

# High dynamic range imaging

Digital Visual Effects

*Yung-Yu Chuang*

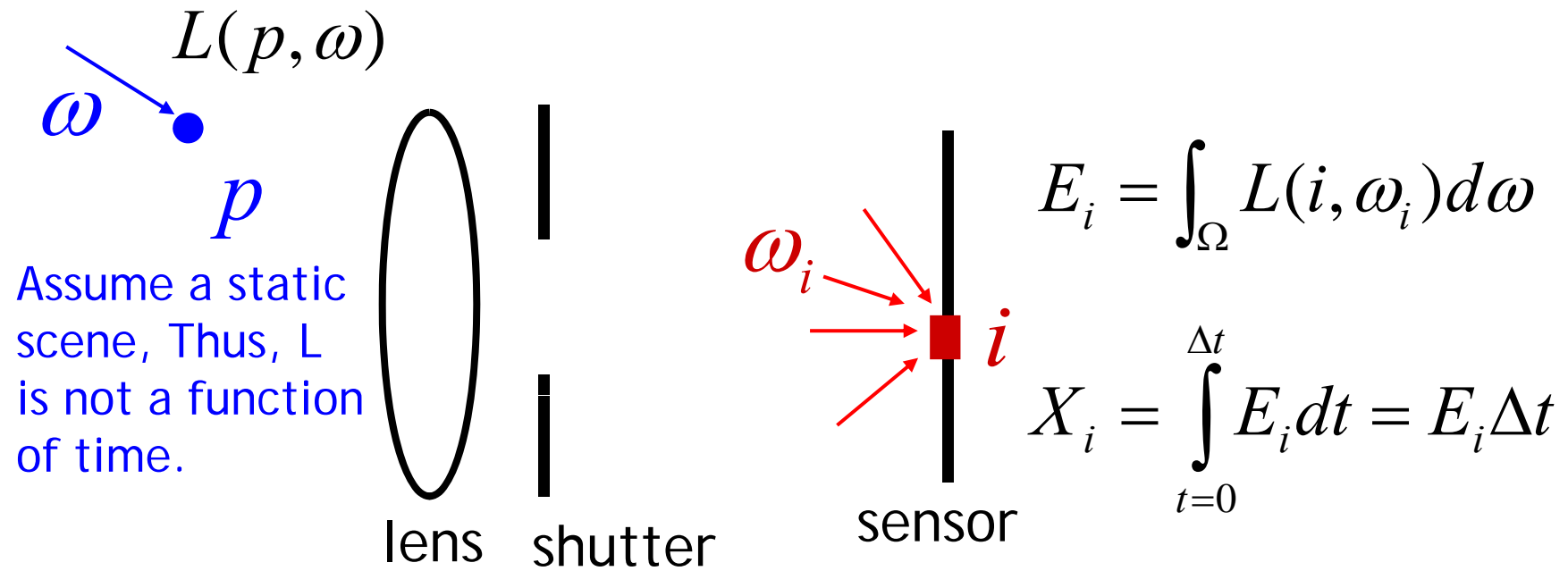
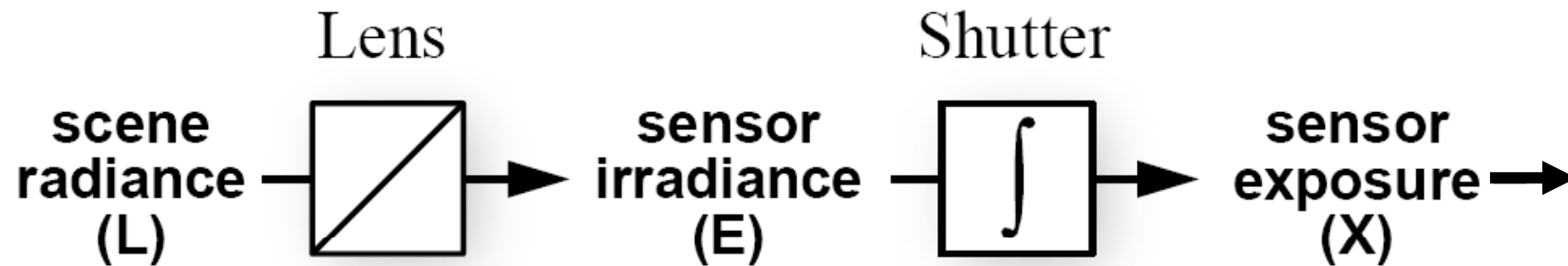
*with slides by Fredo Durand, Brian Curless, Steve Seitz, Paul Debevec and Alexei Efros*

# Camera is an imperfect device

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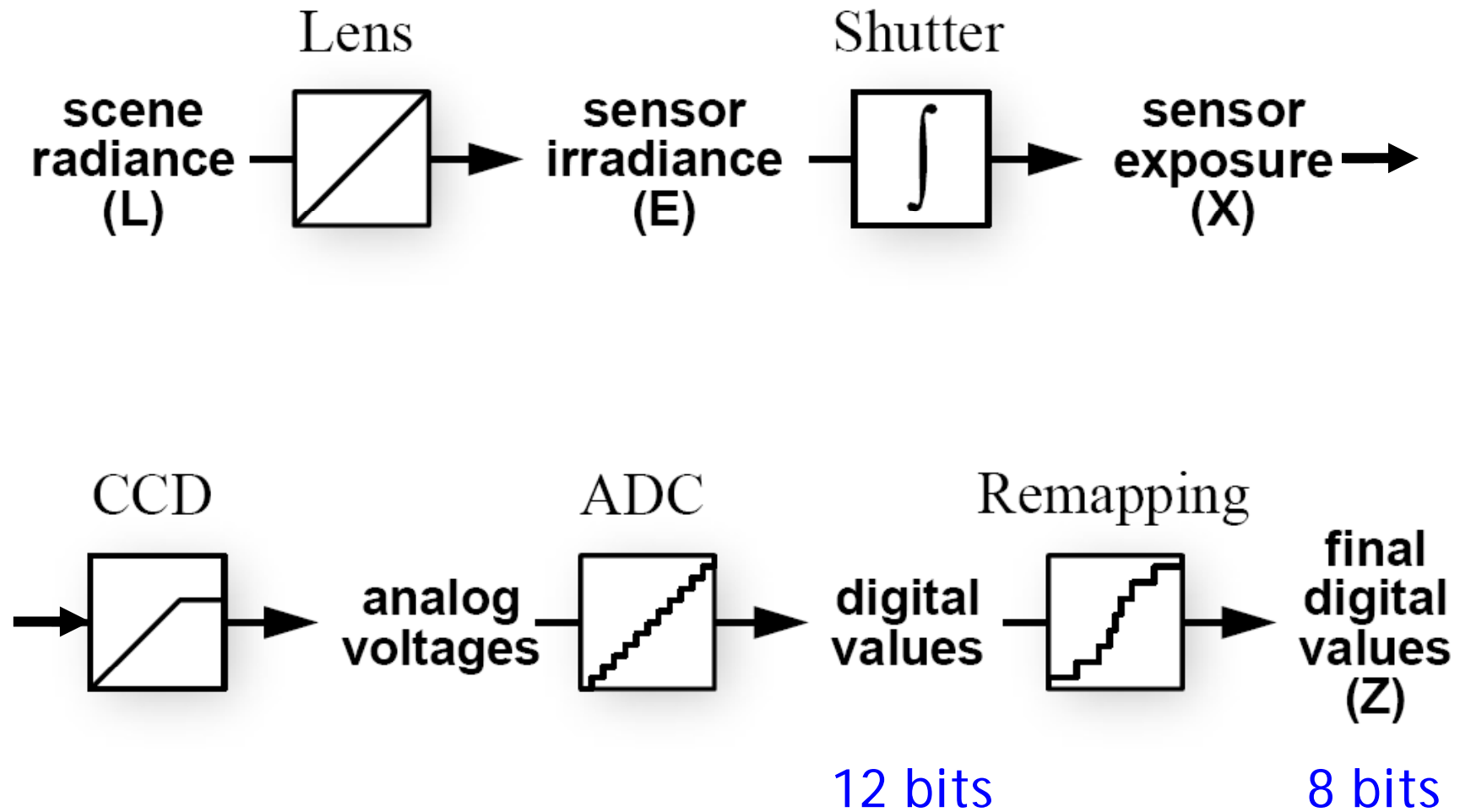
- Camera is an imperfect device for measuring the radiance distribution of a scene because it cannot capture the full spectral content and dynamic range.
- Limitations in sensor design prevent cameras from capturing all information passed by lens.

# Camera pipeline



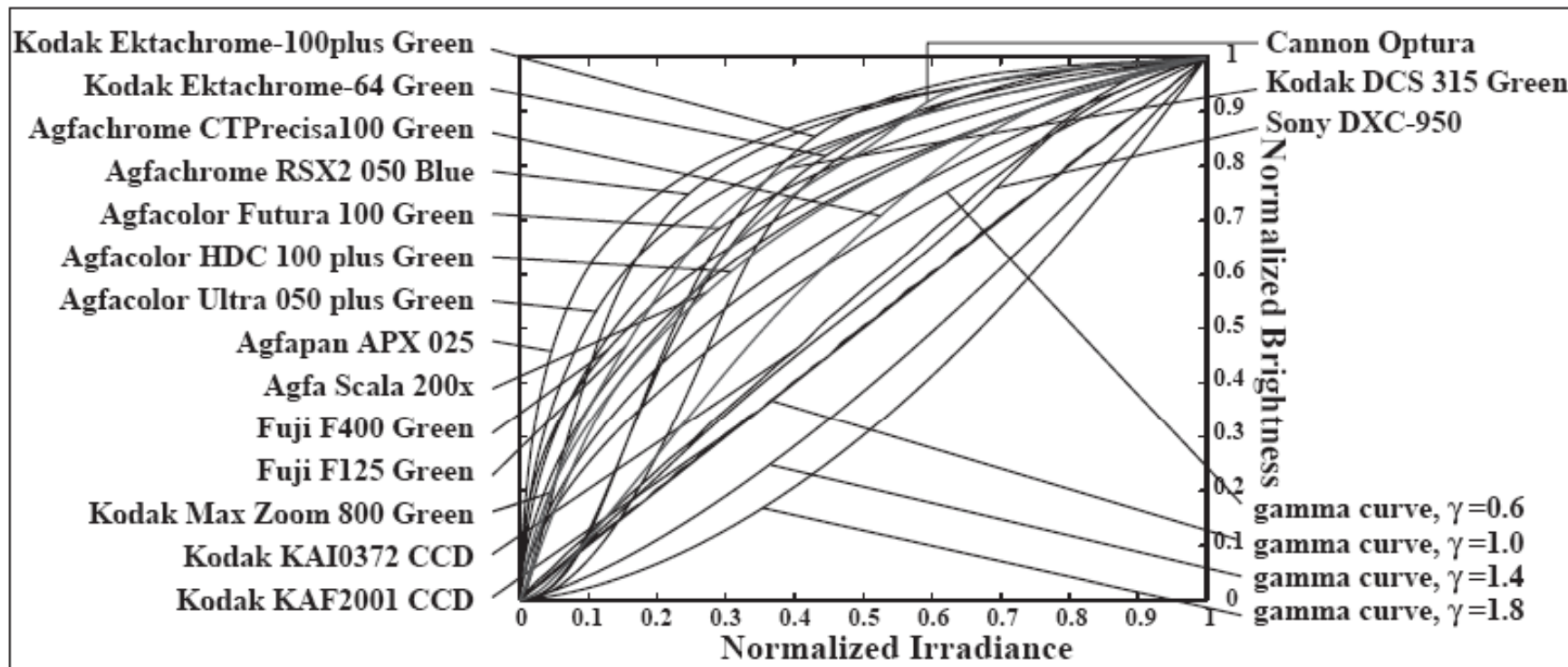
# Camera pipeline

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# Real-world response functions

In general, the response function is not provided by camera makers who consider it part of their proprietary product differentiation. In addition, they are beyond the standard gamma curves.



# The world is high dynamic range

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1



1,500



25,000

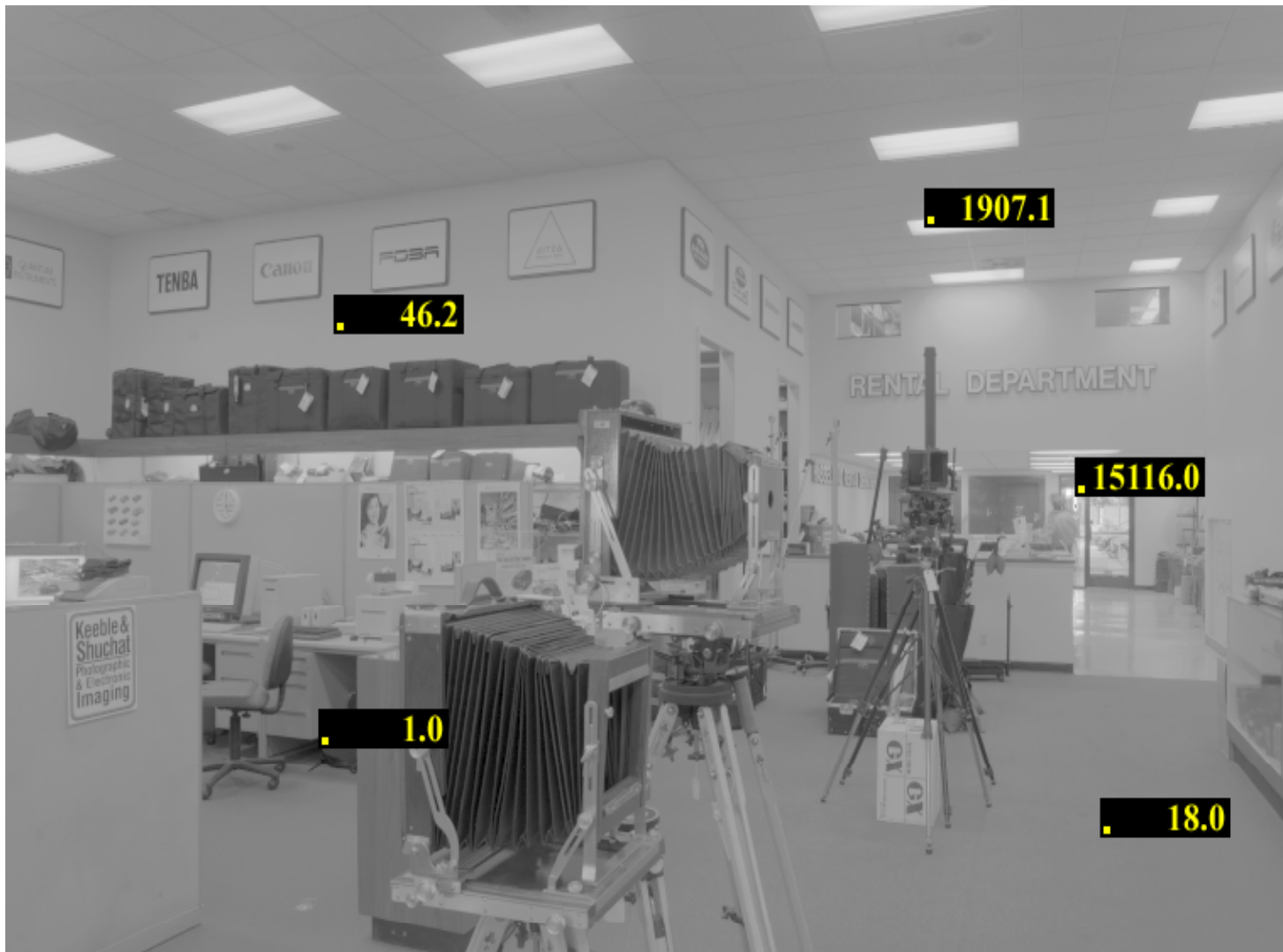


400,000



2,000,000,000

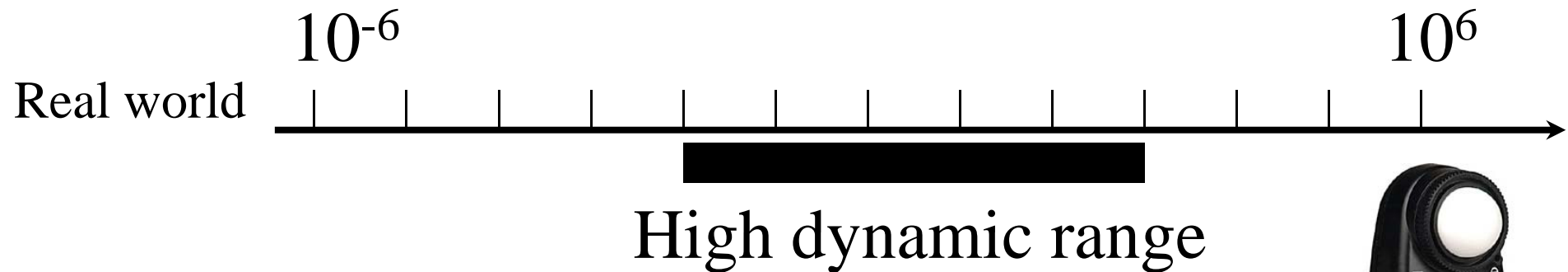
# The world is high dynamic range



# Real world dynamic range

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- Eye can adapt from  $\sim 10^{-6}$  to  $10^6$  cd/m<sup>2</sup>
- Often 1 : 100,000 in a scene
- Typical 1:50, max 1:500 for pictures

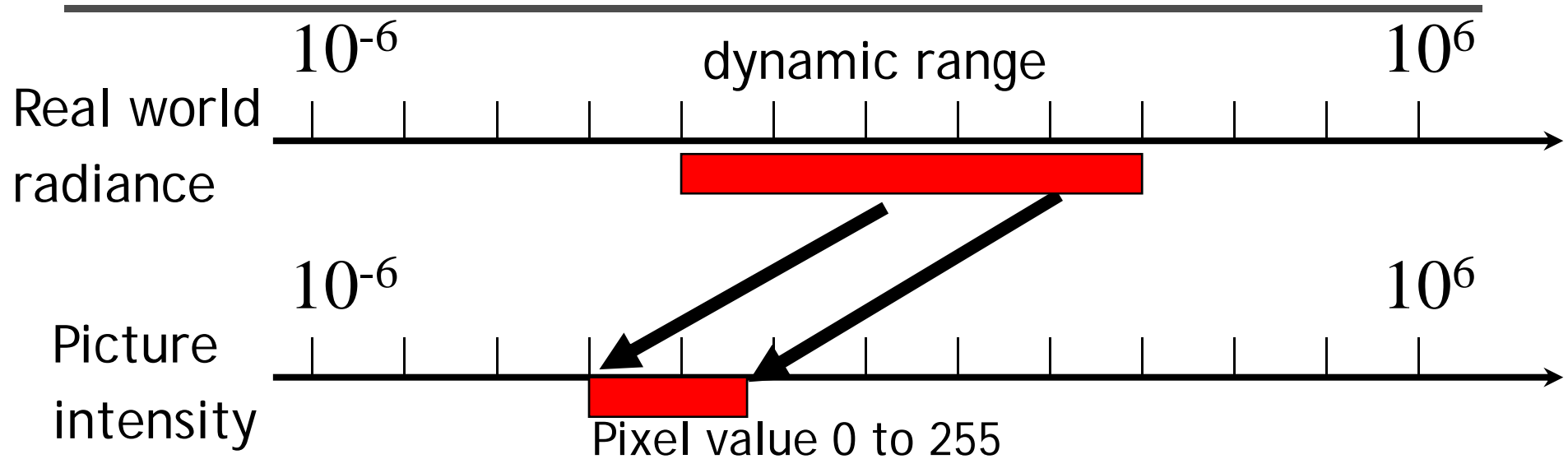


spotmeter

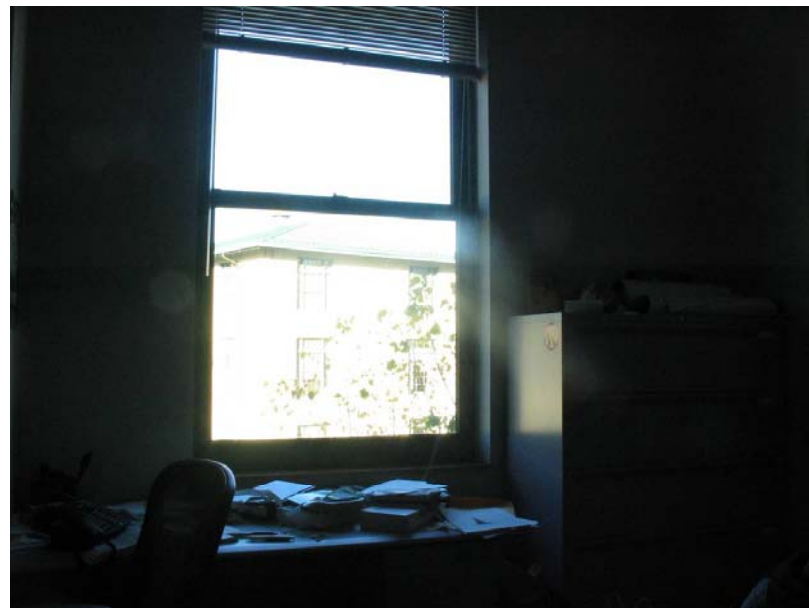
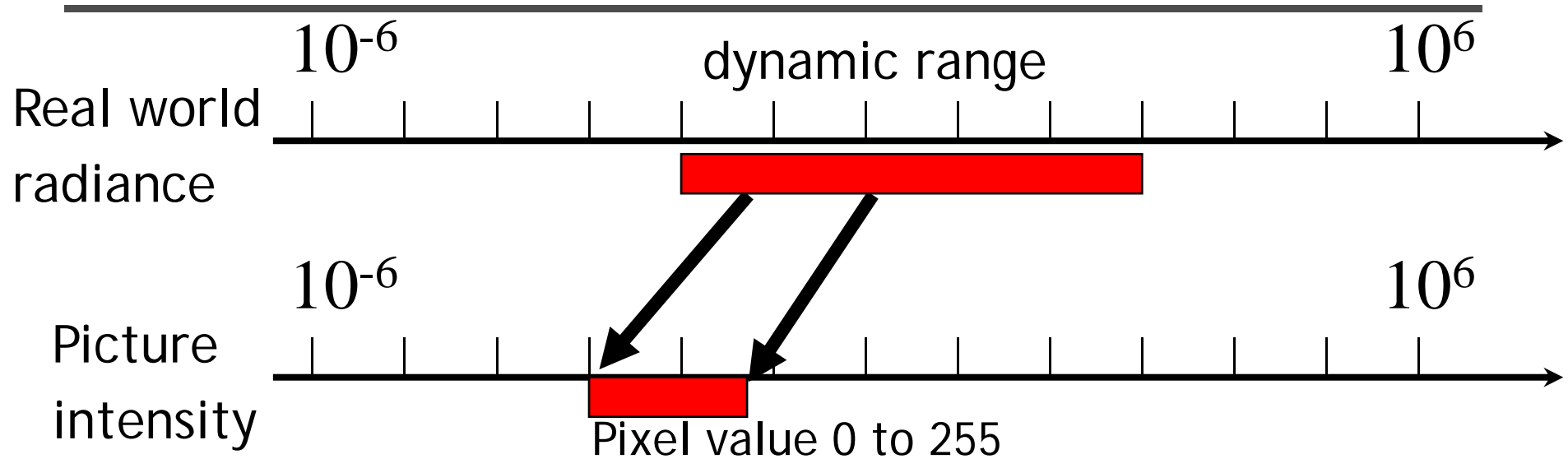




# Short exposure



# Long exposure



# Camera is not a photometer

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- Limited dynamic range
  - ⇒ Perhaps use multiple exposures?
- Unknown, nonlinear response
  - ⇒ Not possible to convert pixel values to radiance
- Solution:
  - Recover response curve from multiple exposures, then reconstruct the *radiance map*

# Varying exposure

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- Ways to change exposure
  - Shutter speed
  - Aperture
  - Neutral density filters



# Shutter speed

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- Note: shutter times usually obey a power series - each "stop" is a factor of 2
- $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{15}$ ,  $\frac{1}{30}$ ,  $\frac{1}{60}$ ,  $\frac{1}{125}$ ,  $\frac{1}{250}$ ,  $\frac{1}{500}$ ,  $\frac{1}{1000}$  sec

Usually really is:

$\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$ ,  $\frac{1}{128}$ ,  $\frac{1}{256}$ ,  $\frac{1}{512}$ ,  $\frac{1}{1024}$  sec

# Varying shutter speeds



# HDRI capturing from multiple exposures

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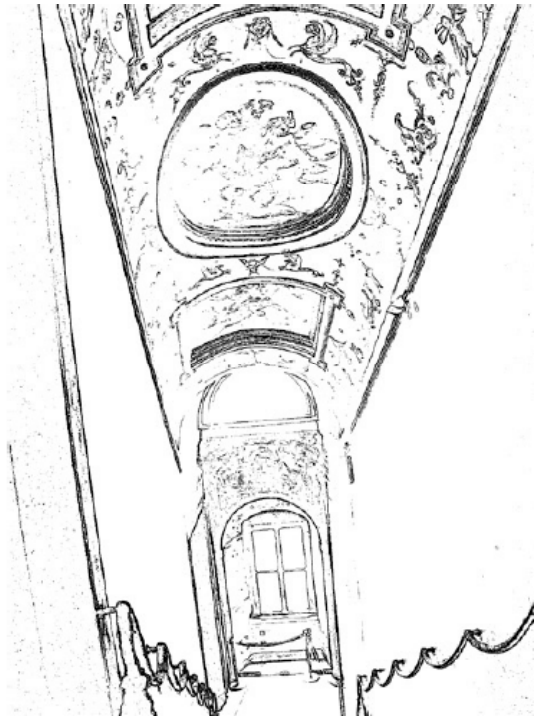
- Capture images with multiple exposures
- Image alignment (even if you use tripod, it is suggested to run alignment)
- Response curve recovery
- Ghost/flare removal

# Image alignment

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- We will introduce a fast and easy-to-implement method for this task, called Median Threshold Bitmap (MTB) alignment technique.
- Consider only integral translations. It is enough empirically.
- The inputs are N grayscale images. (You can either use the green channel or convert into grayscale by  $Y=(54R+183G+19B)/256$ )
- MTB is a binary image formed by thresholding the input image using the median of intensities.





# Why is MTB better than gradient?

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- Edge-detection filters are dependent on image exposures
- Taking the difference of two edge bitmaps would not give a good indication of where the edges are misaligned.

# Search for the optimal offset

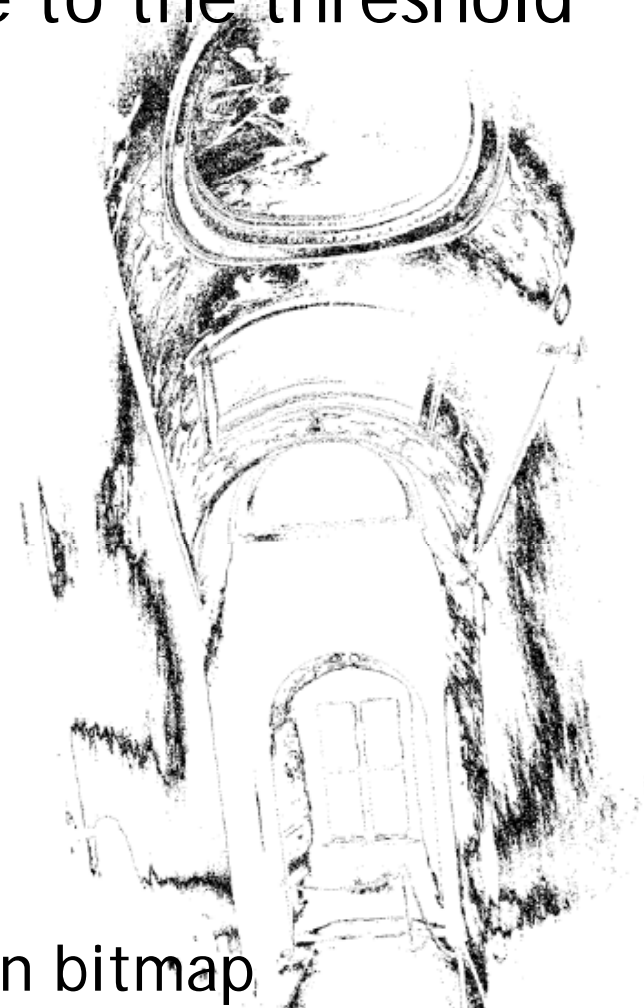
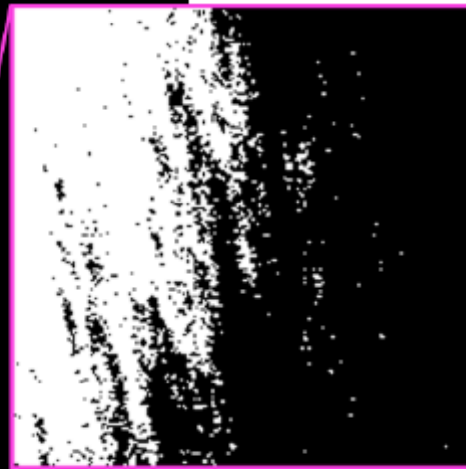
- Try all possible offsets.
- Gradient descent
- Multiscale technique
- $\log(\text{max\_offset})$  levels
- Try 9 possibilities for the top level
- Scale by 2 when passing down; try its 9 neighbors



# Threshold noise



ignore pixels that are close to the threshold



exclusion bitmap

# Efficiency considerations

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- XOR for taking difference
- AND with exclusion maps
- Bit counting by table lookup

# Results

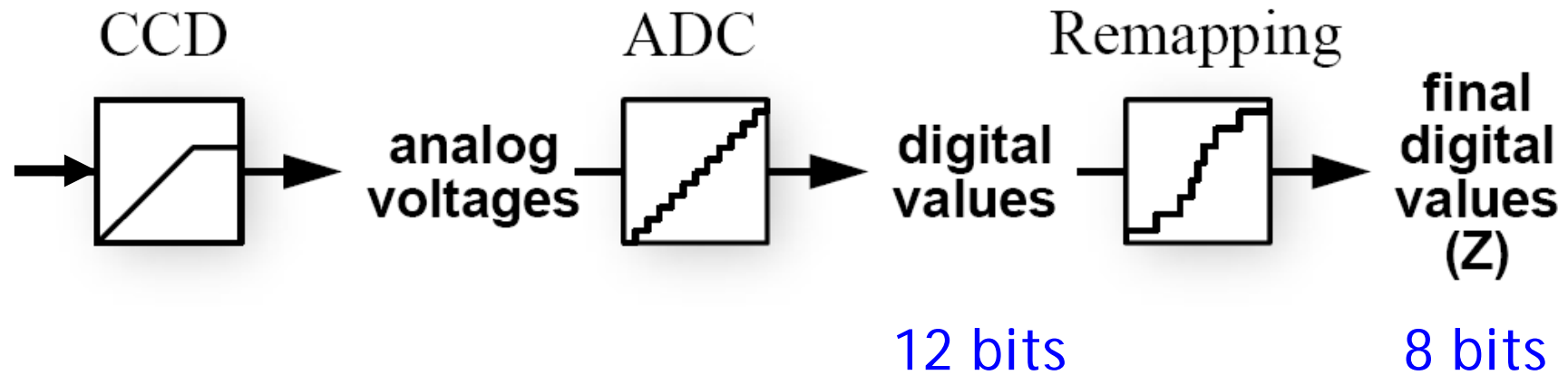
Success rate = 84%. 10% failure due to rotation. 3% for excessive motion and 3% for too much high-frequency content.



# Recovering response curve



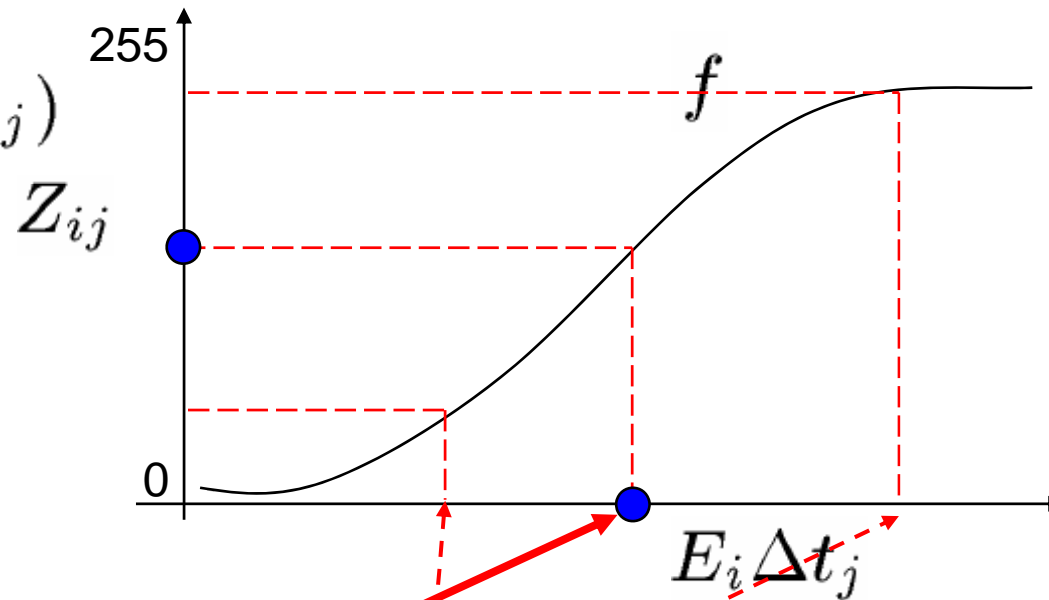
$$Z_{ij} = f(E_i \Delta t_j)$$



# Recovering response curve

- We want to obtain the inverse of the response curve

$$Z_{ij} = f(E_i \Delta t_j)$$





# Recovering response curve

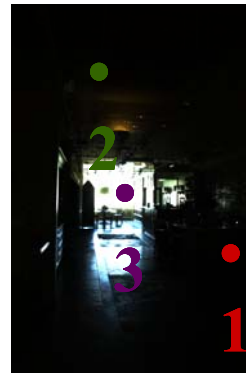
## Image series



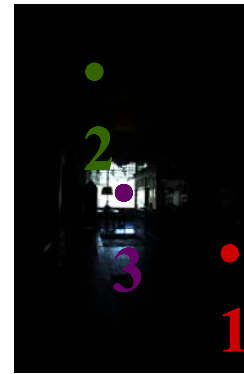
$\Delta t =$   
2 sec



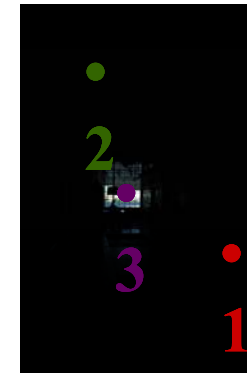
$\Delta t =$   
1 sec



$\Delta t =$   
1/2 sec

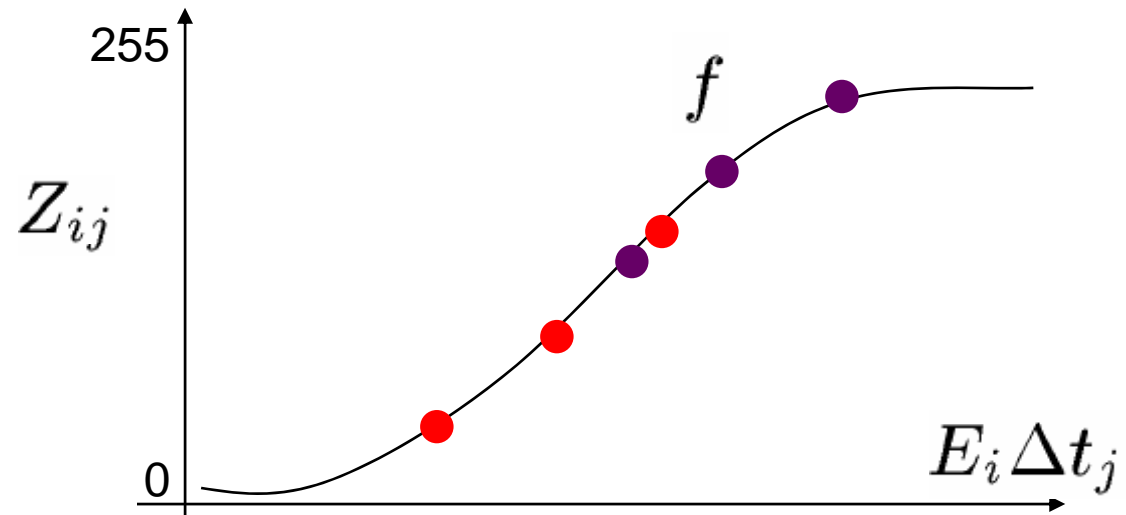


$\Delta t =$   
1/4 sec



$\Delta t =$   
1/8 sec

$$Z_{ij} = f(E_i \Delta t_j)$$



# Recovering response curve

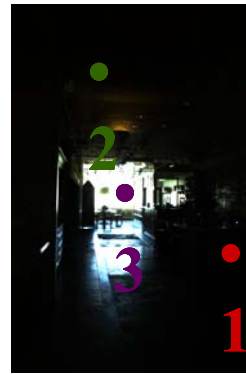
## Image series



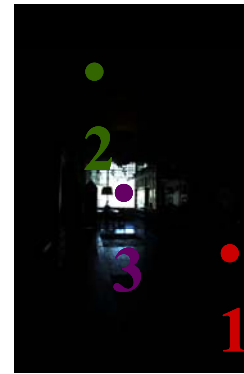
$\Delta t =$   
2 sec



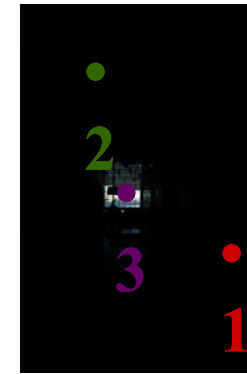
$\Delta t =$   
1 sec



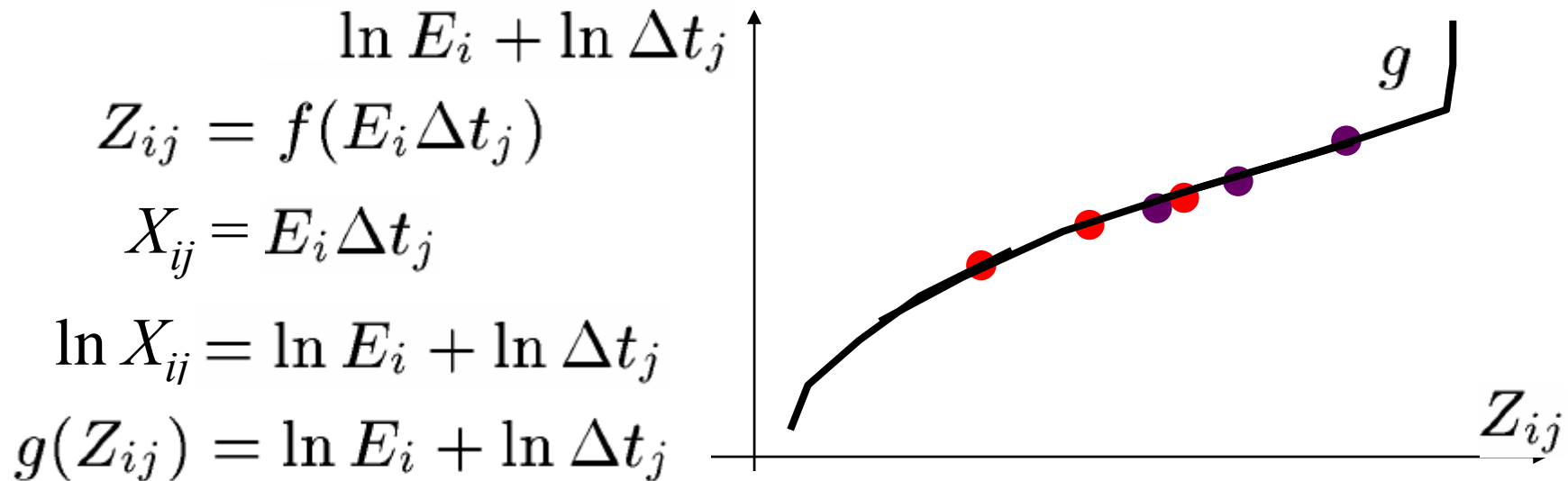
$\Delta t =$   
1/2 sec



$\Delta t =$   
1/4 sec

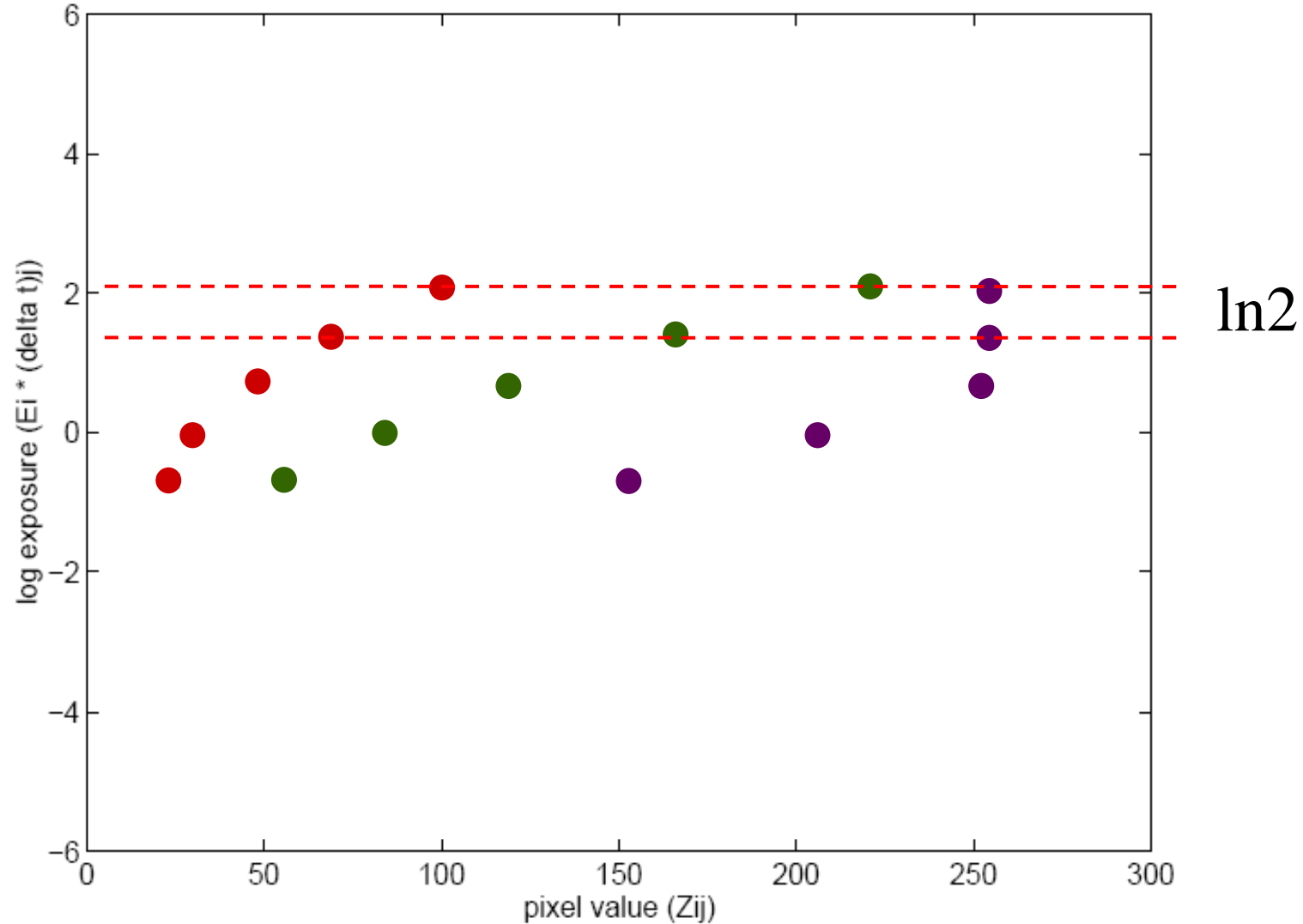


$\Delta t =$   
1/8 sec



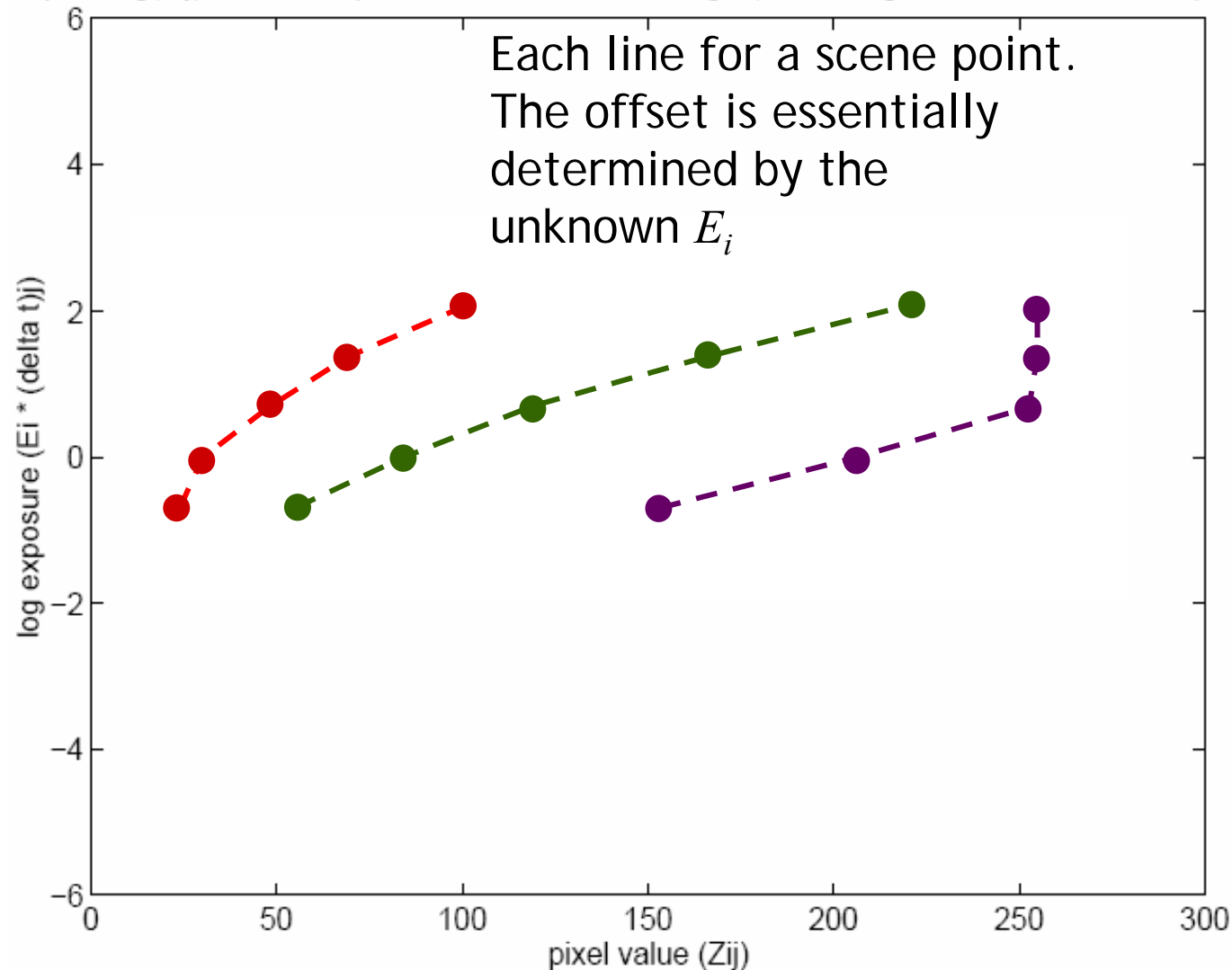
# Idea behind the math

plot of  $g(Z_{ij})$  from three pixels observed in five images, assuming unit radiance at each pixel

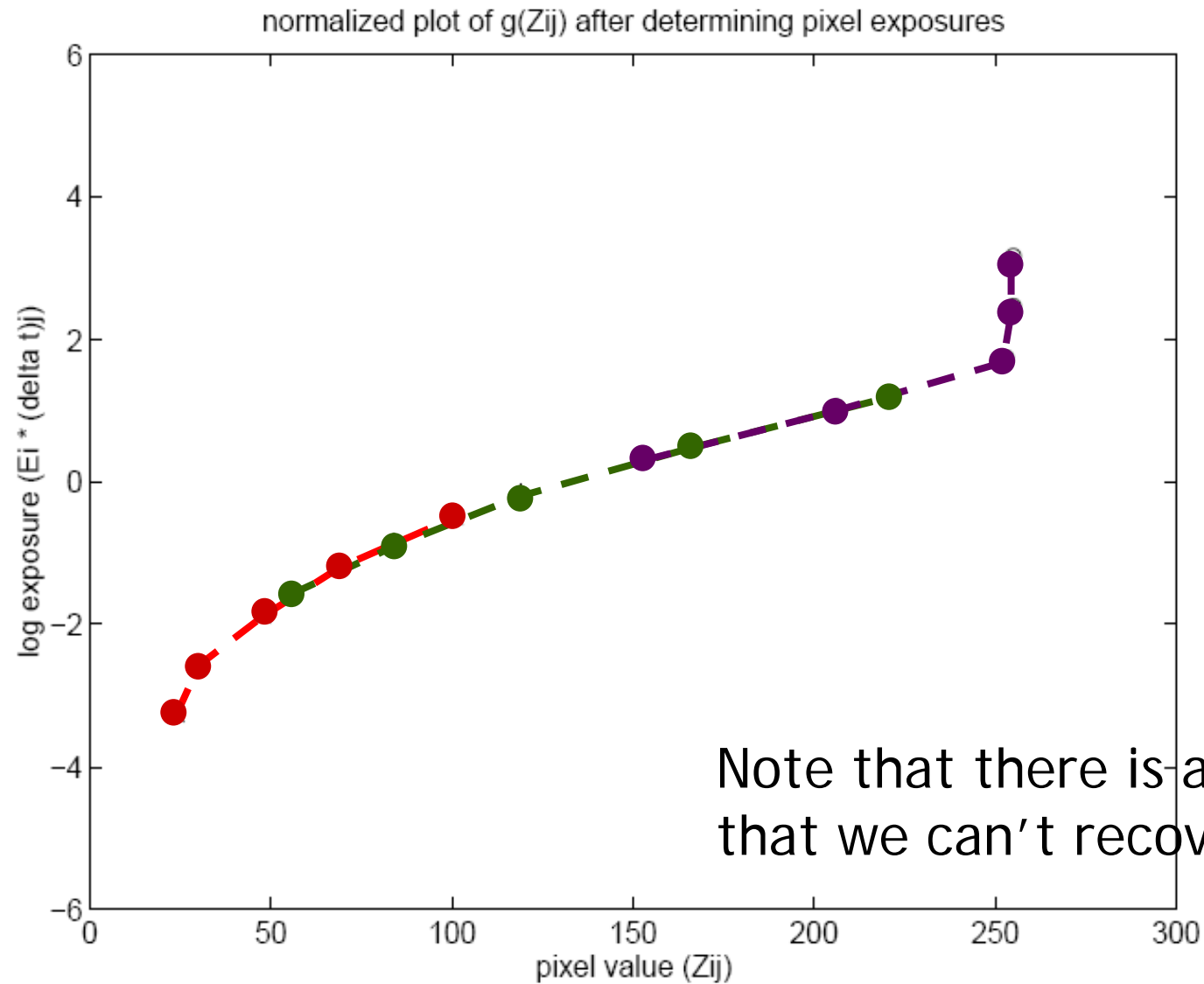


# Idea behind the math

plot of  $g(Z_{ij})$  from three pixels observed in five images, assuming unit radiance at each pixel



# Idea behind the math



# Basic idea

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- Design an objective function
- Optimize it

# Math for recovering response curve

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$$Z_{ij} = f(E_i \Delta t_j)$$

$f$  is monotonic, it is invertible

$$\ln f^{-1}(Z_{ij}) = \ln E_i + \ln \Delta t_j$$

let us define function  $g = \ln f^{-1}$

$$g(Z_{ij}) = \ln E_i + \ln \Delta t_j$$

minimize the following

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]^2 + \lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} g''(z)^2$$

$$g''(z) = g(z-1) - 2g(z) + g(z+1)$$

# Recovering response curve

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- The solution can be only up to a scale, add a constraint

$$g(Z_{mid}) = 0, \text{ where } Z_{mid} = \frac{1}{2}(Z_{min} + Z_{max})$$

- Add a hat weighting function

$$w(z) = \begin{cases} z - Z_{min} & \text{for } z \leq \frac{1}{2}(Z_{min} + Z_{max}) \\ Z_{max} - z & \text{for } z > \frac{1}{2}(Z_{min} + Z_{max}) \end{cases}$$

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$



# Recovering response curve

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- We want  $N(P - 1) > (Z_{max} - Z_{min})$   
If  $P=11$ ,  $N \sim 25$  (typically 50 is used)
- We prefer that selected pixels are well distributed and sampled from constant regions. They picked points by hand.
- It is an overdetermined system of linear equations and can be solved using SVD

# How to optimize?

---

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$

# 1. Set partial derivatives to zero

---

$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$

# How to optimize?

---

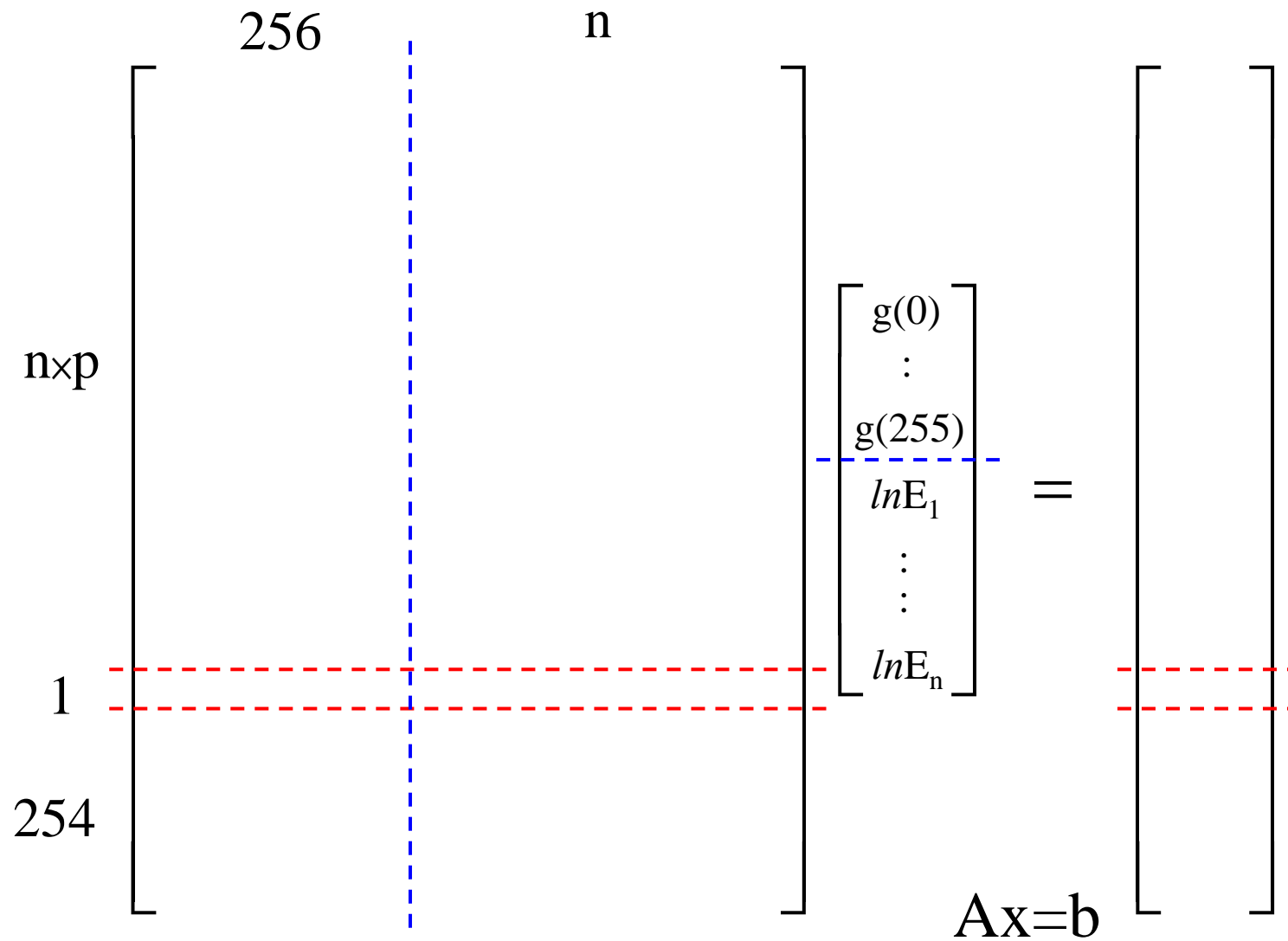
$$\mathcal{O} = \sum_{i=1}^N \sum_{j=1}^P \{w(Z_{ij}) [g(Z_{ij}) - \ln E_i - \ln \Delta t_j]\}^2 +$$

$$\lambda \sum_{z=Z_{min}+1}^{Z_{max}-1} [w(z)g''(z)]^2$$

1. Set partial derivatives to zero
- 2.

$$\min \sum_{i=1}^N (\mathbf{a}_i \mathbf{x} - \mathbf{b}_i)^2 \rightarrow \text{least - square solution of } \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_N \end{bmatrix}$$

# Sparse linear system



# Questions

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- Will  $g(127)=0$  always be satisfied? Why or why not?
- How to find the least-square solution for an over-determined system?

# Least-square solution for a linear system

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$$\mathbf{Ax} = \mathbf{b}$$

$m \times n$     $n$     $m$   
 $m > n$

They are often mutually incompatible. We instead find  $\mathbf{x}$  to minimize the norm  $\|\mathbf{Ax} - \mathbf{b}\|$  of the residual vector  $\mathbf{Ax} - \mathbf{b}$ . If there are multiple solutions, we prefer the one with the minimal length  $\|\mathbf{x}\|$ .

# Least-square solution for a linear system

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If we perform SVD on  $\mathbf{A}$  and rewrite it as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$$

then  $\hat{\mathbf{x}} = \mathbf{V}\mathbf{\Sigma}^+\mathbf{U}^T\mathbf{b}$  is the least-square solution.  
pseudo inverse

$$\mathbf{\Sigma}^+ = \begin{bmatrix} 1/\sigma_1 & & & & & & 0 & \dots & 0 \\ & \ddots & & & & & \vdots & & \vdots \\ & & 1/\sigma_r & & & & \vdots & & \vdots \\ & & & 0 & & & & & \\ & & & & \ddots & & & & \\ & & & & & 0 & 0 & \dots & 0 \end{bmatrix}$$



# Proof

find  $x$  使  $\|Ax - b\|$  最小

$$\|Ax - b\| = \|U \Sigma V^T x - b\|$$

$$= \|U(\Sigma V^T x - U^T b)\|$$

U 是 rotation  
不动长度

$$= \|\Sigma V^T x - U^T b\|$$

$$\text{令 } y = V^T x \quad c = U^T b$$

则 相当于 求  $y$  使  $\|\Sigma y - c\|$  最小

$$\begin{pmatrix} \sigma_1 & & & & & & 0 \\ & \ddots & & & & & \\ & & \sigma_r & & & & 0 \\ & & & 0 & & & \\ & & & & \ddots & & \\ & & & & & \sigma_n & \\ & & & & & & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} - \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$$

# Proof

$$\Rightarrow y_i = \frac{c_i}{\sigma_i} \quad i=1 \dots r \quad y_i = 0 \quad i=r+1 \dots n$$

$$\Rightarrow \tilde{y} = \begin{pmatrix} 1/\sigma_1 & & & 0 \\ & \dots & & \\ & & 1/\sigma_r & \\ 0 & & 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_r \\ \vdots \\ c_n \end{pmatrix} = \Sigma^+ c$$

$$\Rightarrow \tilde{y} = V^T \tilde{x} = \Sigma^+ c = \Sigma^+ U^T b$$

$$\Rightarrow \tilde{x} = V \Sigma^+ U^T b$$

# Libraries for SVD

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- Matlab
- GSL
- Boost
- LAPACK
- ATLAS

# Matlab code

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```
%  
% gsolve.m - Solve for imaging system response function  
%  
% Given a set of pixel values observed for several pixels in several  
% images with different exposure times, this function returns the  
% imaging system's response function g as well as the log film irradiance  
% values for the observed pixels.  
%  
% Assumes:  
%  
%   Zmin = 0  
%   Zmax = 255  
%  
% Arguments:  
%  
%   Z(i,j) is the pixel values of pixel location number i in image j  
%   B(j)   is the log delta t, or log shutter speed, for image j  
%   l      is lambda, the constant that determines the amount of smoothness  
%   w(z)   is the weighting function value for pixel value z  
%  
% Returns:  
%  
%   g(z)   is the log exposure corresponding to pixel value z  
%   lE(i)  is the log film irradiance at pixel location i  
%
```

# Matlab code

---

```
function [g,lE]=gsolve(Z,B,l,w)

n = 256;
A = zeros(size(Z,1)*size(Z,2)+n+1,n+size(Z,1));
b = zeros(size(A,1),1);

k = 1;           %% Include the data-fitting equations
for i=1:size(Z,1)
    for j=1:size(Z,2)
        wij = w(Z(i,j)+1);
        A(k,Z(i,j)+1) = wij; A(k,n+i) = -wij; b(k,1) = wij * B( j);
        k=k+1;
    end
end

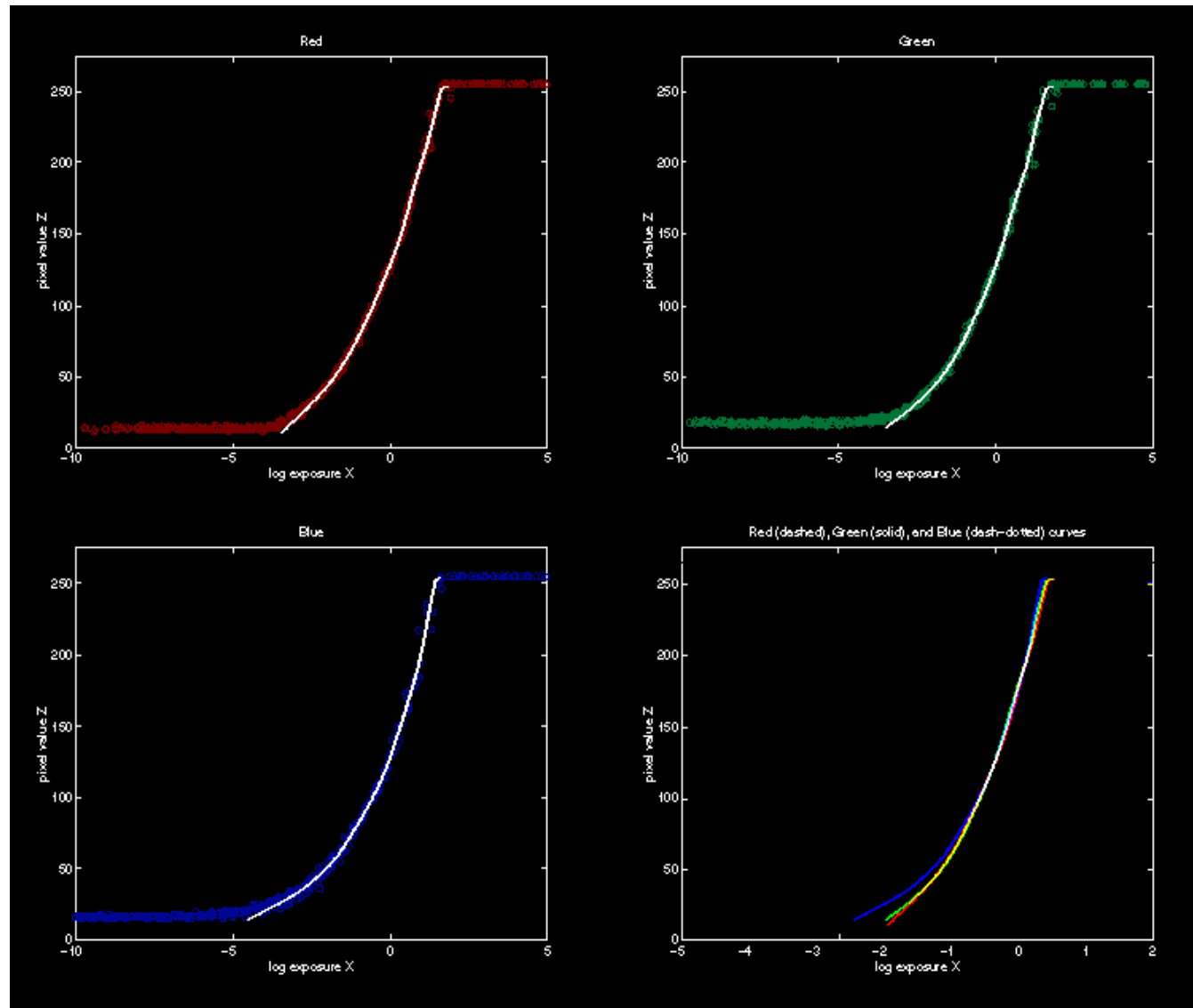
A(k,129) = 1;    %% Fix the curve by setting its middle value to 0
k=k+1;

for i=1:n-2      %% Include the smoothness equations
    A(k,i)=1*w(i+1); A(k,i+1)=-2*1*w(i+1); A(k,i+2)=1*w(i+1);
    k=k+1;
end

x = A\b;        %% Solve the system using SVD

g = x(1:n);
lE = x(n+1:size(x,1));
```

# Recovered response function



# Constructing HDR radiance map

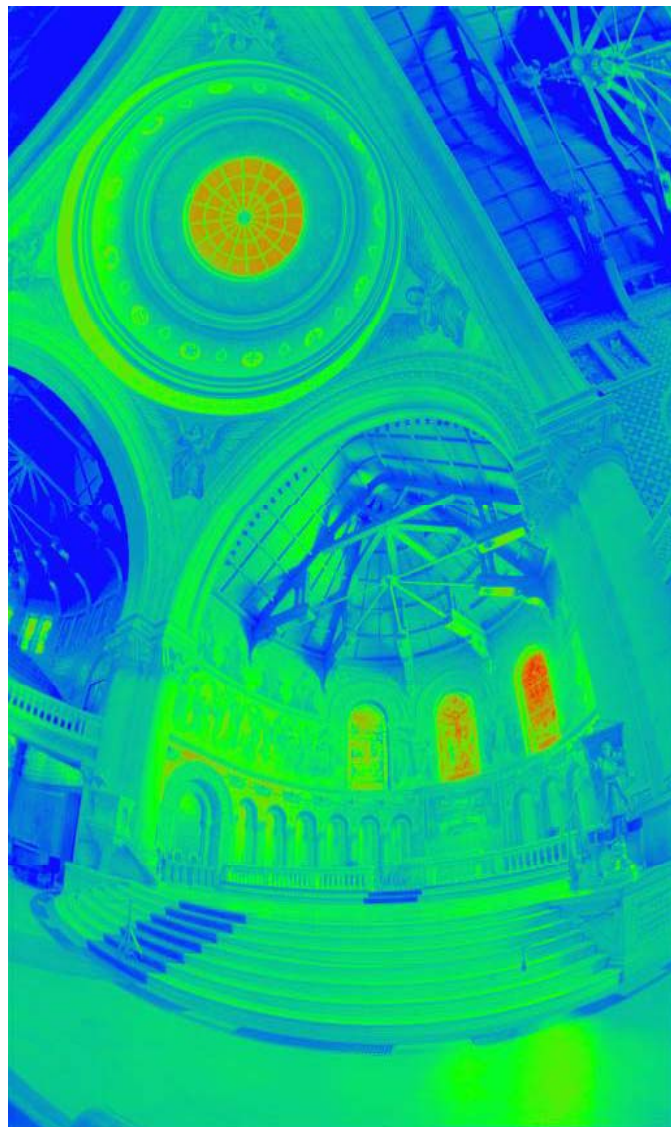
---

$$\ln E_i = g(Z_{ij}) - \ln \Delta t_j$$

combine pixels to reduce noise and obtain a more reliable estimation

$$\ln E_i = \frac{\sum_{j=1}^P w(Z_{ij})(g(Z_{ij}) - \ln \Delta t_j)}{\sum_{j=1}^P w(Z_{ij})}$$

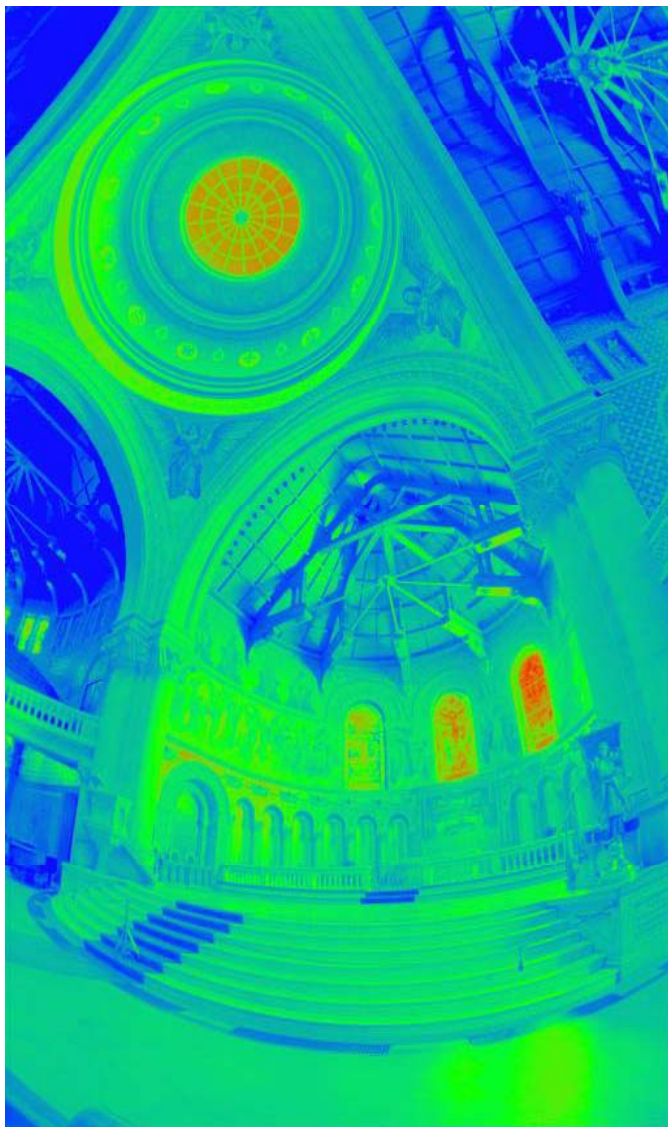
# Reconstructed radiance map





# What is this for?

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- Human perception
- Vision/graphics applications

# Automatic ghost removal

---



before



after

# Weighted variance



Moving objects and high-contrast edges render high variance.

# Region masking



Thresholding; dilation; identify regions;

# Best exposure in each region



# Lens flare removal

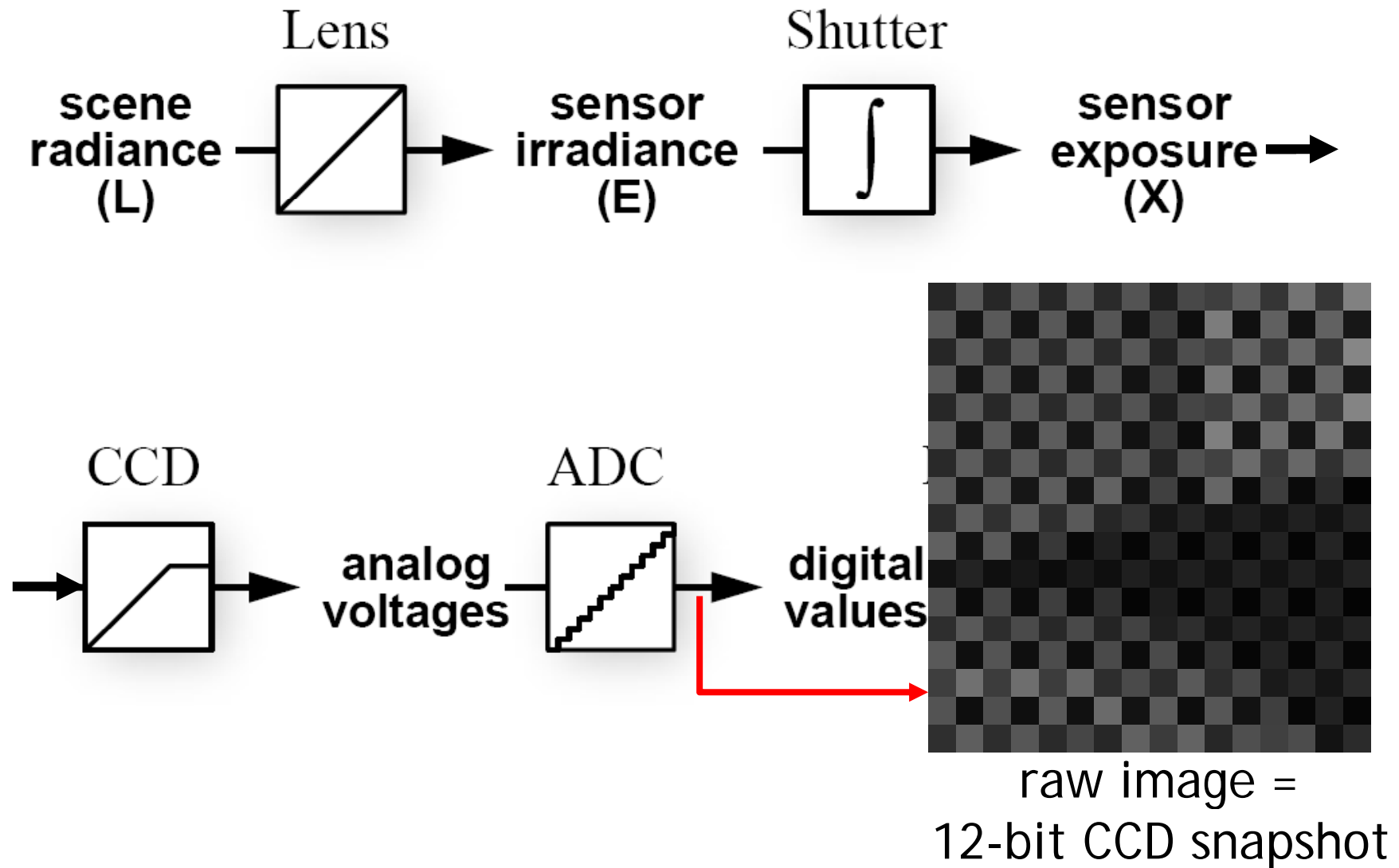


before



after

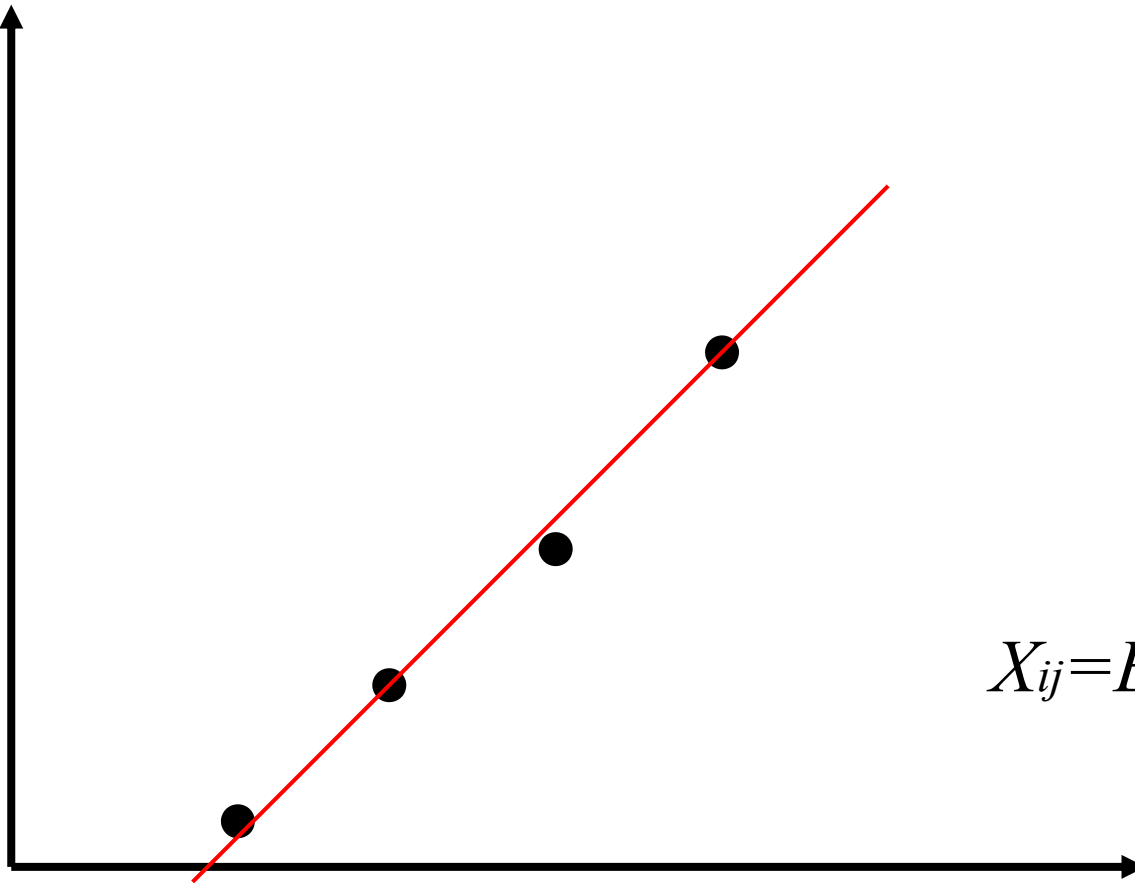
# Easier HDR reconstruction



# Easier HDR reconstruction

---

Exposure ( $X$ )



$$X_{ij} = E_i * \Delta t_j$$

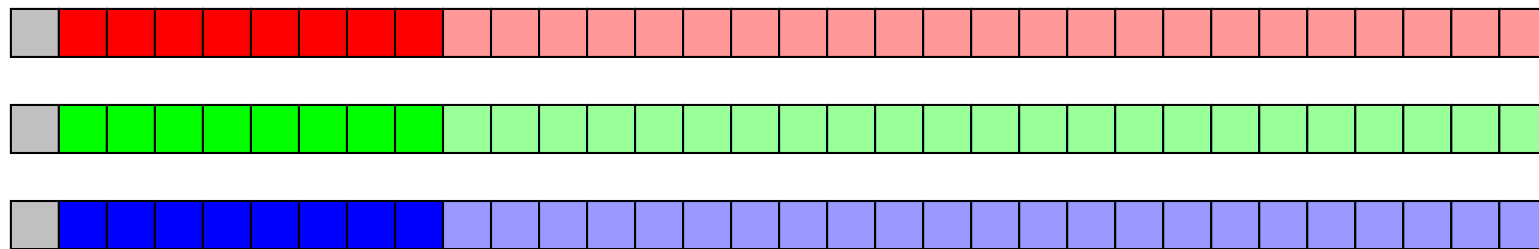
$\Delta t$



# Portable floatMap (.pfm)

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- 12 bytes per pixel, 4 for each channel



sign exponent

mantissa

Text header similar to Jeff Poskanzer's .ppm image format:

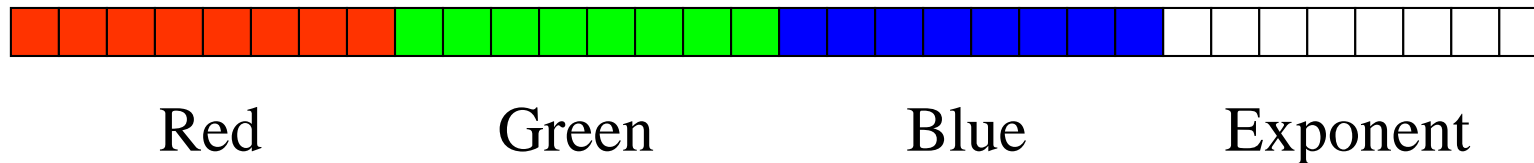
```
PF
768 512
1
<binary image data>
```

Floating Point TIFF similar

# Radiance format (.pic, .hdr, .rad)



32 bits/pixel



$$(145, 215, 87, 149) =$$
$$(145, 215, 87) * 2^{(149-128)} =$$

1190000 1760000 713000

$$(145, 215, 87, 103) =$$
$$(145, 215, 87) * 2^{(103-128)} =$$

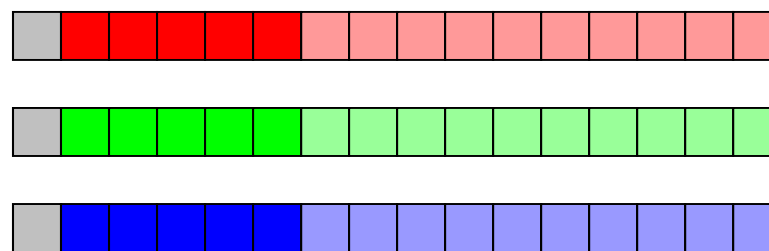
0.00000432 0.00000641 0.00000259

Ward, Greg. "Real Pixels," in Graphics Gems IV, edited by James Arvo, Academic Press, 1994

# ILM's OpenEXR (.exr)

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- 6 bytes per pixel, 2 for each channel, compressed



sign    exponent    mantissa

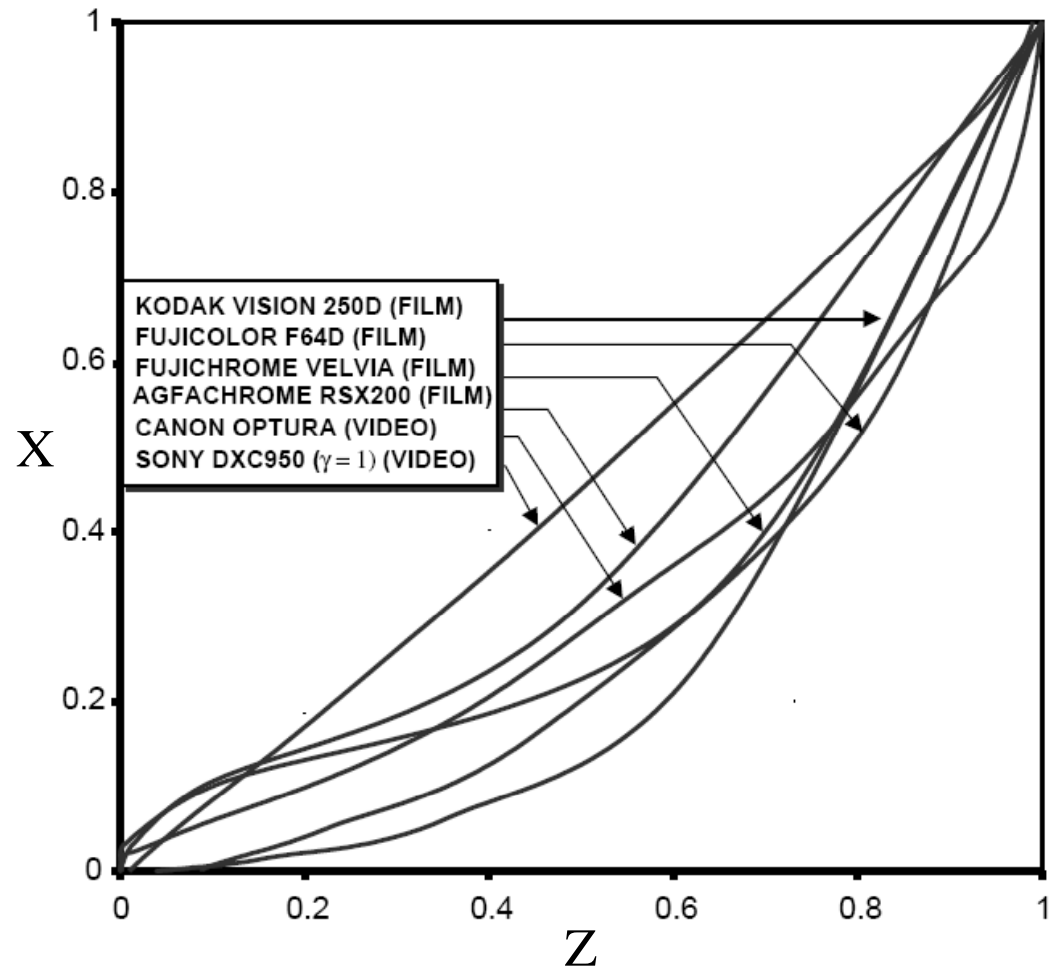
- Several lossless compression options, 2:1 typical
- Compatible with the "half" datatype in NVidia's Cg
- Supported natively on GeForce FX and Quadro FX
- Available at <http://www.openexr.net/>

# Radiometric self calibration

- Assume that any response function can be modeled as a high-order polynomial

$$X = g(Z) = \sum_{m=0}^M c_m Z^m$$

- No need to know exposure time in advance. Useful for cheap cameras



# Mitsunaga and Nayar

- To find the coefficients  $c_m$  to minimize the following

$$\mathcal{E} = \sum_{i=1}^N \sum_{j=1}^P \left[ \sum_{m=0}^M c_m Z_{ij}^m - R_{j,j+1} \sum_{m=0}^M c_m Z_{i,j+1}^m \right]^2$$

A guess for the ratio of

$$\frac{X_{ij}}{X_{i,j+1}} = \frac{E_i \Delta t_j}{E_i \Delta t_{j+1}} = \frac{\Delta t_j}{\Delta t_{j+1}}$$

# Mitsunaga and Nayar

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- Again, we can only solve up to a scale. Thus, add a constraint  $f(1)=1$ . It reduces to  $M-1$  variables.
- How to solve it?

# Mitsunaga and Nayar

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- We solve the above iteratively and update the exposure ratio accordingly

$$R_{j,j+1}^{(k)} = \frac{1}{N} \sum_{i=1}^N \frac{\sum_{m=0}^M c_m^{(k)} Z_{ij}^m}{\sum_{m=0}^M c_m^{(k)} Z_{i,j+1}^m}$$

- How to determine M? Solve up to M=10 and pick up the one with the minimal error. Notice that you prefer to have the same order for all channels. Use the combined error.

# Robertson et. al.

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$$Z_{ij} = f(E_i \Delta t_j)$$

$$g(Z_{ij}) = f^{-1}(Z_{ij}) = E_i \Delta t_j$$

Given  $Z_{ij}$  and  $\Delta t_j$ , the goal is to find both  $E_i$  and  $g(Z_{ij})$

Maximum likelihood

$$\Pr(E_i, g \mid Z_{ij}, \Delta t_j) \propto \exp\left(-\frac{1}{2} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2\right)$$

$$\hat{g}, \hat{E}_i = \arg \min_{g, E_i} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2$$



# Robertson et. al.

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$$\hat{g}, \hat{E}_i = \arg \min_{g, E_i} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2$$

repeat

    assuming  $g(Z_{ij})$  is known, optimize for  $E_i$

    assuming  $E_i$  is known, optimize for  $g(Z_{ij})$

until converge

# Robertson et. al.

---

$$\hat{g}, \hat{E}_i = \arg \min_{g, E_i} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2$$

repeat

assuming  $g(Z_{ij})$  is known, optimize for  $E_i$

assuming  $E_i$  is known, optimize for  $g(Z_{ij})$

until converge

# Robertson et. al.

---

$$\hat{g}, \hat{E}_i = \arg \min_{g, E_i} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2$$

repeat

assuming  $g(Z_{ij})$  is known, optimize for  $E_i$

assuming  $E_i$  is known, optimize for  $g(Z_{ij})$

until converge

$$E_i = \frac{\sum_j w(Z_{ij}) g(Z_{ij}) \Delta t_j}{\sum_j w(Z_{ij}) \Delta t_j^2}$$

# Robertson et. al.

---

$$\hat{g}, \hat{E}_i = \arg \min_{g, E_i} \sum_{ij} w(Z_{ij}) (g(Z_{ij}) - E_i \Delta t_j)^2$$

repeat

assuming  $g(Z_{ij})$  is known, optimize for  $E_i$

assuming  $E_i$  is known, optimize for  $g(Z_{ij})$

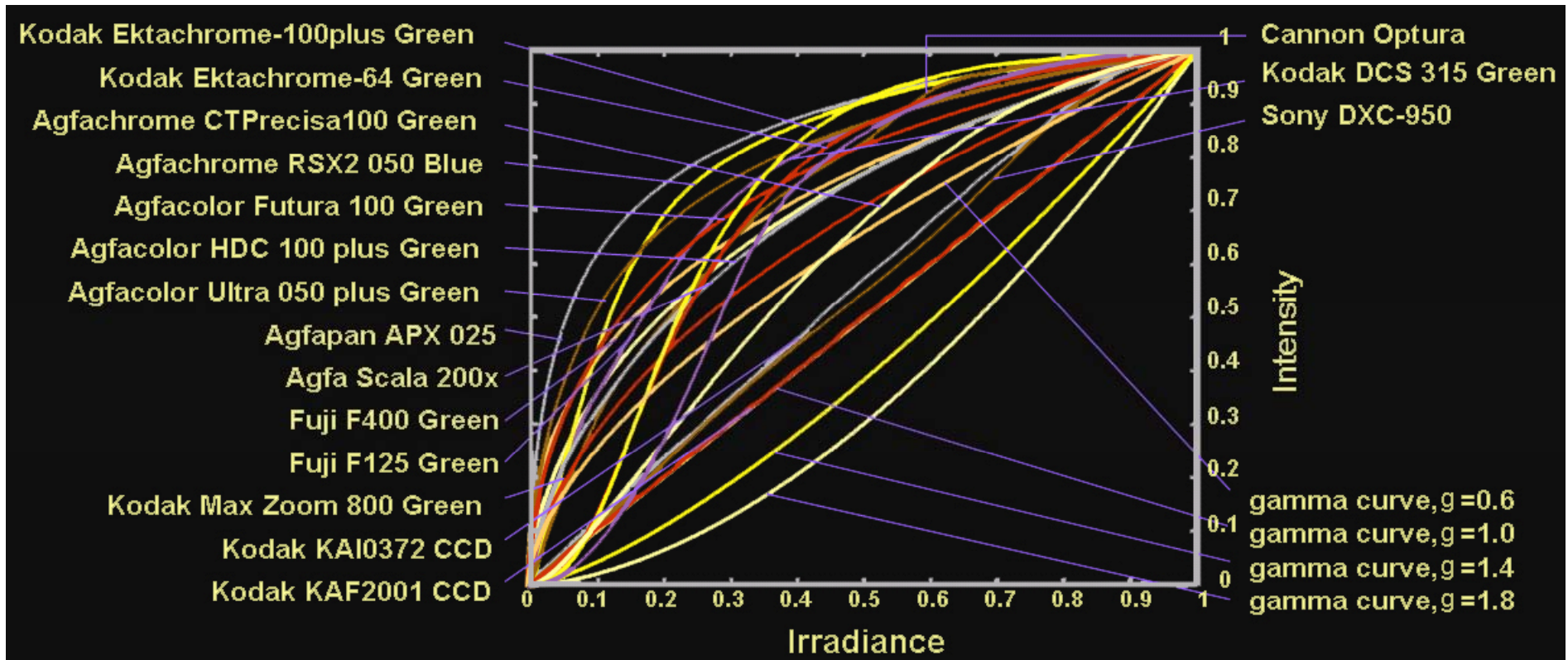
until converge

$$g(m) = \frac{1}{|E_m|} \sum_{ij \in E_m} E_i \Delta t_j$$

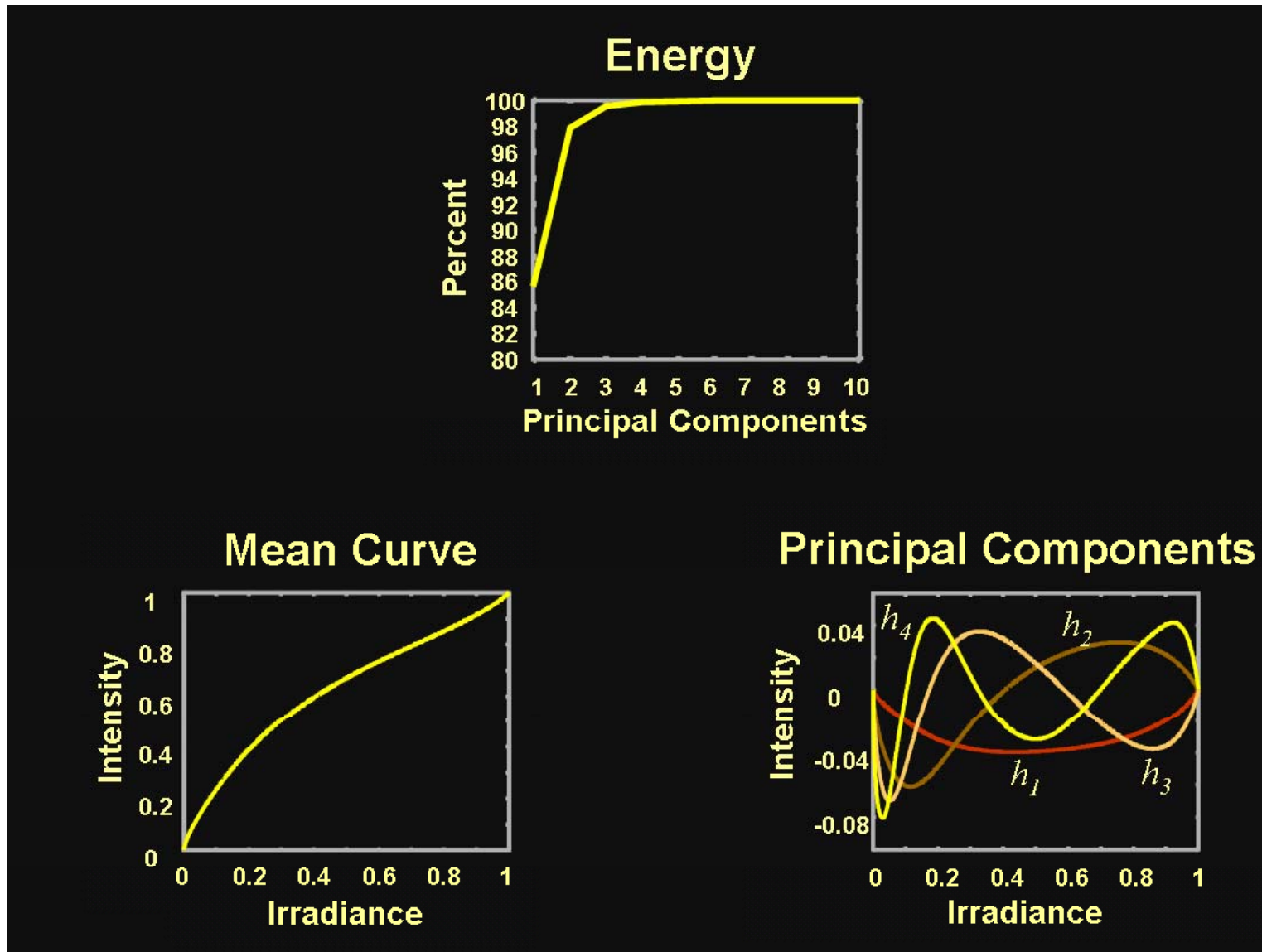
normalize so that

$$g(128) = 1$$

# Space of response curves



# Space of response curves



# Patch-Based HDR



# HDR Video

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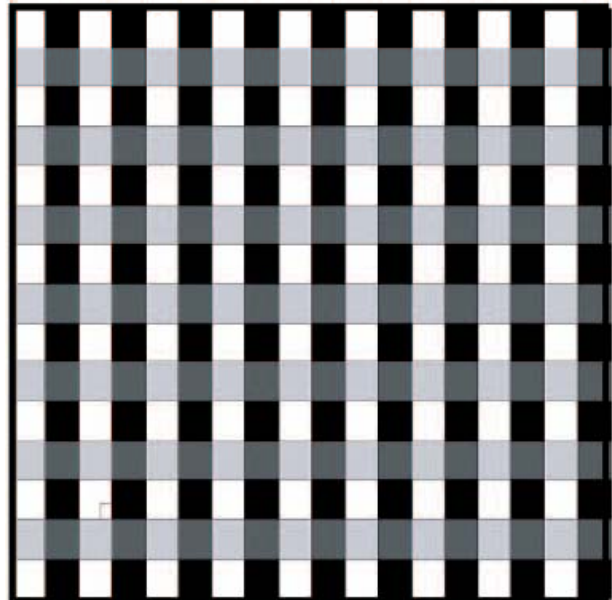
- High Dynamic Range Video  
Sing Bing Kang, Matthew Uyttendaele, Simon Winder, Richard Szeliski  
SIGGRAPH 2003

[video](#)



# Assorted pixel

R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B



R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B
R	G	R	G	R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B	G	B	G	B

# Assorted pixel



# Assorted pixel

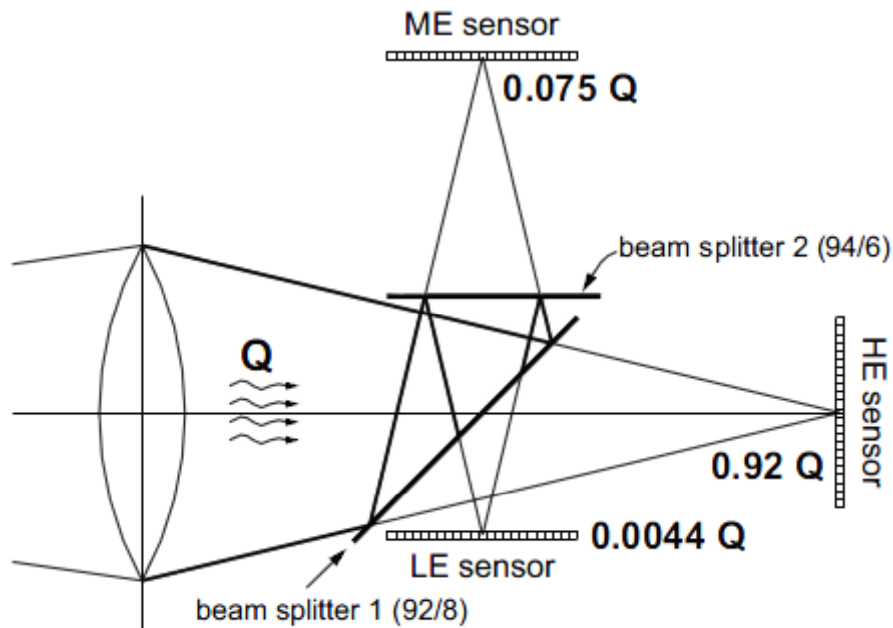
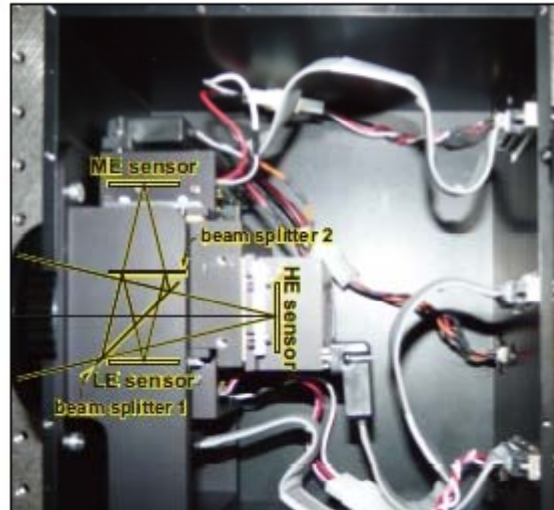
Normal Camera



Assorted Pixel Camera

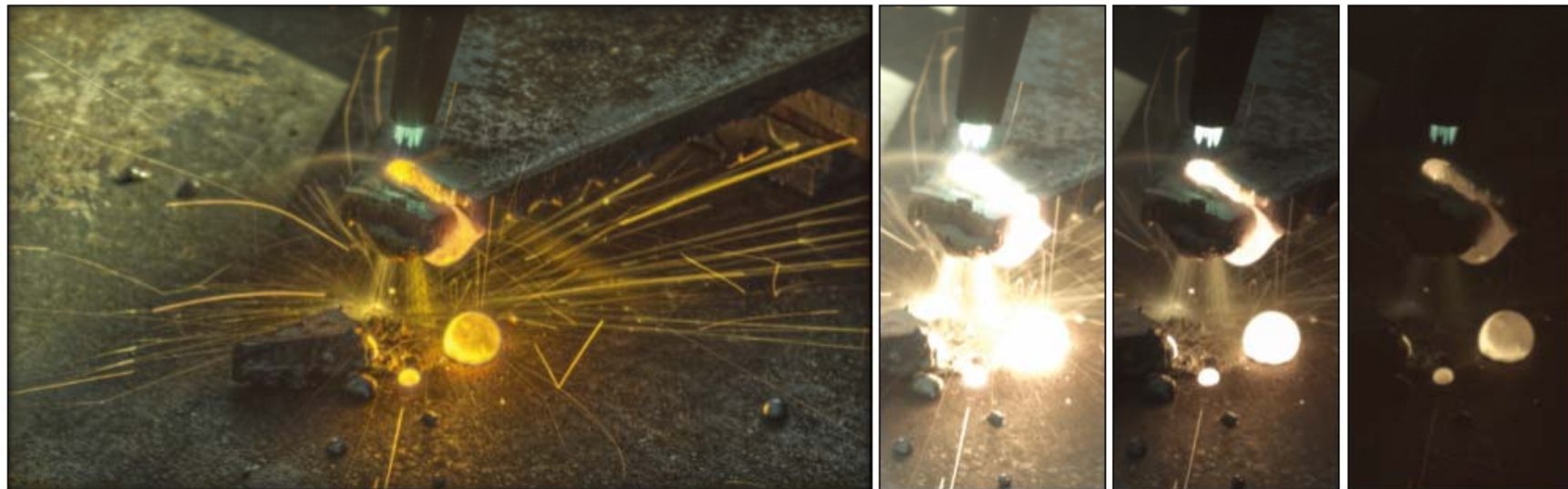
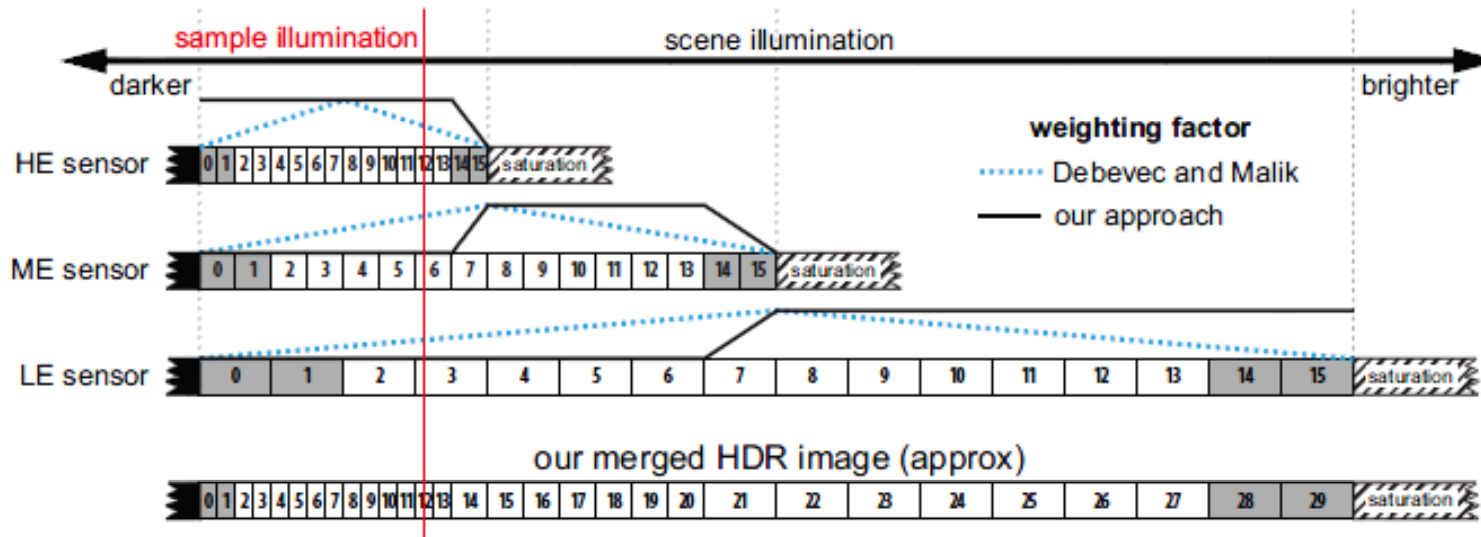


# A Versatile HDR Video System



video

# A Versatile HDR Video System



# HDR becomes common practice

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- Many cameras has bracket exposure modes
- iPhone 4 has HDR option, but it is more exposure blending rather than true HDR.

# Assignment #1 HDR image assemble

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- Work in teams of two
- Taking pictures
- Assemble HDR images and optionally the response curve.
- Develop your HDR using tone mapping

# Taking pictures

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- Use a tripod to take multiple photos with different shutter speeds. Try to fix anything else. Smaller images are probably good enough.
- There are two sets of test images available on the web.
- We have tripods and a Canon PowerShot G7 for you to borrow starting from the next week.
- Try not touching the camera during capturing.  
But, how?



# 1. Taking pictures

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- Use a laptop and a remote capturing program.
  - PSRemote
  - AHDRIA (obsolete since 2007)
- PSRemote
  - Manual (can use auto-bracket mode)
  - Not free
  - Supports both jpg and raw
  - Support most Canon's PowerShot cameras
- AHDRIA
  - Automatic
  - Free
  - Only supports jpg
  - Support less models

## 2. HDR assembling

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- Write a program to convert the captured images into a radiance map and optionally to output the response curve.
- We will provide image I/O library which supports many traditional image formats such as .jpg and .png, and float-point images such as .hdr and .exr.
- Paul Debevec's method. You will need SVD for this method.
- Recover from CCD snapshots. You will need `dcraw.c`.

## 3. Tone mapping

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- Apply some tone mapping operation to develop your photograph.
  - Reinhard's algorithm (HDRShop plugin)
  - Photomatix
  - LogView
  - Fast Bilateral (.exr Linux only)
  - PFStmo (Linux only)  
`pfsin a.hdr | pfs_fattal02 | pfsout o.hdr`

# Bells and Whistles

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- Other methods for HDR assembling algorithms
- Implement tone mapping algorithms
- Implement MTB alignment algorithm
- Others

# Submission

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- You have to turn in your complete source, the executable, a html report, pictures you have taken, HDR image, and an artifact (tone-mapped image).
- Report page contains:  
description of the project, what do you learn, algorithm, implementation details, results, bells and whistles...
- The class will have vote on artifacts.
- Submission mechanism will be announced later.

# Assignment #1 HDR image assemble

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- It is for warming up and considered easier; it is suggested that you implement at least one bonus (MTB/tone mapping/other HDR construction)
- You have a total of 10 days of delay without penalty for assignments; after that, -1 point per day applies in your final grade until reaching zero for each project.

# References



# References

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- Tomoo Mitsunaga, Shree Nayar, [Radiometric Self Calibration](#), CVPR 1999.
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- Srinivasa Narasimhan, Shree Nayar, [Enhancing Resolution Along Multiple Imaging Dimensions Using Assorted Pixels](#), PAMI 2005.
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