

Features

Digital Visual Effects

Yung-Yu Chuang

*with slides by Trevor Darrell, Cordelia Schmid, David Lowe, Darya Frolova, Denis Simakov,
Robert Collins and Jiwon Kim*

Outline

- Features
- Harris corner detector
- SIFT
- Extensions
- Applications

Features

Features

- Also known as interesting points, salient points or keypoints. Points that you can easily point out their correspondences in multiple images using only local information.



Desired properties for features



- Distinctive: a single feature can be correctly matched with high probability.
- Invariant: invariant to scale, rotation, affine, illumination and noise for robust matching across a substantial range of affine distortion, viewpoint change and so on. That is, it is repeatable.

Applications



- Object or scene recognition
- Structure from motion
- Stereo
- Motion tracking
- ...

Components



- **Feature detection** locates where they are
- **Feature description** describes what they are
- **Feature matching** decides whether two are the same one

Harris corner detector

Moravec corner detector (1980)

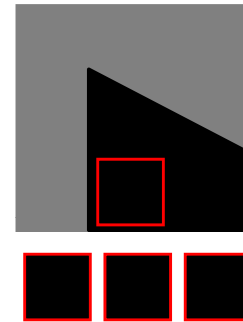
DigiVFX

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give a *large change* in intensity



Moravec corner detector

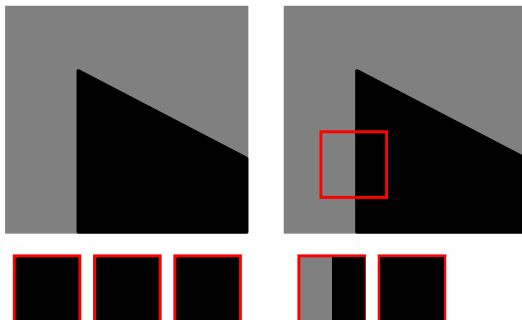
DigiVFX



flat

Moravec corner detector

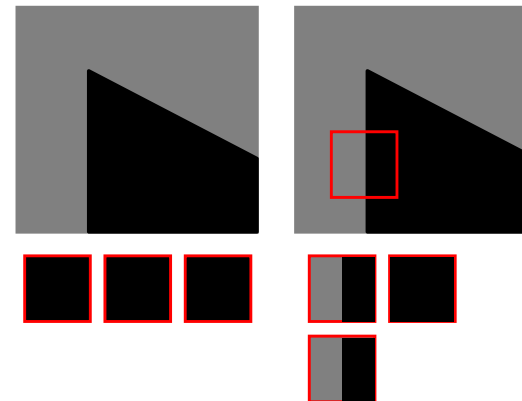
DigiVFX



flat

Moravec corner detector

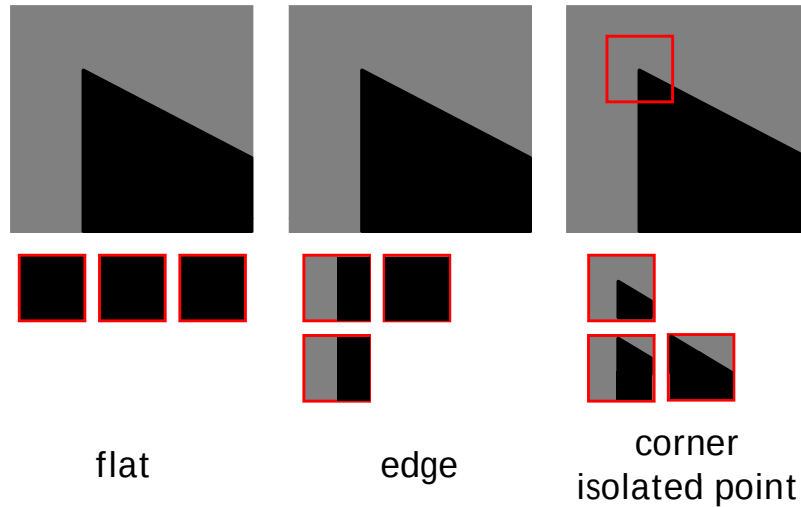
DigiVFX



flat

edge

Moravec corner detector



Moravec corner detector

Change of intensity for the shift $[u, v]$:

$$E(u, v) = \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

Diagram illustrating the Moravec corner detector formula and its components:

- Window function**: A 3x3 grid of weights $w(x, y)$ is shown, with a red box highlighting the center pixel.
- shifted intensity**: A diagram shows a gray-to-black gradient being shifted by $[u, v]$ to compare with the original intensity.
- intensity**: A diagram shows a gray-to-black gradient.
- Window function $w(x, y)$** : A diagram shows a red box on a gray-to-black gradient, with the text "1 in window, 0 outside".

Four shifts: $(u, v) = (1, 0), (1, 1), (0, 1), (-1, 1)$
Look for local maxima in $\min\{E\}$

Problems of Moravec detector

- Noisy response due to a binary window function
- Only a set of shifts at every 45 degree is considered
- Only minimum of E is taken into account

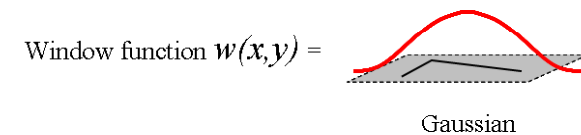
⇒ Harris corner detector (1988) solves these problems.

Harris corner detector

Noisy response due to a binary window function

➤ Use a Gaussian function

$$w(x, y) = \exp\left(-\frac{(x^2 + y^2)}{2\sigma^2}\right)$$



Harris corner detector



Only a set of shifts at every 45 degree is considered

➤ Consider all small shifts by Taylor's expansion

Harris corner detector



Only a set of shifts at every 45 degree is considered

➤ Consider all small shifts by Taylor's expansion

$$\begin{aligned} E(u, v) &= \sum_{x, y} w(x, y) [I(x+u, y+v) - I(x, y)]^2 \\ &= \sum_{x, y} w(x, y) [I_x u + I_y v + O(u^2, v^2)]^2 \end{aligned}$$

$$E(u, v) = Au^2 + 2Cuv + Bv^2$$

$$A = \sum_{x, y} w(x, y) I_x^2(x, y)$$

$$B = \sum_{x, y} w(x, y) I_y^2(x, y)$$

$$C = \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y)$$

Harris corner detector



Equivalently, for small shifts $[u, v]$ we have a *bilinear* approximation:

$$E(u, v) \cong [u \ v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

, where $\mathbf{M}$ is a 2x2 matrix computed from image derivatives:

$$\mathbf{M} = \sum_{x, y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Harris corner detector (matrix form)



$$E(\mathbf{u}) = |I(\mathbf{x}_0 + \mathbf{u}) - I(\mathbf{x}_0)|^2$$

$$= \left| \left(I_0 + \frac{\partial I^T}{\partial \mathbf{x}} \mathbf{u} \right) - I_0 \right|^2$$

$$= \left| \frac{\partial I^T}{\partial \mathbf{x}} \mathbf{u} \right|^2$$

$$= \mathbf{u}^T \frac{\partial I}{\partial \mathbf{x}} \frac{\partial I^T}{\partial \mathbf{x}} \mathbf{u}$$

$$= \mathbf{u}^T \mathbf{M} \mathbf{u}$$

Harris corner detector

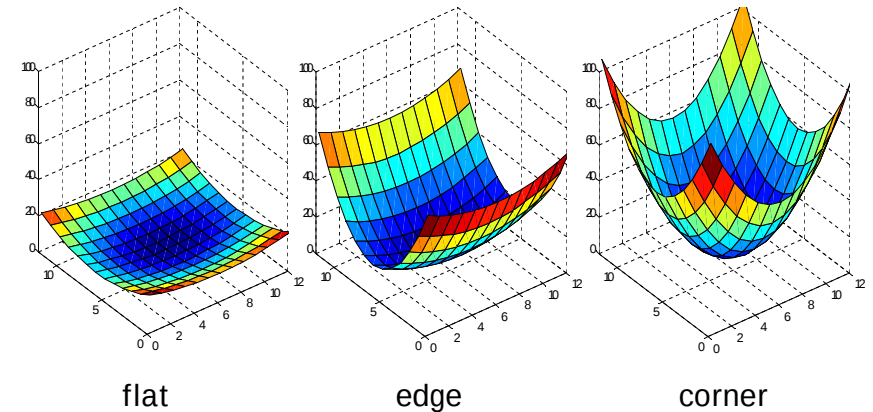
Only minimum of E is taken into account

➤ A new corner measurement by investigating the shape of the error function

$\mathbf{u}^T \mathbf{M} \mathbf{u}$ represents a quadratic function; Thus, we can analyze E 's shape by looking at the property of $\mathbf{M}$

Harris corner detector

High-level idea: what shape of the error function will we prefer for features?



Quadratic forms

- Quadratic form (homogeneous polynomial of degree two) of n variables x_i

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} x_i x_j$$

- Examples

$$4x_1^2 + 5x_2^2 + 3x_3^2 + 2x_1x_2 + 4x_1x_3 + 6x_2x_3$$

$$= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Symmetric matrices

- Quadratic forms can be represented by a real symmetric matrix $\mathbf{A}$ where

$$a_{ij} = \begin{cases} c_{ij} & \text{if } i = j, \\ \frac{1}{2}c_{ij} & \text{if } i < j, \\ \frac{1}{2}c_{ji} & \text{if } i > j. \end{cases}$$

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} x_i x_j = \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j$$

$$= \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \mathbf{x}^t \mathbf{A} \mathbf{x}$$

Eigenvalues of symmetric matrices

DigiVFX

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, i.e., $A = A^T$

fact: the eigenvalues of A are real

suppose $Av = \lambda v$, $v \neq 0$, $v \in \mathbf{C}^n$

$$\bar{v}^T Av = \bar{v}^T (\lambda v) = \lambda \bar{v}^T v = \lambda \sum_{i=1}^n |v_i|^2$$

$$\bar{v}^T Av = \overline{(Av)}^T v = \overline{(\lambda v)}^T v = \bar{\lambda} \sum_{i=1}^n |v_i|^2$$

we have $\lambda = \bar{\lambda}$, i.e., $\lambda \in \mathbf{R}$

(hence, can assume $v \in \mathbf{R}^n$)

Brad Osgood

Eigenvectors of symmetric matrices

DigiVFX

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, i.e., $A = A^T$

fact: there is a set of orthonormal eigenvectors of A
 $A = Q\Lambda Q^T$

Eigenvectors of symmetric matrices

DigiVFX

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, i.e., $A = A^T$

fact: there is a set of orthonormal eigenvectors of A

$$A = Q\Lambda Q^T$$

$$\mathbf{x}^T A \mathbf{x}$$

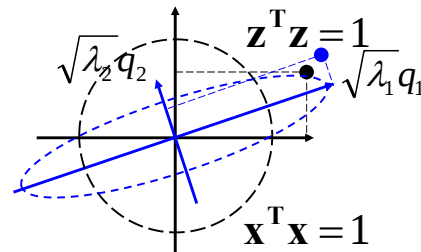
$$= \mathbf{x}^T Q \Lambda Q^T \mathbf{x}$$

$$= (Q^T \mathbf{x})^T \Lambda (Q^T \mathbf{x})$$

$$= \mathbf{y}^T \Lambda \mathbf{y}$$

$$= (\Lambda^{\frac{1}{2}} \mathbf{y})^T (\Lambda^{\frac{1}{2}} \mathbf{y})$$

$$= \mathbf{z}^T \mathbf{z}$$



Harris corner detector

DigiVFX

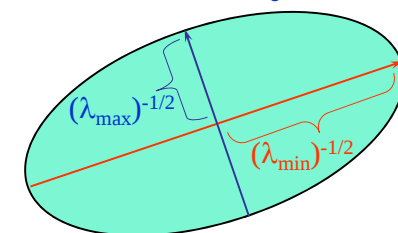
Intensity change in shifting window: eigenvalue analysis

$$E(u, v) \cong [u, v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } \mathbf{M}$$

Ellipse $E(u, v) = \text{const}$

direction of the
fastest change

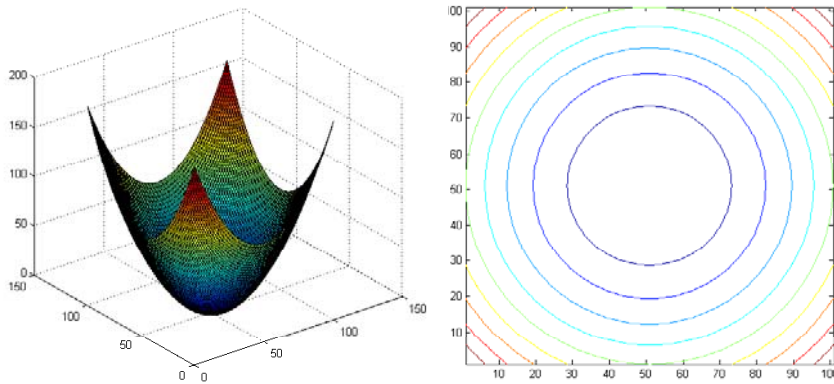
direction of the
slowest change



Visualize quadratic functions

DigiVFX

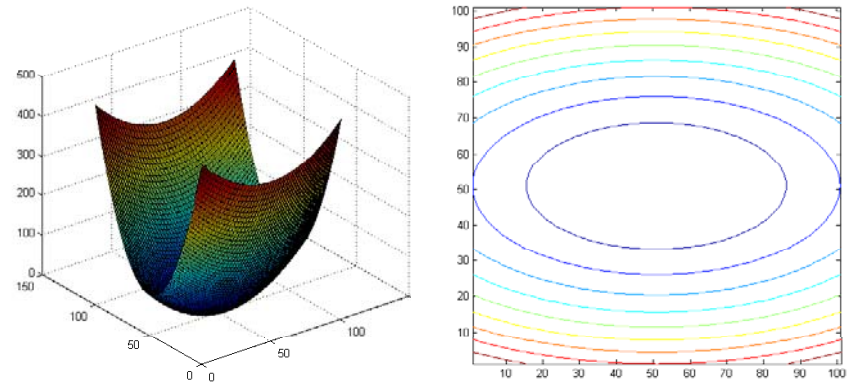
$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$



Visualize quadratic functions

DigiVFX

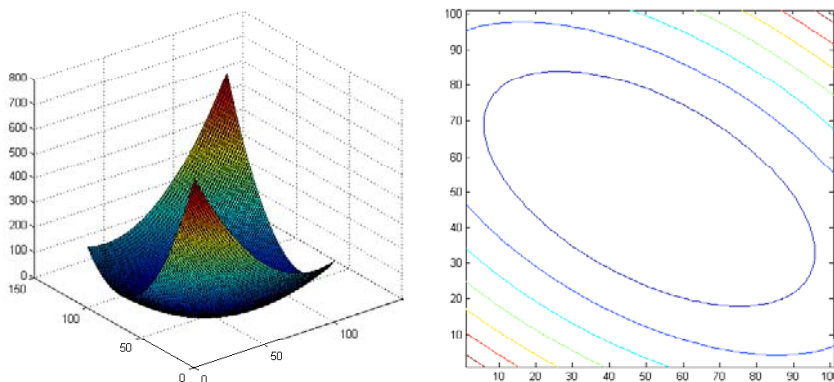
$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$



Visualize quadratic functions

DigiVFX

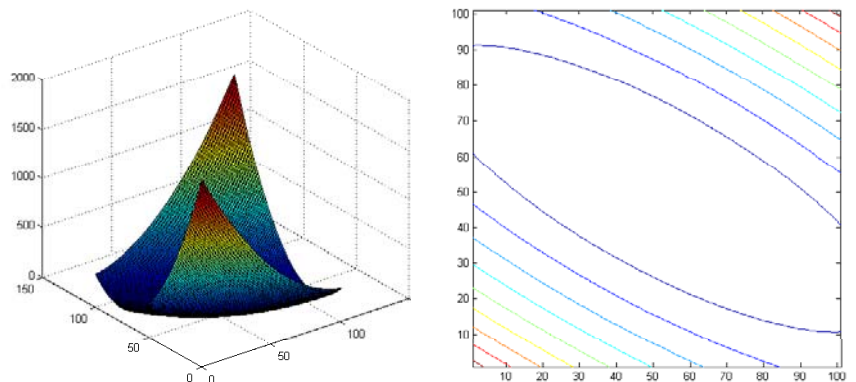
$$\mathbf{A} = \begin{bmatrix} 3.25 & 1.30 \\ 1.30 & 1.75 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$



Visualize quadratic functions

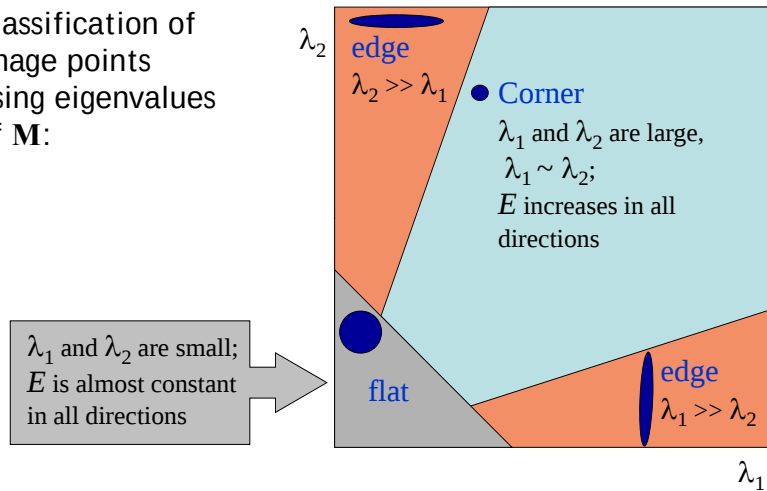
DigiVFX

$$\mathbf{A} = \begin{bmatrix} 7.75 & 3.90 \\ 3.90 & 3.25 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 0.50 & -0.87 \\ -0.87 & -0.50 \end{bmatrix}^T$$



Harris corner detector

Classification of image points using eigenvalues of $\mathbf{M}$:



Harris corner detector

$$\lambda = \frac{a_{00} + a_{11} \pm \sqrt{(a_{00} - a_{11})^2 + 4a_{10}a_{01}}}{2}$$

Only for reference, you do not need them to compute R

Measure of corner response:

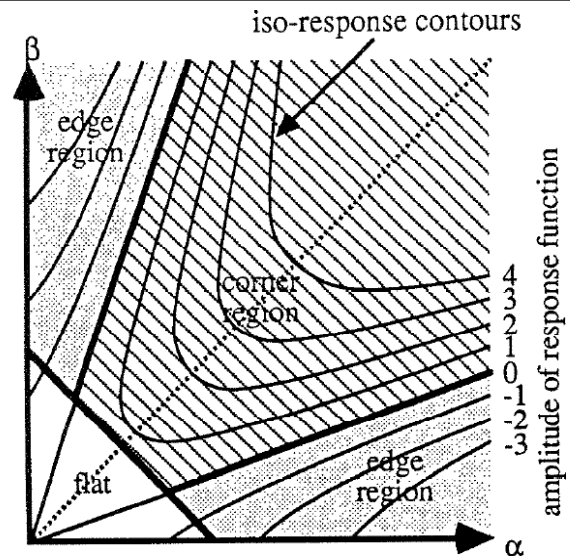
$$R = \det \mathbf{M} - k(\text{trace} \mathbf{M})^2$$

$$\det \mathbf{M} = \lambda_1 \lambda_2$$

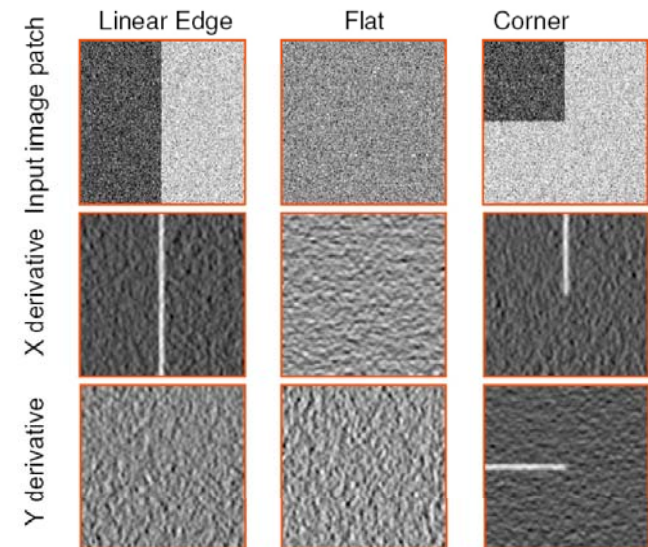
$$\text{trace} \mathbf{M} = \lambda_1 + \lambda_2$$

(k - empirical constant, $k = 0.04-0.06$)

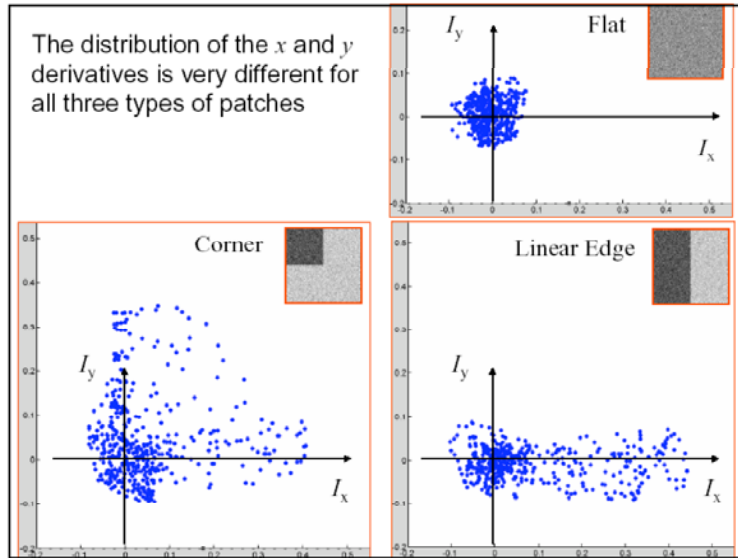
Harris corner detector



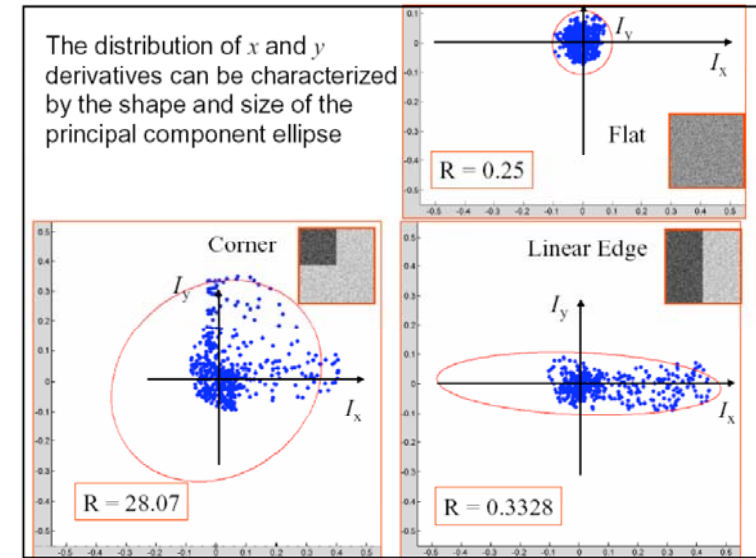
Another view



Another view



Another view



Summary of Harris detector

1. Compute x and y derivatives of image

$$I_x = G_x^* I \quad I_y = G_y^* I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \quad I_{y^2} = I_y \cdot I_y \quad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma^2} * I_{x^2} \quad S_{y^2} = G_{\sigma^2} * I_{y^2} \quad S_{xy} = G_{\sigma^2} * I_{xy}$$

Summary of Harris detector

4. Define the matrix at each pixel

$$M(x, y) = \begin{bmatrix} S_{x^2}(x, y) & S_{xy}(x, y) \\ S_{xy}(x, y) & S_{y^2}(x, y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = \det M - k(\text{trace} M)^2$$

6. Threshold on value of R ; compute nonmax suppression.

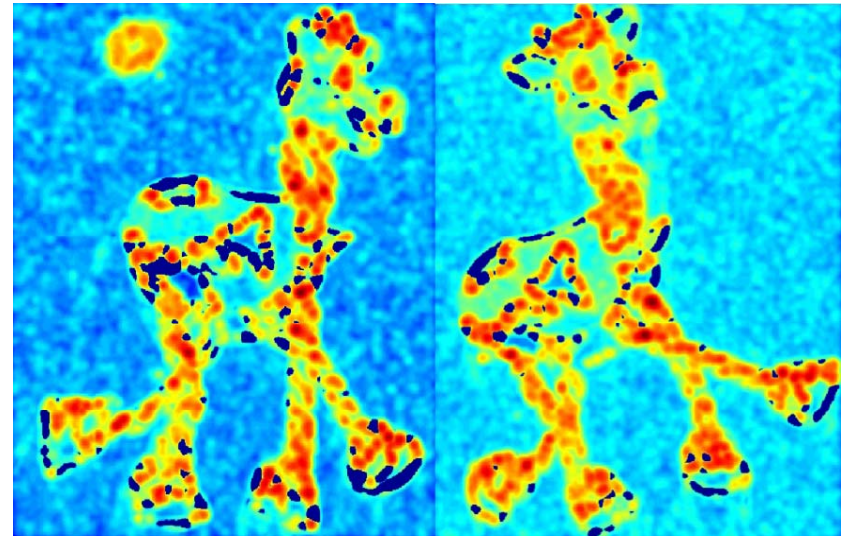
Harris corner detector (input)

DigiVFX



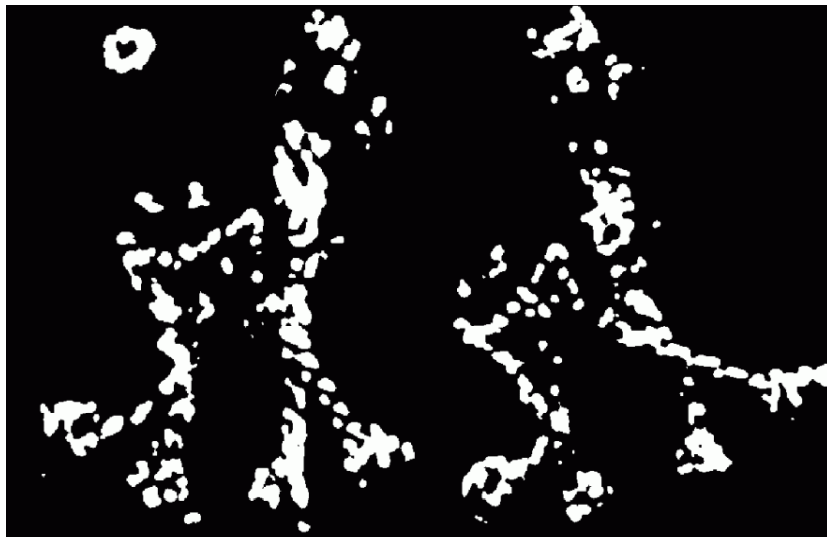
Corner response R

DigiVFX



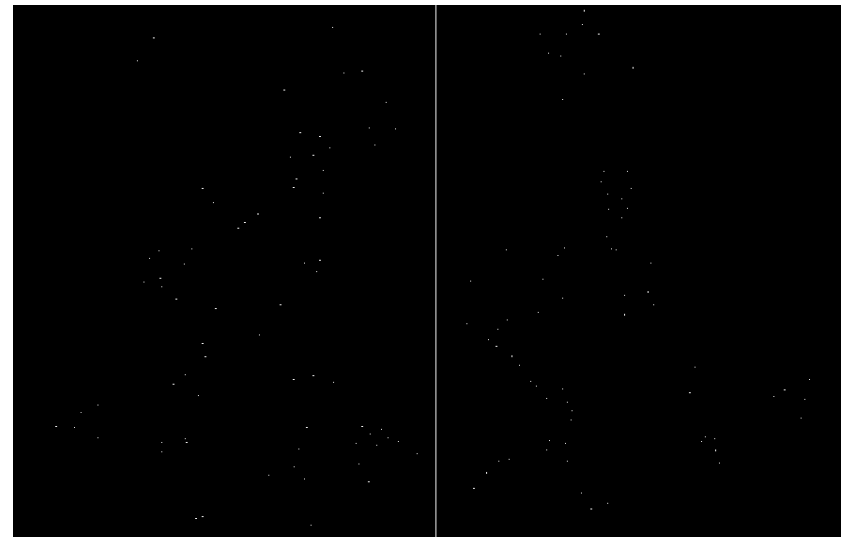
Threshold on R

DigiVFX



Local maximum of R

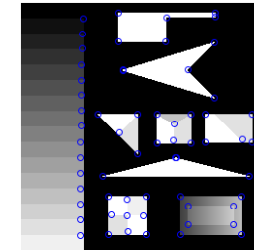
DigiVFX



Harris corner detector



Corner detection demo



<http://www.cim.mcgill.ca/~dparks/CornerDetector/mainApplet.htm>

Harris detector: summary

- Average intensity change in direction $[u, v]$ can be expressed as a bilinear form:

$$E(u, v) \cong [u, v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

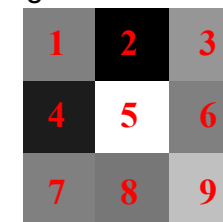
- Describe a point in terms of eigenvalues of M :
measure of corner response

$$R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

- A good (corner) point should have a *large intensity change in all directions*, i.e. R should be large positive

Now we know where features are

- But, how to match them?
- What is the descriptor for a feature? The simplest solution is the intensities of its spatial neighbors. This might not be robust to brightness change or small shift/rotation.

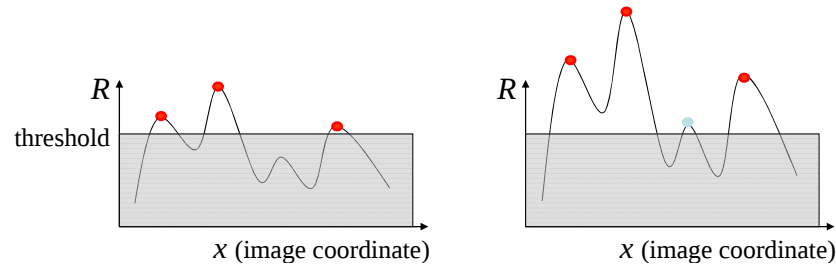


Harris detector: some properties

- Partial invariance to *affine intensity* change

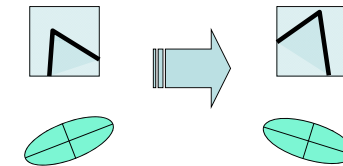
✓ Only derivatives are used =>
invariance to intensity shift $I \rightarrow I + b$

✓ Intensity scale: $I \rightarrow a I$



Harris Detector: Some Properties

- Rotation invariance



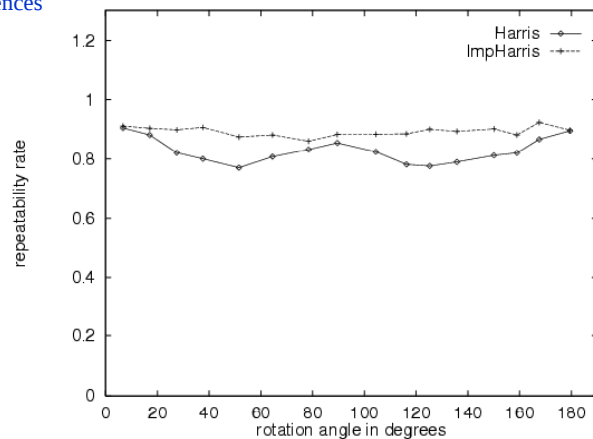
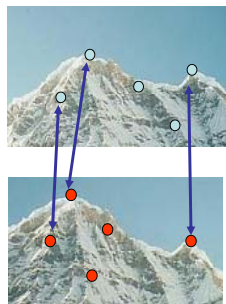
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response R is invariant to image rotation

Harris Detector is rotation invariant

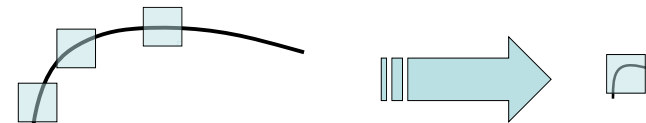
Repeatability rate:

$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$



Harris Detector: Some Properties

- But: not invariant to *image scale*!



All points will be classified as *edges*

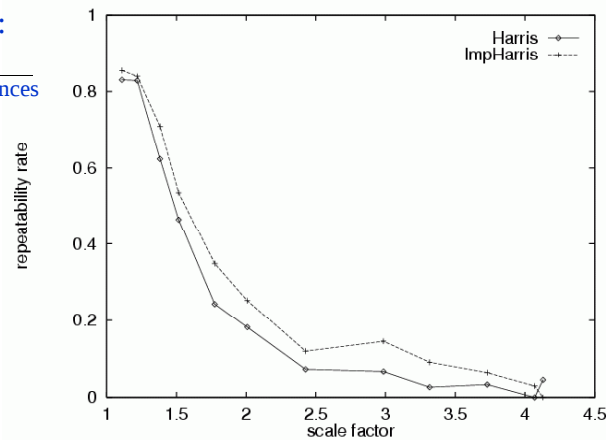
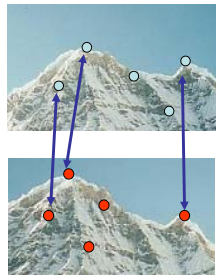
Corner !

Harris detector: some properties

- Quality of Harris detector for different scale changes

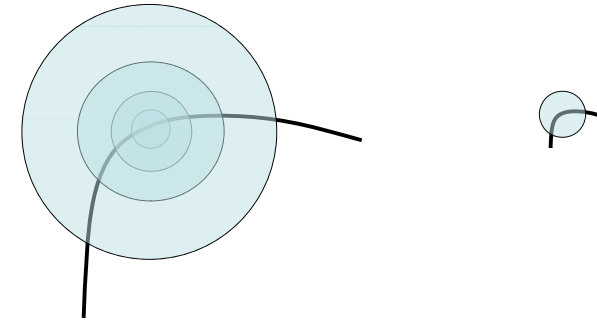
Repeatability rate:

$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



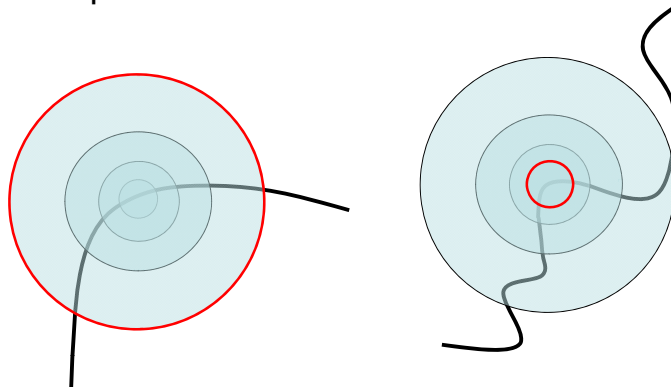
Scale invariant detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale invariant detection

- The problem: how do we choose corresponding circles **independently** in each image?
- Aperture problem



SIFT
(**Scale Invariant** Feature Transform)

SIFT

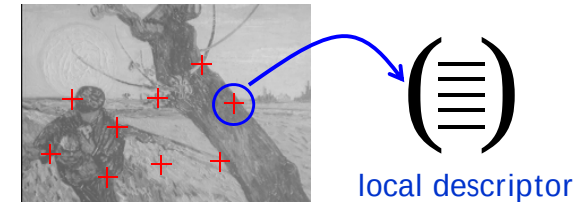
DigiVFX

- SIFT is an carefully designed procedure with empirically determined parameters for the invariant and distinctive features.

SIFT stages:

DigiVFX

- | | |
|---------------------------------|-------------------|
| • Scale-space extrema detection | detector |
| • Keypoint localization | |
| • Orientation assignment | descriptor |
| • Keypoint descriptor | |



A 500x500 image gives about 2000 features

1. Detection of scale-space extrema

DigiVFX

- For scale invariance, search for stable features across all possible scales using a continuous function of scale, scale space.
- SIFT uses DoG filter for scale space because it is efficient and as stable as scale-normalized Laplacian of Gaussian.

DoG filtering

DigiVFX

Convolution with a variable-scale Gaussian

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$

$$G(x, y, \sigma) = 1/(2\pi\sigma^2) \exp^{-(x^2+y^2)/\sigma^2}$$

Difference-of-Gaussian (DoG) filter

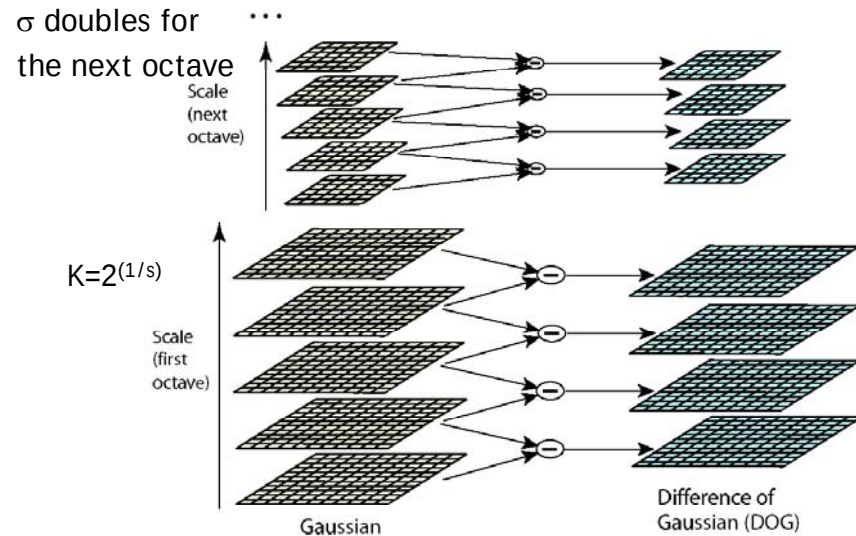
$$G(x, y, k\sigma) - G(x, y, \sigma)$$

Convolution with the DoG filter

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

Scale space

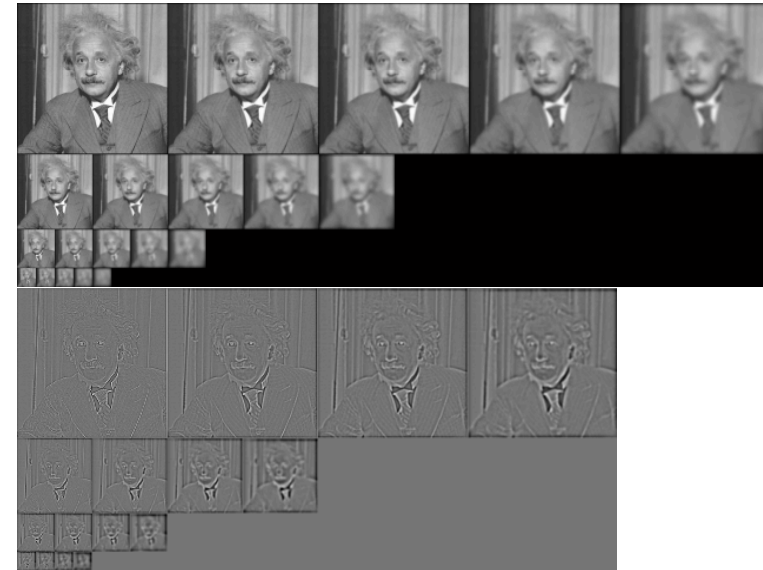
DigiVFX



Dividing into octave is for efficiency only.

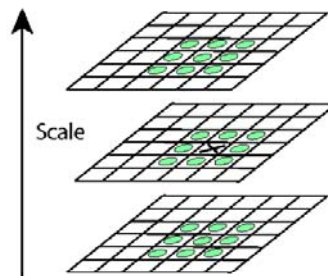
Detection of scale-space extrema

DigiVFX



Keypoint localization

DigiVFX



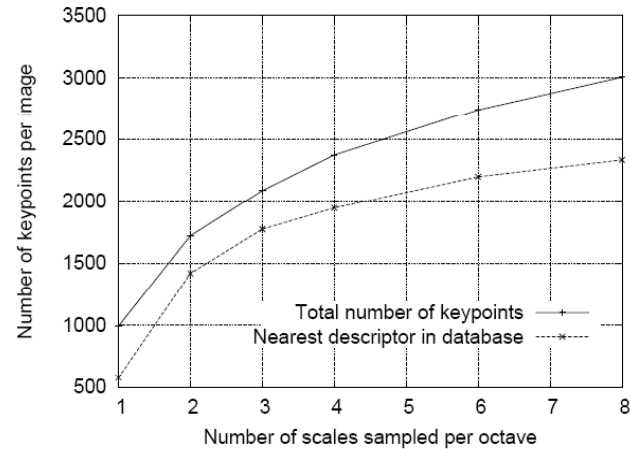
X is selected if it is larger or smaller than all 26 neighbors

Decide scale sampling frequency

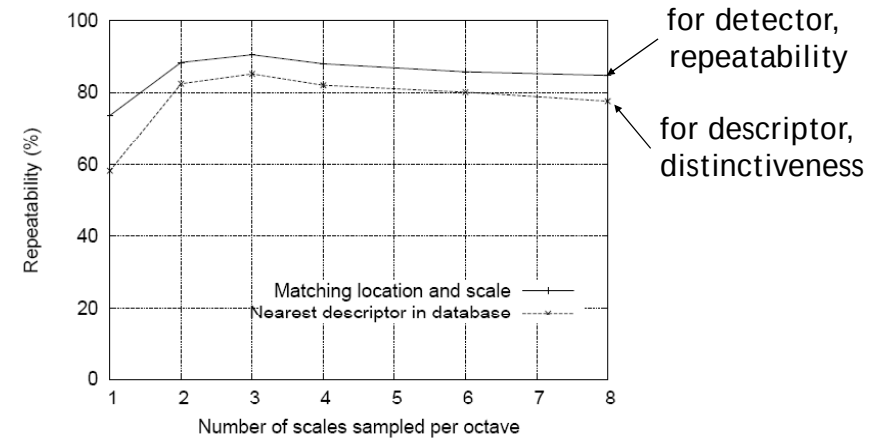
DigiVFX

- It is impossible to sample the whole space, tradeoff efficiency with completeness.
- Decide the best sampling frequency by experimenting on 32 real image subject to synthetic transformations. (rotation, scaling, affine stretch, brightness and contrast change, adding noise...)

Decide scale sampling frequency

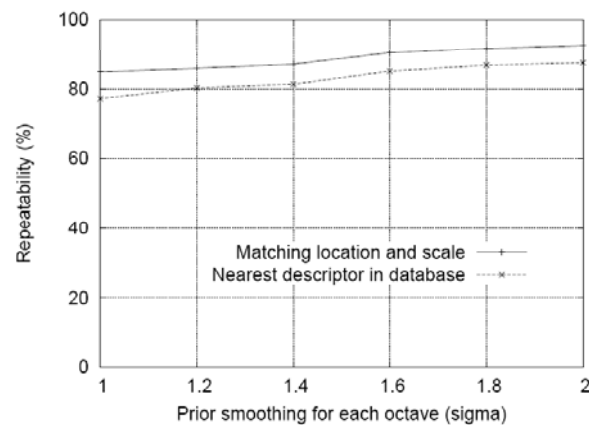


Decide scale sampling frequency



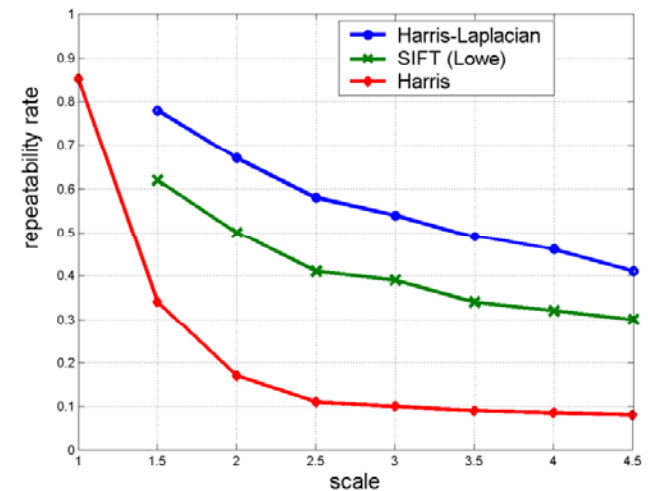
s=3 is the best, for larger s, too many unstable features

Pre-smoothing



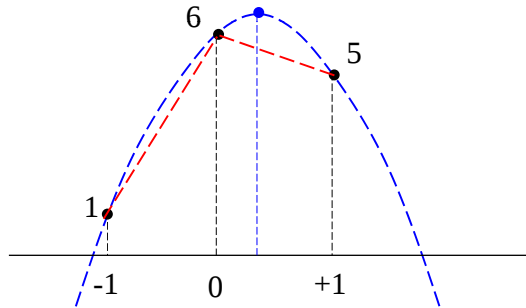
$\sigma = 1.6$, plus a double expansion

Scale invariance



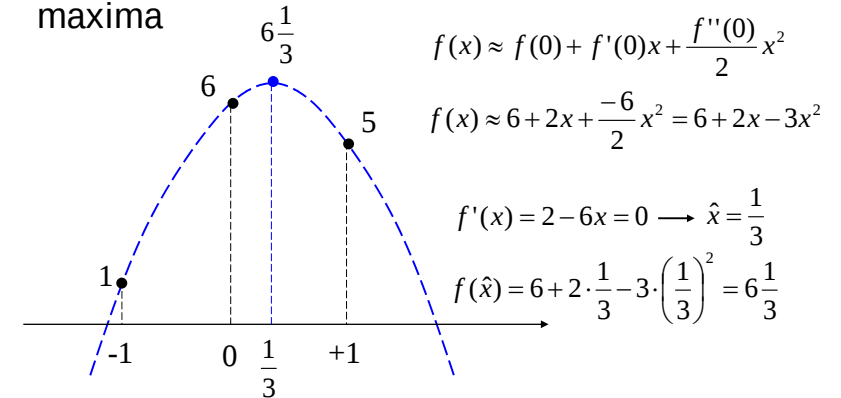
2. Accurate keypoint localization

- Reject points with low contrast (flat) and poorly localized along an edge (edge)
- Fit a 3D quadratic function for sub-pixel maxima



2. Accurate keypoint localization

- Reject points with low contrast (flat) and poorly localized along an edge (edge)
- Fit a 3D quadratic function for sub-pixel maxima



2. Accurate keypoint localization

- Taylor series of several variables

$$T(x_1, \dots, x_d) = \sum_{n_1=0}^{\infty} \dots \sum_{n_d=0}^{\infty} \frac{\partial^{n_1}}{\partial x_1^{n_1}} \dots \frac{\partial^{n_d}}{\partial x_d^{n_d}} \frac{f(a_1, \dots, a_d)}{n_1! \dots n_d!} (x_1 - a_1)^{n_1} \dots (x_d - a_d)^{n_d}$$

- Two variables

$$f(x, y) \approx f(0, 0) + \left(\frac{\partial f}{\partial x} x + \frac{\partial f}{\partial y} y \right) + \frac{1}{2} \left(\frac{\partial^2 f}{\partial x \partial x} x^2 + 2 \frac{\partial^2 f}{\partial x \partial y} xy + \frac{\partial^2 f}{\partial y \partial y} y^2 \right)$$

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \approx f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) + \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \frac{\partial^2 f}{\partial x \partial x} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y \partial y} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f(\mathbf{x}) \approx f(\mathbf{0}) + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

Accurate keypoint localization

- Taylor expansion in a matrix form, $\mathbf{x}$ is a vector, f maps $\mathbf{x}$ to a scalar

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

Hessian matrix
(often symmetric)

gradient $\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$

$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \dots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \dots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \dots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$

2D illustration

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\begin{aligned} \frac{\partial f}{\partial x} &= (f_{1,0} - f_{-1,0})/2 \\ \frac{\partial f}{\partial y} &= (f_{0,1} - f_{0,-1})/2 \\ \frac{\partial^2 f}{\partial x^2} &= f_{1,0} - 2f_{0,0} + f_{-1,0} \\ \frac{\partial^2 f}{\partial y^2} &= f_{0,1} - 2f_{0,0} + f_{0,-1} \\ \frac{\partial^2 f}{\partial x \partial y} &= (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4 \end{aligned}$$

2D example

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

-17	-1	-1
-9	7	7
-9	7	7

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{g}^T \mathbf{x} \quad \frac{\partial h}{\partial \mathbf{x}} =$$

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$\begin{aligned} h(\mathbf{x}) &= \mathbf{g}^T \mathbf{x} \\ &= (g_1 \quad \cdots \quad g_n) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \\ &= \sum_{i=1}^n g_i x_i \end{aligned} \quad \frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial h}{\partial x_1} \\ \vdots \\ \frac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} g_1 \\ \vdots \\ g_n \end{pmatrix} = \mathbf{g}$$

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x}$$

$$\frac{\partial h}{\partial \mathbf{x}} =$$

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x} = \begin{pmatrix} x_1 & \cdots & x_n \end{pmatrix} \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j$$

$$\frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial h}{\partial x_1} \\ \vdots \\ \frac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n a_{i1} x_i + \sum_{j=1}^n a_{1j} x_j \\ \vdots \\ \sum_{i=1}^n a_{in} x_i + \sum_{j=1}^n a_{nj} x_j \end{pmatrix} = \mathbf{A}^T \mathbf{x} + \mathbf{A} \mathbf{x}$$

$$= (\mathbf{A}^T + \mathbf{A}) \mathbf{x}$$

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$\frac{\partial f}{\partial \mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}} + \frac{1}{2} \left(\frac{\partial^2 f}{\partial \mathbf{x}^2} + \frac{\partial^2 f^T}{\partial \mathbf{x}^2} \right) \mathbf{x} = \frac{\partial f}{\partial \mathbf{x}} + \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$\mathbf{x}_m = - \frac{\partial^2 f^{-1}}{\partial \mathbf{x}^2} \frac{\partial f}{\partial \mathbf{x}}$$

Accurate keypoint localization

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

- $\mathbf{x}$ is a 3-vector
- Change sample point if offset is larger than 0.5
- Throw out low contrast (<0.03)

Accurate keypoint localization

- Throw out low contrast $|D(\hat{\mathbf{x}})| < 0.03$

$$\begin{aligned}
 D(\hat{\mathbf{x}}) &= D + \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}} + \frac{1}{2} \hat{\mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \hat{\mathbf{x}} \\
 &= D + \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}} + \frac{1}{2} \left(-\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}} \right)^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \left(-\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}} \right) \\
 &= D + \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D^T}{\partial \mathbf{x}} \frac{\partial^2 D^{-T}}{\partial \mathbf{x}^2} \frac{\partial^2 D}{\partial \mathbf{x}^2} \frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}} \\
 &= D + \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D^T}{\partial \mathbf{x}} \frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2} \frac{\partial D}{\partial \mathbf{x}} \\
 &= D + \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D^T}{\partial \mathbf{x}} (-\hat{\mathbf{x}}) \\
 &= D + \frac{1}{2} \frac{\partial D^T}{\partial \mathbf{x}} \hat{\mathbf{x}}
 \end{aligned}$$

Eliminating edge responses

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \quad \text{Hessian matrix at keypoint location}$$

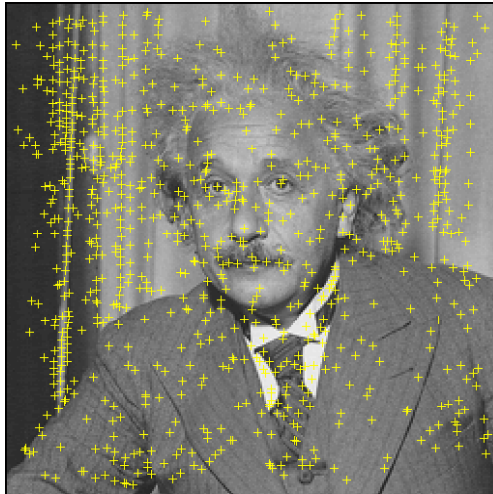
$$\text{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\text{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

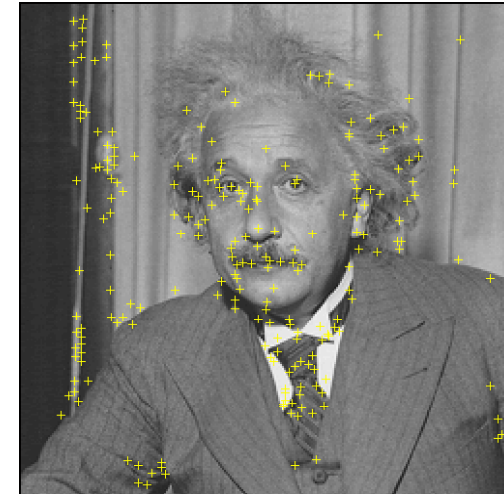
$$\text{Let } \alpha = r\beta \quad \frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(r\beta + \beta)^2}{r\beta^2} = \frac{(r + 1)^2}{r}$$

$$\text{Keep the points with } \frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} < \frac{(r + 1)^2}{r}. \quad r=10$$

Maxima in D



Remove low contrast and edges



Keypoint detector

233x89



832 extrema



729 after contrast filtering



536 after curvature filtering

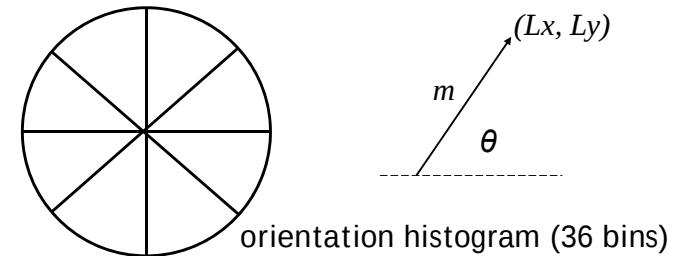


3. Orientation assignment

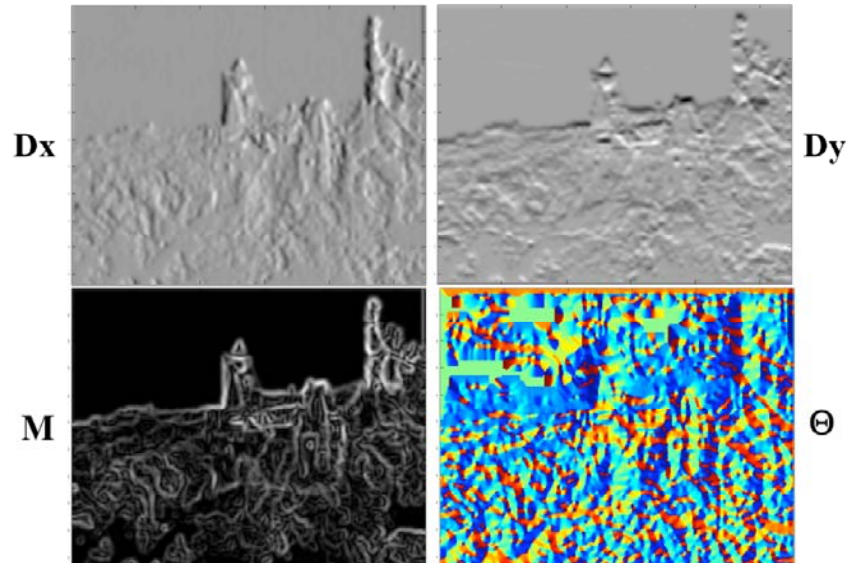
- By assigning a consistent orientation, the keypoint descriptor can be orientation invariant.
- For a keypoint, L is the Gaussian-smoothed image with the closest scale,

$$m(x, y) = \sqrt{(L(x+1, y) - L(x-1, y))^2 + (L(x, y+1) - L(x, y-1))^2}$$

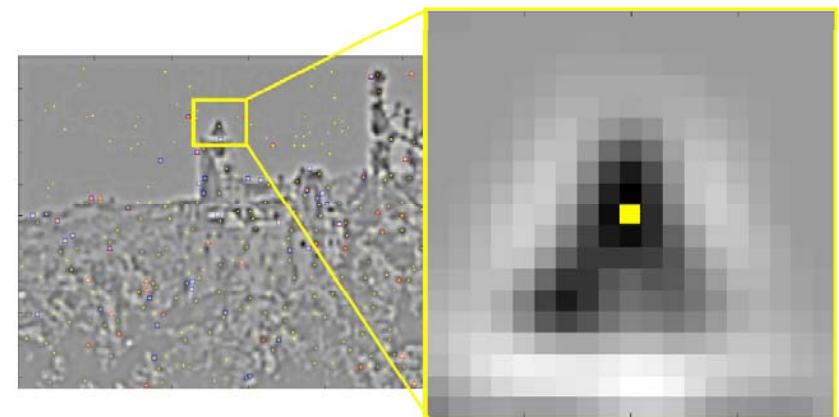
$$\theta(x, y) = \tan^{-1}((L(x, y+1) - L(x, y-1)) / (L(x+1, y) - L(x-1, y)))$$



Orientation assignment



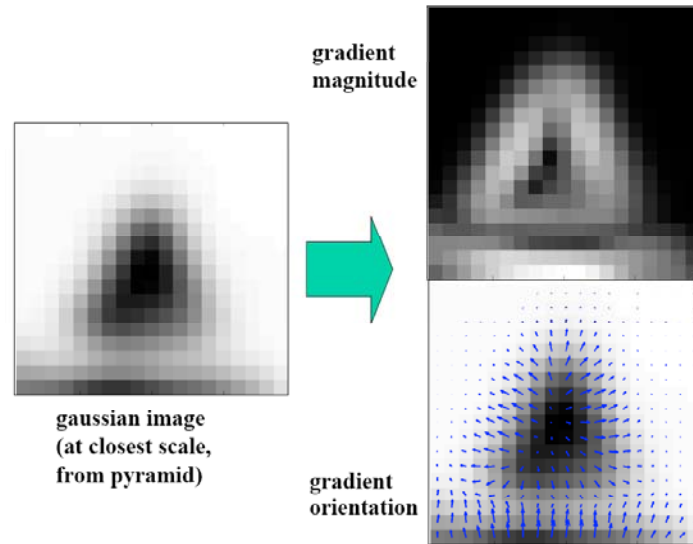
Orientation assignment



- Keypoint location = extrema location
- Keypoint scale is scale of the DOG image

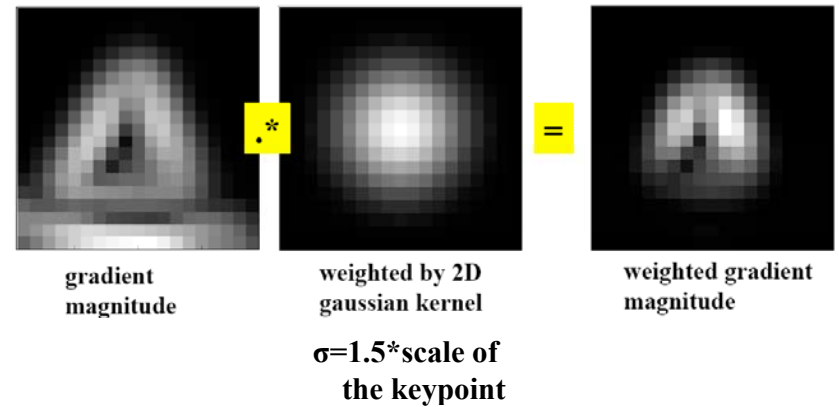
Orientation assignment

DigiVFX



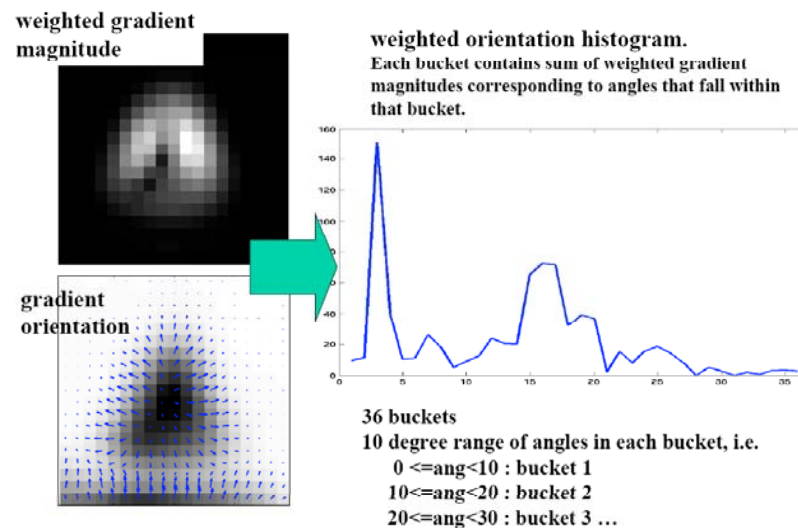
Orientation assignment

DigiVFX



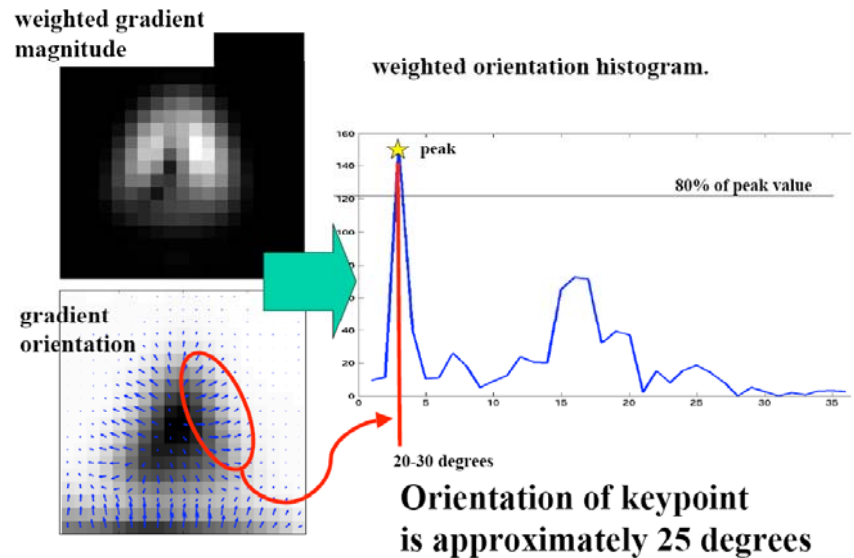
Orientation assignment

DigiVFX



Orientation assignment

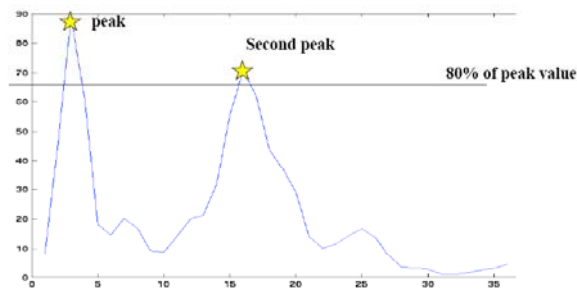
DigiVFX



Orientation assignment

There may be multiple orientations.

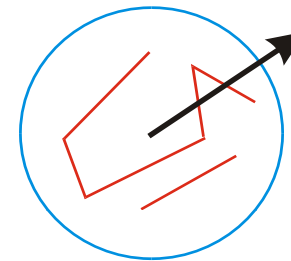
**accurate peak position
is determined by fitting**



In this case, generate duplicate keypoints, one with orientation at 25 degrees, one at 155 degrees.

Design decision: you may want to limit number of possible multiple peaks to two.

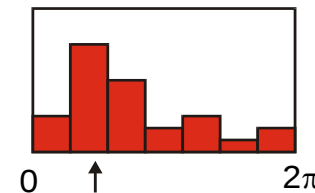
Orientation assignment



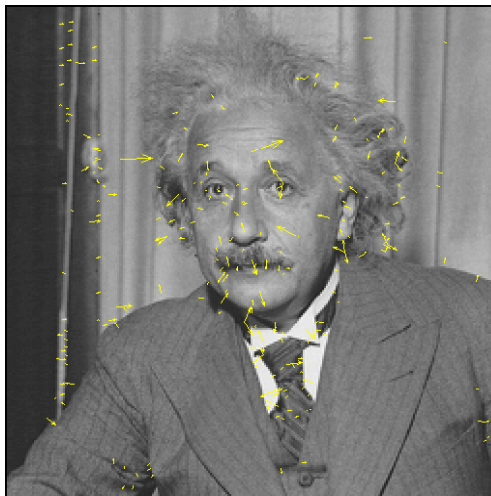
36-bin orientation histogram over 360° , weighted by m and $1.5 \cdot \text{scale}$ falloff
Peak is the orientation

Local peak within 80% creates multiple orientations

About 15% has multiple orientations and they contribute a lot to stability

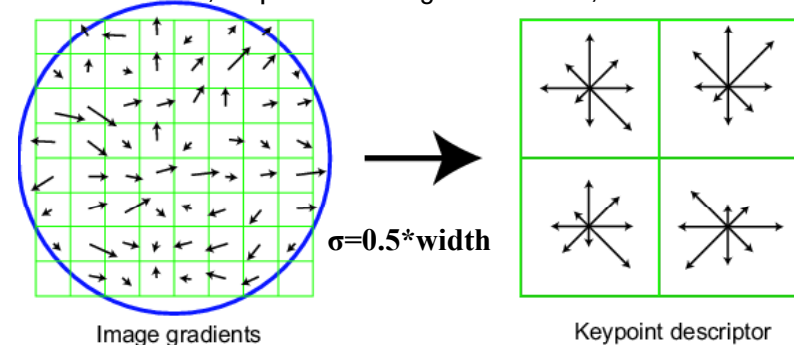


SIFT descriptor

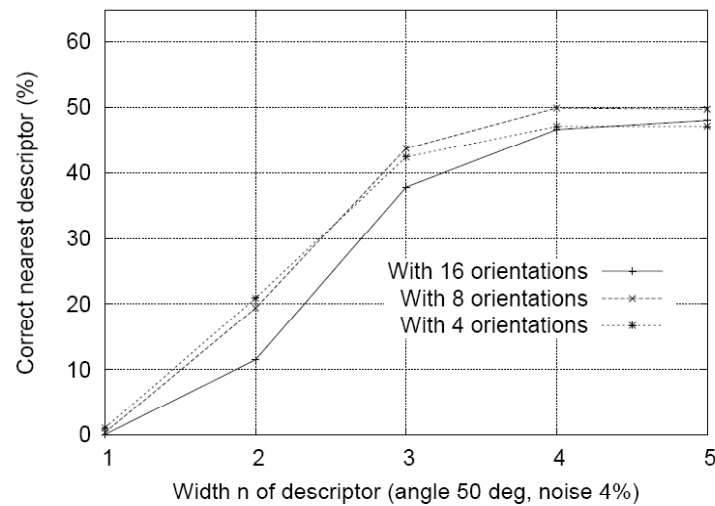


4. Local image descriptor

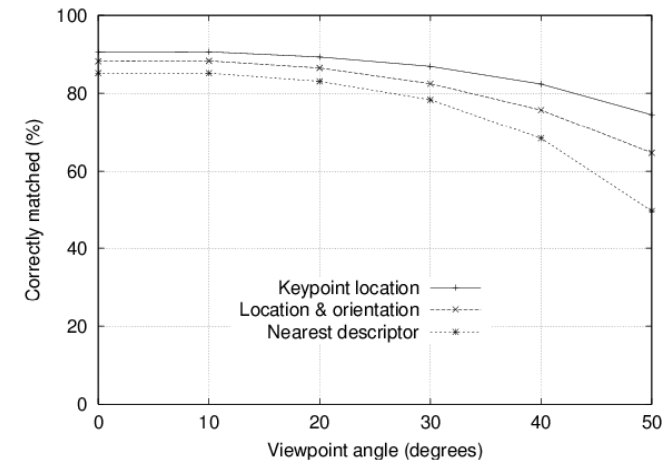
- Thresholded image gradients are sampled over 16×16 array of locations in scale space
- Create array of orientation histograms (w.r.t. key orientation)
- 8 orientations $\times$ 4×4 histogram array = 128 dimensions
- Normalized, clip values larger than 0.2, renormalize



Why 4x4x8?



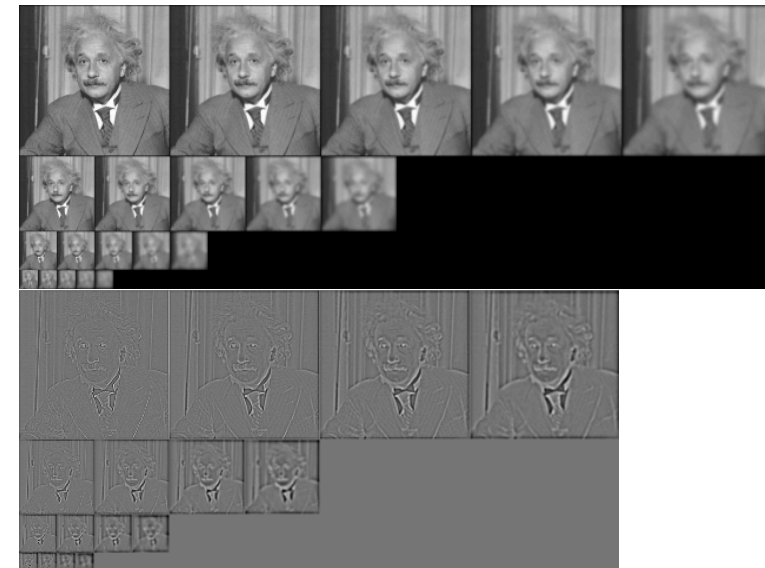
Sensitivity to affine change



Feature matching

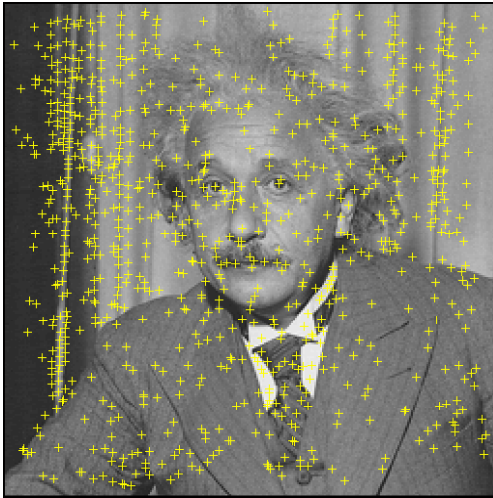
- for a feature x , he found the closest feature x_1 and the second closest feature x_2 . If the distance ratio of $d(x, x_1)$ and $d(x, x_2)$ is smaller than 0.8, then it is accepted as a match.

SIFT flow



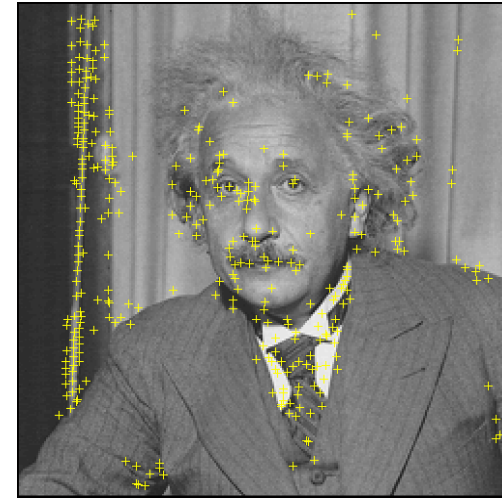
Maxima in D

DigiVFX



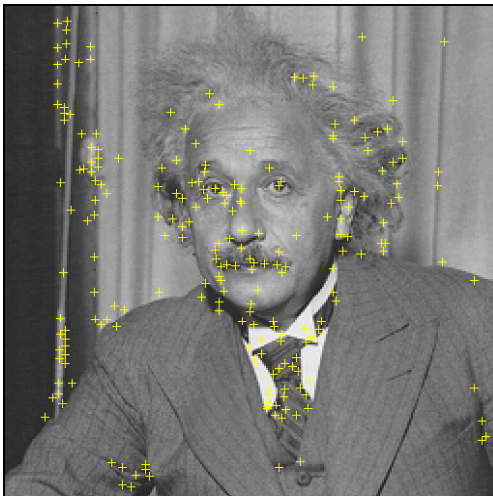
Remove low contrast

DigiVFX



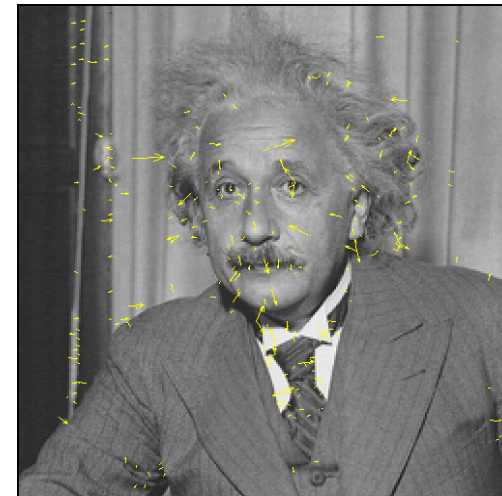
Remove edges

DigiVFX



SIFT descriptor

DigiVFX





SIFT extensions

Estimated rotation

DigiVFX

- Computed affine transformation from rotated image to original image:

0.7060	-0.7052	128.4230
0.7057	0.7100	-128.9491
0	0	1.0000

- Actual transformation from rotated image to original image:

0.7071	-0.7071	128.6934
0.7071	0.7071	-128.6934
0	0	1.0000

PCA

DigiVFX

Average face:



Top ten eigenfaces (left = highest eigenvalue, right = lowest eigenvalue):



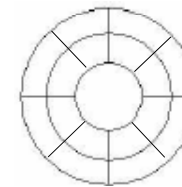
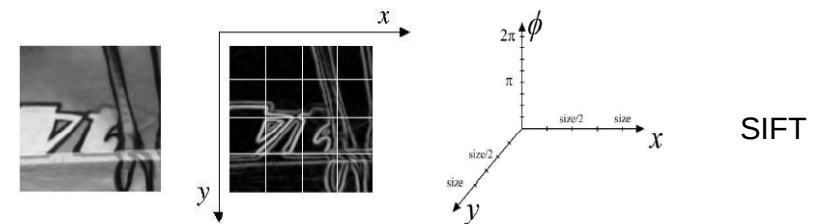
PCA-SIFT

DigiVFX

- Only change step 4
- Pre-compute an eigen-space for local gradient patches of size 41x41
- 2x39x39=3042 elements
- Only keep 20 components
- A more compact descriptor

GLOH (Gradient location-orientation histogram)

DigiVFX



17 location bins
16 orientation bins
Analyze the 17x16=272-d
eigen-space, keep 128 components

SIFT is still considered the best.

Multi-Scale Oriented Patches

DigiVFX

- Simpler than SIFT. Designed for image matching. [Brown, Szeliski, Winder, CVPR'2005]
- Feature detector
 - Multi-scale Harris corners
 - Orientation from blurred gradient
 - Geometrically invariant to rotation
- Feature descriptor
 - Bias/gain normalized sampling of local patch (8x8)
 - Photometrically invariant to affine changes in intensity

Multi-Scale Harris corner detector

DigiVFX

$$P_0(x, y) = I(x, y)$$

$$P'_l(x, y) = P_l(x, y) * g_{\sigma_p}(x, y)$$

$$P_{l+1}(x, y) = P'_l(sx, sy)$$

$$s = 2 \quad \sigma_p = 1.0$$

- Image stitching is mostly concerned with matching images that have the same scale, so sub-octave pyramid might not be necessary.

Multi-Scale Harris corner detector



$$\mathbf{H}_l(x, y) = \nabla_{\sigma_d} P_l(x, y) \nabla_{\sigma_d} P_l(x, y)^T * g_{\sigma_i}(x, y)$$

$$\nabla_{\sigma} f(x, y) \triangleq \nabla f(x, y) * g_{\sigma}(x, y)$$

smoother version of gradients

$$\sigma_i = 1.5 \quad \sigma_d = 1.0$$

Corner detection function:

$$f_{HM}(x, y) = \frac{\det \mathbf{H}_l(x, y)}{\text{tr} \mathbf{H}_l(x, y)} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Pick local maxima of 3x3 and larger than 10

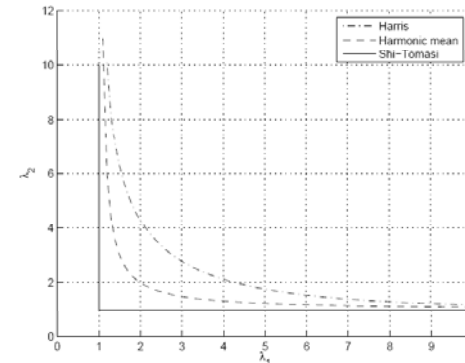
Keypoint detection function



$$\text{Harris } f_H = \lambda_1 \lambda_2 - 0.04(\lambda_1 + \lambda_2)^2 = \det \mathbf{H} - 0.04(\text{tr} \mathbf{H})^2$$

$$\text{Harmonic mean } f_{HM} = \lambda_1 \lambda_2 / (\lambda_1 + \lambda_2) = \det \mathbf{H} / \text{tr} \mathbf{H}$$

$$\text{Shi-Tomasi } f_{ST} = \min(\lambda_1, \lambda_2)$$



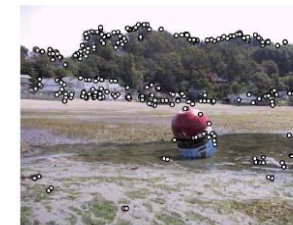
Experiments show roughly the same performance.

Non-maximal suppression



- Restrict the maximal number of interest points, but also want them spatially well distributed
- Only retain maximums in a neighborhood of radius r .
- Sort them by strength, decreasing r from infinity until the number of keypoints (500) is satisfied.

Non-maximal suppression



(a) Strongest 250



(b) Strongest 500



(c) ANMS 250, $r = 24$



(d) ANMS 500, $r = 16$

Sub-pixel refinement

$$f(\mathbf{x}) = f + \frac{\partial f^T}{\partial \mathbf{x}} \mathbf{x} + \frac{1}{2} \mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2} \mathbf{x}$$

$$\mathbf{x}_m = - \frac{\partial^2 f}{\partial \mathbf{x}^2}^{-1} \frac{\partial f}{\partial \mathbf{x}}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\begin{aligned} \frac{\partial f}{\partial x} &= (f_{1,0} - f_{-1,0})/2 \\ \frac{\partial f}{\partial y} &= (f_{0,1} - f_{0,-1})/2 \\ \frac{\partial^2 f}{\partial x^2} &= f_{1,0} - 2f_{0,0} + f_{-1,0} \\ \frac{\partial^2 f}{\partial y^2} &= f_{0,1} - 2f_{0,0} + f_{0,-1} \\ \frac{\partial^2 f}{\partial x \partial y} &= (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4 \end{aligned}$$

Orientation assignment

- Orientation = blurred gradient

$$\mathbf{u}_l(x, y) = \nabla_{\sigma_o} P_l(x, y)$$

$$\sigma_o = 4.5$$

$$[\cos \theta, \sin \theta] = \mathbf{u}/|\mathbf{u}|$$

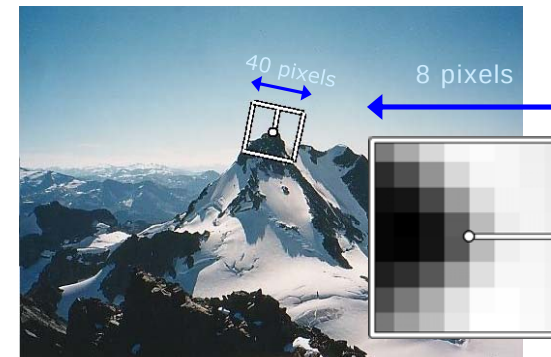
Descriptor Vector

- Rotation Invariant Frame
 - Scale-space position (x, y, s) + orientation (θ)



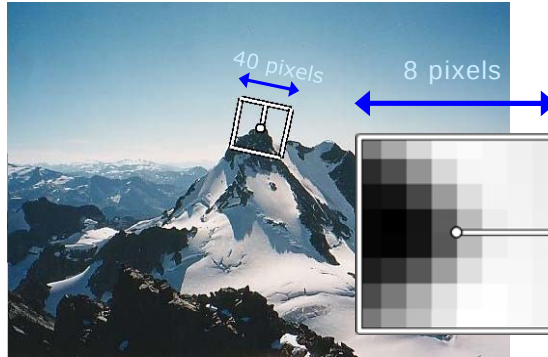
MOPS descriptor vector

- 8x8 oriented patch sampled at 5 x scale. See TR for details.
- Sampled from $P_l(x, y) * g_{2 \times \sigma_p}(x, y)$ with spacing=5



MOPS descriptor vector

- 8x8 oriented patch sampled at 5 x scale. See TR for details.
- Bias/gain normalisation: $I' = (I - \mu) / \sigma$
- Wavelet transform



Detections at multiple scales

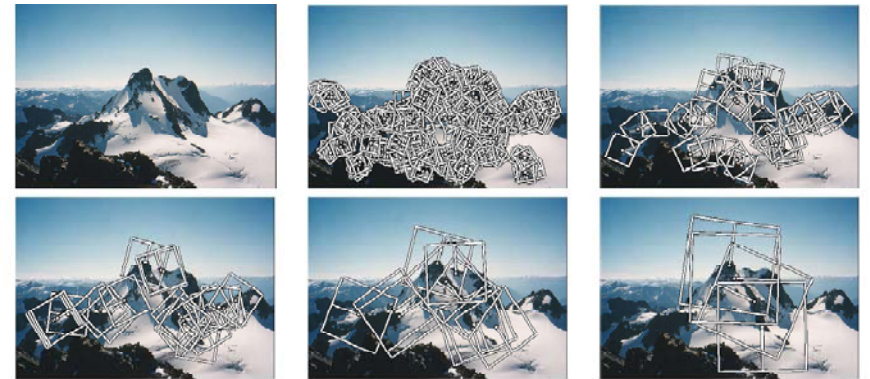


Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.

Summary

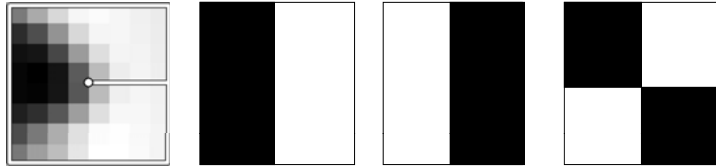
- Multi-scale Harris corner detector
- Sub-pixel refinement
- Orientation assignment by gradients
- Blurred intensity patch as descriptor

Feature matching

- Exhaustive search
 - for each feature in one image, look at *all* the other features in the other image(s)
- Hashing
 - compute a short descriptor from each feature vector, or hash longer descriptors (randomly)
- Nearest neighbor techniques
 - *k*-trees and their variants (Best Bin First)

Wavelet-based hashing

- Compute a short (3-vector) descriptor from an 8x8 patch using a Haar “wavelet”



- Quantize each value into 10 (overlapping) bins (10^3 total entries)
- [Brown, Szeliski, Winder, CVPR'2005]

Applications

Nearest neighbor techniques

- k -D tree and
- Best Bin First (BBF)

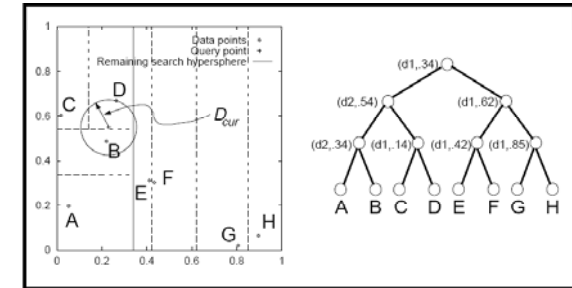
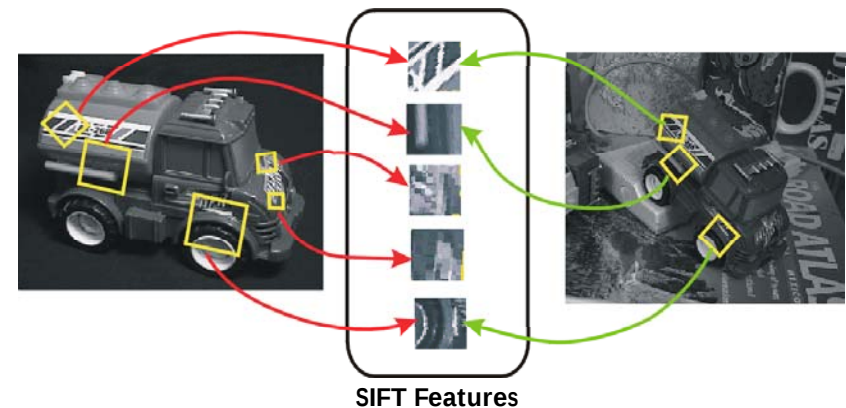


Figure 6: k -d-tree with 8 data points labelled A-H, dimension of space $k=2$. On the right is the full tree, the leaf nodes containing the data points. Internal node information consists of the dimension of the cut plane and the value of the cut in that dimension. On the left is the 2D feature space carved into various sizes and shapes of bin, according to the distribution of the data points. The two representations are isomorphic. The situation shown on the left is after initial tree traversal to locate the bin for query point q (contains point D). In standard search, the closest nodes in the tree are examined first (starting at C). In BBF search, the closest bins to query point q are examined first (starting at B). The latter is more likely to maximize the overlap of (i) the hypersphere centered on q with radius D_{cur} , and (ii) the hyperrectangle of the bin to be searched. In this case, BBF search reduces the number of leaves to examine, since once point B is discovered, all other branches can be pruned.

Indexing Without Invariants in 3D Object Recognition, Beis and Lowe, PAMI'99

Recognition



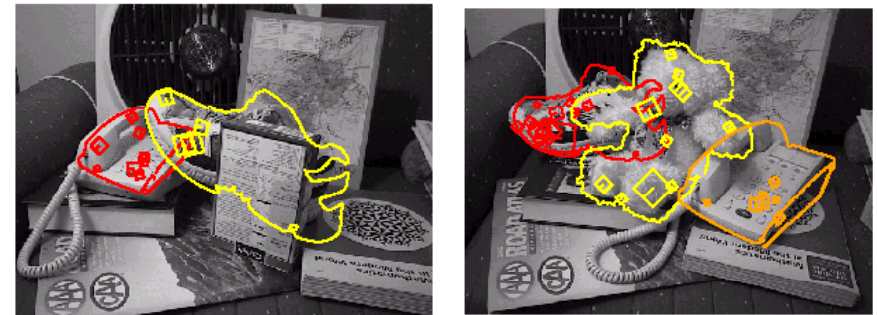
3D object recognition

DigiVFX



3D object recognition

DigiVFX



Office of the past

DigiVFX

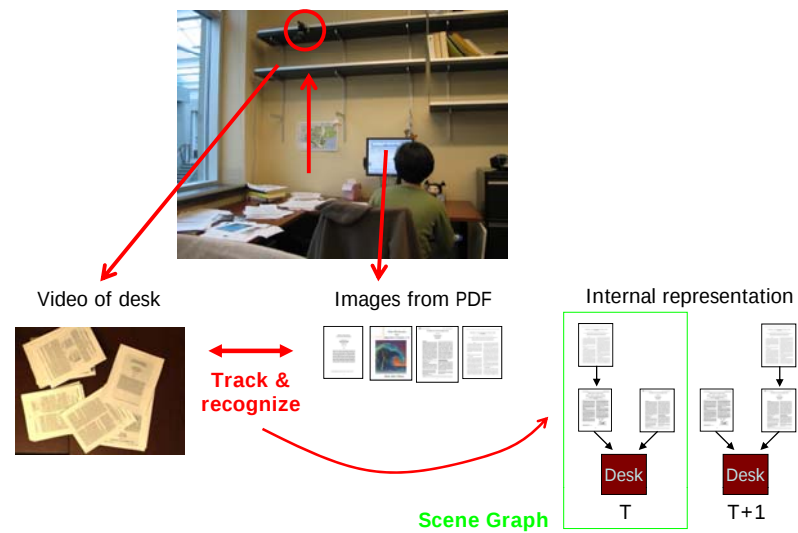


Image retrieval

DigiVFX

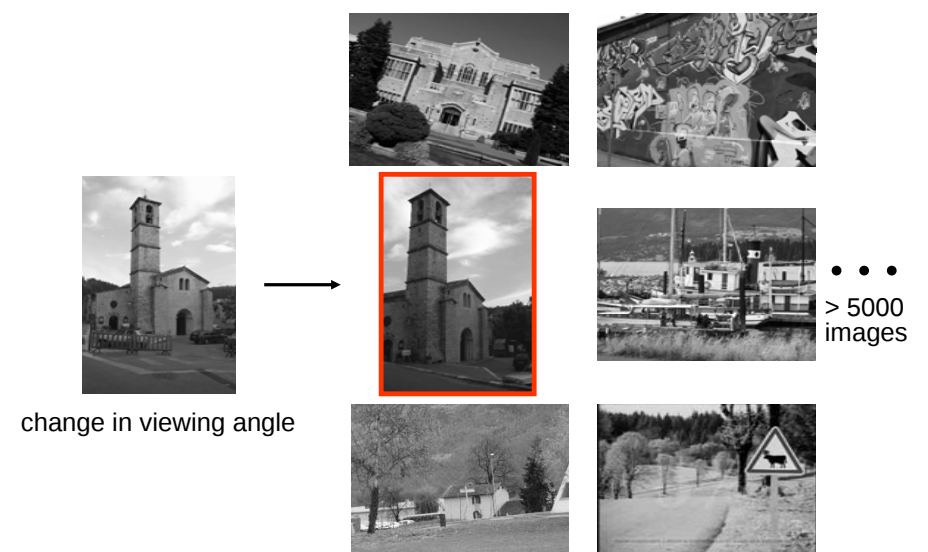
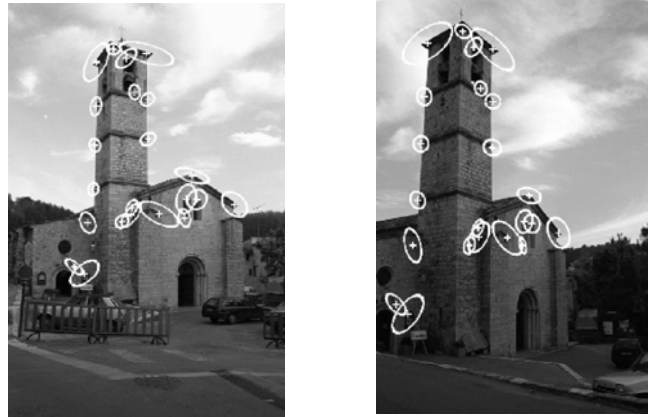


Image retrieval

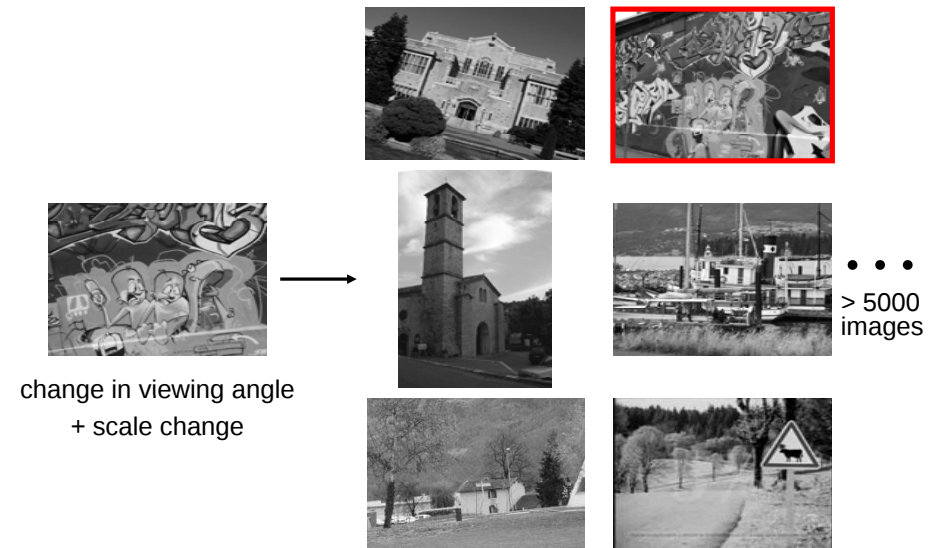
DigiVFX



22 correct matches

Image retrieval

DigiVFX



Robot location

DigiVFX



Robotics: Sony Aibo

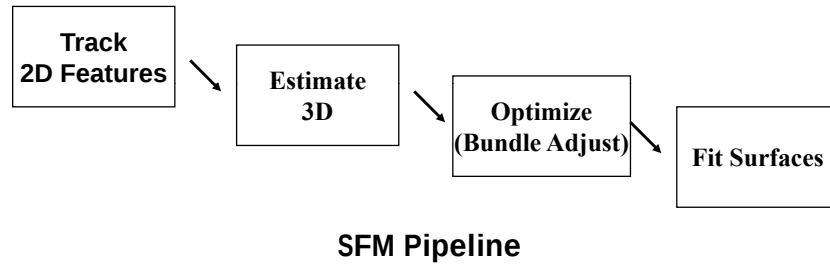
DigiVFX

- SIFT is used for
- Recognizing charging station
 - Communicating with visual cards
 - Teaching object recognition
 - soccer

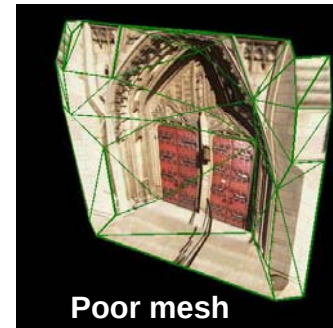


Structure from Motion

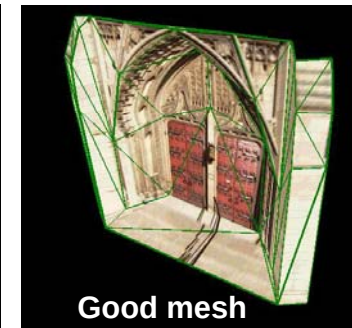
- The SFM Problem
 - Reconstruct scene geometry and camera motion from two or more images



Structure from Motion



Poor mesh

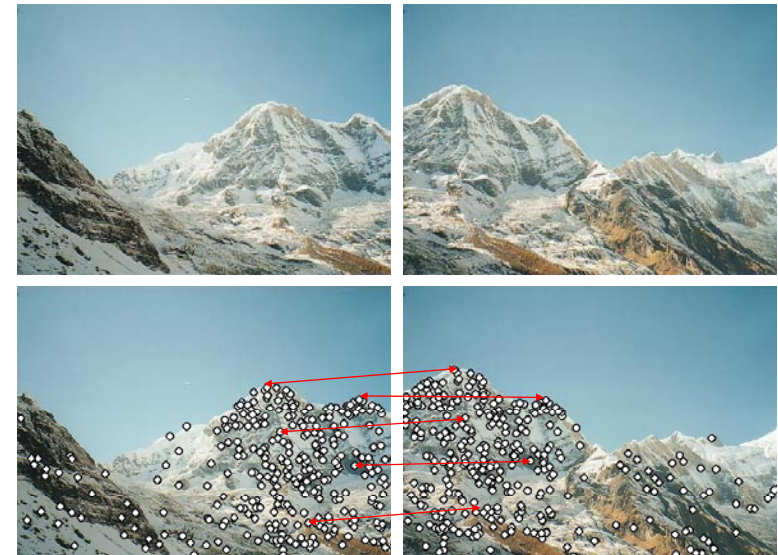


Good mesh

Augmented reality

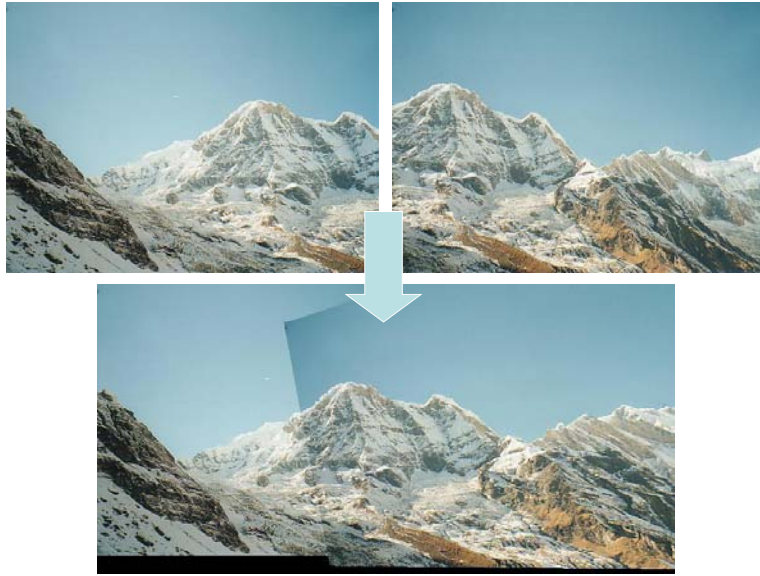


Automatic image stitching



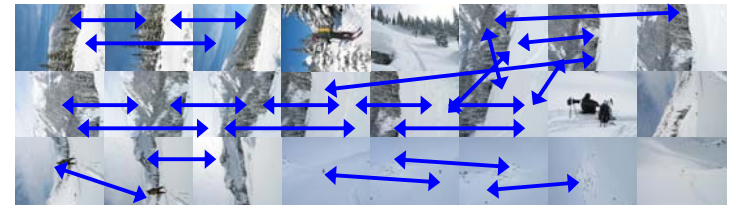
Automatic image stitching

DigiVFX



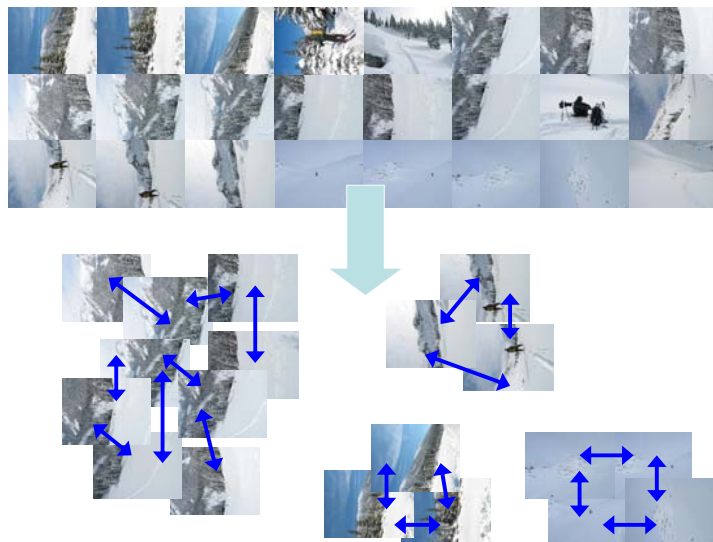
Automatic image stitching

DigiVFX



Automatic image stitching

DigiVFX



Automatic image stitching

DigiVFX

