More on natural image matting

Digital Visual Effects

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with slides by

- A closed form solution to natural alpha matting, CVPR 2006
- With slides from Prof. Hwann-Tzong Chen

Linear relation (grayscale for now)

$$I_i = \alpha_i F_i + (1 - \alpha_i) B_i$$

Assumption: both F and B are approximately constant over a small window around each pixel

$$I_i \approx \alpha_i F + (1 - \alpha_i) B$$
$$I_i \approx \alpha_i (F - B) + B$$
$$\alpha_i \approx \frac{1}{F-B} I_i - \frac{B}{F-B}$$

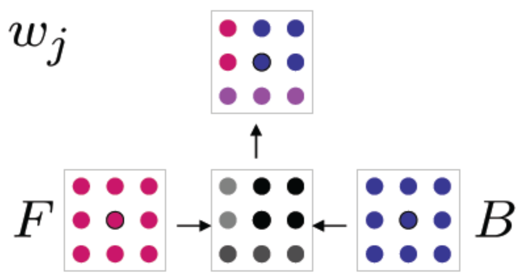
$$\alpha_i \approx a I_i + b, \quad \forall i \in w$$ ← a small window

$$a = \frac{1}{F-B} \qquad b = -\frac{B}{F-B}$$

Linear relation

$$\alpha_i \approx a I_i + b, \quad \forall i \in w$$ ← a small window

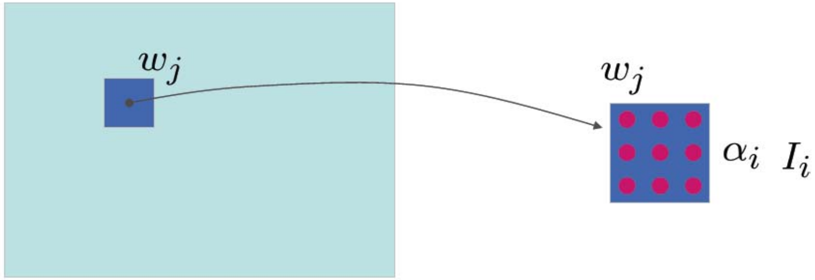
$$a = \frac{1}{F-B} \qquad b = -\frac{B}{F-B}$$



Optimization

$$J(\alpha, a, b) = \sum_{j \in I} \left(\sum_{i \in w_j} (\alpha_i - a_j I_i - b_j)^2 + \epsilon a_j^2 \right)$$

where w_j is a small window around pixel j



Optimization

$$J(\alpha, a, b) = \sum_{j \in I} \left(\sum_{i \in w_j} (\alpha_i - a_j I_i - b_j)^2 + \epsilon a_j^2 \right)$$

where w_j is a small window around pixel j

a regularization term on a:

minimizing the norm of a biases the solution towards smoother α mattes $\alpha_i \approx a\cancel{I_i} + b, \quad \forall i \in w$

$a \ll 0$ implies that F and B are very different

Optimization

$$J(\alpha, a, b) = \sum_{j \in I} \left(\sum_{i \in w_j} (\alpha_i - a_j I_i - b_j)^2 + \epsilon a_j^2 \right)$$

$$J(\alpha, a, b) = \sum_k \left\| \begin{pmatrix} I_1 & 1 \\ \vdots & \vdots \\ I_{|w_k|} & 1 \\ \sqrt{\epsilon} & 0 \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix} - \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{|w_k|} \\ 0 \end{pmatrix} \right\|^2$$

$$J(\alpha, a, b) = \sum_k \left\| G_k \begin{bmatrix} a_k \\ b_k \end{bmatrix} - \bar{\alpha}_k \right\|^2$$

Optimization

$$(a_k^*, b_k^*) = \operatorname{argmin} \left\| G_k \begin{bmatrix} a_k \\ b_k \end{bmatrix} - \bar{\alpha}_k \right\|^2$$

$$\begin{pmatrix} a_k^* \\ b_k^* \end{pmatrix} = (G_k^T G_k)^{-1} G_k^T \bar{\alpha}_k$$

Optimization

$$\begin{pmatrix} a_k^* \\ b_k^* \end{pmatrix} = (G_k^T G_k)^{-1} G_k^T \bar{\alpha}_k$$

$$J(\alpha, a^*, b^*) = \sum_k \left\| G_k \begin{pmatrix} a_k^* \\ b_k^* \end{pmatrix} - \bar{\alpha}_k \right\|^2$$

$$J(\alpha) = \sum_k \left\| \left(G_k (G_k^T G_k)^{-1} G_k^T - \mathbf{I} \right) \bar{\alpha}_k \right\|^2$$
$$= \sum_k \bar{\alpha}_k^T \bar{G}_k^T \bar{G}_k \bar{\alpha}_k$$

$$\bar{G}_k = \mathbf{I} - G_k (G_k^T G_k)^{-1} G_k^T$$

Optimization

$$\bar{G}_k^T \bar{G}_k = (\mathbf{I} - G_k (G_k^T G_k)^{-1} G_k^T)^T (\mathbf{I} - G_k (G_k^T G_k)^{-1} G_k^T)$$
$$= \mathbf{I} - G_k (G_k^T G_k)^{-1} G_k^T$$

$$G_k = \begin{pmatrix} I_1 & 1 \\ \vdots & \vdots \\ I_{|w_k|} & 1 \\ \sqrt{\epsilon} & 0 \end{pmatrix}$$

the (i,j)-th element of $\mathbf{I} - G_k (G_k^T G_k)^{-1} G_k^T$ is

$$\delta_{ij} - \frac{1}{|w_k|} \left(1 + \frac{1}{\frac{\epsilon}{|w_k|} + \sigma_k^2} (I_i - \mu_k)(I_j - \mu_k) \right)$$

Optimization

The (i,j) element

$$\left(\mathbf{I} - G_k \left(G_k^T G_k \right)^{-1} G_k^T \right)_{ij}$$
$$= \delta_{ij} - (I_i \; 1) \begin{pmatrix} \sum_n^{|w_k|} I_n^2 + \epsilon & \sum_n^{|w_k|} I_n \\ \sum_n^{|w_k|} I_n & |w_k| \end{pmatrix}^{-1} \begin{pmatrix} I_j \\ 1 \end{pmatrix}$$

$$\mathbf{I} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

$$\begin{pmatrix} I_1 & 1 \\ I_2 & 1 \\ \vdots & \\ I_{|w_k|} & 1 \\ \sqrt{\epsilon} & 0 \end{pmatrix} \left\{ \begin{pmatrix} I_1 & I_2 & \cdots & I_{|w_k|} & \sqrt{\epsilon} \\ 1 & 1 & & 1 & 0 \end{pmatrix} \begin{pmatrix} I_1 & 1 \\ I_2 & 1 \\ \vdots & \\ I_{|w_k|} & 1 \\ \sqrt{\epsilon} & 0 \end{pmatrix} \right\}^{-1} \begin{pmatrix} I_1 & I_2 & \cdots & I_{|w_k|} & \sqrt{\epsilon} \\ 1 & 1 & & 1 & 0 \end{pmatrix}$$

The inverse

$$\begin{pmatrix} \sum_n^{|w_k|} I_n^2 + \epsilon & \sum_n^{|w_k|} I_n \\ \sum_n^{|w_k|} I_n & |w_k| \end{pmatrix}^{-1}$$

$$= \frac{\begin{pmatrix} |w_k| & -\sum_n^{|w_k|} I_n \\ -\sum_n^{|w_k|} I_n & \sum_n^{|w_k|} I_n^2 + \epsilon \end{pmatrix}}{|w_k| \sum_n^{|w_k|} I_n^2 + \epsilon |w_k| - (\sum_n^{|w_k|} I_n)^2}$$

$$= \frac{|w_k| \begin{pmatrix} 1 & -\mu_k \\ -\mu_k & \sum_n^{|w_k|} I_n^2 / |w_k| + \epsilon / |w_k| \end{pmatrix}}{|w_k|^2 \sigma_k^2 + \epsilon |w_k|}$$

$$= \frac{1}{|w_k| \sigma_k^2 + \epsilon} \begin{pmatrix} 1 & -\mu_k \\ -\mu_k & \sum_l^{|w_k|} I_n^2 / |w_k| + \epsilon / |w_k| \end{pmatrix}$$

Optimization

The (i, j) element

$$\left(\mathbf{I} - G_k\left(G_k^T G_k\right)^{-1} G_k^T\right)_{ij}$$

$$= \delta_{ij} - (I_i\ 1)\begin{pmatrix}\sum_n^{|w_k|} I_n^2+\epsilon & \sum_n^{|w_k|} I_n \\ \sum_n^{|w_k|} I_n & |w_k|\end{pmatrix}^{-1}\begin{pmatrix}I_j\\1\end{pmatrix}$$

$$= \delta_{ij} - (I_i\ 1)\frac{1}{|w_k|\sigma_k^2+\epsilon}\begin{pmatrix}1 & -\mu_k \\ -\mu_k & \sum_n^{|w_k|} I_n^2/|w_k| + \epsilon/|w_k|\end{pmatrix}\begin{pmatrix}I_j\\1\end{pmatrix}$$

$$= \delta_{ij} - \frac{1}{|w_k|\sigma_k^2+\epsilon}\left(I_iI_j - I_i\mu_k - I_j\mu_k + \frac{\sum_n^{|w_k|} I_n^2+\epsilon}{|w_k|}\right)$$

$$= \delta_{ij} - \frac{1}{|w_k|\sigma_k^2+\epsilon}\left(I_iI_j - I_i\mu_k - I_j\mu_k + \mu_k^2 + \frac{\sum_n^{|w_k|} I_n^2}{|w_k|} - \mu_k^2 + \frac{\epsilon}{|w_k|}\right)$$

$$= \delta_{ij} - \frac{1}{|w_k|\sigma_k^2+\epsilon}\left((I_i-\mu_k)(I_j-\mu_k) + \sigma_k^2 + \frac{\epsilon}{|w_k|}\right)$$

$$= \delta_{ij} - \frac{1}{|w_k|}\left(1 + \frac{1}{\sigma_k^2+\epsilon/|w_k|}(I_i-\mu_k)(I_j-\mu_k)\right)$$

Optimization

$$J(\alpha) = \sum_k \bar{\alpha}_k^T \bar{G}_k^T \bar{G}_k \bar{\alpha}_k$$

$$J(\alpha) = \alpha^T L\ \alpha$$

L is a large sparse N-by-N matrix whose (i, j) element is

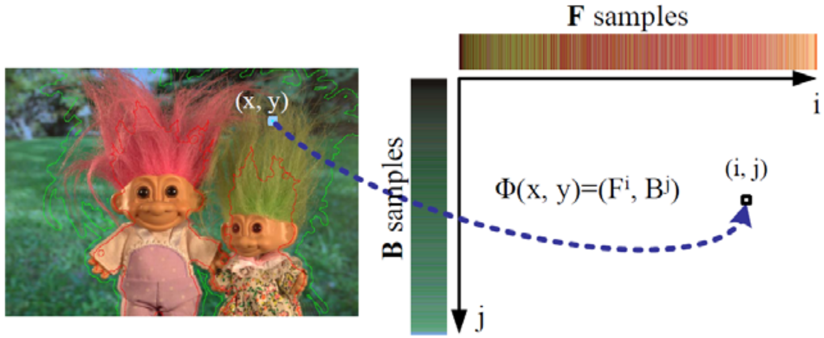
$$\sum_{k|(i,j)\in w_k}\left(\delta_{ij} - \frac{1}{|w_k|}\left(1 + \frac{1}{\sigma_k^2+\epsilon/|w_k|}(I_i-\mu_k)(I_j-\mu_k)\right)\right)$$

N is the number of pixels in the image

- A global sampling method for alpha matting, CVPR 2011

Idea

Search space



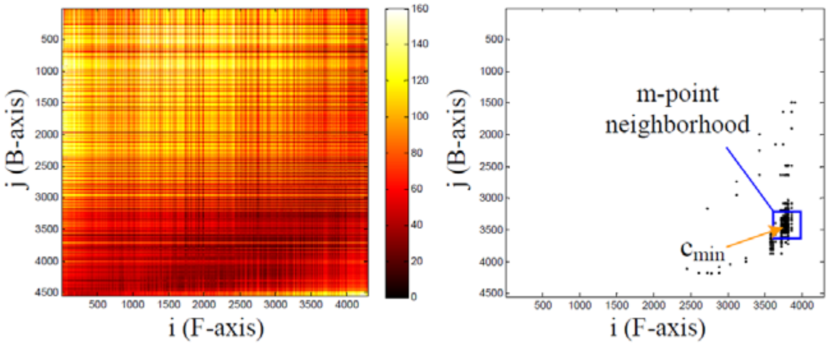
Propagation and random search

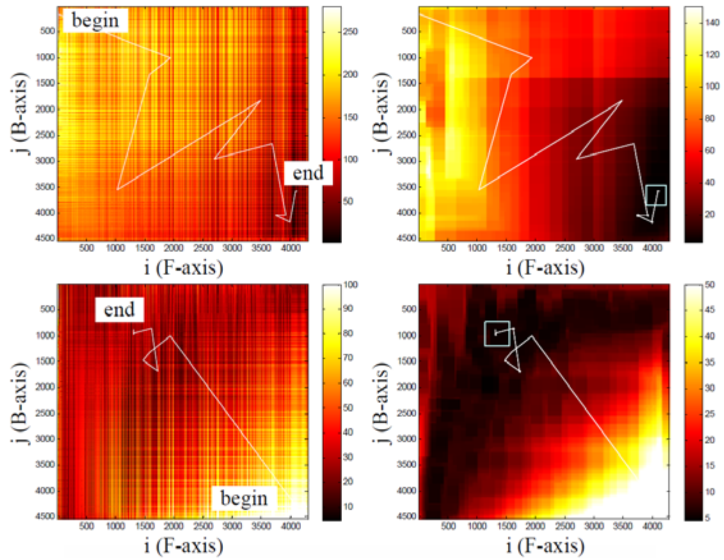
- Propagation

$$\Phi(x,y) \leftarrow \arg \min_{\Phi(x',y')} \mathcal{E}(\Phi(x',y'))$$

- Random search

$$(i_k, j_k) = (i,j) + \omega \beta^k \mathbf{R}_k$$





- KNN Matting, CVPR 2012

Nonlocal principle

$$E[X(i)] \approx \sum_j X(j)k(i,j)\frac{1}{\mathcal{D}_i},$$

$$k(i,j) = \exp(-\frac{1}{h_1^2}\|X(i)-X(j)\|_g^2 - \frac{1}{h_2^2}d_{ij}^2)$$

$$\mathcal{D}_i = \sum_j k(i,j).$$

Nonlocal principle for mattes

$$E[\alpha_i] \approx \sum_j \alpha_j k(i,j)\frac{1}{\mathcal{D}_i}$$

$$\mathcal{D}_i\alpha_i \approx k(i,\cdot)^T\boldsymbol{\alpha}$$

$$\mathcal{D}\boldsymbol{\alpha} \approx \mathcal{A}\boldsymbol{\alpha}$$

$$(\mathcal{D}-\mathcal{A})\boldsymbol{\alpha} \approx \mathbf{0}$$

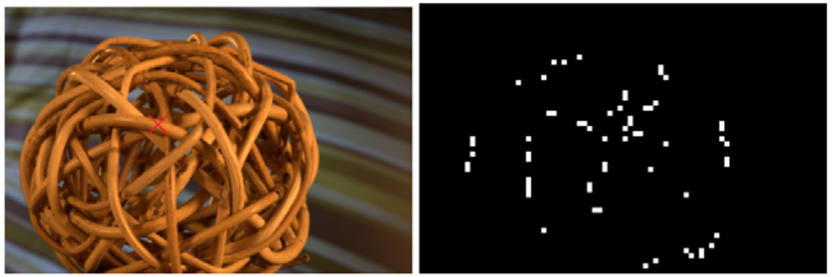
Kernels and features

$$X(i) = (\cos(h), \sin(h), s, v, x, y)_i$$

$$k(i,j) = 1 - \frac{||X(i)-X(j)||}{C}$$

C is the least upper bound of $||X(i)-X(j)||$

KNN matting

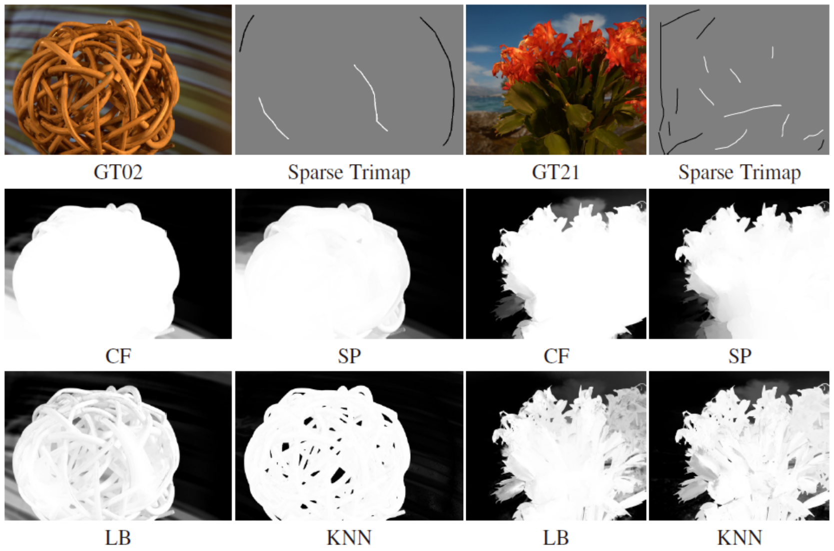


input pixel (red) KNN (10^{-5} sec)

Results

	overall	avg user	pine-apple	plastic bag	normalized score(%)
Shared	3.6	3.5	2	7	79.6
Segmentation	4.2	4	5	9	77.2
KNN	**4.3**	**3.6**	**1**	**1**	**84.6**
Improved color	4.4	4	4	3	75.7
Learning-based	5.9	6.4	12	2	67.8
Closed-Form	6	7.4	10	5	66.1
Shared (real time)	6.1	5.8	3	8	65.4
Large Kernel	6.8	6.5	6	4	62.4
Robust	7.5	8.1	8	6	55.9
High-res	8.5	8.1	9	13	51.5

Results



Results