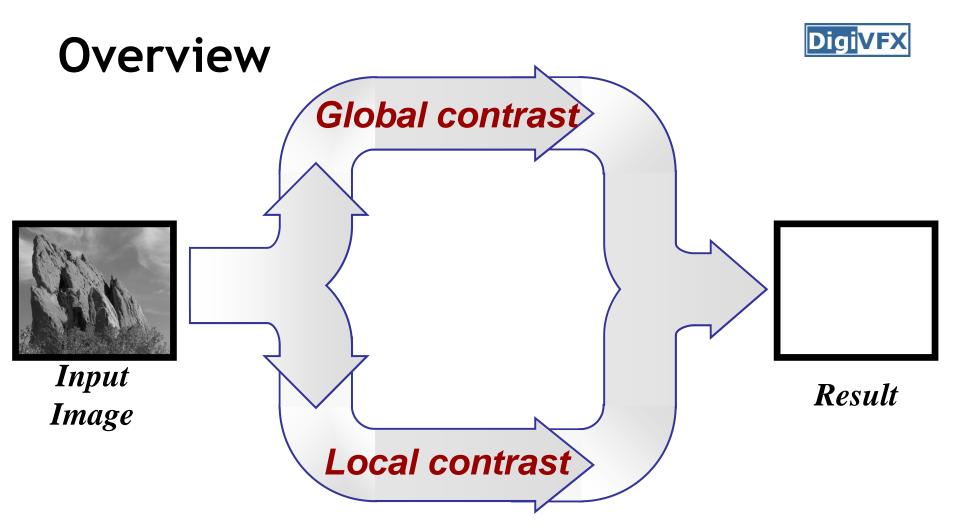




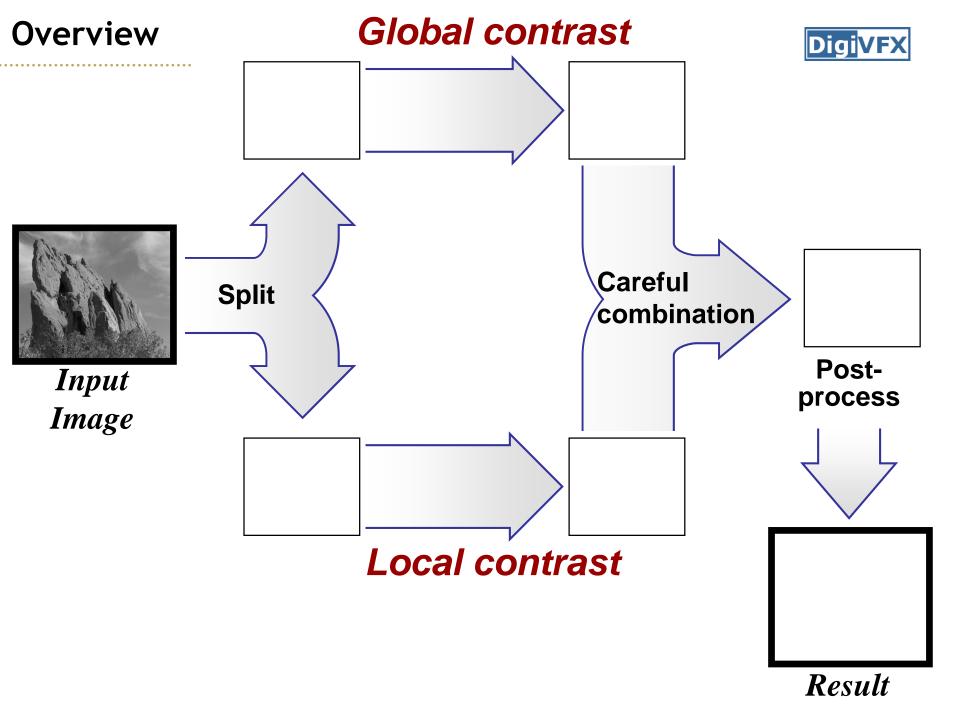


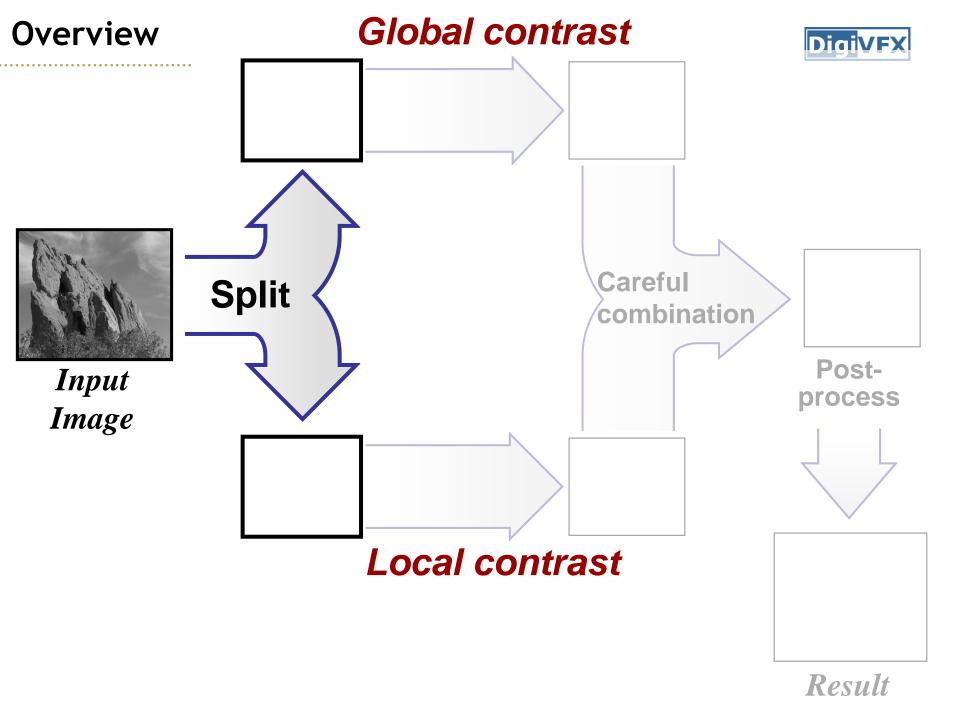


- Transfer look between photographs
  - Tonal aspects



Separate global and local contrast

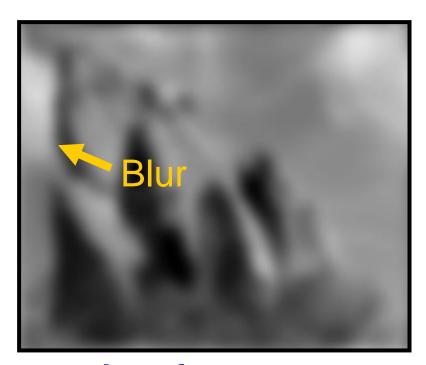






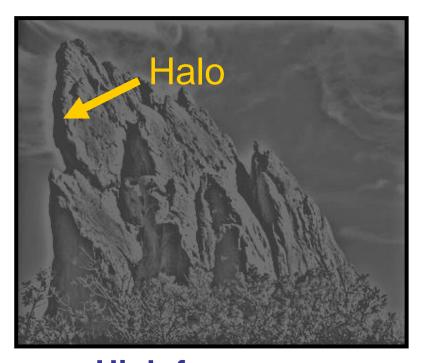
### Split Global vs. Local Contrast

- Naïve decomposition: low vs. high frequency
  - Problem: introduce blur & halos



Low frequency

Global contrast



High frequency Local contrast

#### Bilateral Filter



- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]



After bilateral filtering

Global contrast



Residual after filtering Local contrast

#### Bilateral Filter



- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]

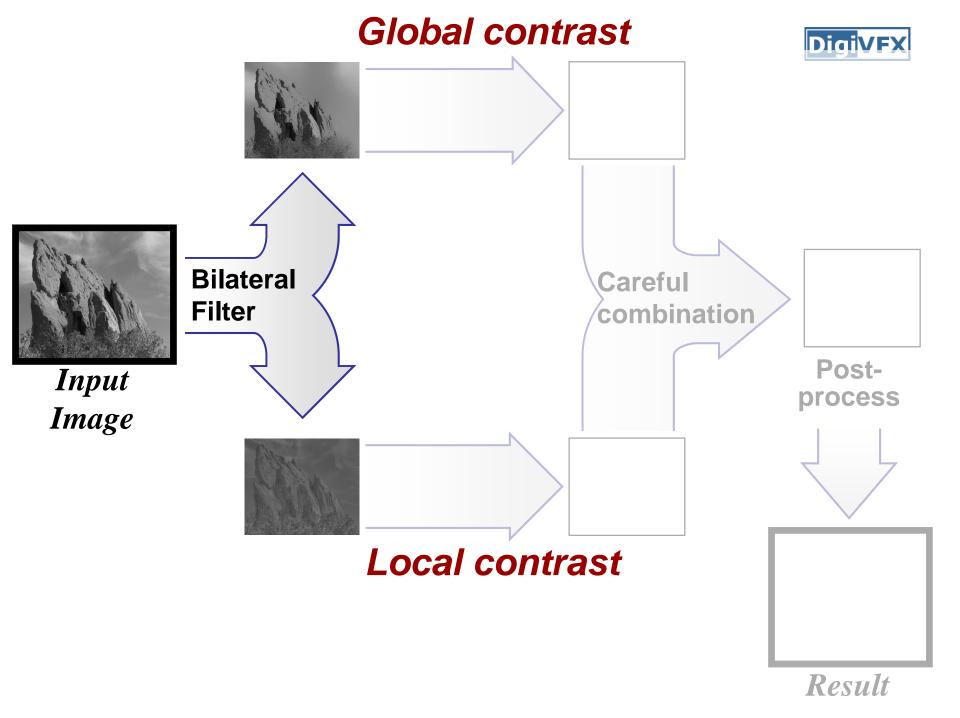


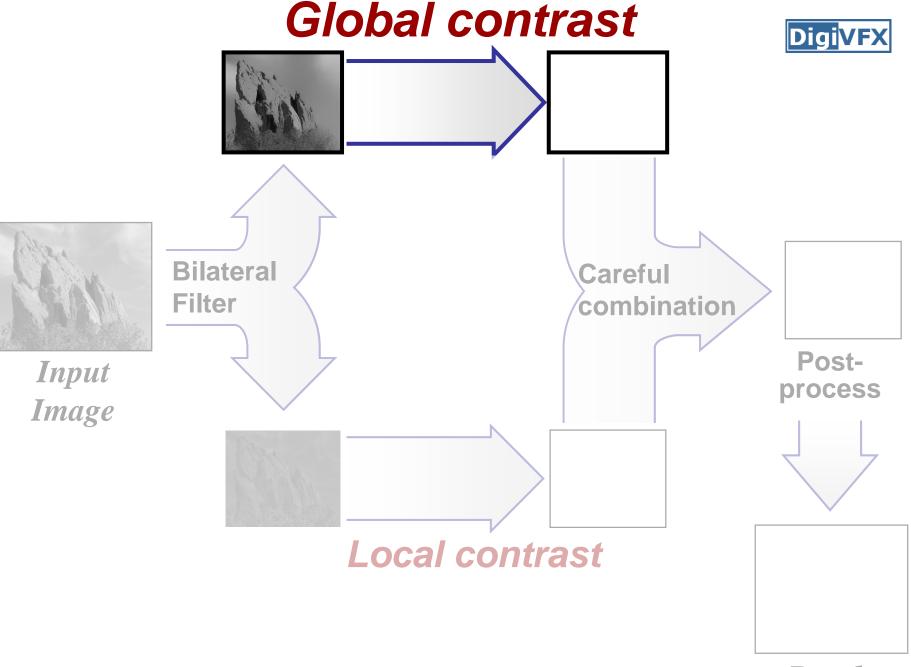
After bilateral filtering

Global contrast



Residual after filtering Local contrast





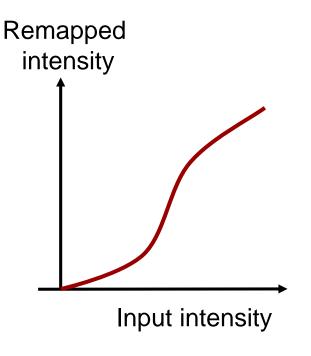
#### **Global Contrast**



Intensity remapping of base layer



Input base



After remapping

# Global Contrast (Model Transfer)





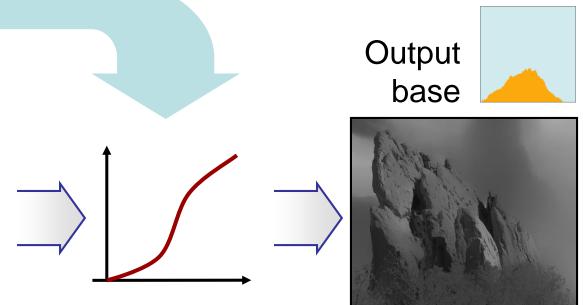
Model base

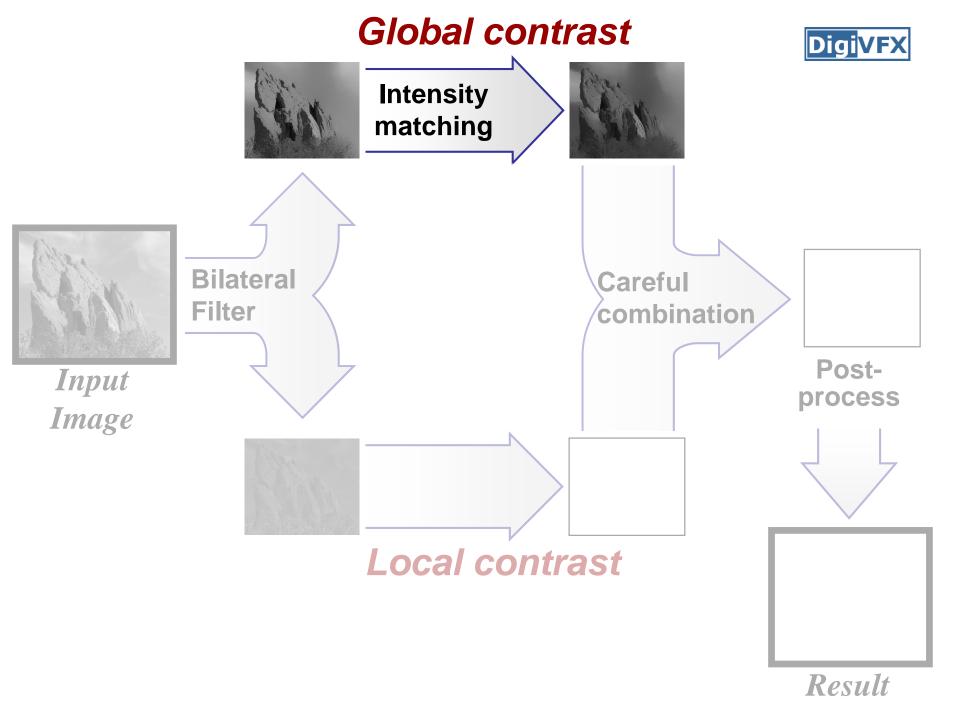
Input base

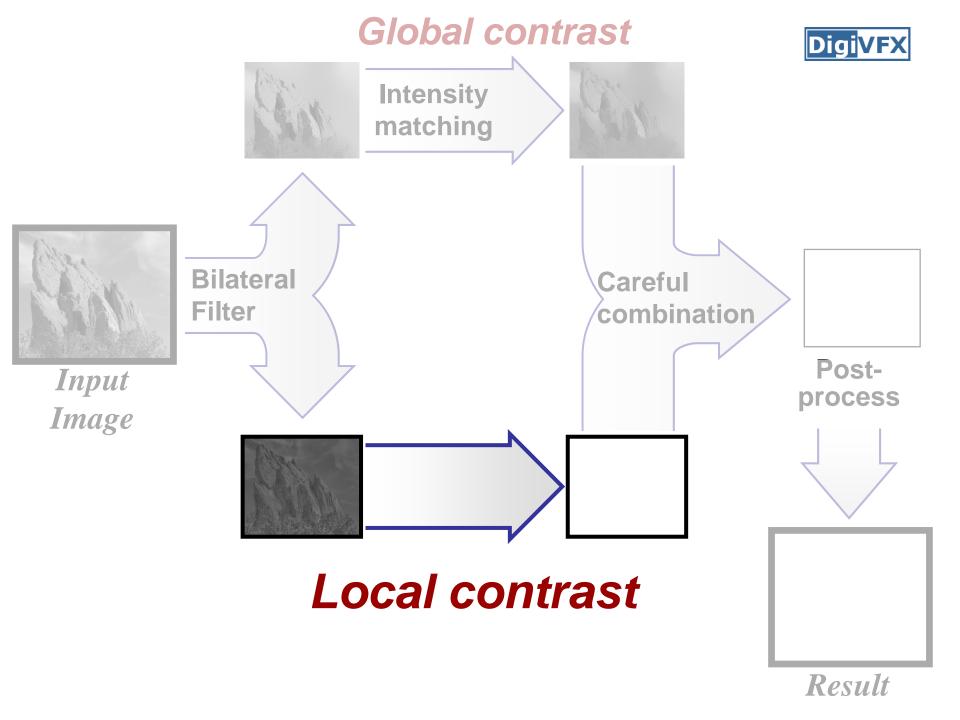


Histogram matching

 Remapping function given input and model histogram



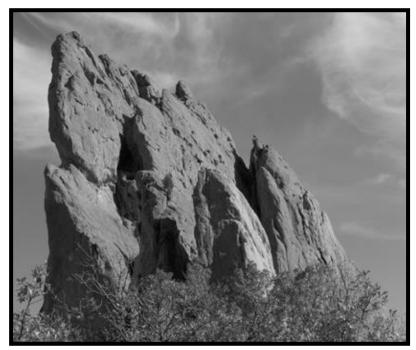




## Local Contrast: Detail Layer



- Uniform control:
  - Multiply all values in the detail layer



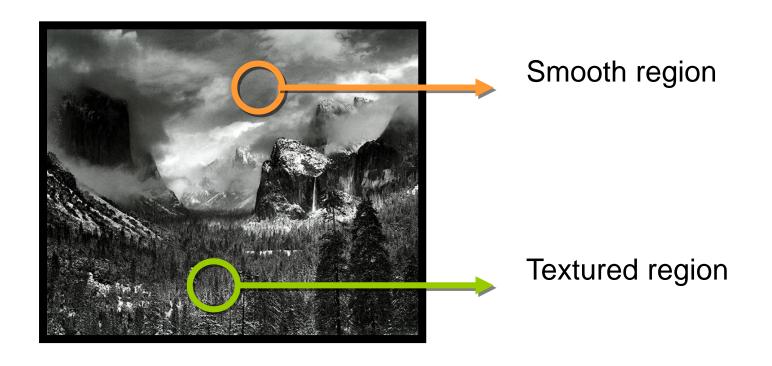
Input



Base + 3 × Detail

# The amount of local contrast is not uniform

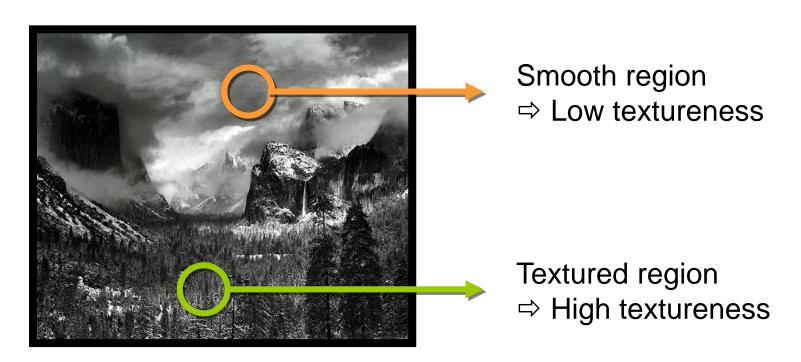


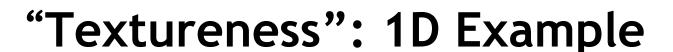


#### **Local Contrast Variation**

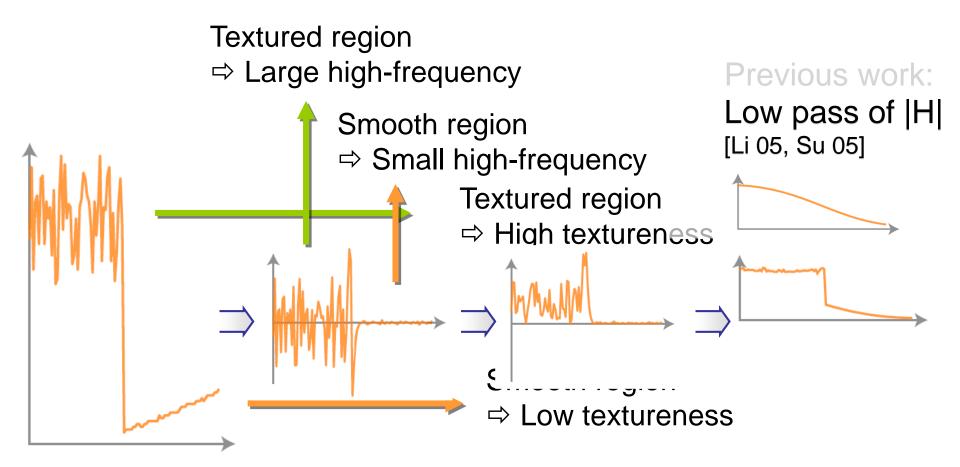


- We define "textureness": amount of local contrast
  - at each pixel based on surrounding region









Input signal High frequency H Amplitude |H| Edge-preserving filter

### **Textureness**





Input Textureness

#### **Textureness Transfer**



Step 1: Histogram transfer Model textureness

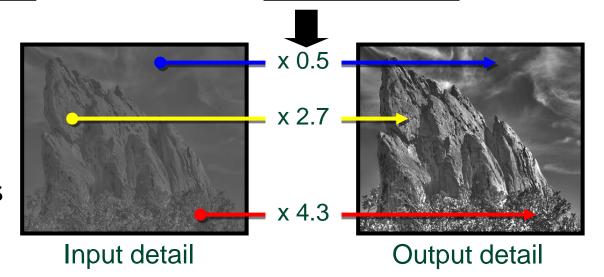
Input textureness

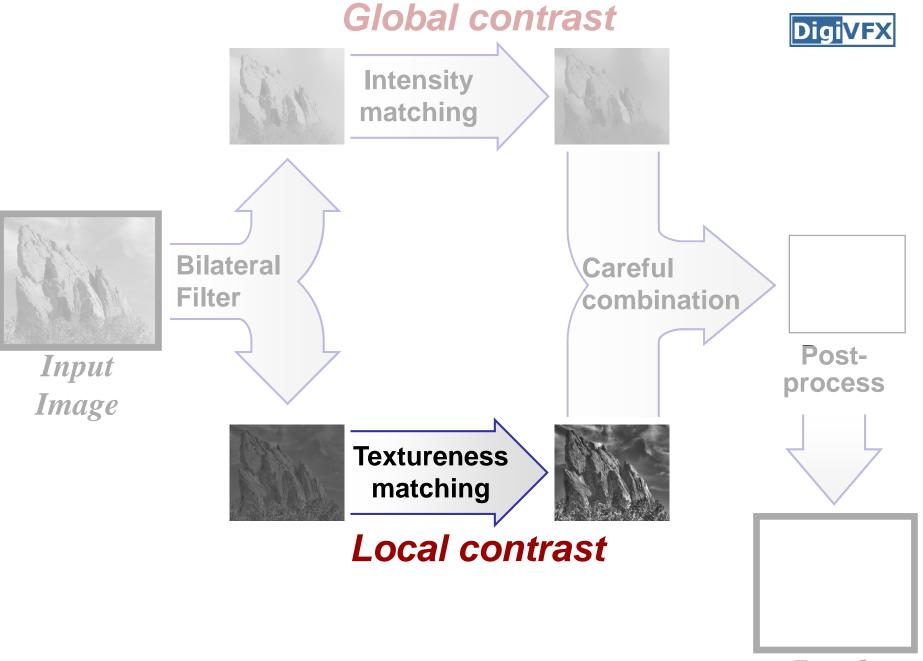
**Hist. transfer** 

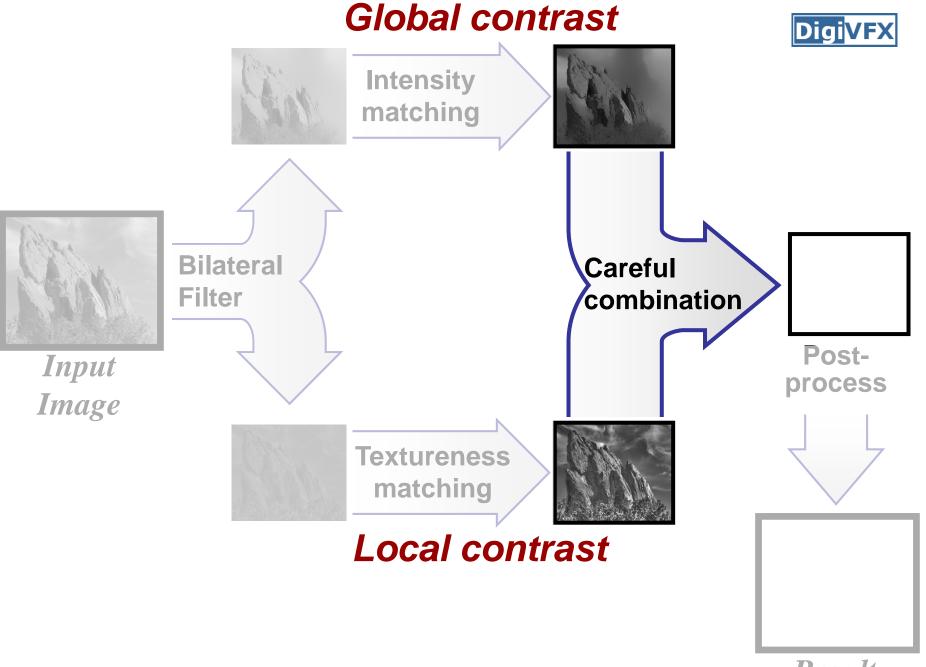
textureness

**Desired** 

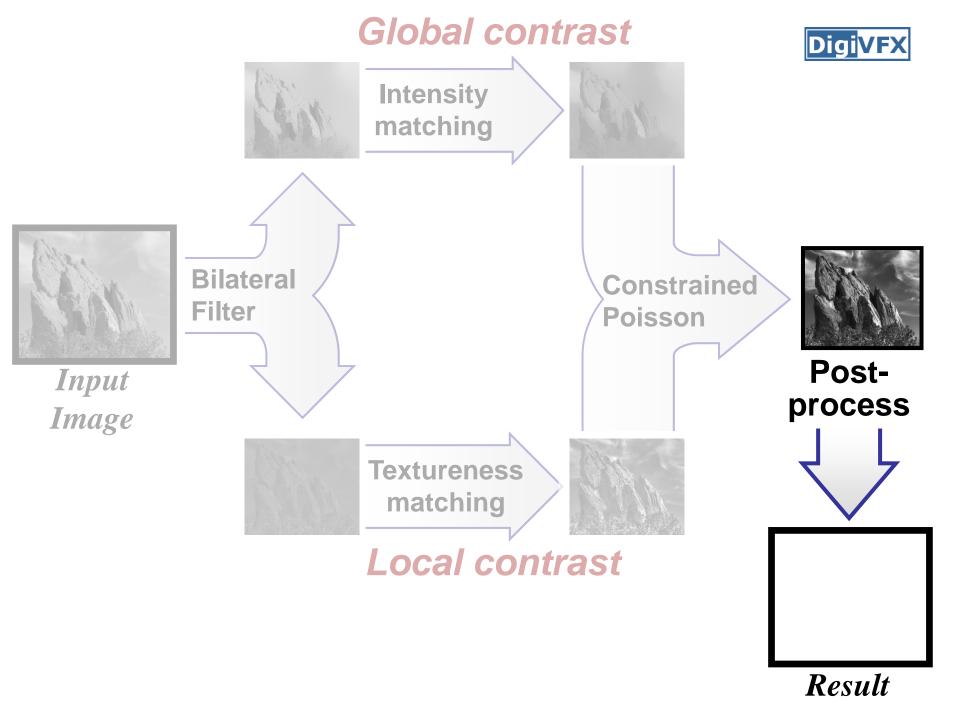
Step 2: Scaling detail layer (per pixel) to match desired textureness







### Global contrast **DigiVFX Intensity** matching **Bilateral** Constrained **Filter Poisson** Post-Input process *Image* **Textureness** matching Local contrast



#### model

#### **Additional Effects**

- Soft focus (high frequency manipulation)
- Film grain (texture synthesis [Heeger 95])
- Color toning (chrominance = f (luminance))



before effects

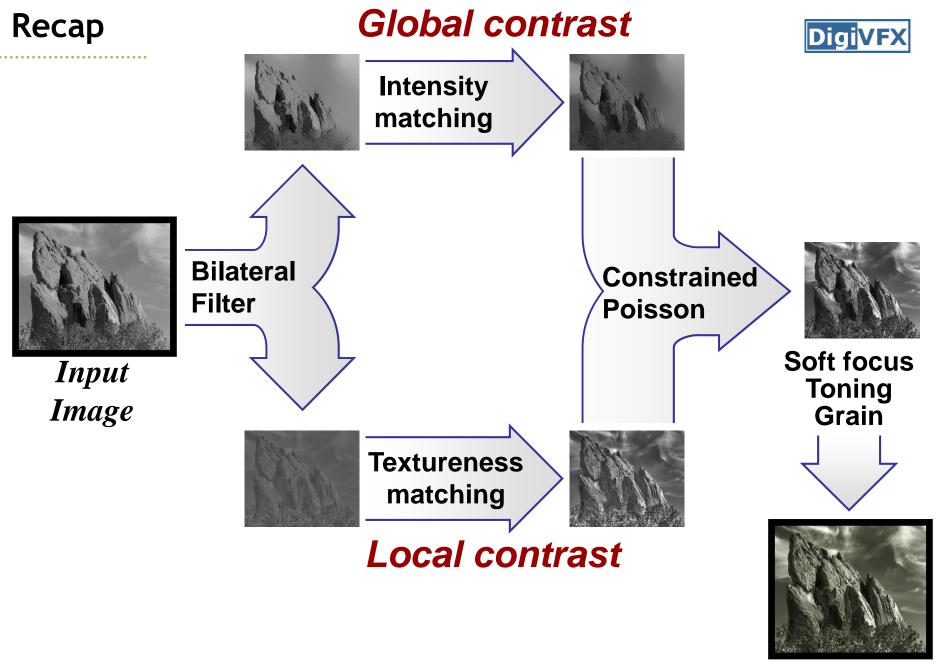




after effects

### Global contrast **DigiVFX Intensity** matching **Bilateral** Constrained **Filter** Poisson Soft focus Input **Toning** *Image* Grain **Textureness** matching Local contrast

Result



Result

#### Results



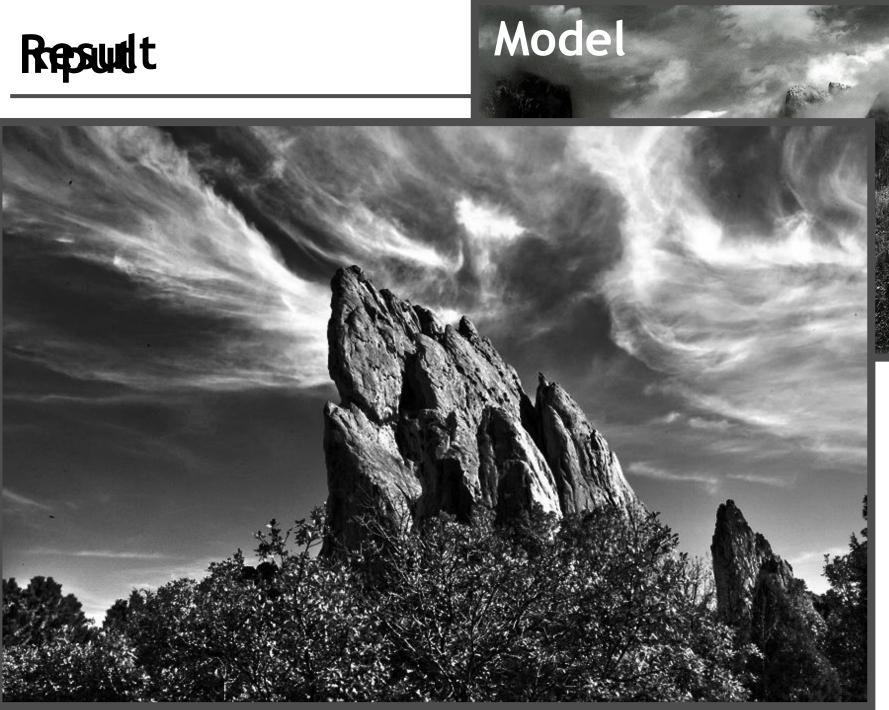
User provides input and model photographs.

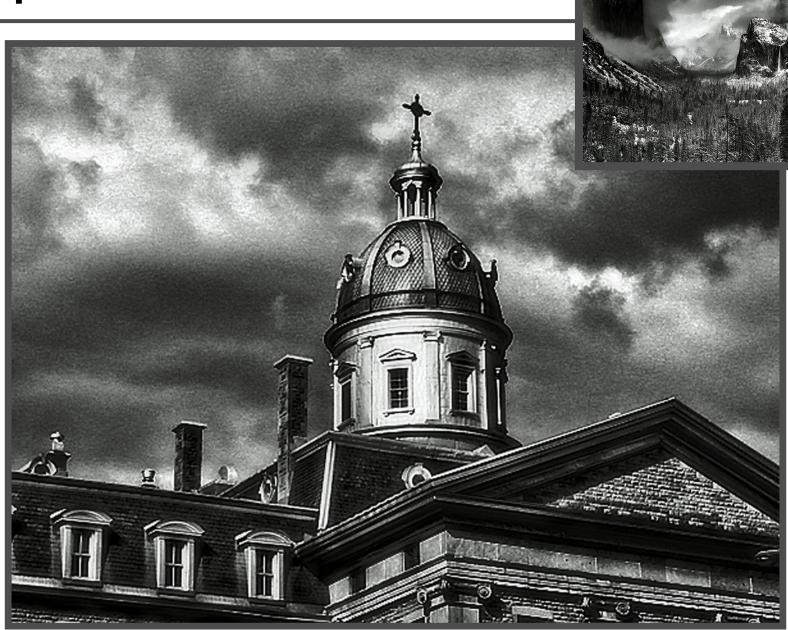
→ Our system automatically produces the result.

#### Running times:

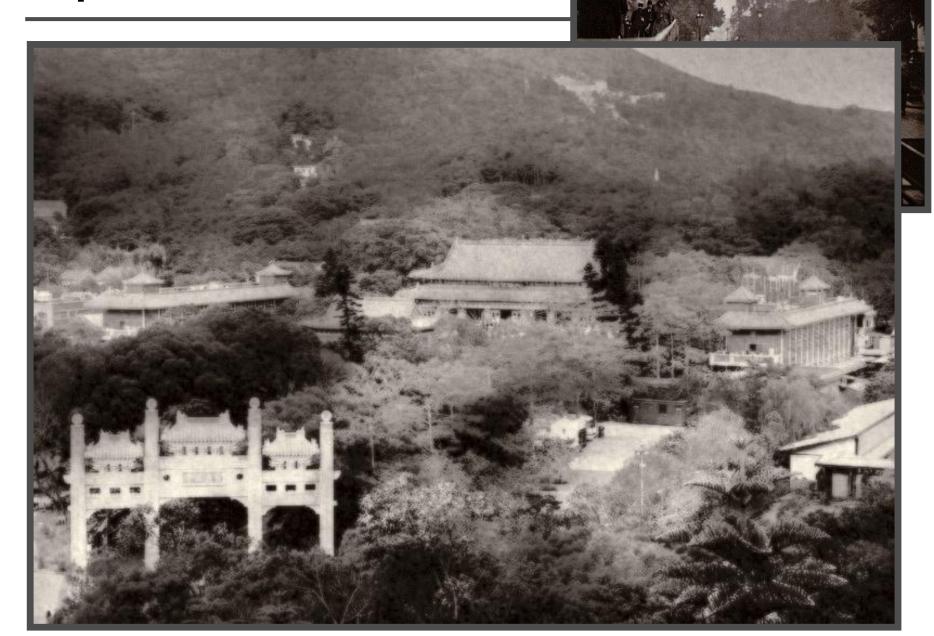
- 6 seconds for 1 MPixel or less
- 23 seconds for 4 MPixels
- multi-grid Poisson solver and fast bilateral filter [Paris 06]

# Repoudt



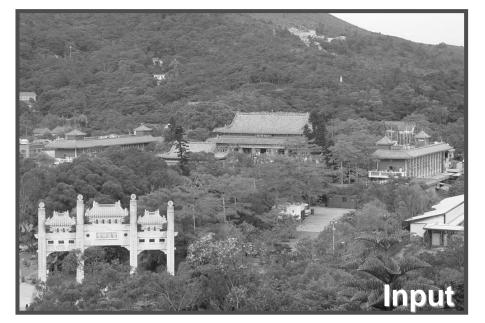


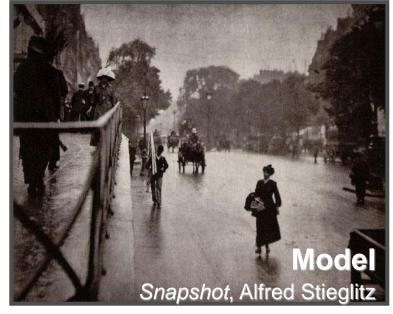
# Repoundt



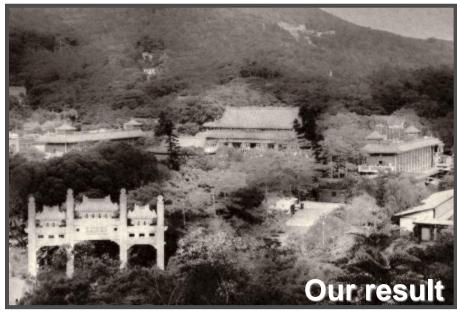
Model

### Comparison with Naïve Histogram Matching









Local contrast, sharpness unfaithful

# Comparison with Naive Histogram Matching







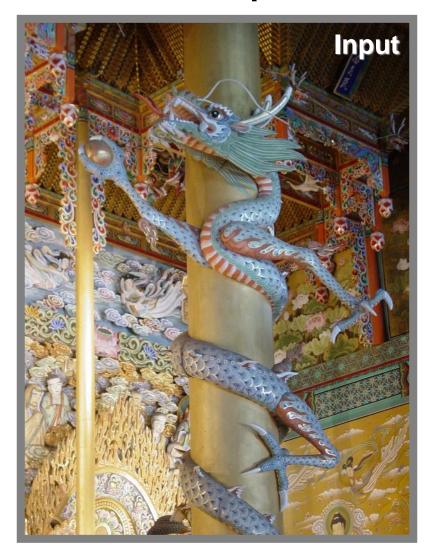


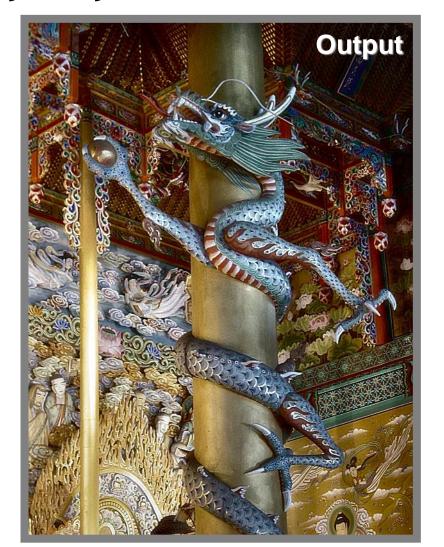
Local contrast too low

# **Color Images**



• Lab color space: modify only luminance

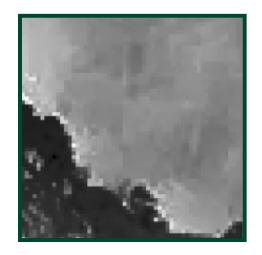




#### Limitations



- Noise and JPEG artifacts
  - amplified defects



- Can lead to unexpected results if the image content is too different from the model
  - Portraits, in particular, can suffer



#### **Conclusions**



• Transfer "look" from a model photo

- Two-scale tone management
  - Global and local contrast
  - New edge-preserving textureness
  - Constrained Poisson reconstruction
  - Additional effects

### Joint bilateral filtering



$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||I_p - I_q||)$$

$$J_{p} = \frac{1}{k_{p}} \sum_{q \in \Omega} I_{q} f(||p - q||) g(||\tilde{I}_{p} - \tilde{I}_{q}||)$$

# Flash / No-Flash Photo Improvement Digivex (Petschnigg04) (Eisemann04)



Merge best features: warm, cozy candle light (no-flash) low-noise, detailed flash image



#### Overview



#### Basic approach of both flash/noflash papers

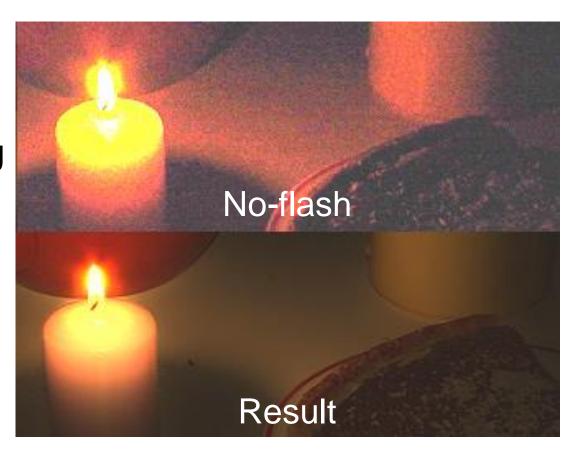
Remove noise + details from image A,

Keep as image A Lighting

-----

Obtain noise-free details from image B,

Discard Image B Lighting



# Petschnigg:

• Flash



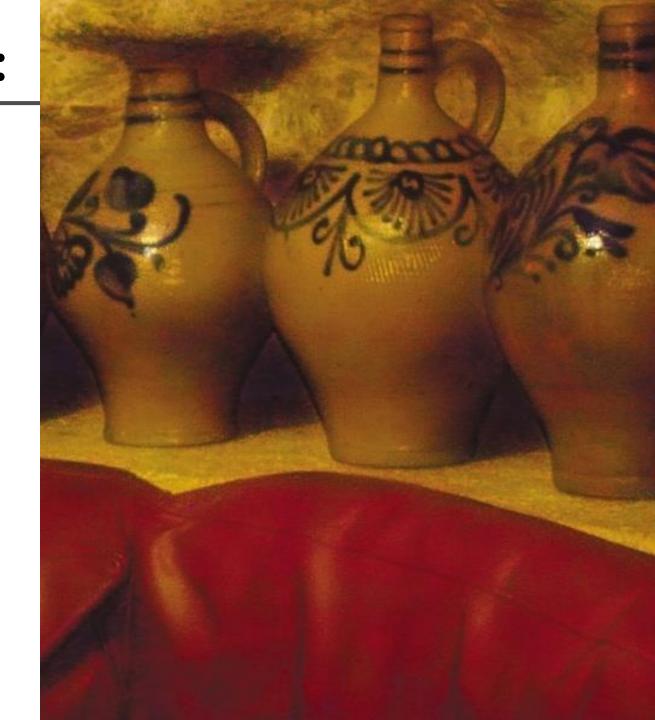
# Petschnigg:

• No Flash,



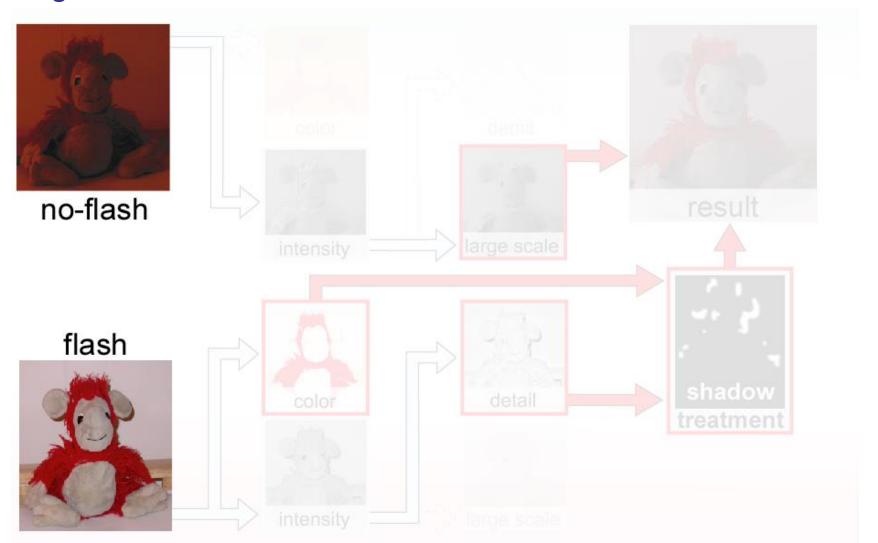
# Petschnigg:

• Result



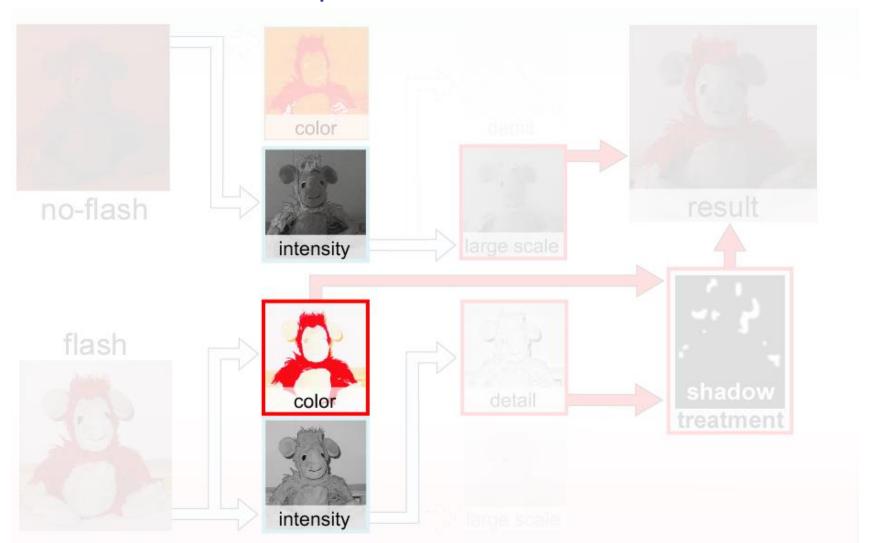


#### Registration





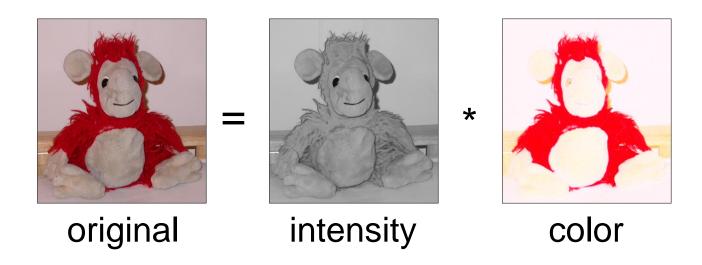
#### **Decomposition**



### **Decomposition**

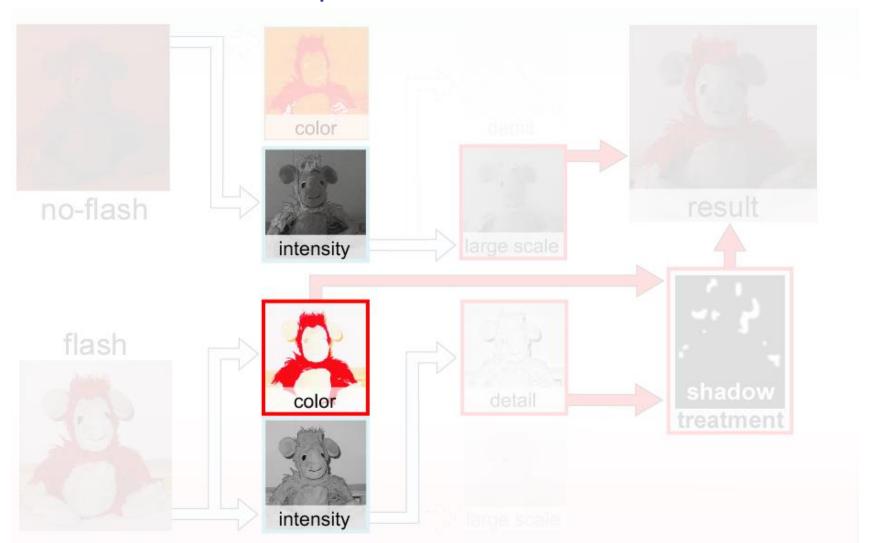


### Color / Intensity:



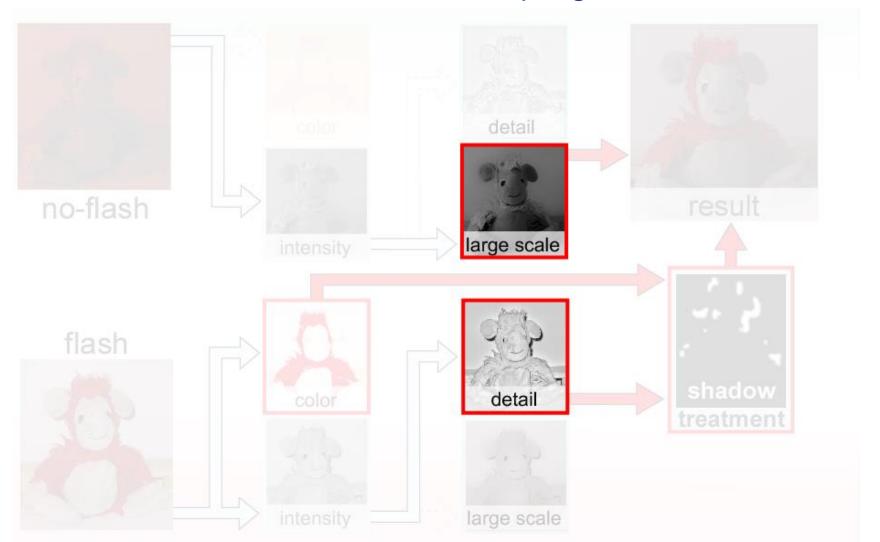


#### **Decomposition**





#### Decoupling



### Decoupling



- Lighting: Large-scale variation
- Texture: Small-scale variation



Lighting



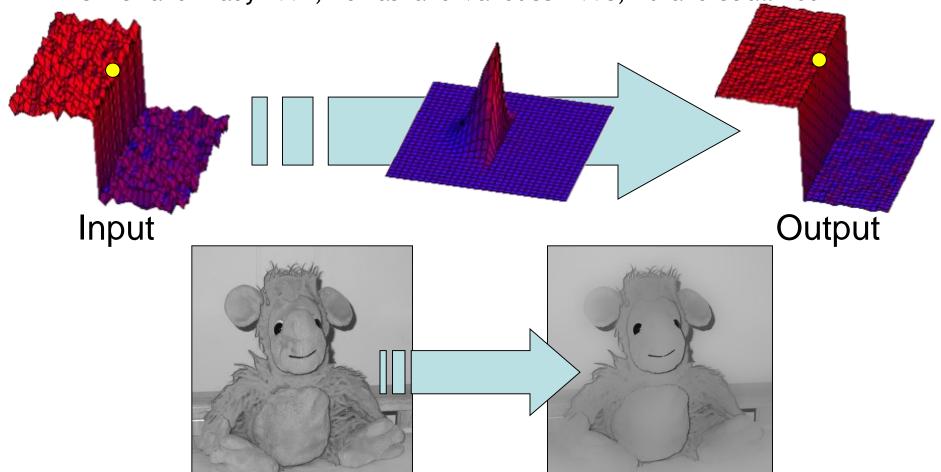
**Texture** 

### Large-scale Layer



Bilateral filter – edge preserving filter

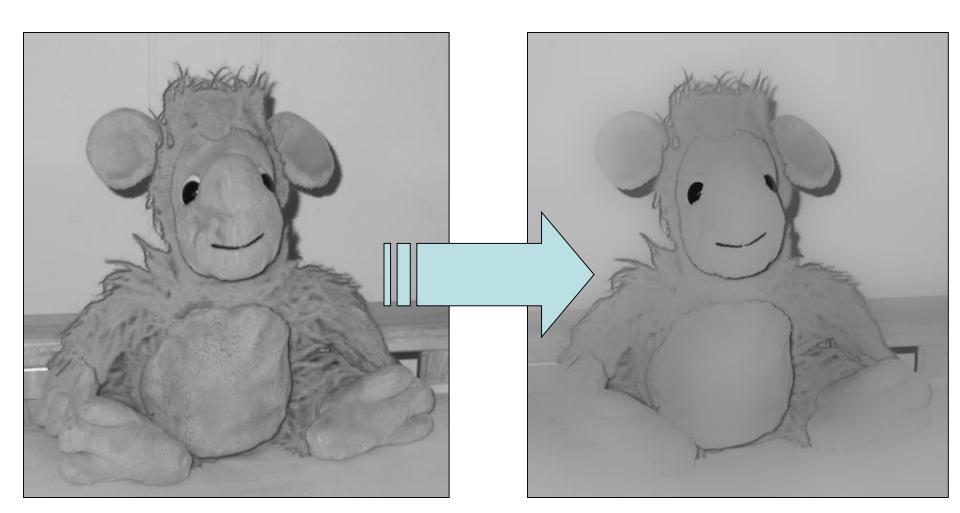
Smith and Brady 1997; Tomasi and Manducci 1998; Durand et al. 2002







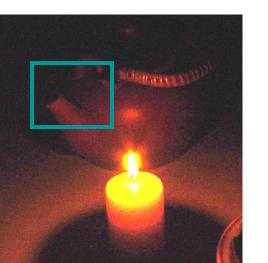
• Bilateral filter







- Similar to joint bilateral filter by Petschnigg et al.
- When no-flash image is too noisy
- Borrow similarity from flash image
  - > edge stopping from flash image

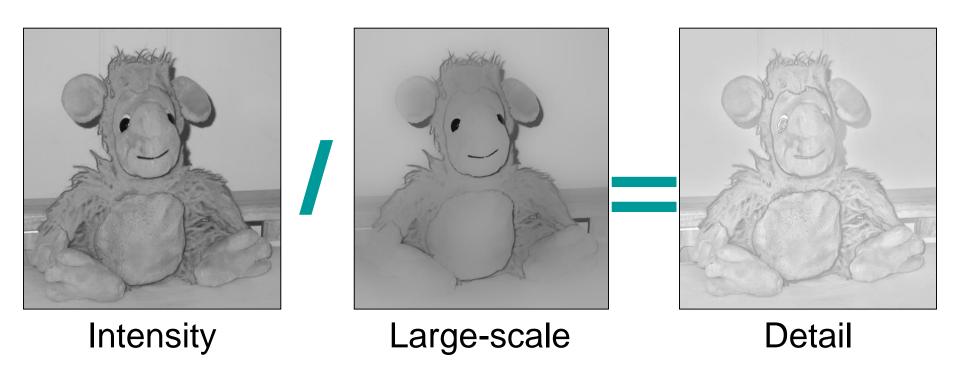






### **Detail Layer**

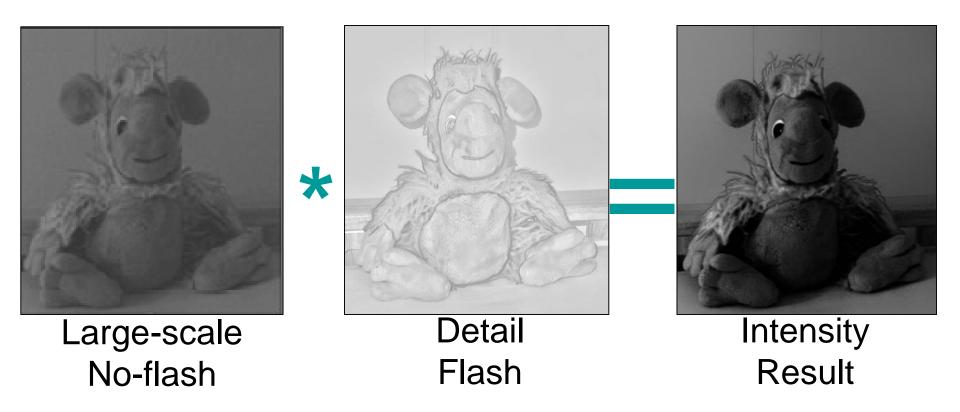




Recombination: Large scale \* Detail = Intensity

#### Recombination





Recombination: Large scale \* Detail = Intensity

#### Recombination



#### shadows



Intensity Result



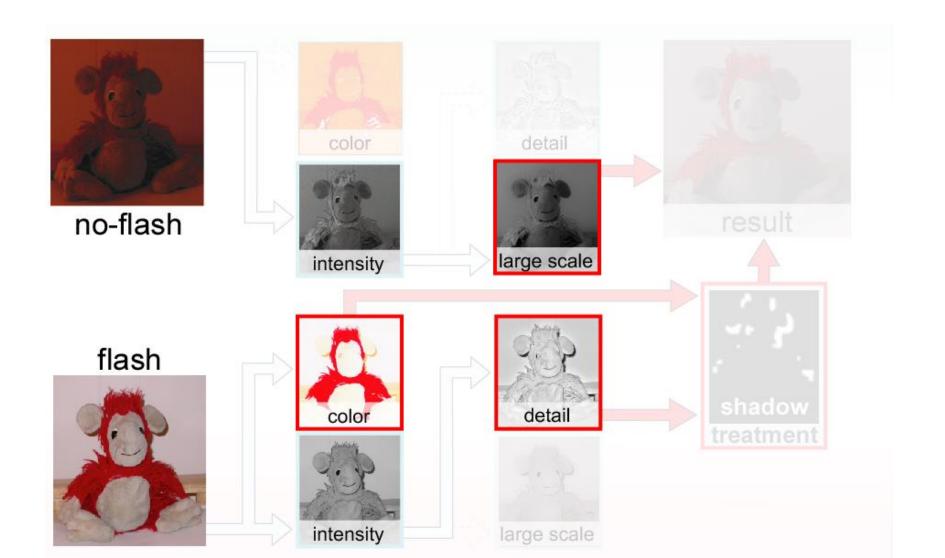
Color Flash



Result

Recombination: Intensity \* Color = Original





### Results



No-flash



Flash

