Multi-view 3D Reconstruction for Dummies

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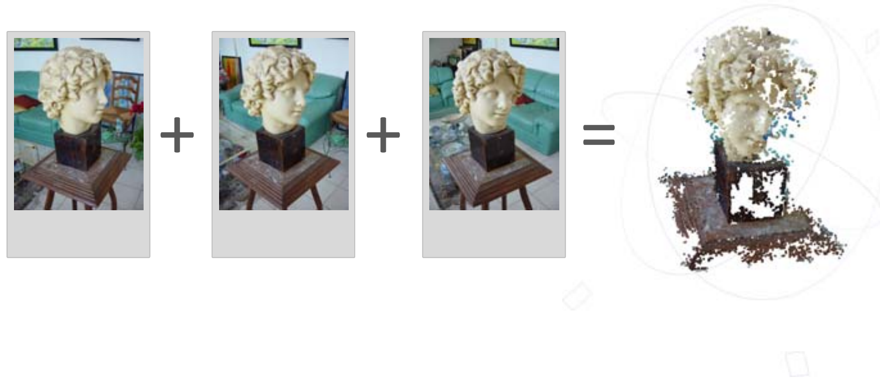
SFMedu Program with Code

Download from:

http://mit.edu/jxiao/Public/software/SFMedu/



Camera projection

- When people take a picture of a point:

$$\mathbf{x} = \mathbf{K}\left[\mathbf{R} \middle| \mathbf{t}\right]\mathbf{X}$$

Camera projection

- When people take two pictures with same camera setting:

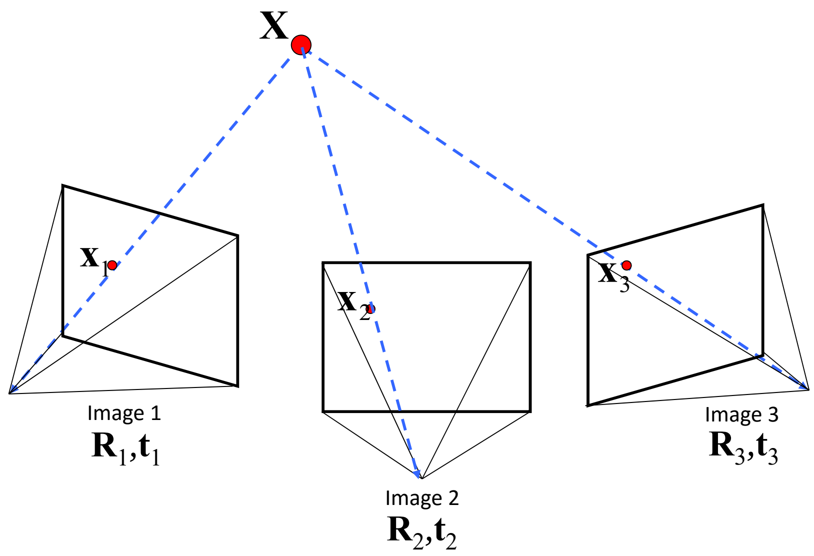
$$\mathbf{x}_1 = \mathbf{K}\left[\mathbf{R}_1 \middle| \mathbf{t}_1\right]\mathbf{X}$$
$$\mathbf{x}_2 = \mathbf{K}\left[\mathbf{R}_2 \middle| \mathbf{t}_2\right]\mathbf{X}$$

Camera projection

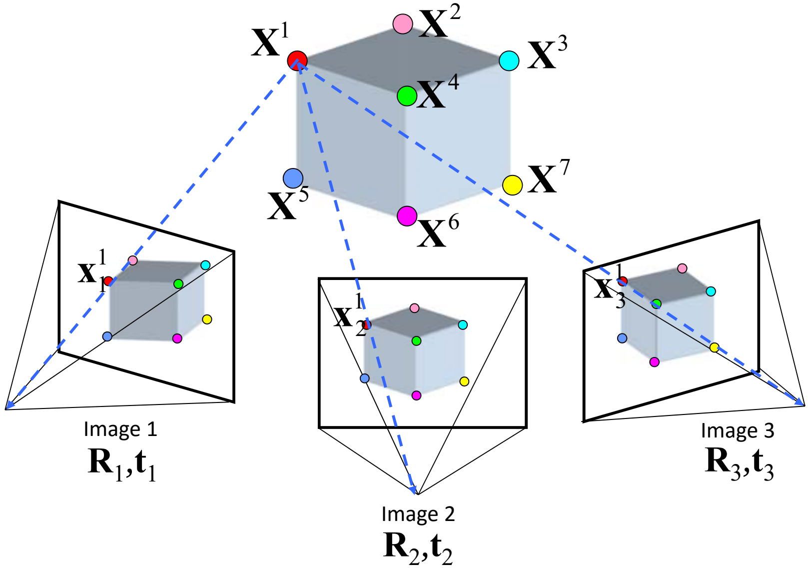
- When people take three pictures with same camera setting:

$$x_1 = K\left[R_1 | t_1\right]X$$
$$x_2 = K\left[R_2 | t_2\right]X$$
$$x_3 = K\left[R_3 | t_3\right]X$$

Camera projection



Camera projection

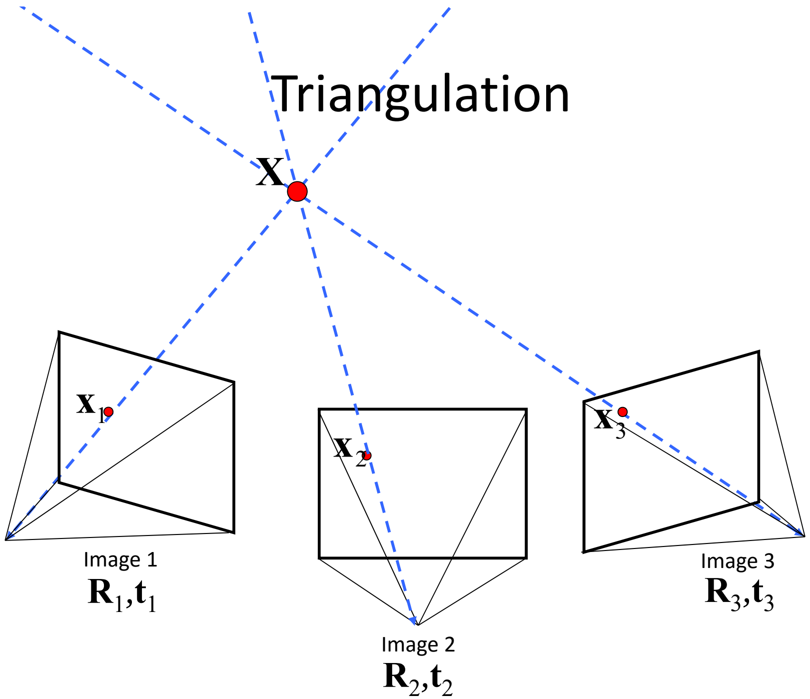


Camera projection

	Point 1	Point 2	Point 3
Image 1	$x_1^1 = K\left[R_1 \| t_1\right]X^1$	$x_1^2 = K\left[R_1 \| t_1\right]X^2$	
Image 2	$x_2^1 = K\left[R_2 \| t_2\right]X^1$	$x_2^2 = K\left[R_2 \| t_2\right]X^2$	$x_2^3 = K\left[R_2 \| t_2\right]X^3$
Image 3	$x_3^1 = K\left[R_3 \| t_3\right]X^1$		$x_3^3 = K\left[R_3 \| t_3\right]X^3$

Same Camera Same Setting = Same K

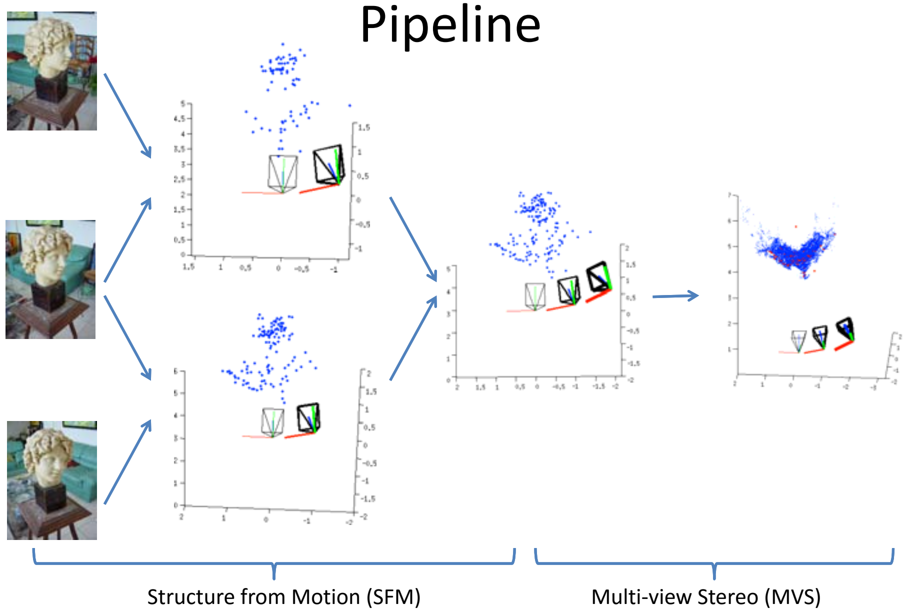
Triangulation



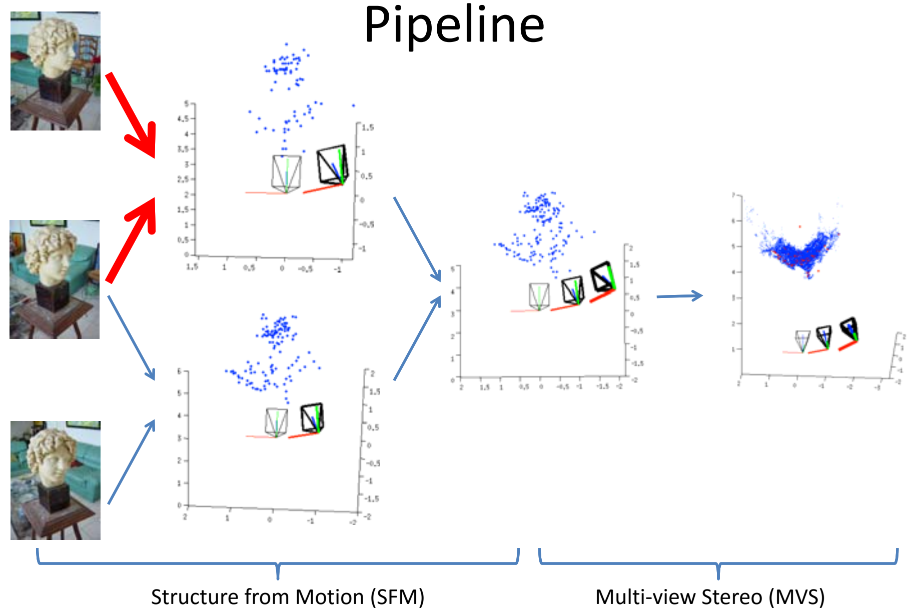
Structure From Motion

- Structure = 3D Point Cloud of the Scene
- Motion = Camera Location and Orientation
- SFM = Get the Point Cloud from Moving Cameras
- Structure and Motion: Joint Problems to Solve

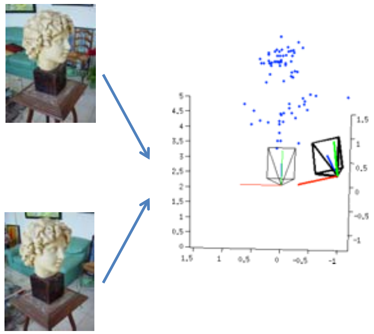
Pipeline



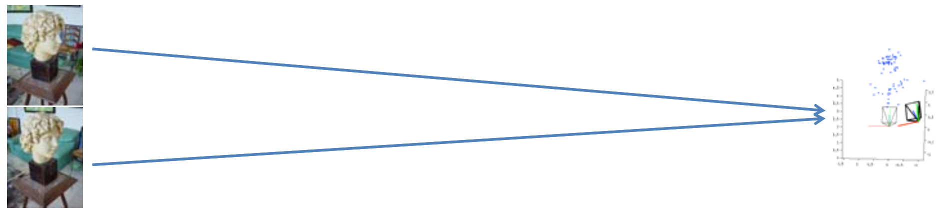
Pipeline



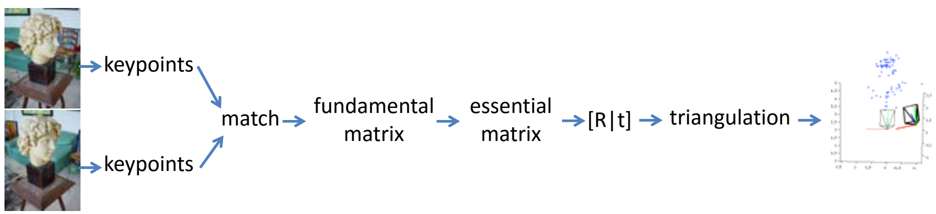
Two-view Reconstruction



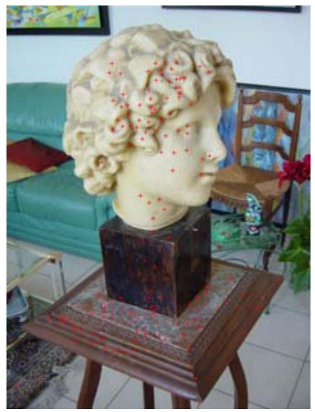
Two-view Reconstruction

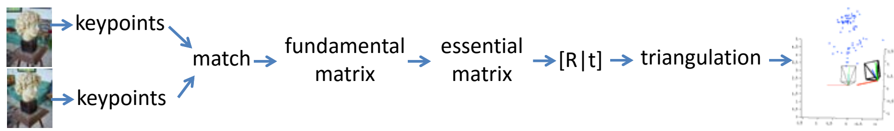


Two-view Reconstruction

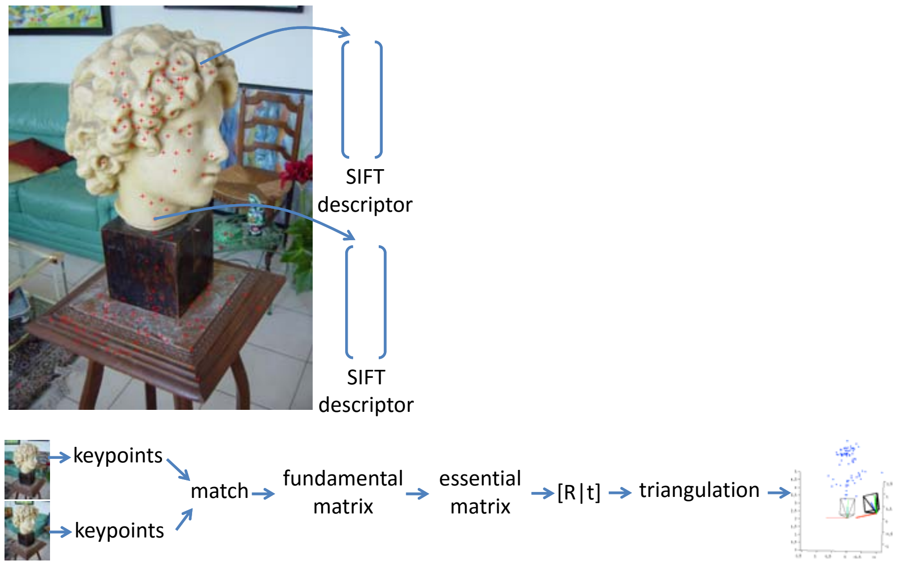


Keypoints Detection

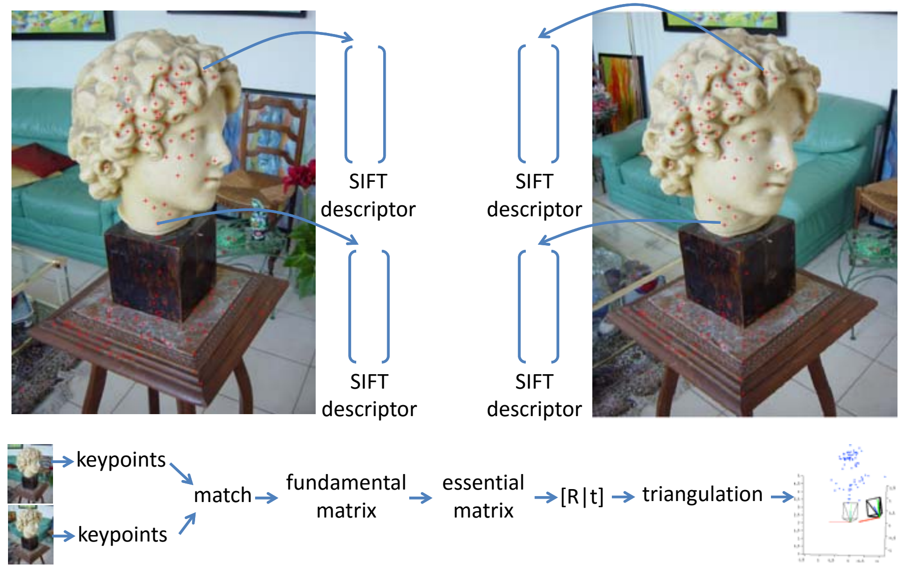




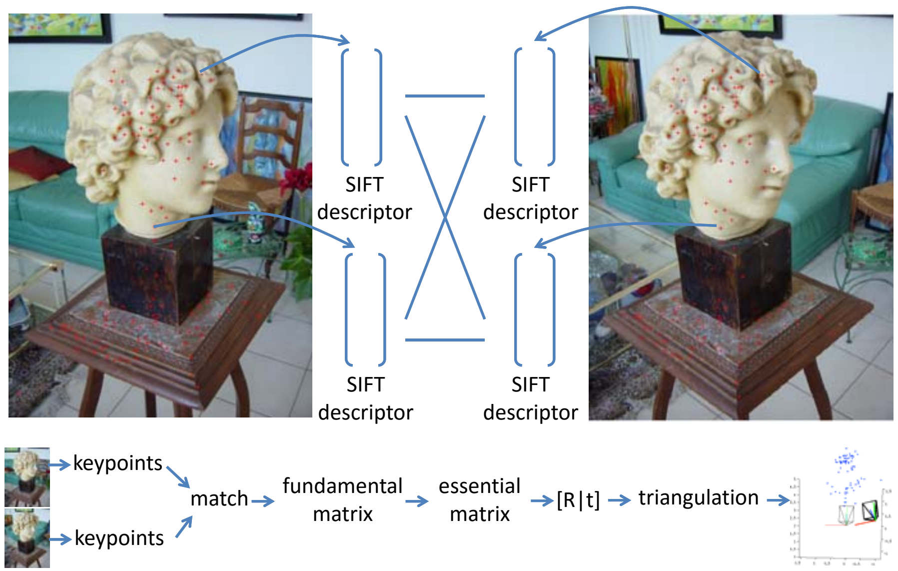
Descriptor for each point



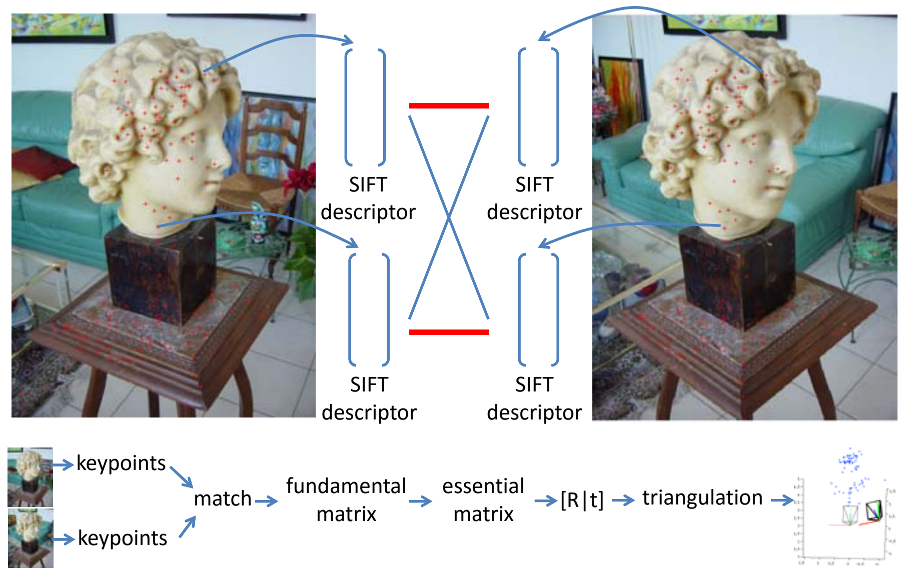
Same for the other images



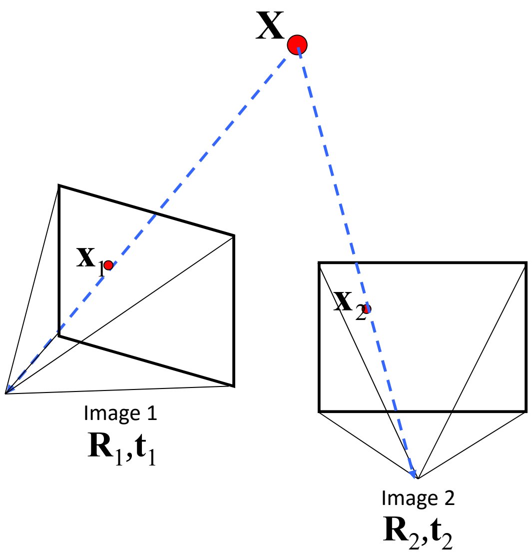
Point Match for correspondences



Point Match for correspondences



Fundamental Matrix



$$\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$$

$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

Estimating Fundamental Matrix

- Given a correspondence

$$\mathbf{x}_1 \leftrightarrow \mathbf{x}_2$$

- The basic incidence relation is

$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0 \quad \Leftrightarrow \quad [x_1 x_2, x_1 y_2, x_1, y_1 x_2, y_1 y_2, y_1, x_2, y_2, 1] \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

Need 8 points

Estimating Fundamental Matrix

$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$ for 8 point correspondences:

$$\mathbf{x}_1^1 \leftrightarrow \mathbf{x}_2^1, \mathbf{x}_1^2 \leftrightarrow \mathbf{x}_2^2, \mathbf{x}_1^3 \leftrightarrow \mathbf{x}_2^3, \mathbf{x}_1^4 \leftrightarrow \mathbf{x}_2^4, \mathbf{x}_1^5 \leftrightarrow \mathbf{x}_2^5, \mathbf{x}_1^6 \leftrightarrow \mathbf{x}_2^6, \mathbf{x}_1^7 \leftrightarrow \mathbf{x}_2^7, \mathbf{x}_1^8 \leftrightarrow \mathbf{x}_2^8$$

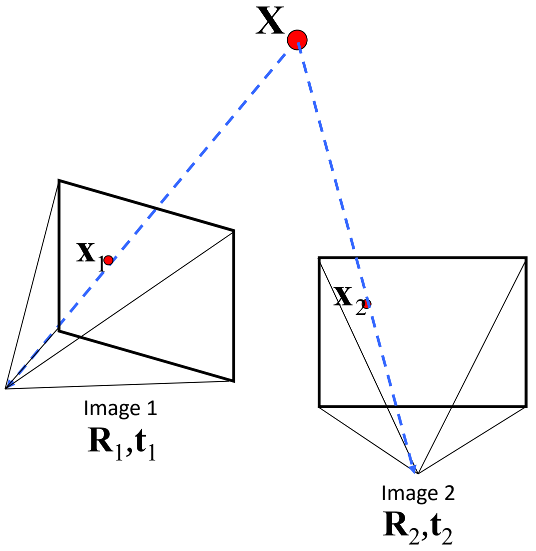
$$\begin{bmatrix}
x_1^1 x_2^1 & x_1^1 y_2^1 & x_1^1 & y_1^1 x_2^1 & y_1^1 y_2^1 & y_1^1 & x_2^1 & y_2^1 & 1 \\
x_1^2 x_2^2 & x_1^2 y_2^2 & x_1^2 & y_1^2 x_2^2 & y_1^2 y_2^2 & y_1^2 & x_2^2 & y_2^2 & 1 \\
x_1^3 x_2^3 & x_1^3 y_2^3 & x_1^3 & y_1^3 x_2^3 & y_1^3 y_2^3 & y_1^3 & x_2^3 & y_2^3 & 1 \\
x_1^4 x_2^4 & x_1^4 y_2^4 & x_1^4 & y_1^4 x_2^4 & y_1^4 y_2^4 & y_1^4 & x_2^4 & y_2^4 & 1 \\
x_1^5 x_2^5 & x_1^5 y_2^5 & x_1^5 & y_1^5 x_2^5 & y_1^5 y_2^5 & y_1^5 & x_2^5 & y_2^5 & 1 \\
x_1^6 x_2^6 & x_1^6 y_2^6 & x_1^6 & y_1^6 x_2^6 & y_1^6 y_2^6 & y_1^6 & x_2^6 & y_2^6 & 1 \\
x_1^7 x_2^7 & x_1^7 y_2^7 & x_1^7 & y_1^7 x_2^7 & y_1^7 y_2^7 & y_1^7 & x_2^7 & y_2^7 & 1 \\
x_1^8 x_2^8 & x_1^8 y_2^8 & x_1^8 & y_1^8 x_2^8 & y_1^8 y_2^8 & y_1^8 & x_2^8 & y_2^8 & 1
\end{bmatrix}
\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0 \quad \Rightarrow \quad \begin{matrix} \mathbf{Ax} = \mathbf{b} \\ \mathbf{Af} = \mathbf{0} \end{matrix}$$

Direct Linear Transformation (DLT)

RANSAC to Estimate Fundamental Matrix

- For many times
  - Pick 8 points
  - Compute a solution for $\mathbf{F}$ using these 8 points
  - Count number of inliers
- Pick the one with the largest number of inliers

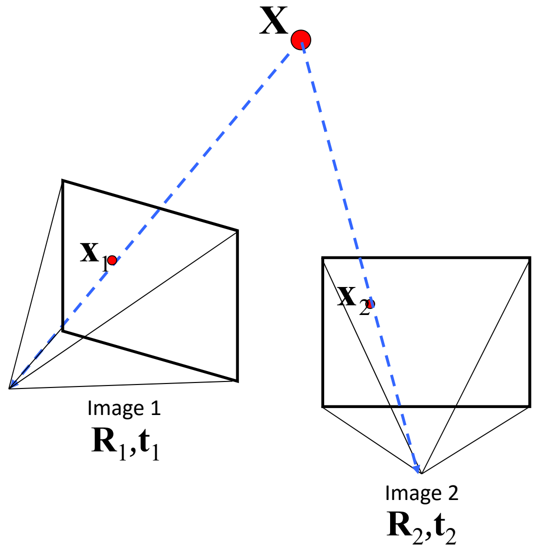
Fundamental Matrix → Essential Matrix



$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

$$\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2$$

Essential Matrix → $\left[\mathbf{R} \middle| \mathbf{t}\right]$



$$\mathbf{x}_1^T \mathbf{F} \mathbf{x}_2 = 0$$

$$\mathbf{E} = \mathbf{K}_1^T \mathbf{F} \mathbf{K}_2$$

Essential Matrix → $\left[\mathbf{R} \middle| \mathbf{t}\right]$

Result 9.19. For a given essential matrix

$$\mathbf{E} = \mathbf{U}\mathrm{diag}(1,1,0)\mathbf{V}^T,$$

and the first camera matrix $\mathbf{P}_1 = \left[\mathbf{I} \middle| \mathbf{0}\right]$, there are four possible choices for the second camera matrix $\mathbf{P}_2$:

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}\mathbf{V}^T \middle| +\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}\mathbf{V}^T \middle| -\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}^T\mathbf{V}^T \middle| +\mathbf{u}_3\right]$$

$$\mathbf{P}_2 = \left[\mathbf{U}\mathbf{W}^T\mathbf{V}^T \middle| -\mathbf{u}_3\right]$$

$$\mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Four Possible Solutions

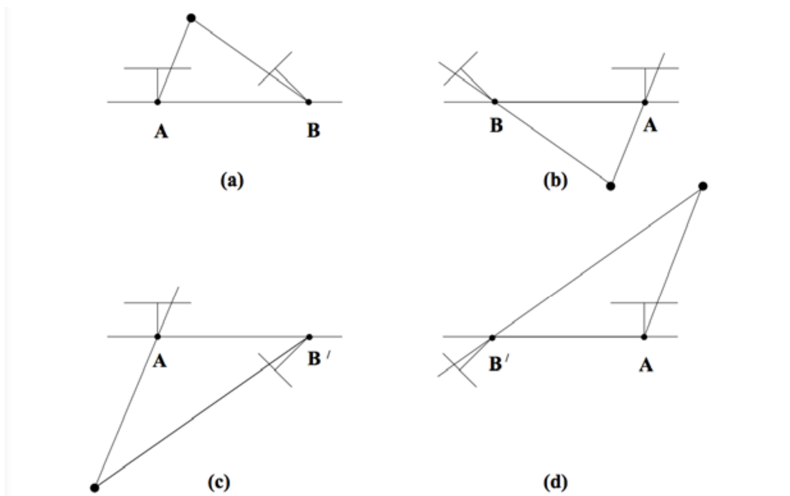

Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates 180° about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

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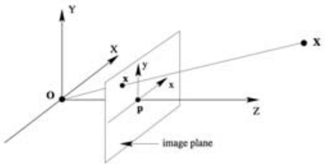
In front of the camera?

- Camera Extrinsic $\left[\mathbf{R}|\mathbf{t}\right]$

$$\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} = \mathbf{R} \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} + \mathbf{t} \Leftrightarrow \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} = \mathbf{R}^{-1} \left(\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} - \mathbf{t} \right) = \mathbf{R}^T \begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} - \mathbf{R}^T \mathbf{t}$$

- Camera Center

$$\begin{bmatrix} X_{cam} \\ Y_{cam} \\ Z_{cam} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \mathbf{C} = \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} = \mathbf{R}^T \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \mathbf{R}^T \mathbf{t} = -\mathbf{R}^T \mathbf{t}$$



- View Direction

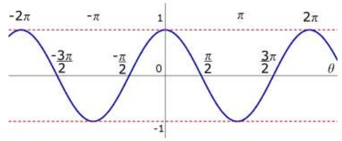
$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left(\mathbf{R}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \mathbf{R}^T \mathbf{t} \right) - (\mathbf{C}) = \left(\mathbf{R}(3,:)^T - \mathbf{R}^T \mathbf{t} \right) - \left(-\mathbf{R}^T \mathbf{t} \right) = \mathbf{R}(3,:)^T$$

Camera Coordinate System World Coordinate System

In front of the camera?

- A point $\mathbf{X}$
- Direction from camera center to point $\mathbf{X} - \mathbf{C}$
- Angle Between Two Vectors

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \|\mathbf{B}\| \cos\theta$$



- Angle Between $\mathbf{X} - \mathbf{C}$ and View Direction
- Just need to test

$$\left(\mathbf{X} - \mathbf{C} \right) \cdot \mathbf{R}(3,:)^T > 0 ?$$

Pick the Solution

With maximal number of points in front of both cameras.

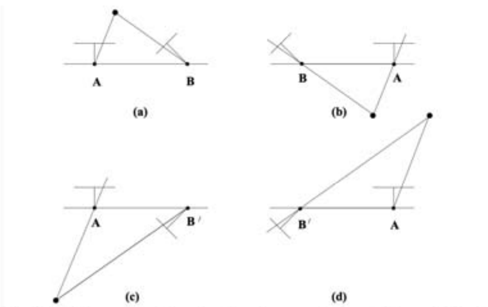


Fig. 9.12. **The four possible solutions for calibrated reconstruction from E.** *Between the left and right sides there is a baseline reversal. Between the top and bottom rows camera B rotates 180° about the baseline. Note, only in (a) is the reconstructed point in front of both cameras.*

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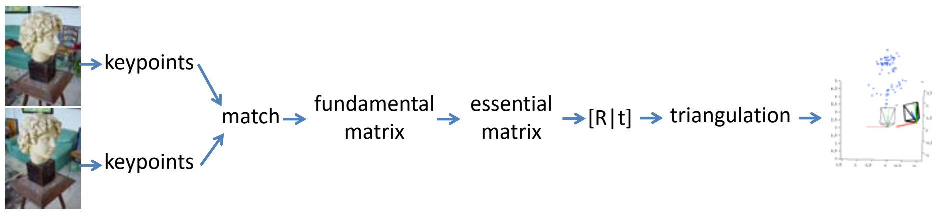
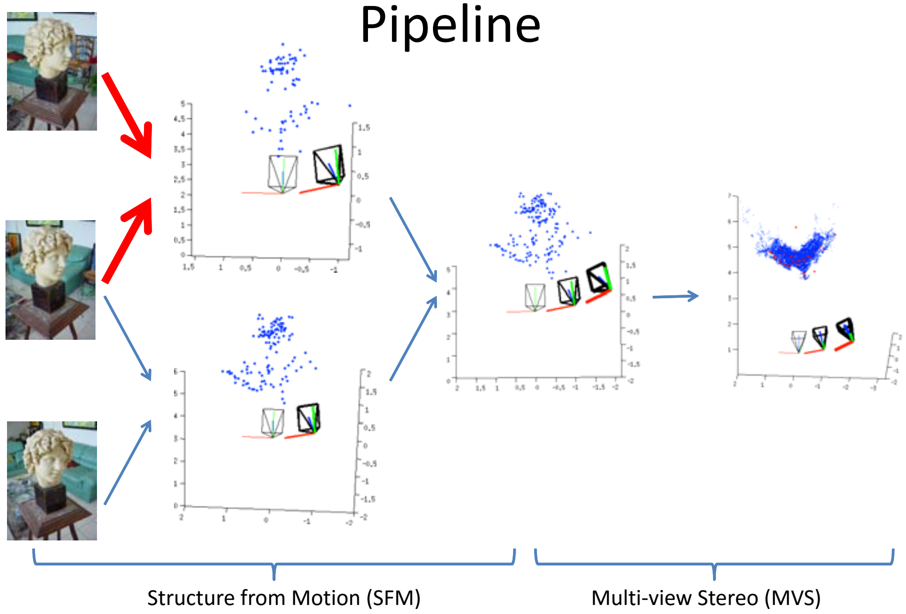
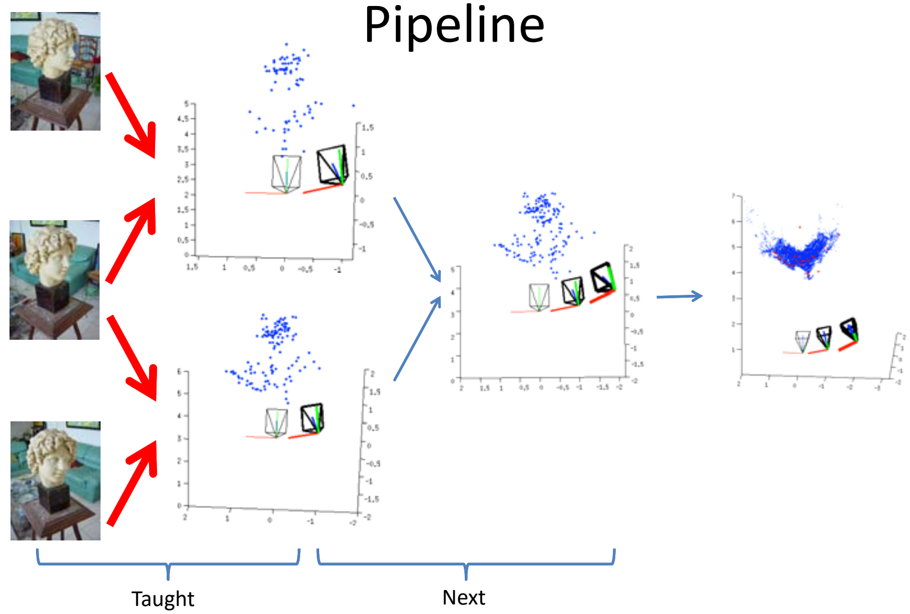
Two-view Reconstruction



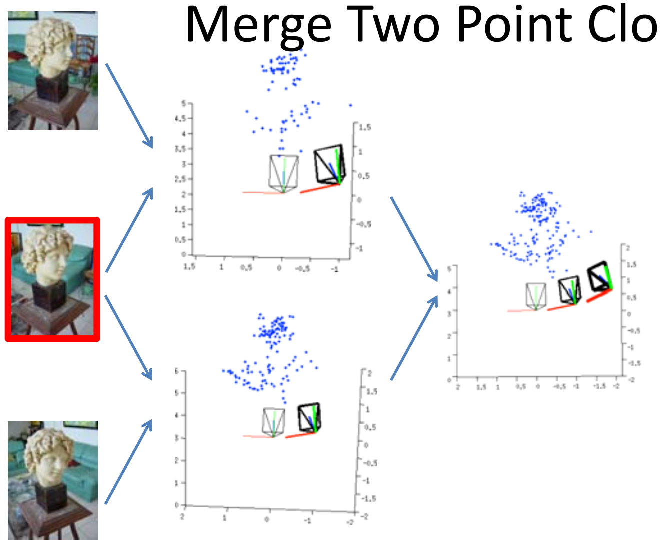
Pipeline



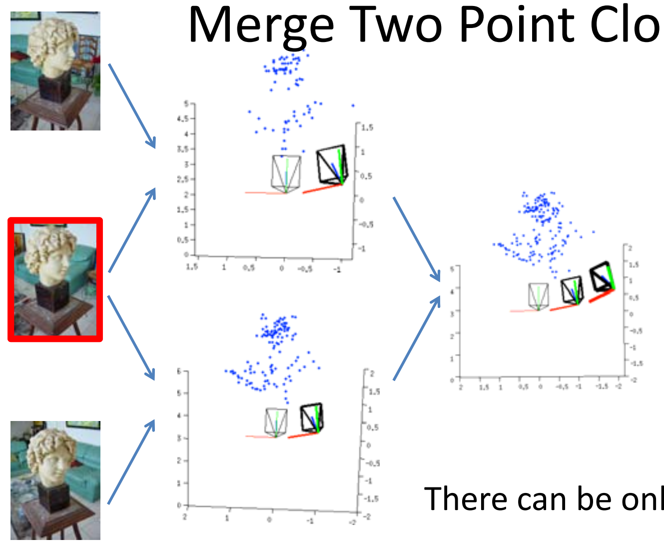
Pipeline



Merge Two Point Cloud



Merge Two Point Cloud

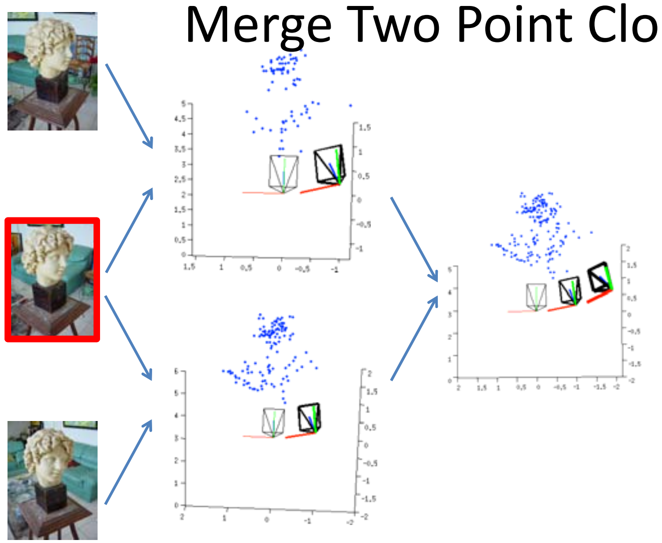


There can be only one $\left[\mathbf{R}_2 | \mathbf{t}_2\right]$

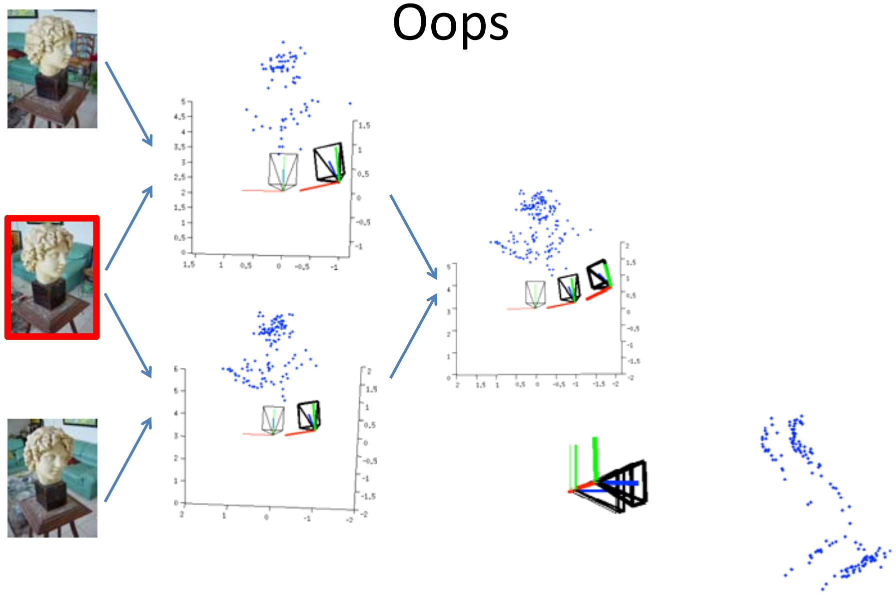
Merge Two Point Cloud

- From the 1st and 2nd images, we have $\left[\mathbf{R}_1|\mathbf{t}_1\right]$ and $\left[\mathbf{R}_2|\mathbf{t}_2\right]$
- From the 2nd and 3rd images, we have $\left[\mathbf{R}_2|\mathbf{t}_2\right]$ and $\left[\mathbf{R}_3|\mathbf{t}_3\right]$
- **Exercise**: How to transform the coordinate system of the second point cloud to align with the first point cloud so that there is only one $\left[\mathbf{R}_2|\mathbf{t}_2\right]$?

Merge Two Point Cloud

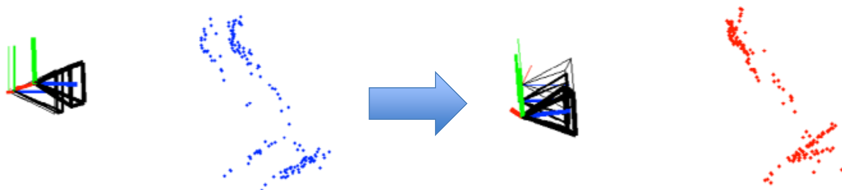


Oops

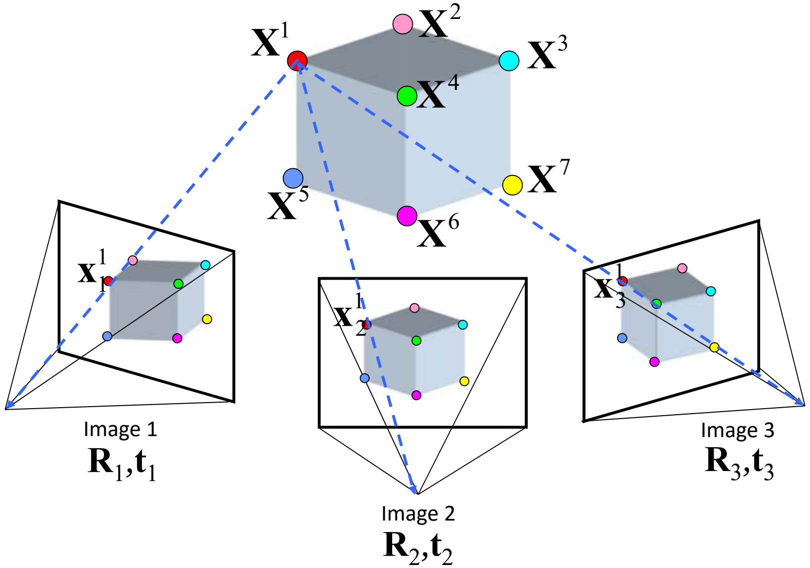


See From a Different Angle

Bundle Adjustment



Camera projection



Camera projection

	Point 1	Point 2	Point 3
Image 1	$\mathbf{x}_1^1 = \mathbf{K}[\mathbf{R}_1 \mid \mathbf{t}_1]\mathbf{X}^1$	$\mathbf{x}_1^2 = \mathbf{K}[\mathbf{R}_1 \mid \mathbf{t}_1]\mathbf{X}^2$	
Image 2	$\mathbf{x}_2^1 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^1$	$\mathbf{x}_2^2 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^2$	$\mathbf{x}_2^3 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^3$
Image 3	$\mathbf{x}_3^1 = \mathbf{K}[\mathbf{R}_3 \mid \mathbf{t}_3]\mathbf{X}^1$		$\mathbf{x}_3^3 = \mathbf{K}[\mathbf{R}_3 \mid \mathbf{t}_3]\mathbf{X}^3$

Same Camera Same Setting = Same $\mathbf{K}$

Rethinking the SFM problem

- Input: Observed 2D image position

$$\begin{matrix} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{matrix}$$

- Output:

  Unknown Camera Parameters (with some guess)

  $$[\mathbf{R}_1 \mid \mathbf{t}_1], [\mathbf{R}_2 \mid \mathbf{t}_2], [\mathbf{R}_3 \mid \mathbf{t}_3]$$

  Unknown Point 3D coordinate (with some guess)

  $$\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3, \cdots$$

Bundle Adjustment

A valid solution $[\mathbf{R}_1 \mid \mathbf{t}_1], [\mathbf{R}_2 \mid \mathbf{t}_2], [\mathbf{R}_3 \mid \mathbf{t}_3]$ and $\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3, \cdots$ must let

Re-projection
$$\left\{ \begin{matrix} \mathbf{x}_1^1 = \mathbf{K}[\mathbf{R}_1 \mid \mathbf{t}_1]\mathbf{X}^1 & \mathbf{x}_1^2 = \mathbf{K}[\mathbf{R}_1 \mid \mathbf{t}_1]\mathbf{X}^2 & \\ \mathbf{x}_2^1 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^1 & \mathbf{x}_2^2 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^2 & \mathbf{x}_2^3 = \mathbf{K}[\mathbf{R}_2 \mid \mathbf{t}_2]\mathbf{X}^3 \\ \mathbf{x}_3^1 = \mathbf{K}[\mathbf{R}_3 \mid \mathbf{t}_3]\mathbf{X}^1 & & \mathbf{x}_3^3 = \mathbf{K}[\mathbf{R}_3 \mid \mathbf{t}_3]\mathbf{X}^3 \end{matrix} \right.$$

=

Observation
$$\left\{ \begin{matrix} \tilde{\mathbf{x}}_1^1 & \tilde{\mathbf{x}}_1^2 & \\ \tilde{\mathbf{x}}_2^1 & \tilde{\mathbf{x}}_2^2 & \tilde{\mathbf{x}}_2^3 \\ \tilde{\mathbf{x}}_3^1 & & \tilde{\mathbf{x}}_3^3 \end{matrix} \right.$$

Bundle Adjustment

A valid solution $[\mathbf{R}_1|\mathbf{t}_1],[\mathbf{R}_2|\mathbf{t}_2],[\mathbf{R}_3|\mathbf{t}_3]$ and $\mathbf{X}^1,\mathbf{X}^2,\mathbf{X}^3,\cdots$ must let the Re-projection close to the Observation, i.e. to minimize the reprojection error

$$\min \sum_i \sum_j \left(\tilde{\mathbf{x}}_i^j - \mathbf{K}\left[\mathbf{R}_i|\mathbf{t}_i\right]\mathbf{X}^j\right)^2$$

Solving This Optimization Problem

- Theory:

  The Levenberg–Marquardt algorithm

  http://en.wikipedia.org/wiki/Levenberg-Marquardt_algorithm

- Practice:

  The Ceres-Solver from Google

  http://code.google.com/p/ceres-solver/