Gradient domain operations

Digital Visual Effects

Yung-Yu Chuang

with slides by Fredo Durand, Ramesh Raskar, Amit Agrawal

Gradient Domain Manipulations

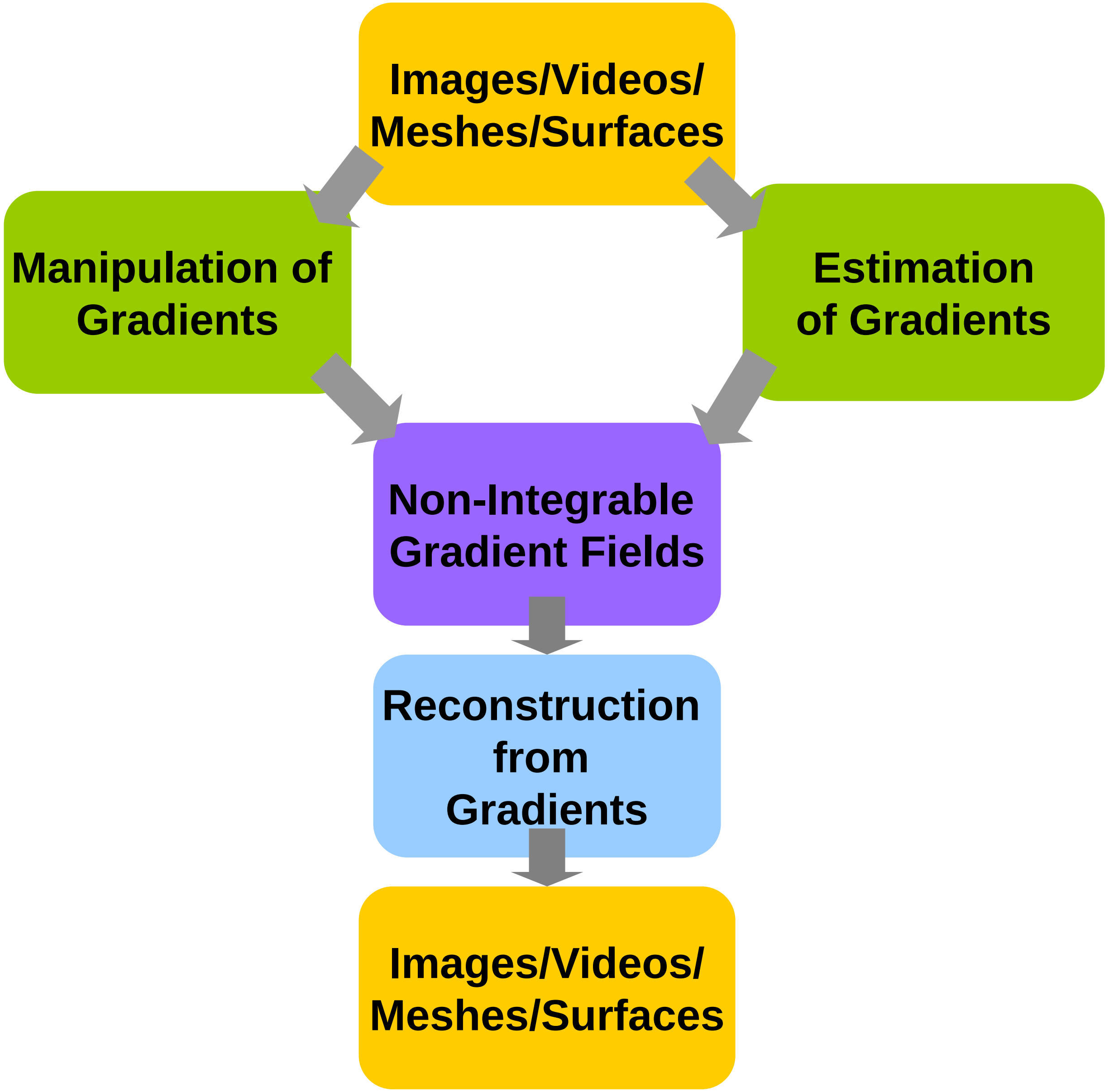
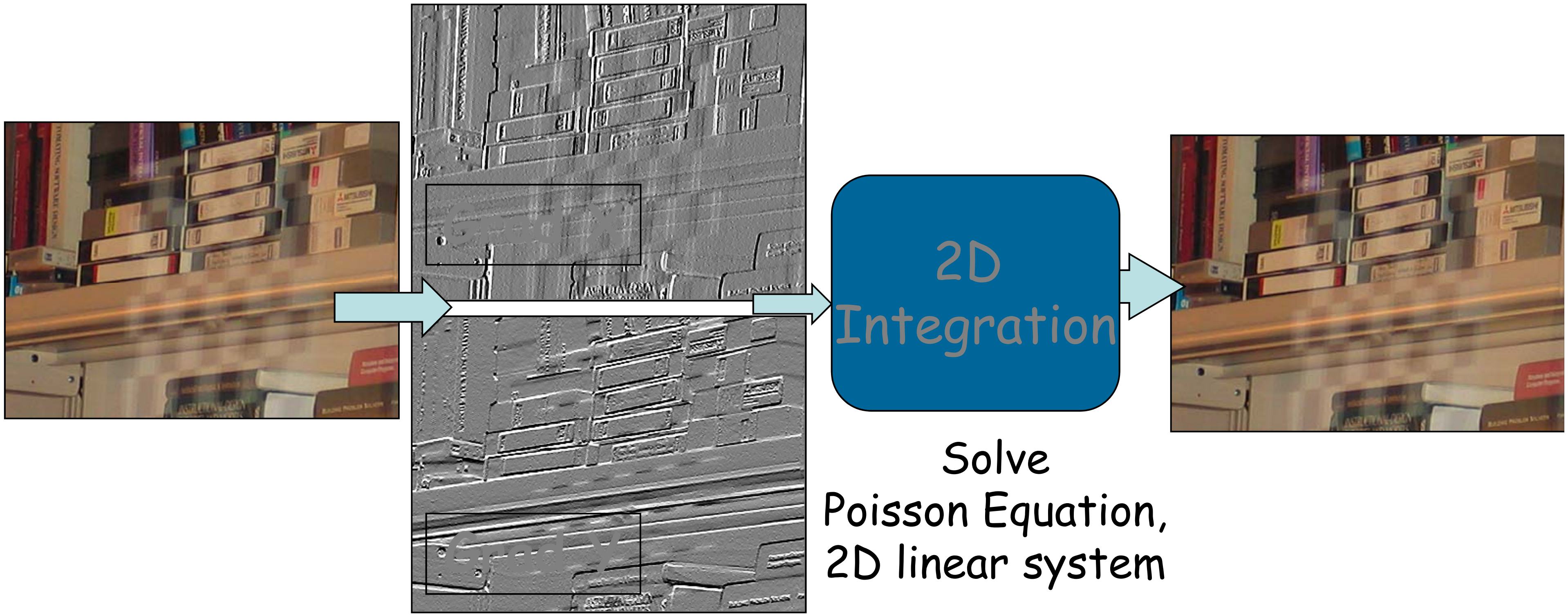
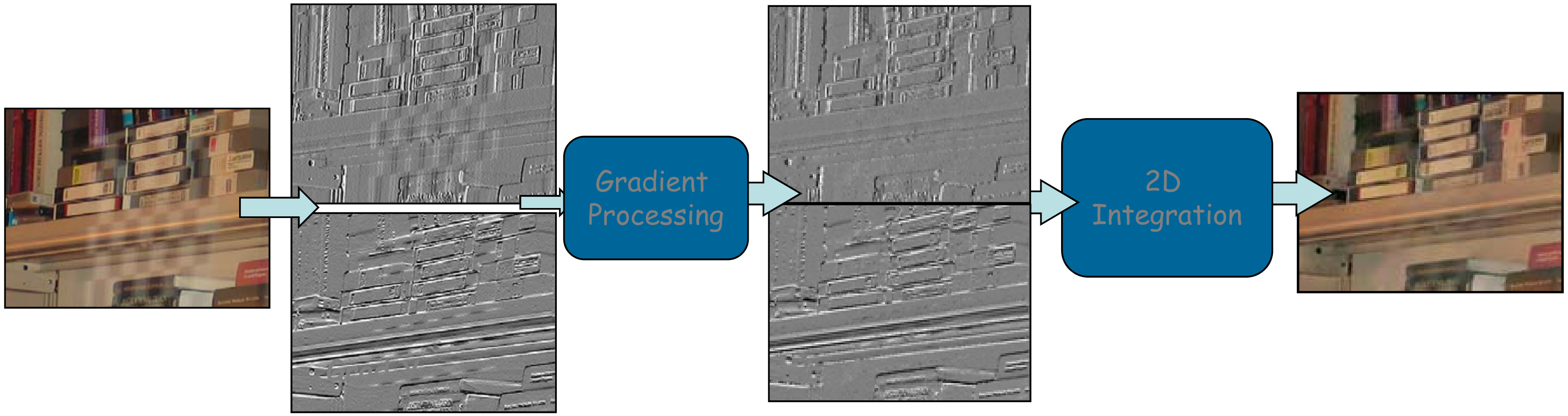


Image Intensity Gradients in 2D



Intensity Gradient Manipulation

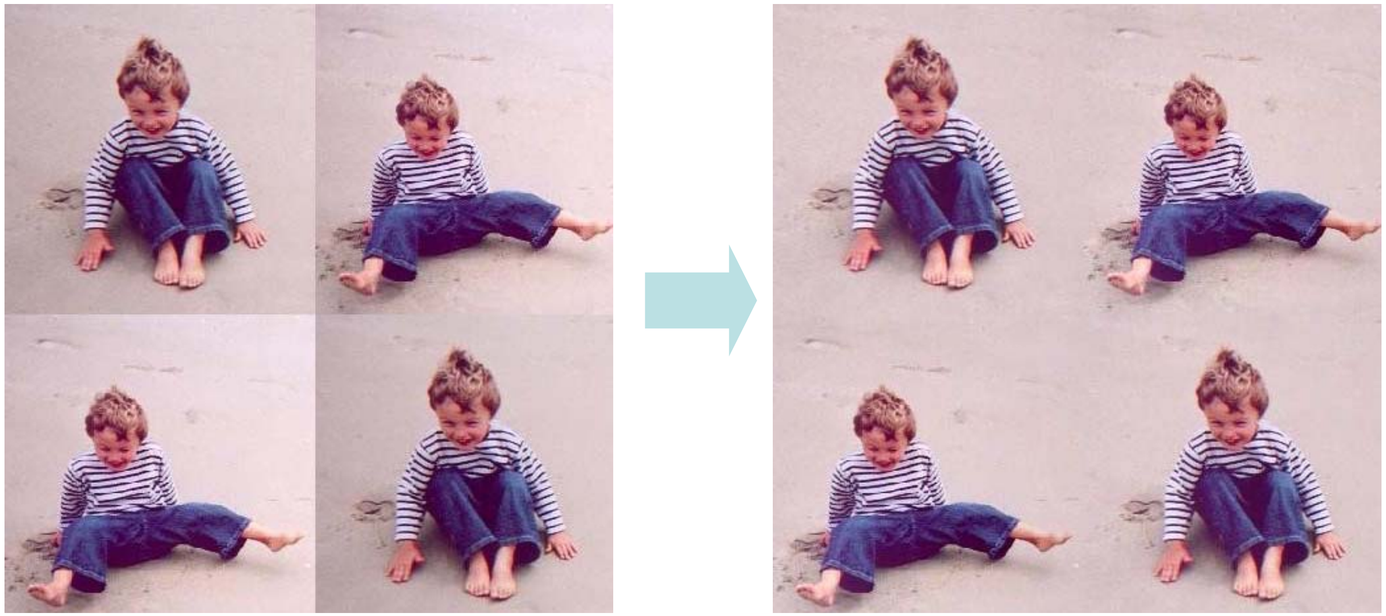
A Common Pipeline



1. Gradient manipulation
2. Reconstruction from gradients

Example Applications

Removing Glass Reflections



Seamless Image Stitching

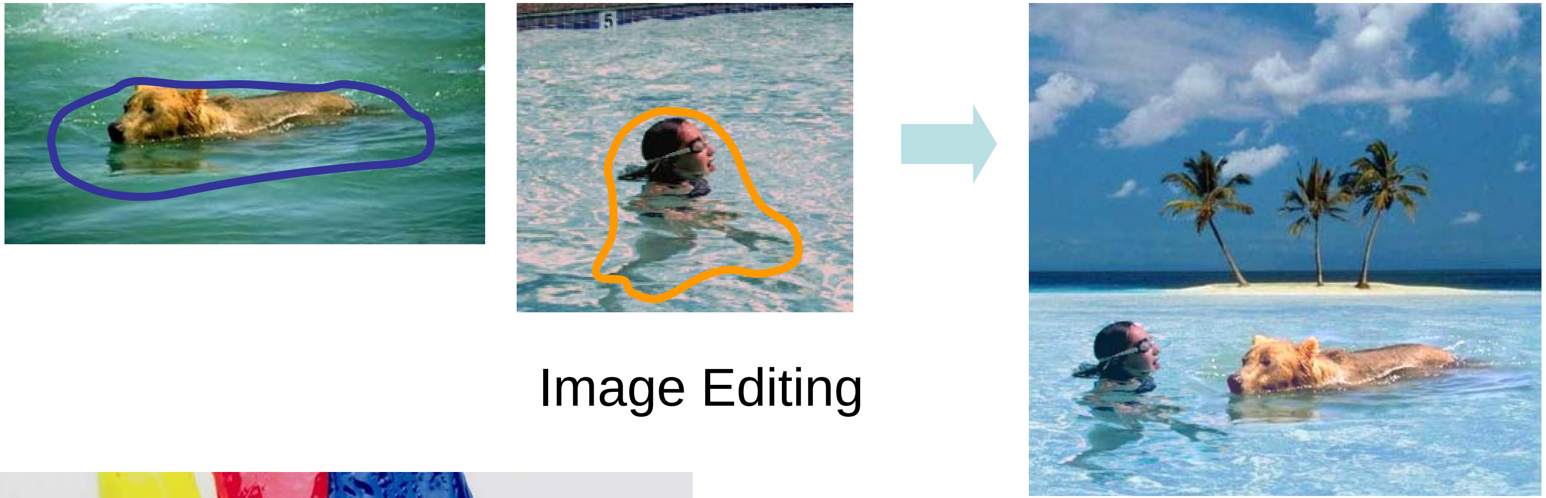
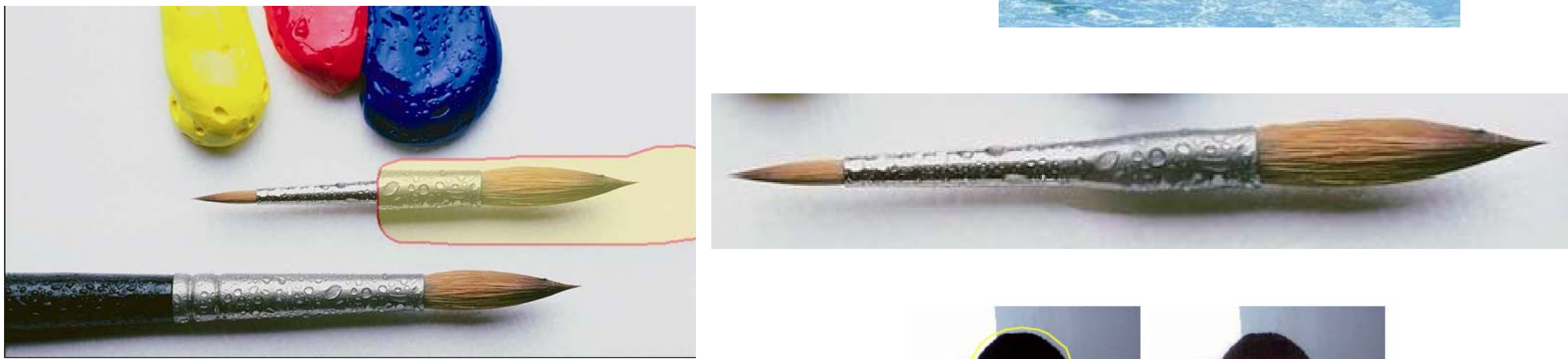
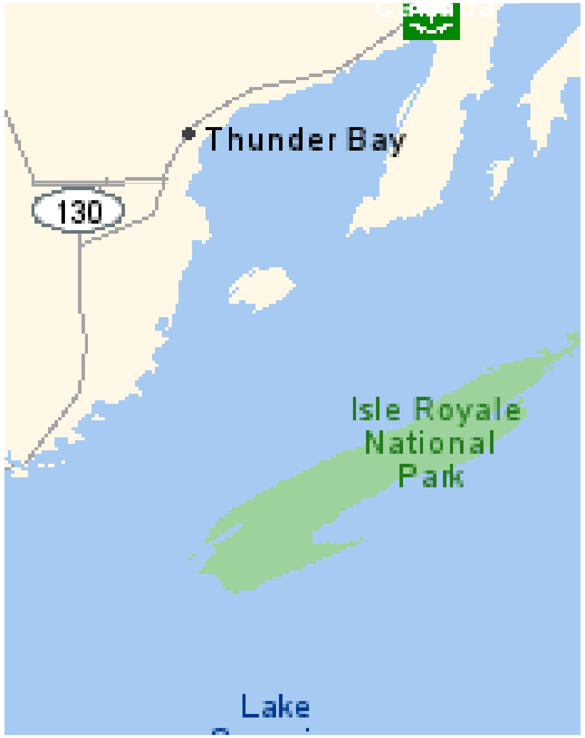


Image Editing

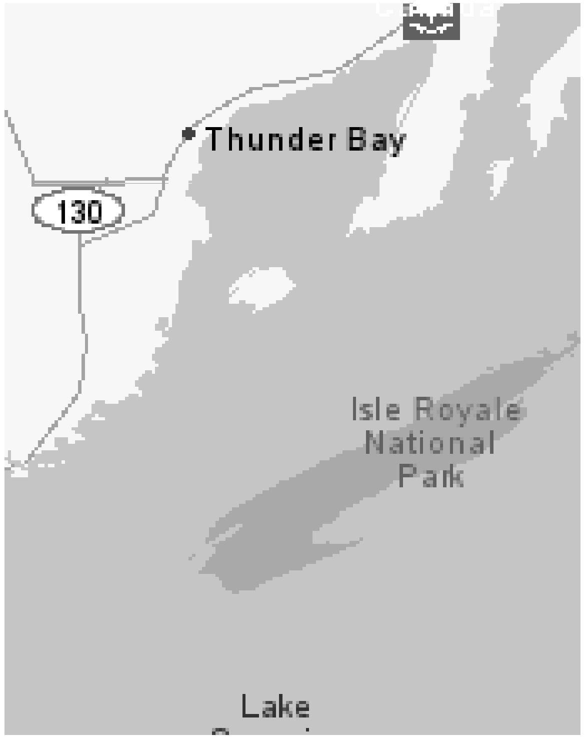


Changing Local Illumination



Original

PhotoshopGrey



Color2Gray

Color to Gray Conversion

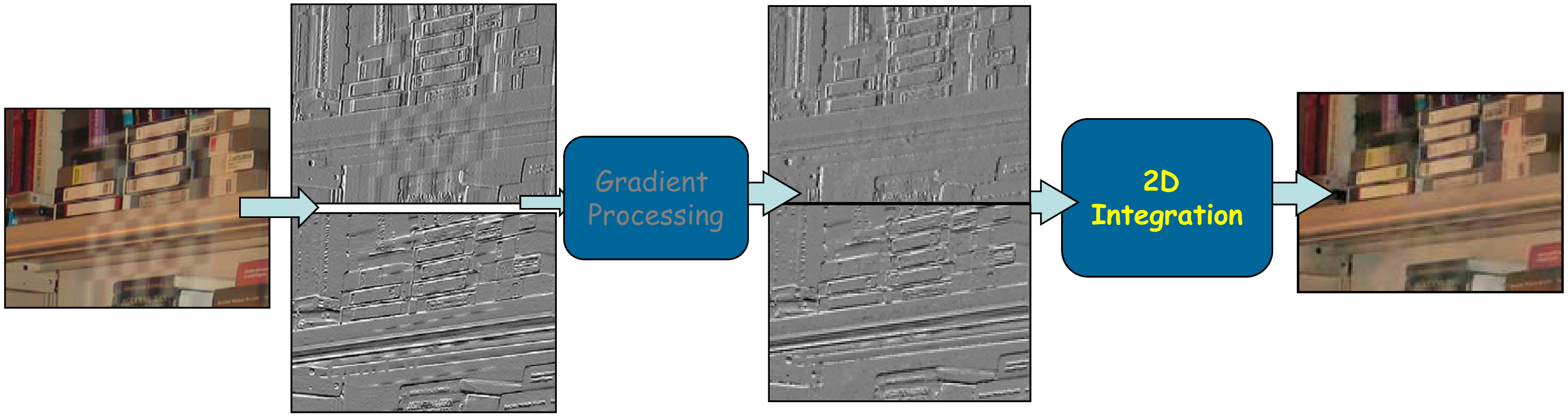
High Dynamic Range Compression

Edge Suppression under Significant Illumination Variations

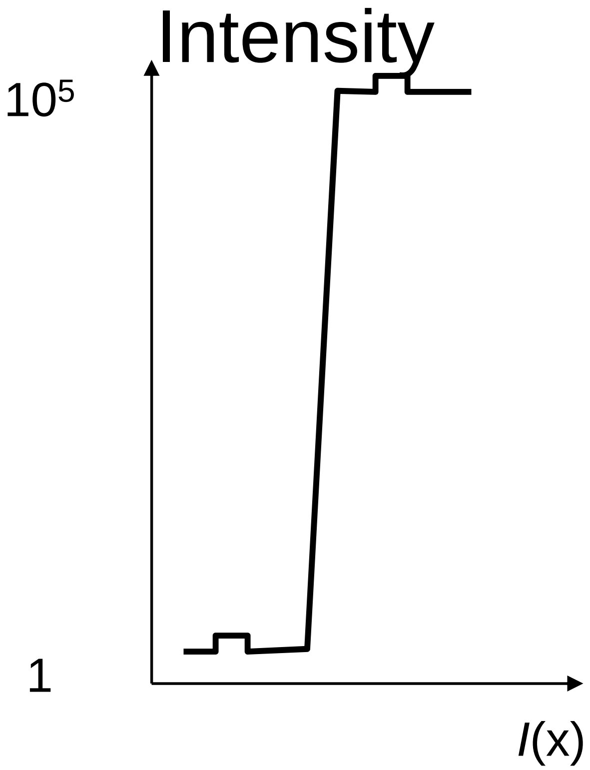
Fusion of day and night images

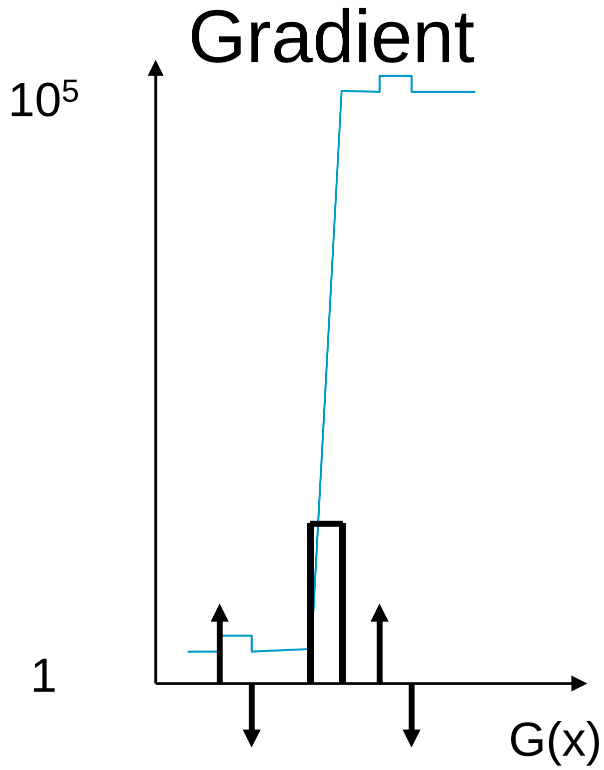
Intensity Gradient Manipulation

A Common Pipeline



Intensity Gradient in 1D



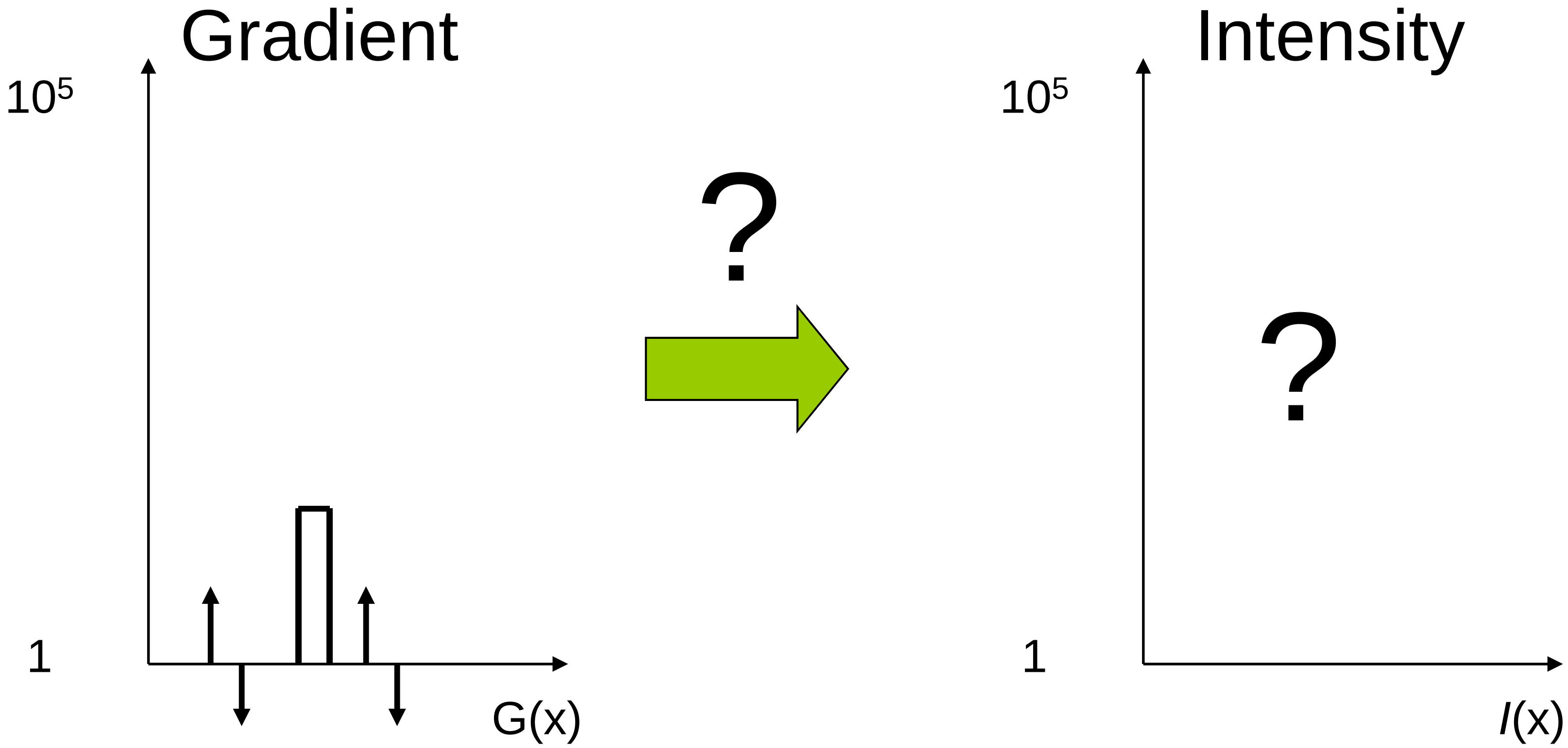


Gradient at x,

$$G(x) = I(x+1) - I(x)$$

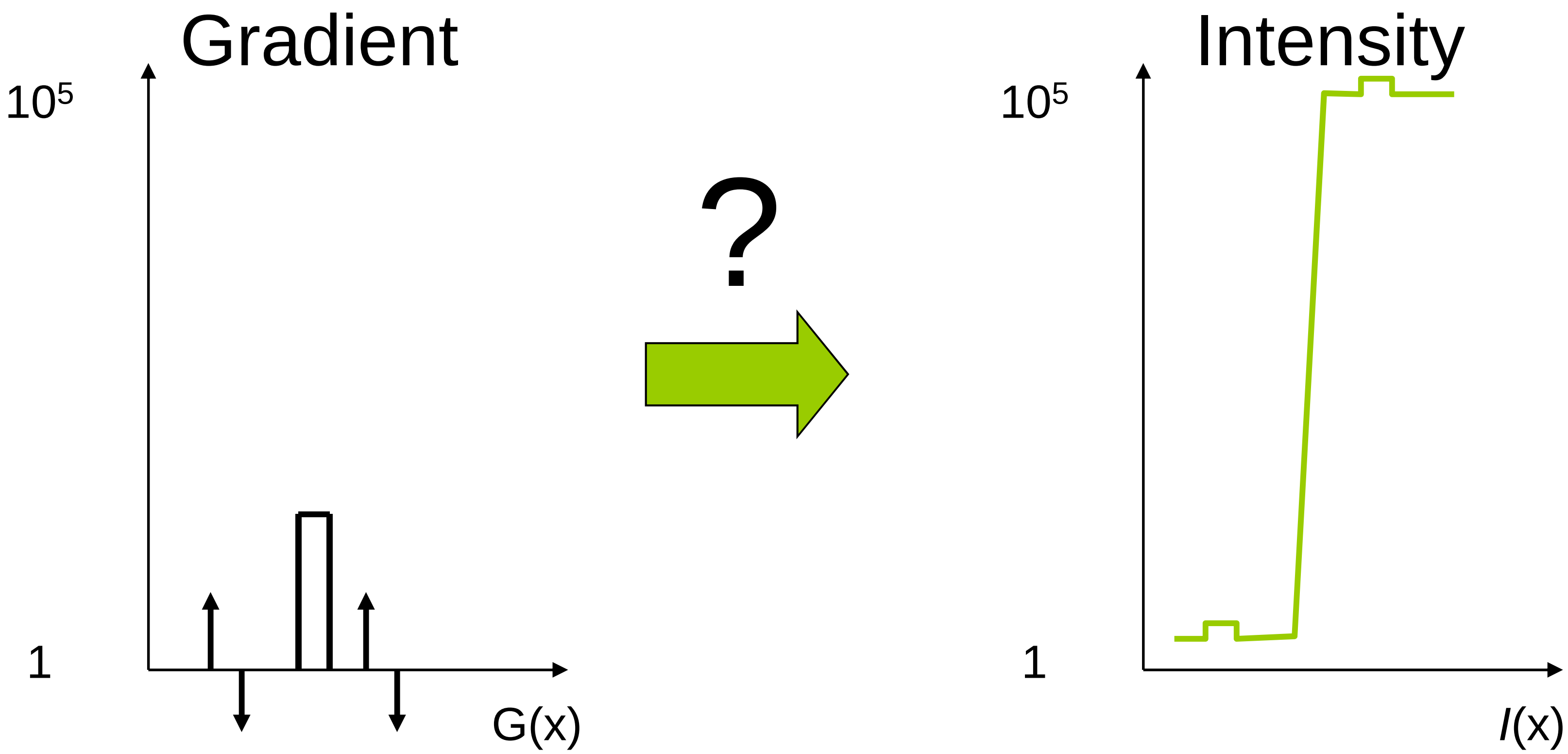
Forward Difference

Reconstruction from Gradients



For *n* intensity values, about *n* gradients

Reconstruction from Gradients



1D Integration

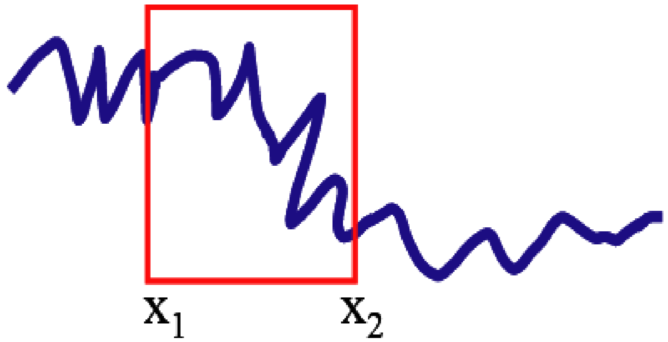
$$I(x) = I(x-1) + G(x)$$

Cumulative sum

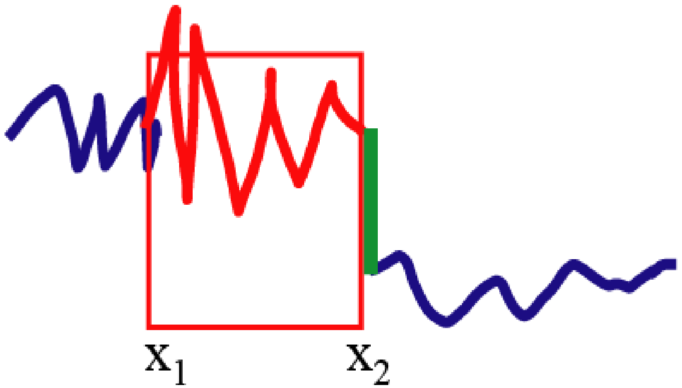
1D case with constraints

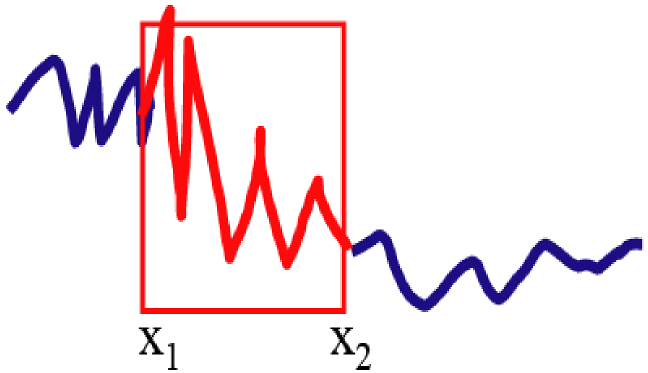
Seamlessly paste

onto



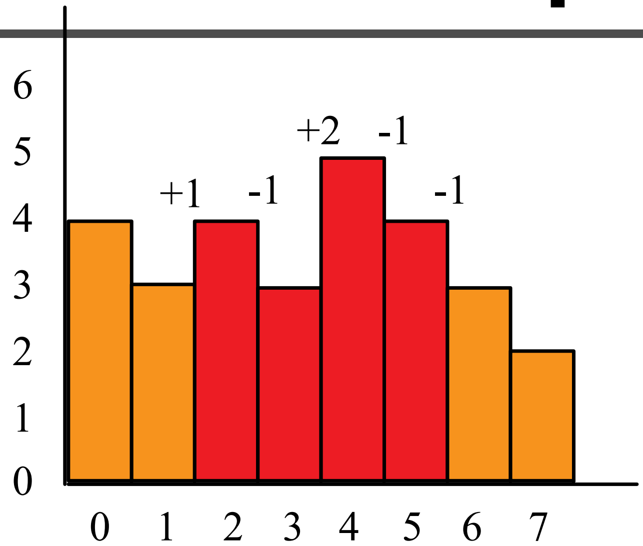
Just add a linear function so that the boundary condition is respected



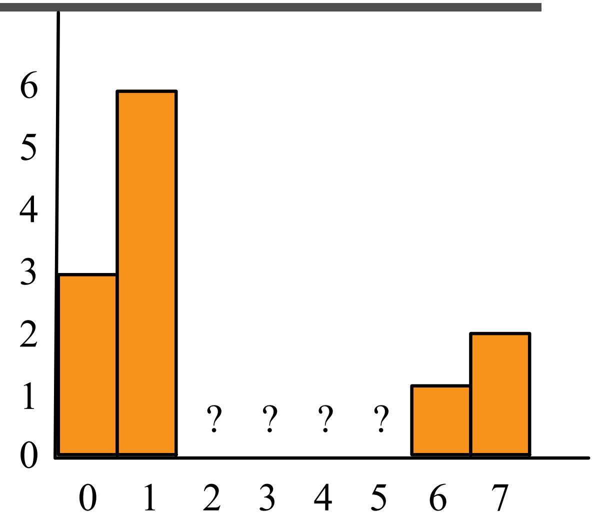


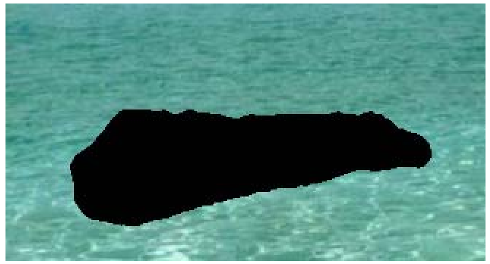
Discrete 1D example: minimization

- Copy



to

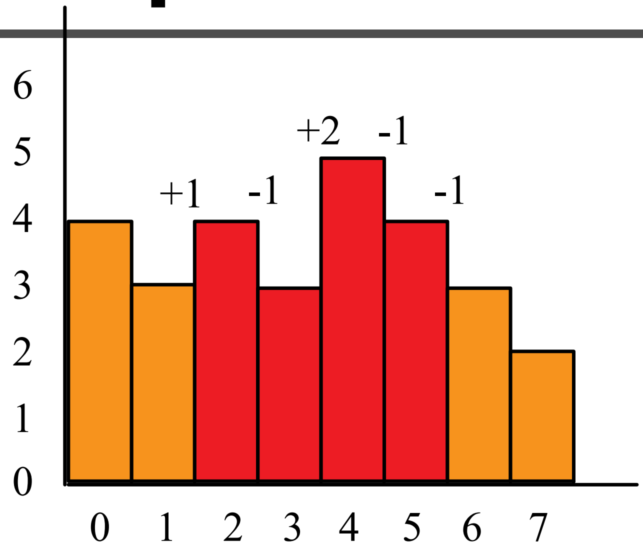
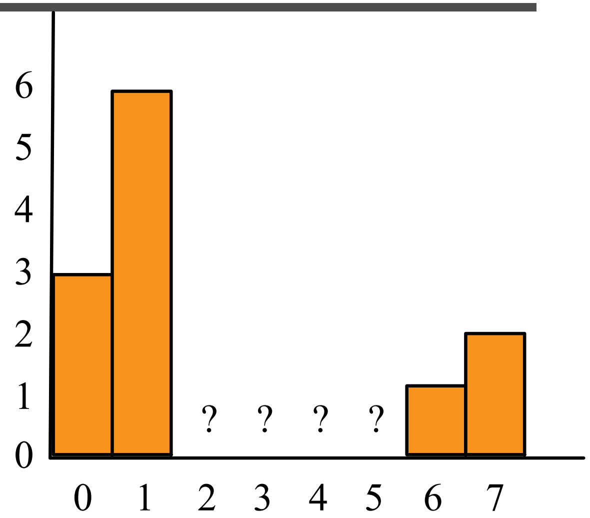




- Min $((f_2-f_1)-1)^2$
- Min $((f_3-f_2)-(-1))^2$
- Min $((f_4-f_3)-2)^2$
- Min $((f_5-f_4)-(-1))^2$
- Min $((f_6-f_5)-(-1))^2$

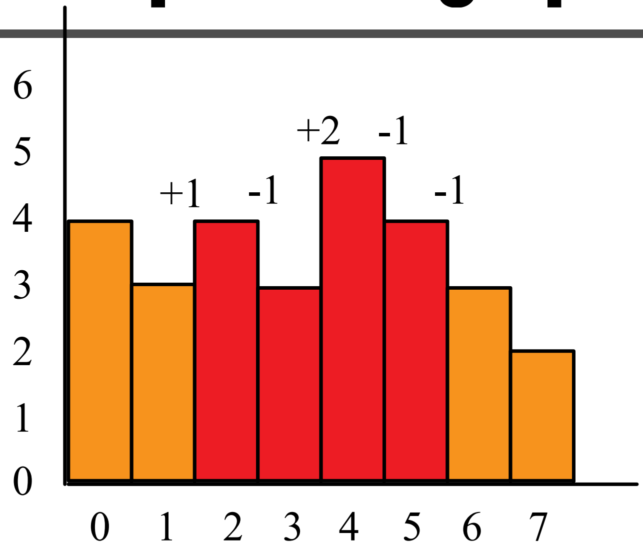
With
$f_1=6$
$f_6=1$

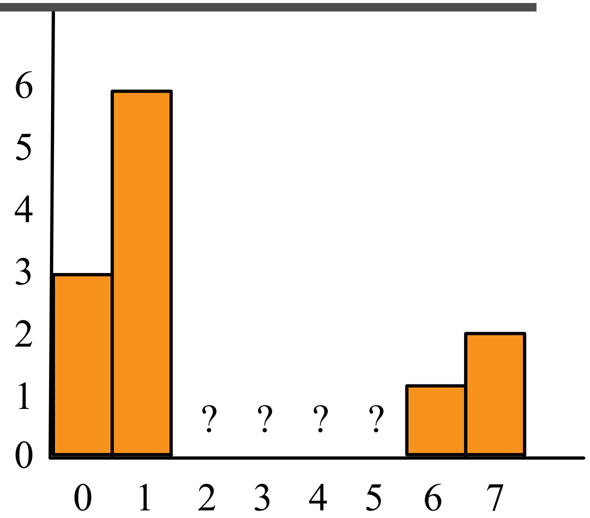
1D example: minimization

- Copy  to 

- Min $((f_2-6)-1)^2$ ==> $f_2^2+49-14f_2$
- Min $((f_3-f_2)-(-1))^2$ ==> $f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$
- Min $((f_4-f_3)-2)^2$ ==> $f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$
- Min $((f_5-f_4)-(-1))^2$ ==> $f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$
- Min $((1-f_5)-(-1))^2$ ==> $f_5^2+4-4f_5$

1D example: big quadratic

- Copy 

to 

- Min $(f_2^2+49-14f_2$

  $+ f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$

  $+ f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$

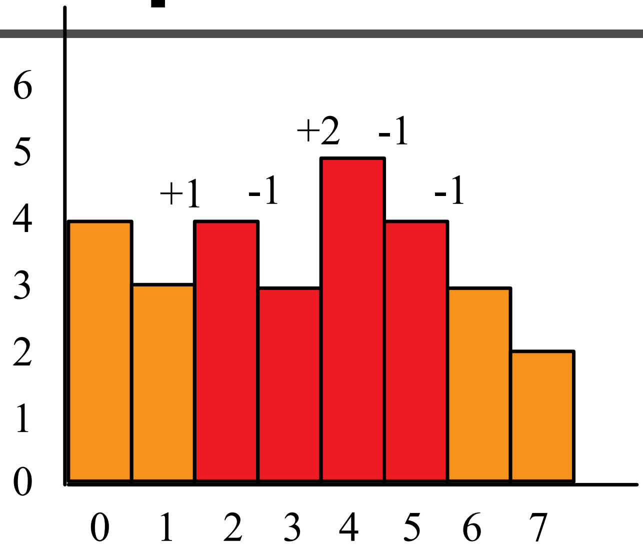
  $+ f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$

  $+ f_5^2+4-4f_5)$

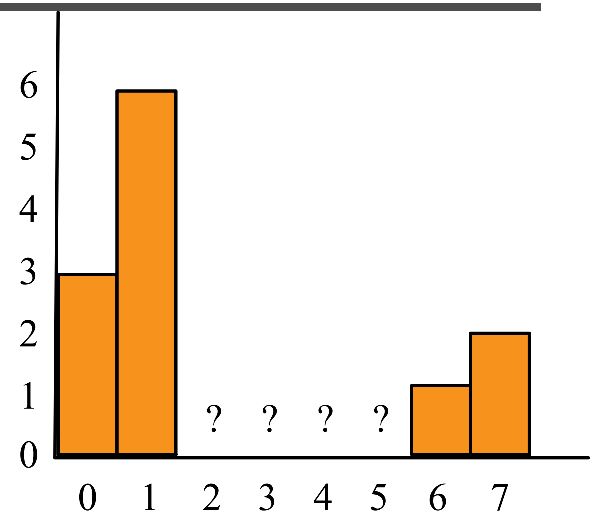
  Denote it Q

1D example: derivatives

- Copy



to



Min $(f_2^2+49-14f_2$

$+ f_3^2+f_2^2+1-2f_3f_2 +2f_3-2f_2$

$+ f_4^2+f_3^2+4-2f_3f_4 -4f_4+4f_3$

$+ f_5^2+f_4^2+1-2f_5f_4 +2f_5-2f_4$

$+ f_5^2+4-4f_5)$

Denote it Q

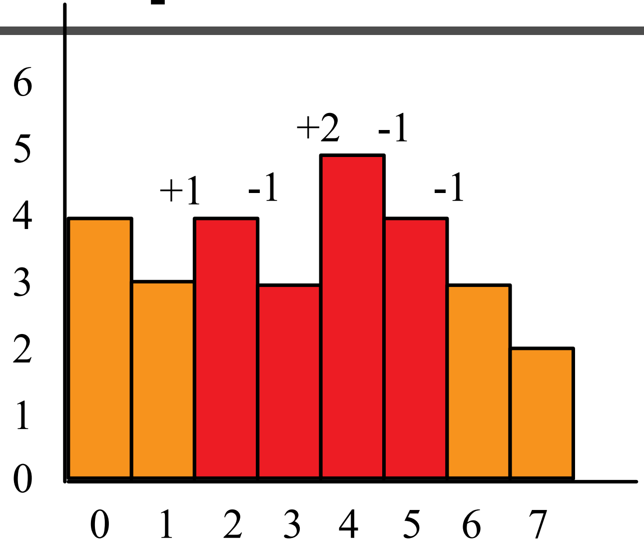
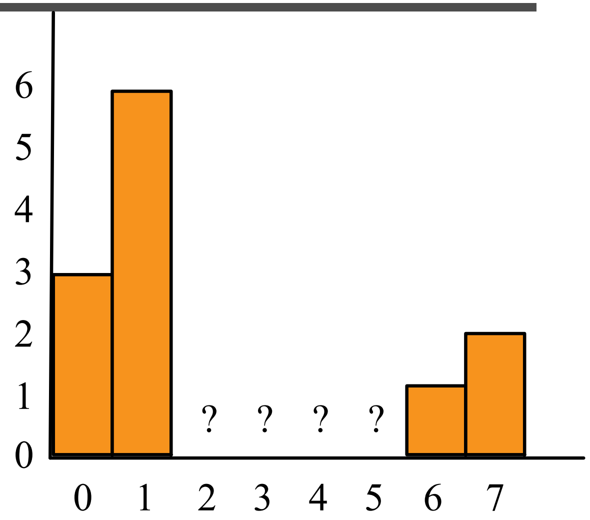
$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$

$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$

$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$

$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

1D example: set derivatives to zero

- Copy  to 

$$\frac{dQ}{df_2} = 2f_2 + 2f_2 - 2f_3 - 16$$

$$\frac{dQ}{df_3} = 2f_3 - 2f_2 + 2 + 2f_3 - 2f_4 + 4$$

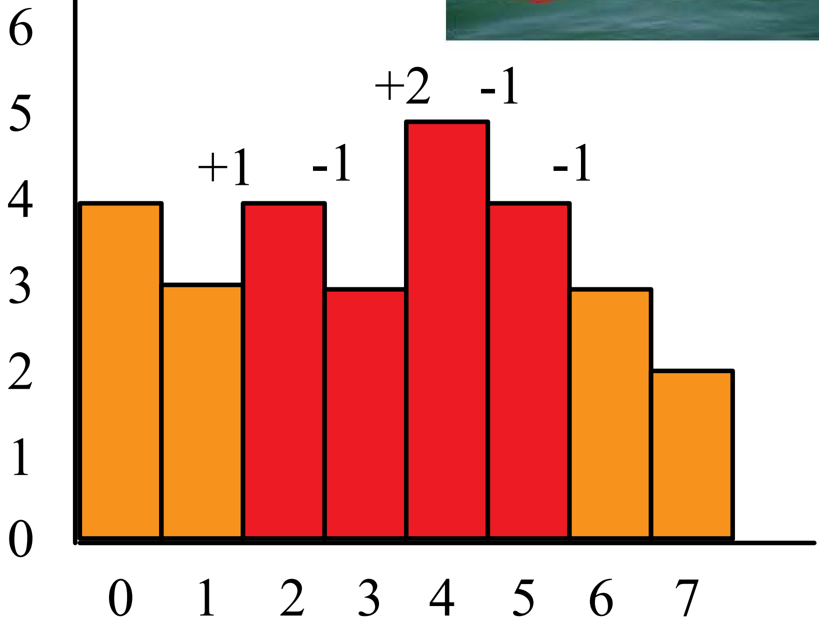
$$\frac{dQ}{df_4} = 2f_4 - 2f_3 - 4 + 2f_4 - 2f_5 - 2$$

$$\frac{dQ}{df_5} = 2f_5 - 2f_4 + 2 + 2f_5 - 4$$

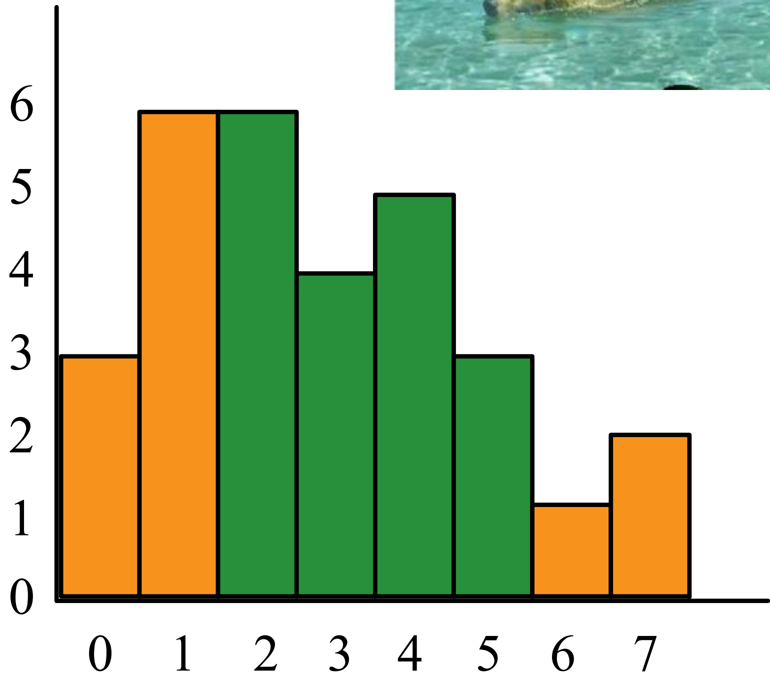
$$==> \begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

1D example

- Copy



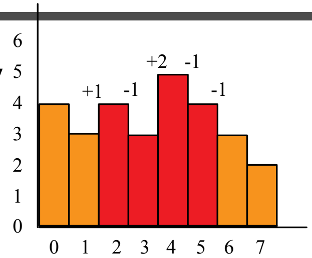
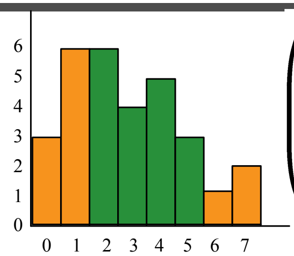
to



$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \\ 3 \end{pmatrix}$$

1D example: remarks

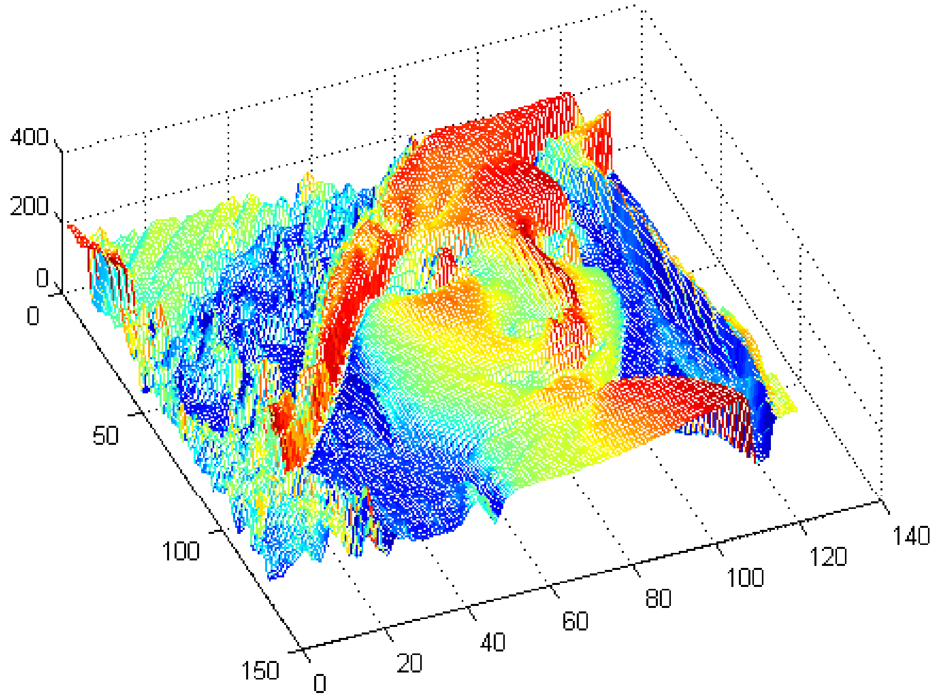
- Copy  to 

$$\begin{pmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \\ 6 \\ 2 \end{pmatrix}$$

- Matrix is sparse
- Matrix is symmetric
- Everything is a multiple of 2
  - because square and derivative of square
- Matrix is a convolution (kernel -2 4 -2)
- Matrix is independent of gradient field. Only RHS is
- Matrix is a second derivative

2D example: images

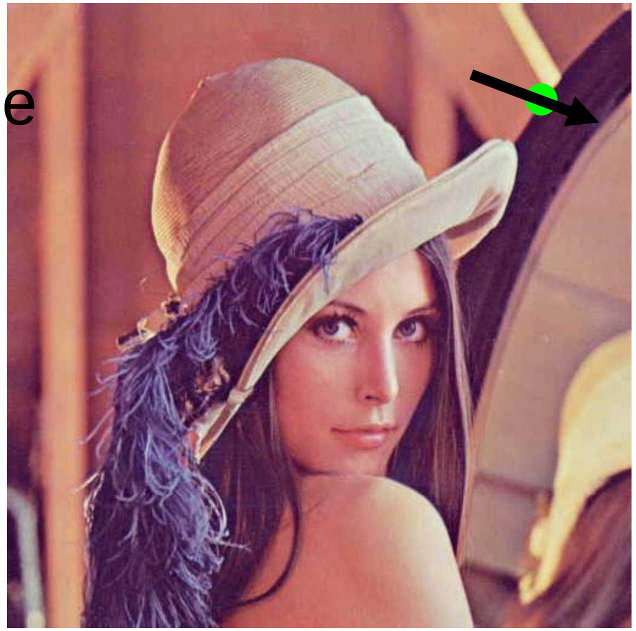
- Images as scalar fields
  - R^2 -> R

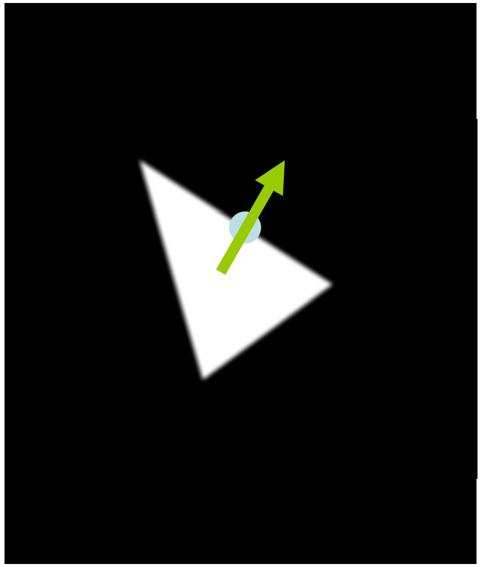


Gradients

- Vector field (gradient field)
  - Derivative of a scalar field

- Direction
  - Maximum rate of change of scalar field
- Magnitude
  - Rate of change





Gradient Field

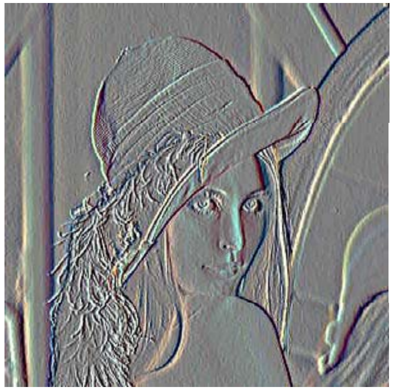
- Components of gradient
  - Partial derivatives of scalar field

$$I(x, y) \qquad \nabla I = \{\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\}$$

$$I(x, y, t) \qquad \nabla I = \{\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}, \frac{\partial I}{\partial t}\}$$

Example

Image
I(x,y)



I_x

I_y

Gradient at x,y as Forward Differences

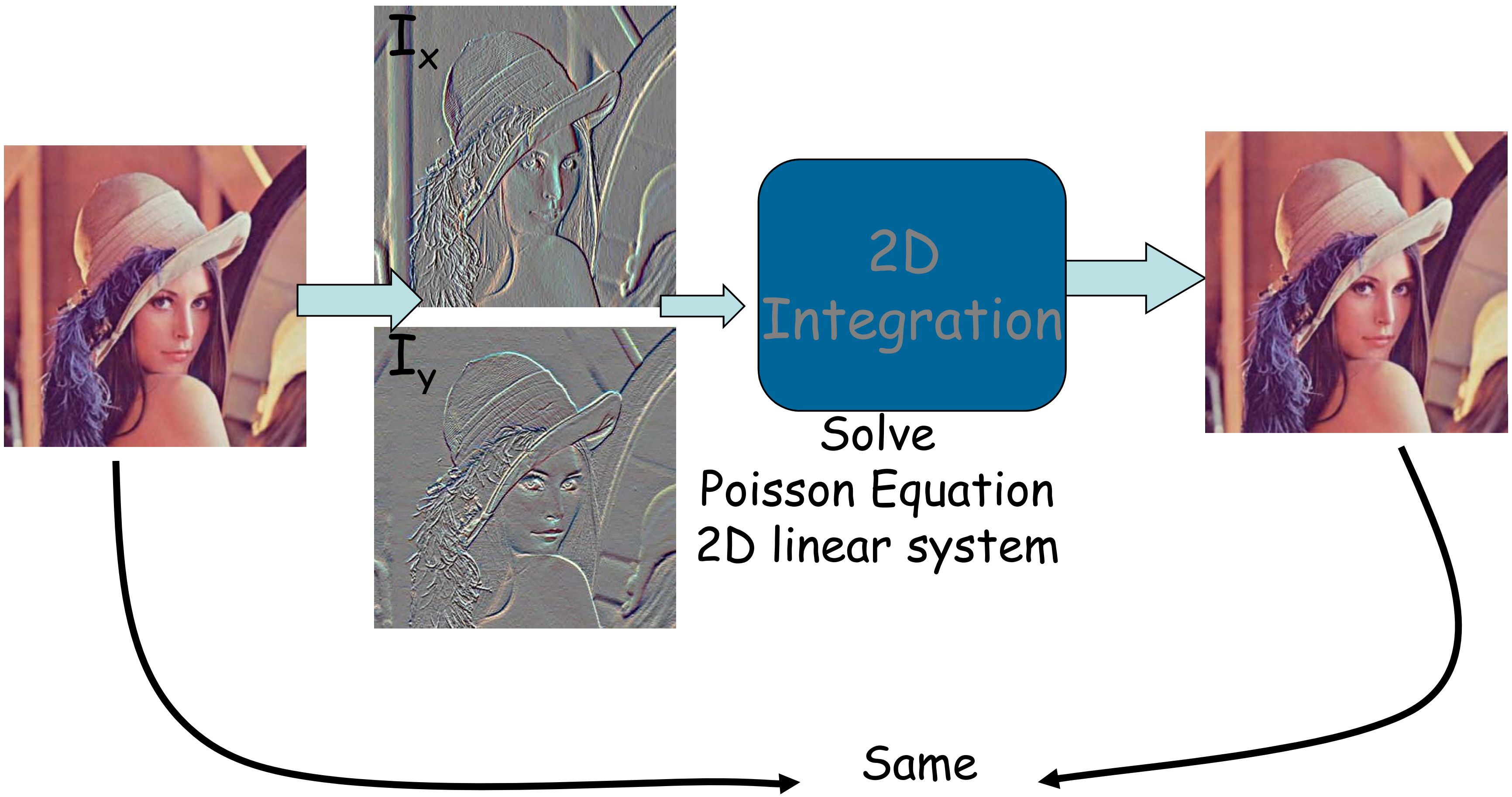
$$G_x(x,y) = I(x+1\ ,\ y) - I(x,y)$$

$$G_y(x,y) = I(x\ ,\ y+1) - I(x,y)$$

$$G(x,y) = (G_x\ ,\ G_y)$$

Reconstruction from Gradients

Sanity Check:
Recovering Original Image

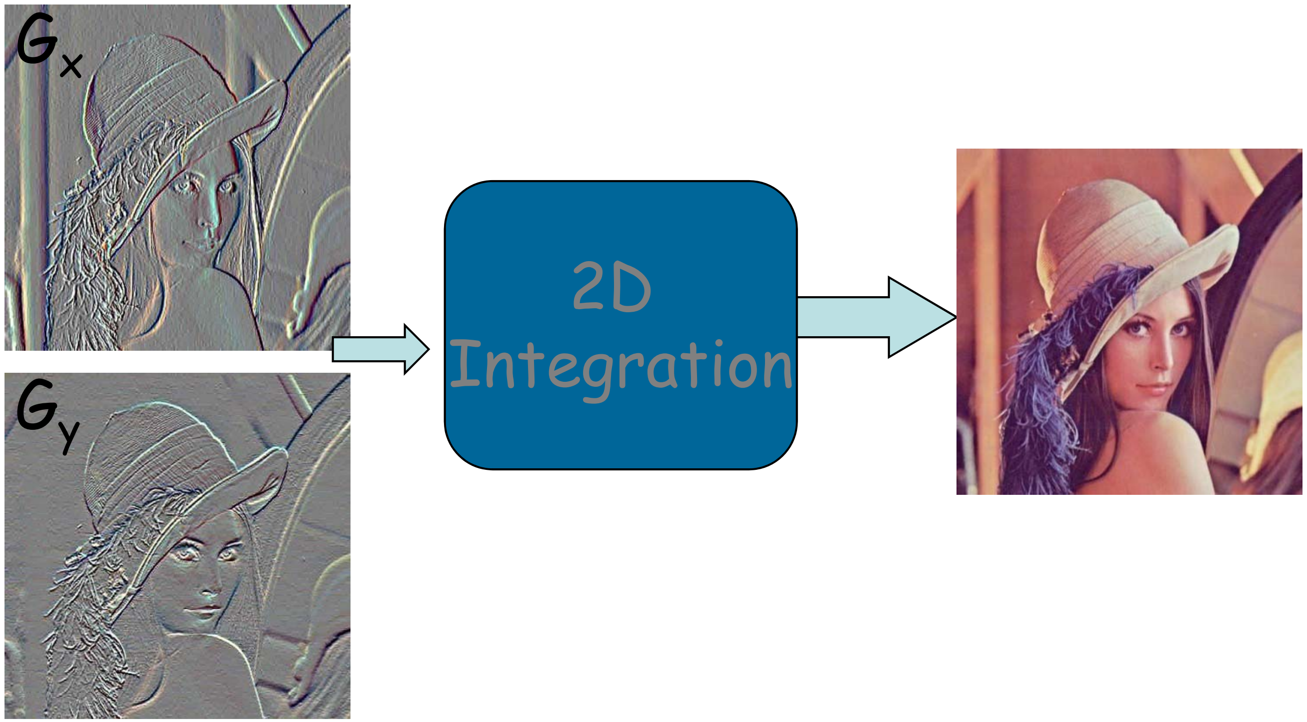


Reconstruction from Gradients

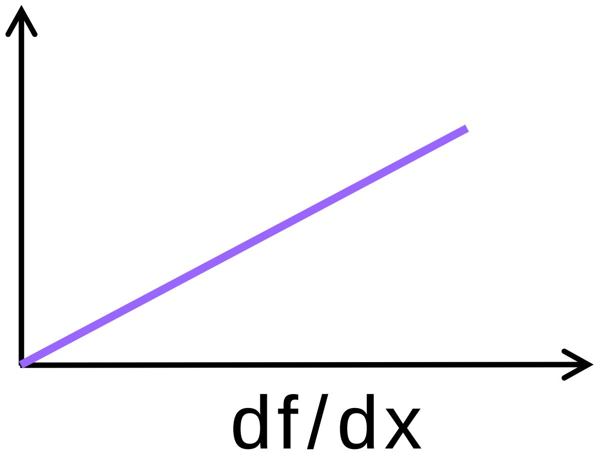
Given $G(x,y) = (G_x , G_y)$

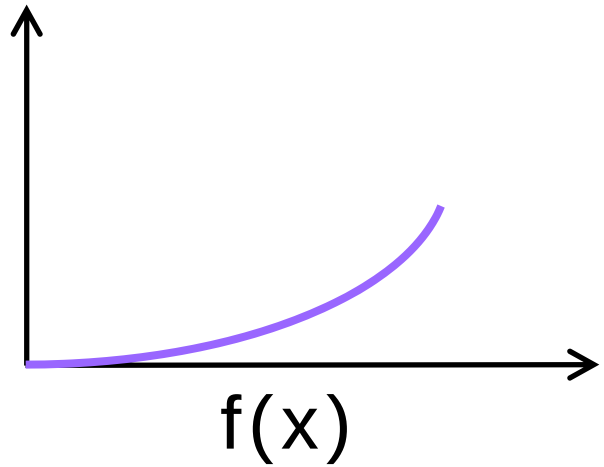
How to compute $I(x,y)$ for the image ?

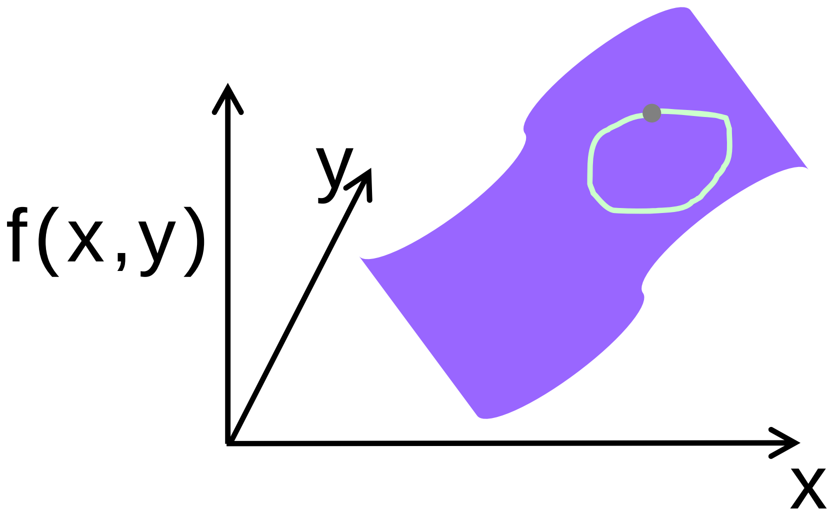
For n^2 image pixels, $2n^2$ gradients !



2D Integration is non-trivial







Reconstruction depends on chosen path

Reconstruction from Gradient Field G

- Look for image ***I*** with gradient closest to ***G*** in the least squares sense.
- ***I*** minimizes the integral: $\iint F(\nabla I, G)\,dxdy$

$$F(\nabla I, G) = \|\nabla I - G\|^2 = \left(\frac{\partial I}{\partial x} - G_x\right)^2 + \left(\frac{\partial I}{\partial y} - G_y\right)^2$$

$$\Rightarrow \quad \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

Solve $$\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y}$$

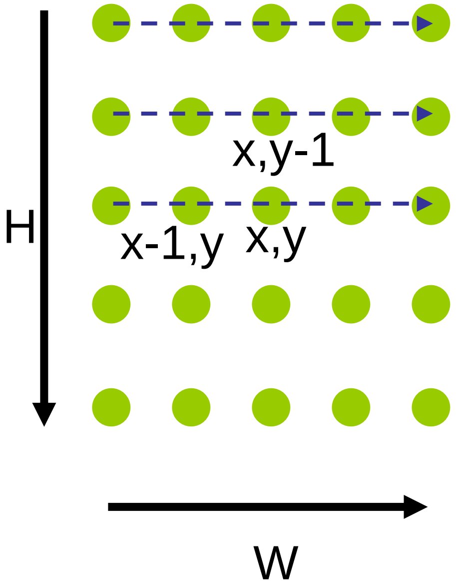
$$G_x(x, y) - G_x(x-1, y) + G_y(x, y) - G_y(x, y-1)$$

$$I(x+1, y) + I(x-1, y) + I(x, y+1) + I(x, y-1) - 4I(x, y)$$

$$\begin{bmatrix} \\ ..\ 1 \ldots\ \ 1\ {-4}\ 1 \ldots\ \ 1\ .. \\ \\ \\ \end{bmatrix} \begin{bmatrix} \\ \mathbf{I} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Linear System

$$-4I(x,y)+I(x,y+1)+I(x,y-1)+I(x+1,y)+I(x-1,y)=u(x,y)$$



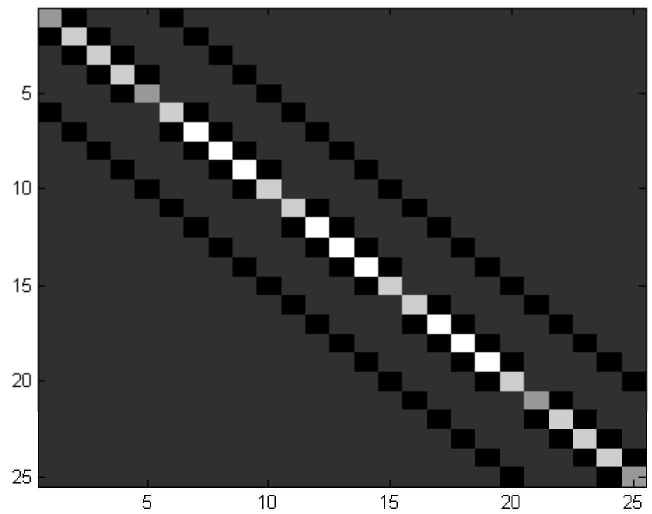
$$\underbrace{\left[\; . \;\; \overbrace{1 \;\; . \;\; . \;\; . \;\; 1}^{H} \;\; -4 \;\; 1 \;\; . \;\; . \;\; . \;\; 1 \;\; . \;\right]}_{A} \underbrace{\begin{bmatrix} . \\ . \\ I(x-1,y) \\ . \\ . \\ . \\ I(x,y-1) \\ I(x,y) \\ I(x,y+1) \\ . \\ . \\ . \\ I(x+1,y) \\ . \\ . \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} . \\ . \\ . \\ . \\ . \\ . \\ . \\ u(x,y) \\ . \\ . \\ . \\ . \\ . \\ . \\ . \end{bmatrix}}_{b}$$

(The first H bracket spans the entries from 1 to 1 in A; the second H bracket spans the x entries from $I(x-1,y)$ to $I(x,y)$.)

Sparse Linear system

$$
\begin{bmatrix}
 & & 1 & -4 & 1 & & & & 1 & & & & \\
 & & & 1 & -4 & 1 & & & & 1 & & & \\
1 & & & & 1 & -4 & 1 & & & & 1 & & \\
 & 1 & & & & 1 & -4 & 1 & & & & 1 & \\
 & & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & & 1 & & & & 1 & -4 & 1 & & & \\
 & & & & 1 & & & & 1 & -4 & 1 & &
\end{bmatrix}
$$

A matrix



Solving Linear System

- Image size N*N
- Size of A ~ N^2 by N^2
- Impractical to form and store A

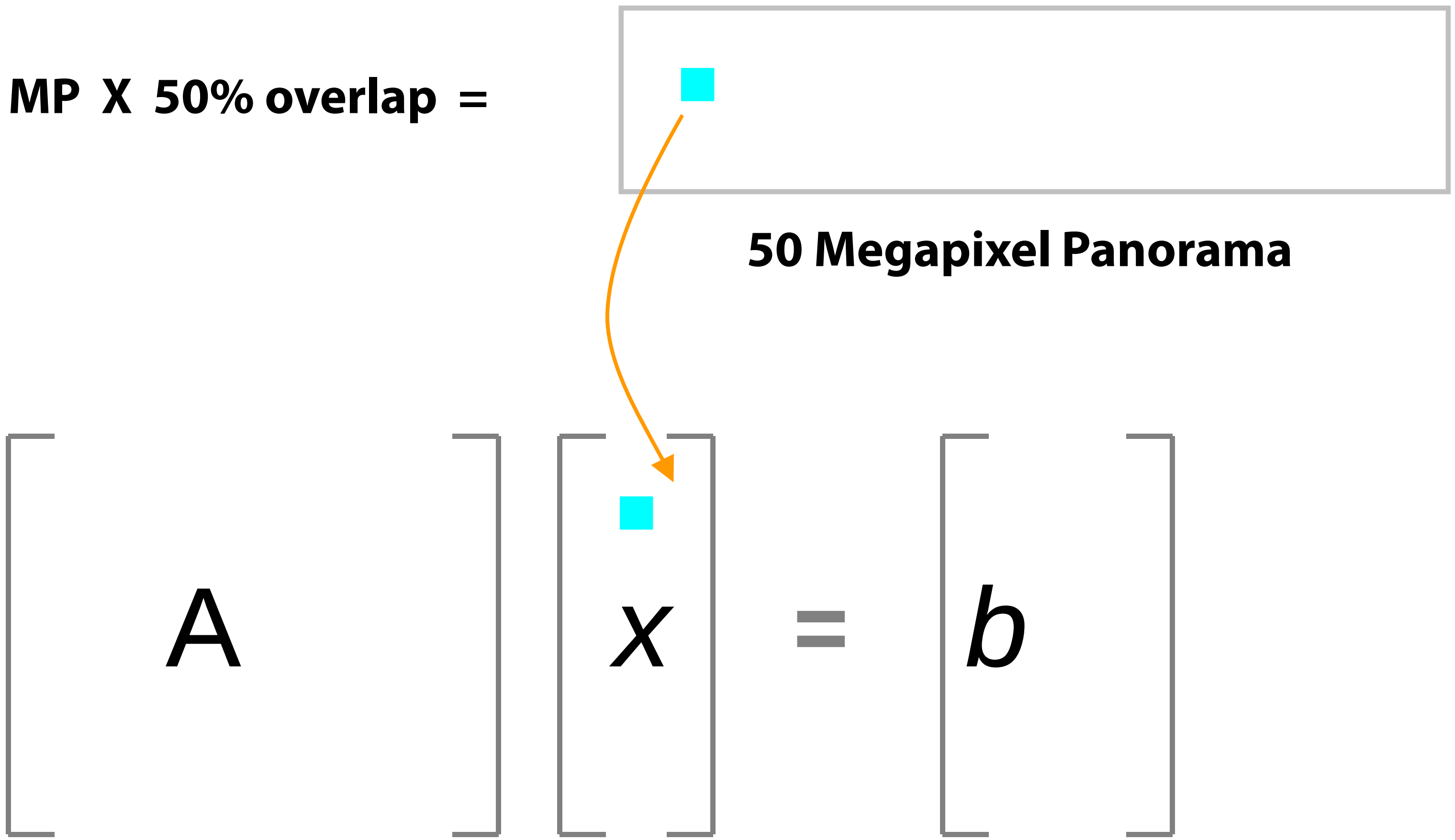
- Direct Solvers
- Basis Functions
- Multigrid
- Conjugate Gradients

Approximate Solution for Large Scale Problems

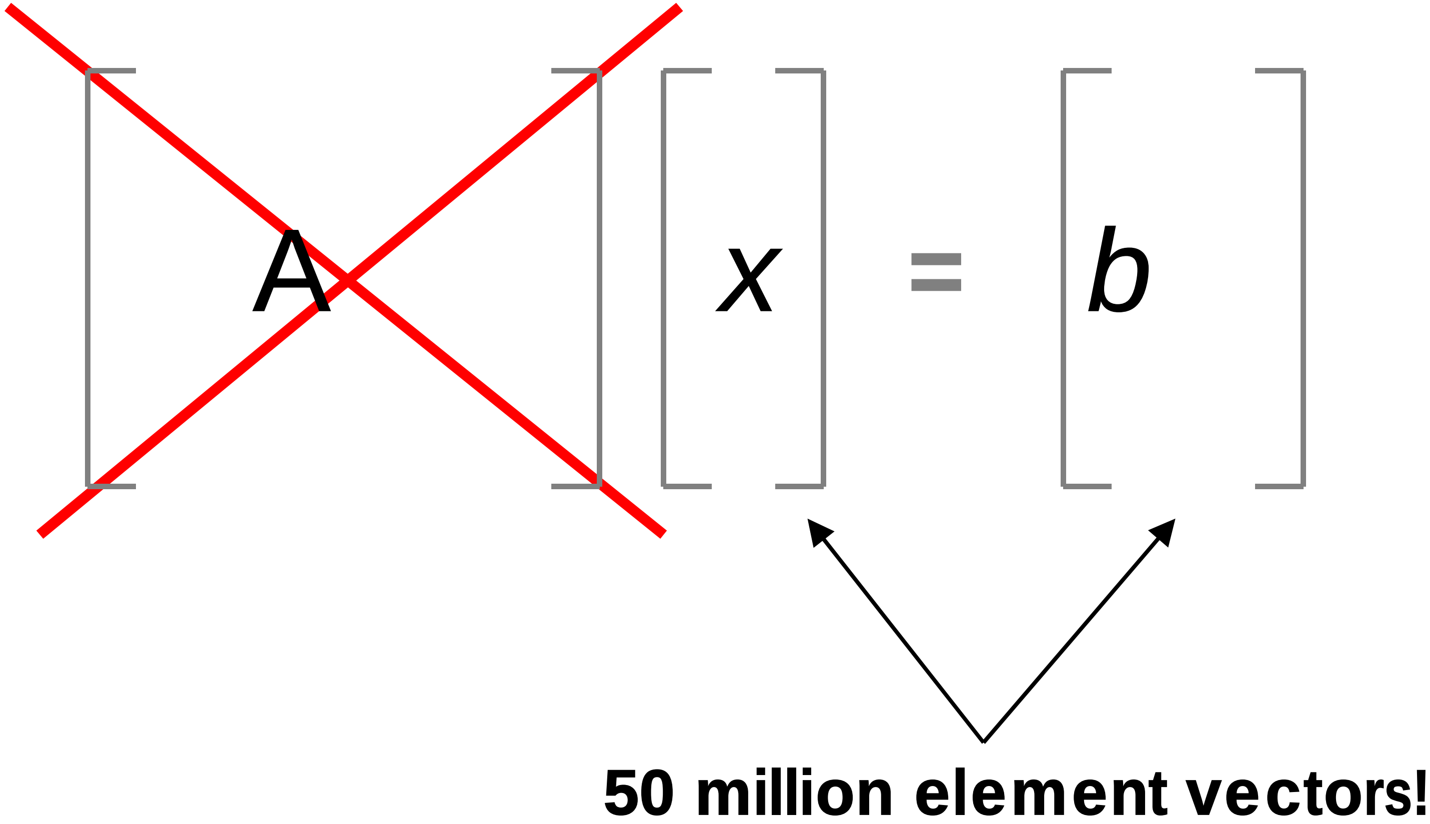
- Resolution is increasing in digital cameras
- Stitching, Alignment requires solving large linear system

Scalability problem

10 X 10 MP X 50% overlap =



Scalability problem



Approximate Solution

- Reduce size of linear system
- Handle high resolution images

- Part of Photoshop CS3

Aseem Agarwala. "Efficient gradient-domain compositing using quadtrees," ACM Transactions on Graphics (Proceedings of SIGGRAPH 2007)

The key insight

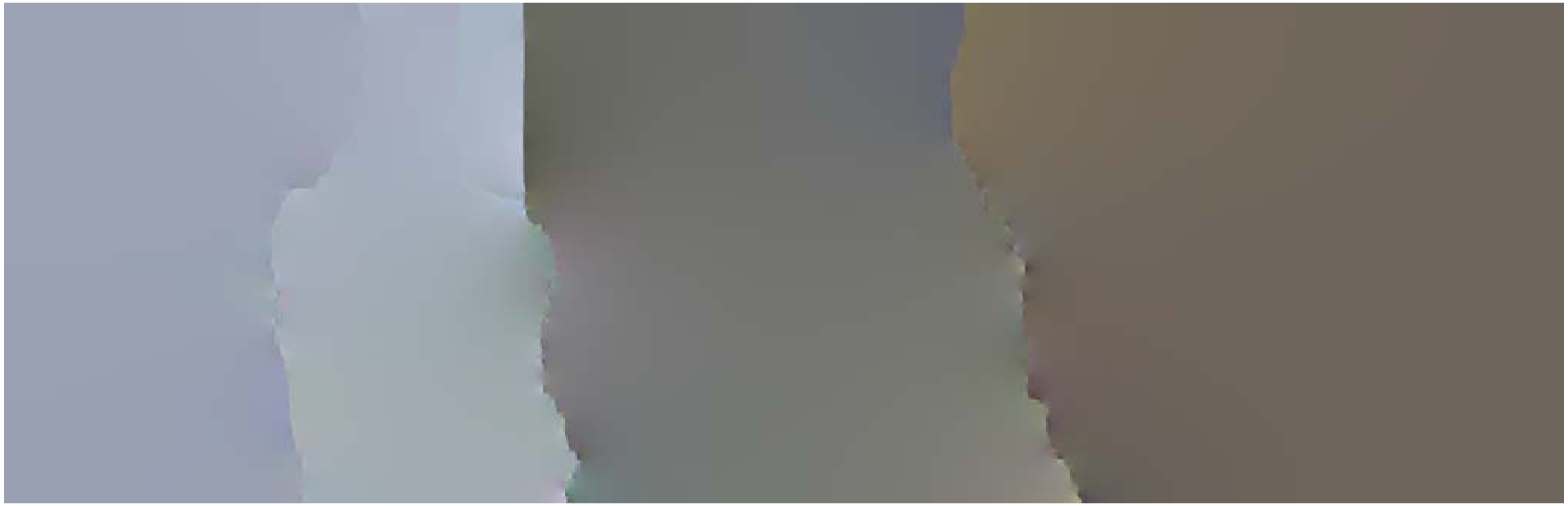
Desired solution x

−

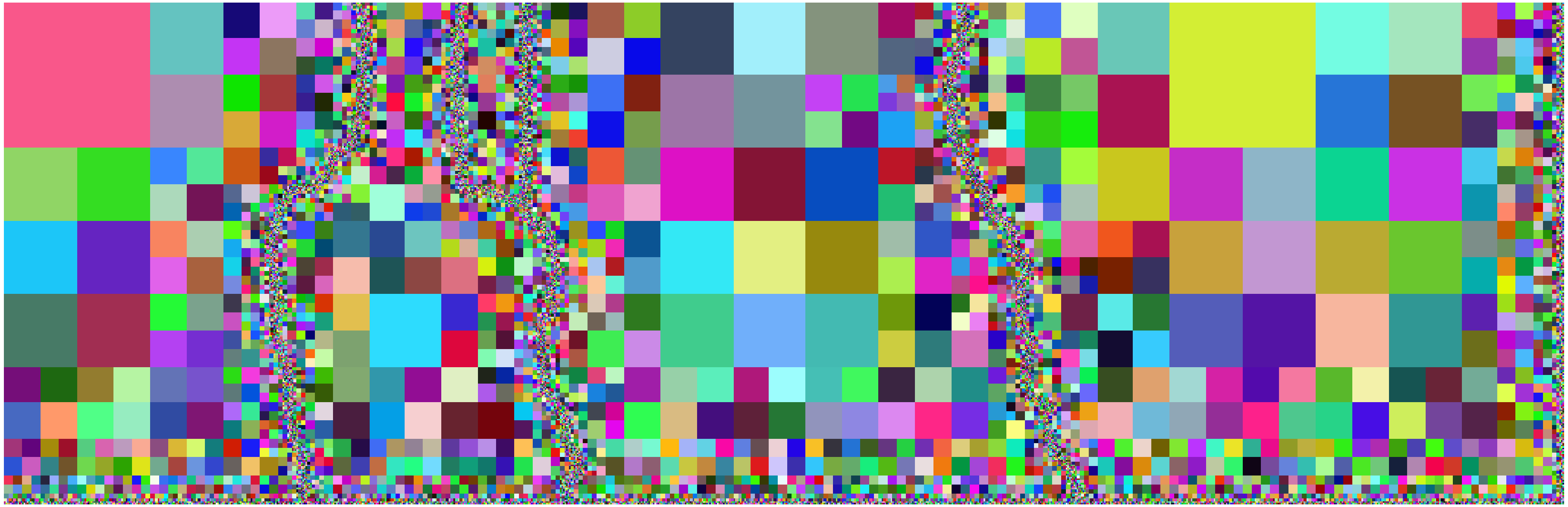
Initial Solution x_0

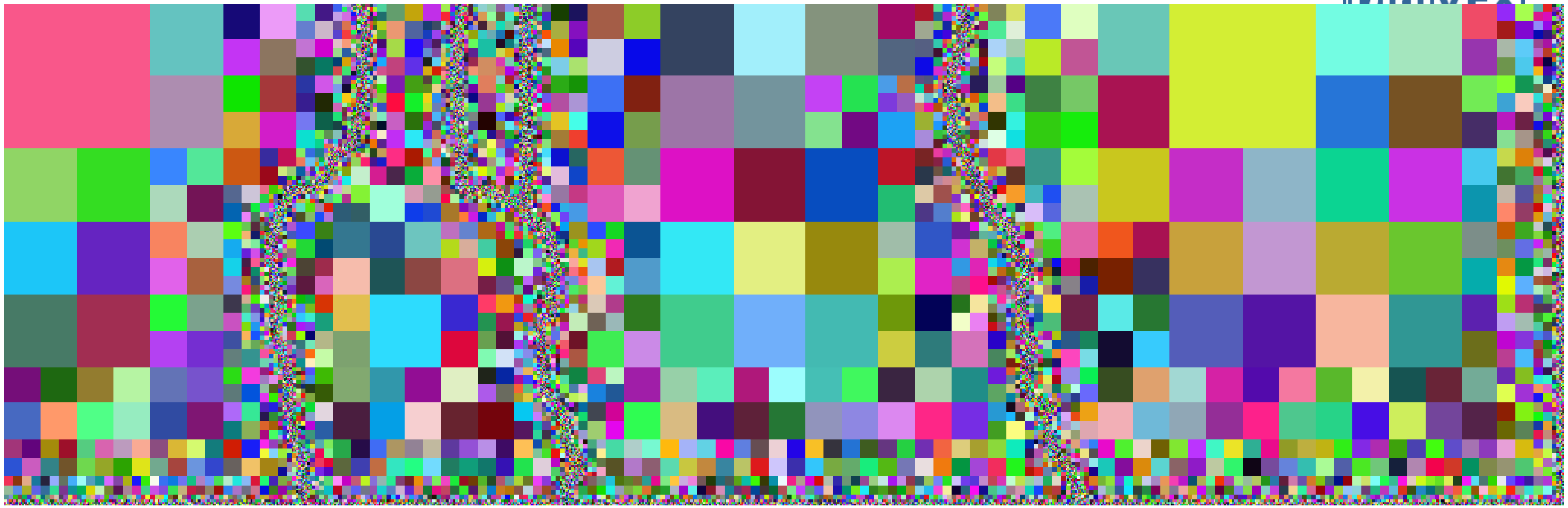
=

Difference x_δ



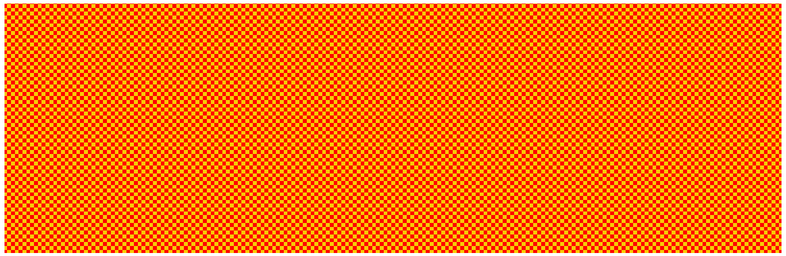
Quadtree decomposition

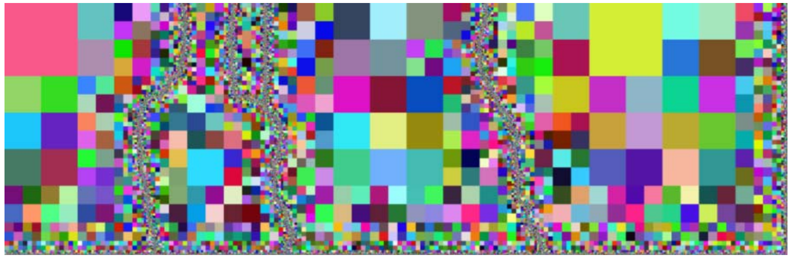




- Maximally subdivide quadtree along seams
- Variables placed at node corners
- Restricted quadtree
- Bi-linear interpolation reconstructs full solution
- Square nodes

Reduced space





x

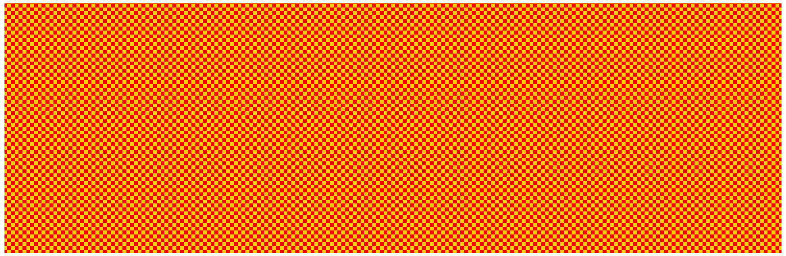
n variables

y

m variables

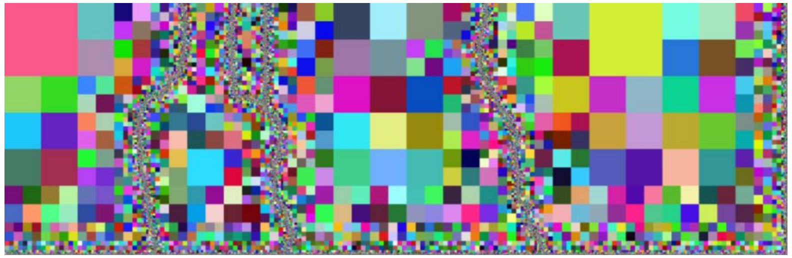
$$m << n$$

Reduced space



x

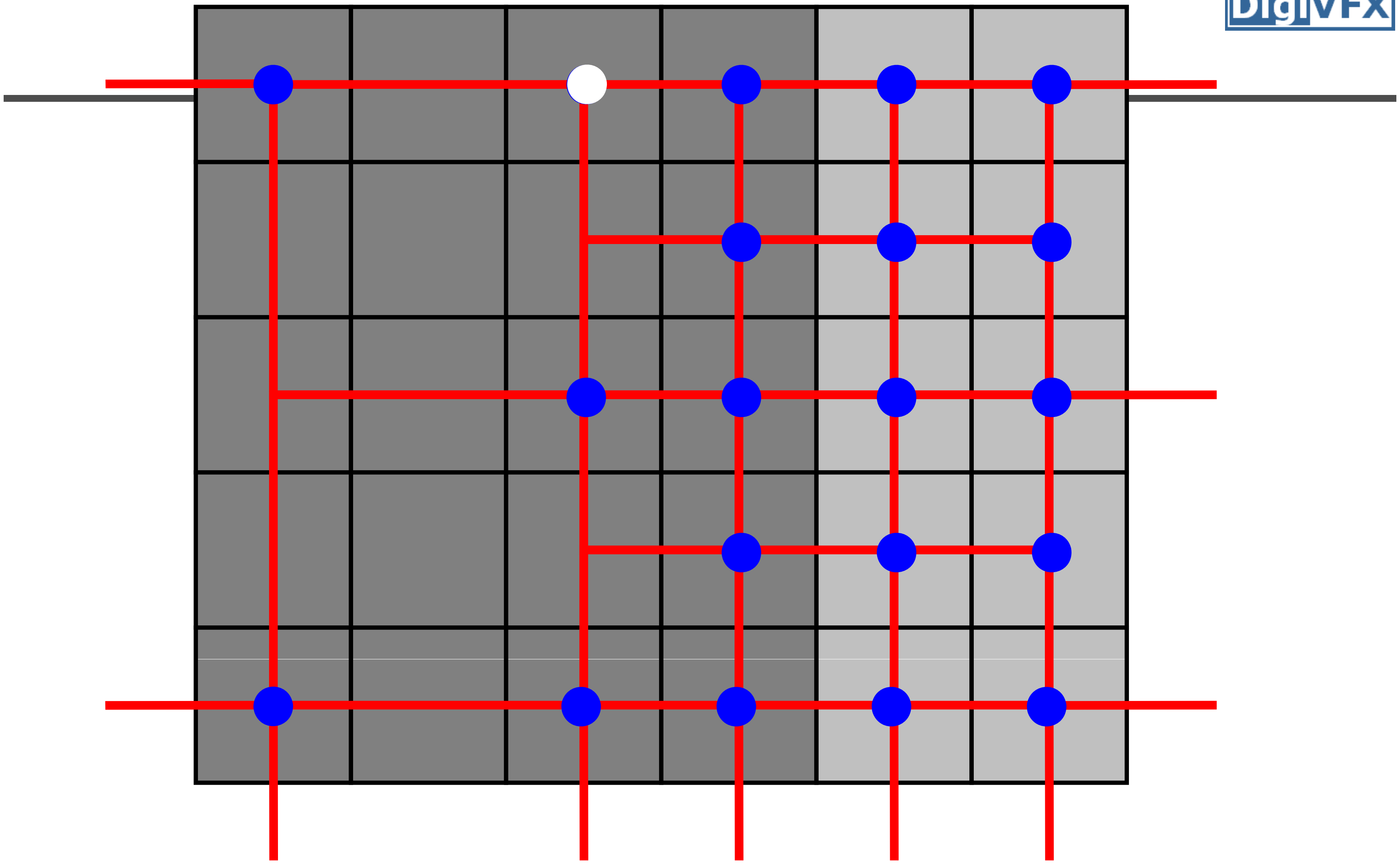
n variables



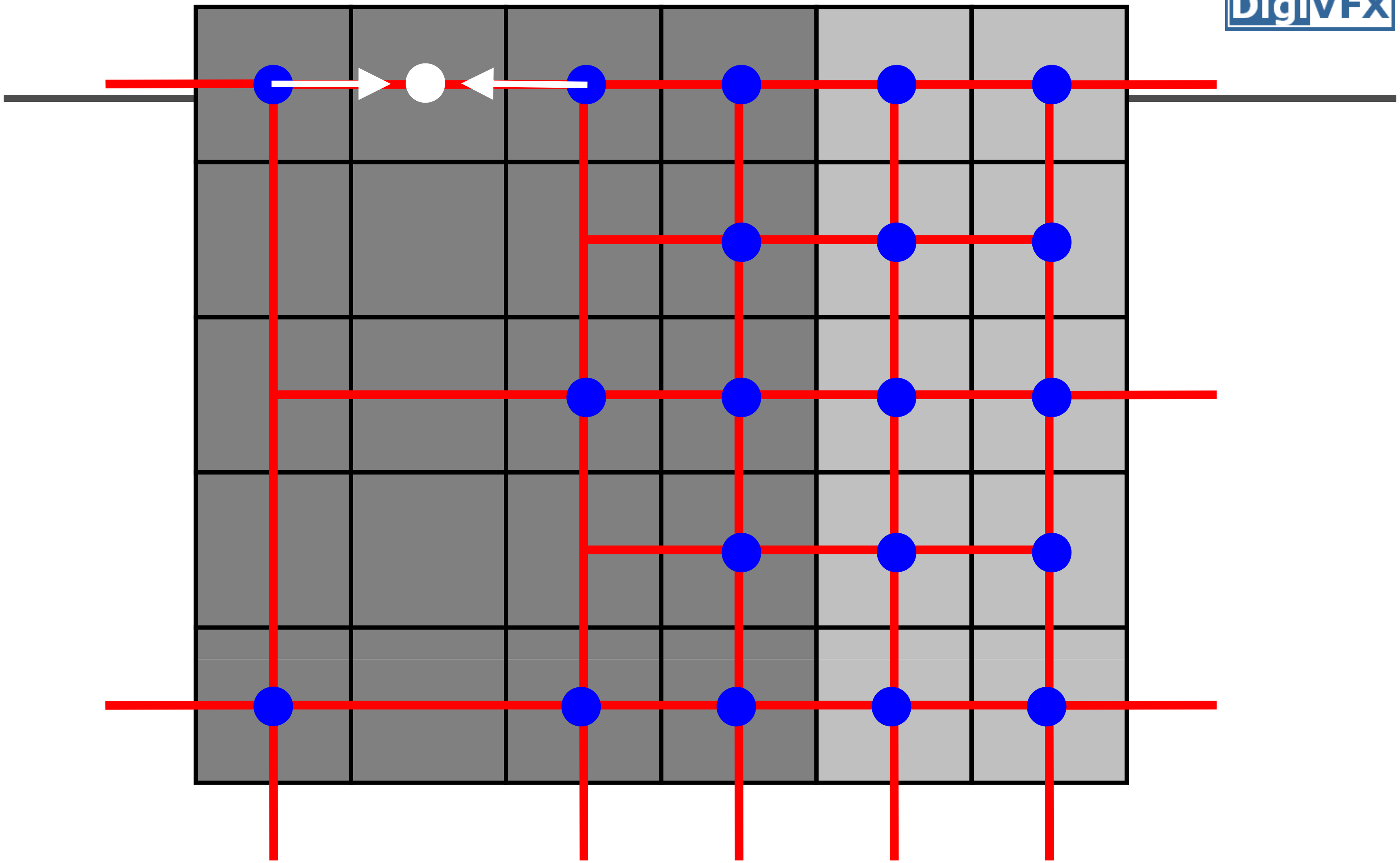
y

m variables

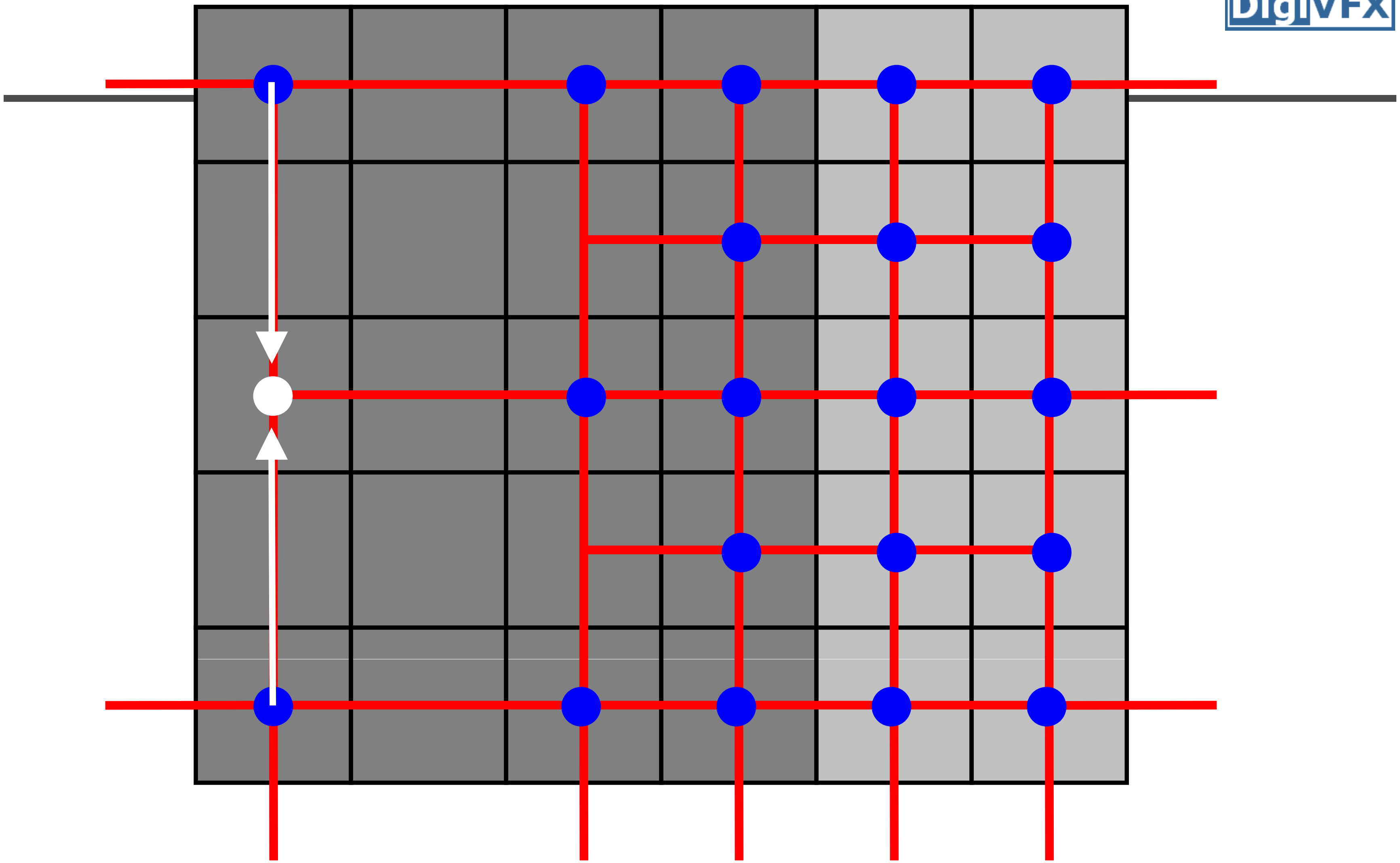
$$\mathrm{x} = \mathrm{S}\mathrm{y}$$



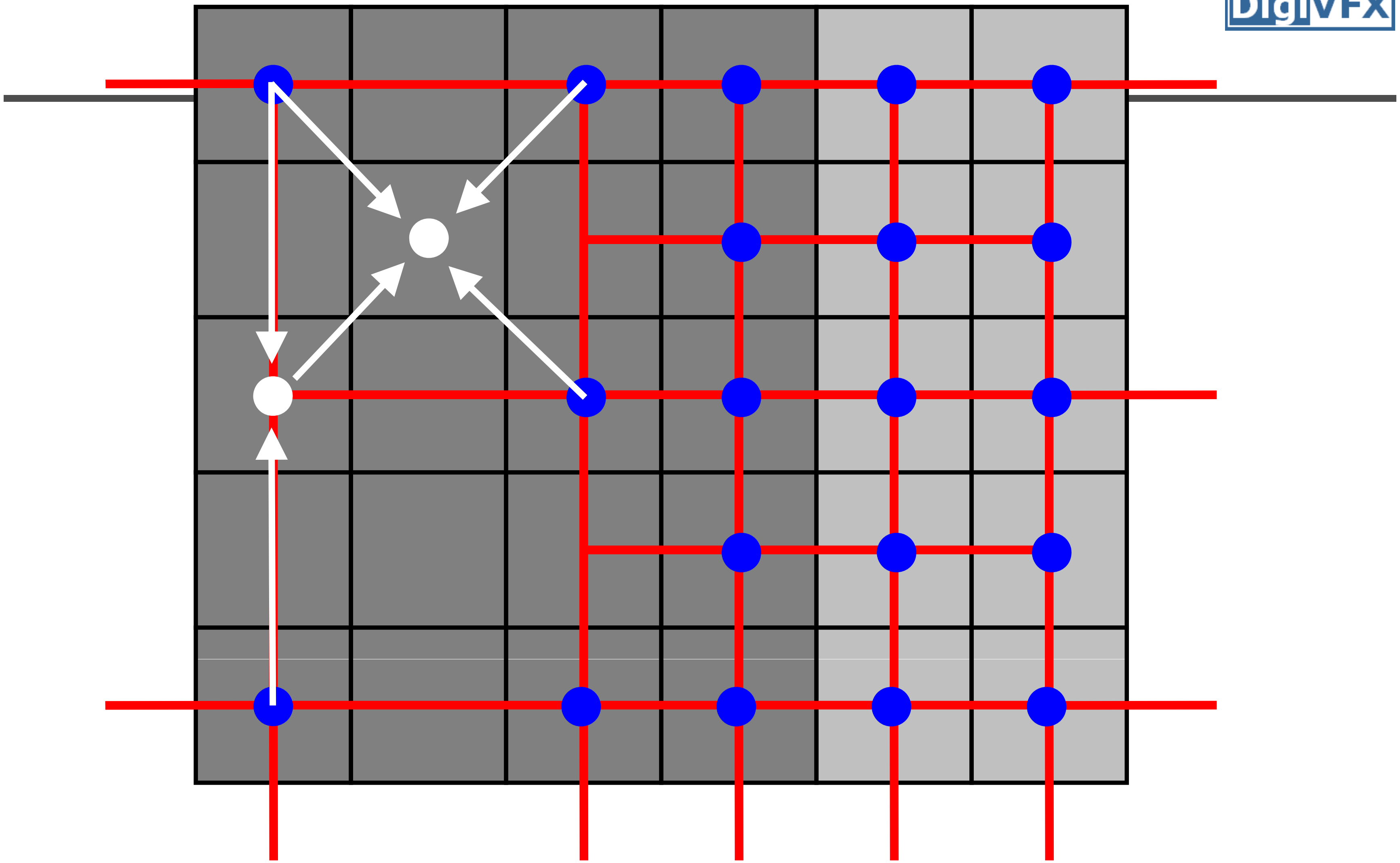
$$x = Sy$$



$$x = Sy$$

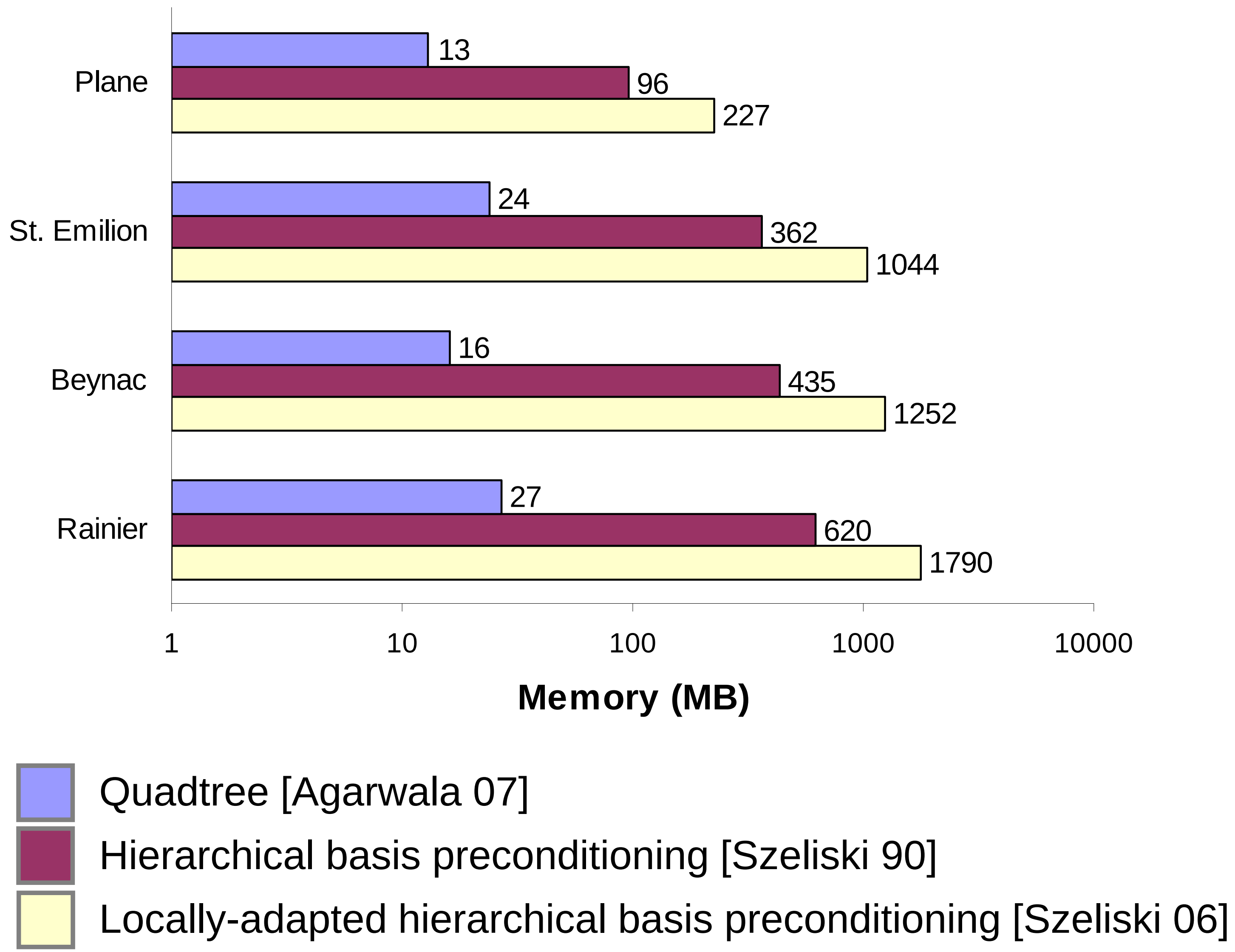


$$x = Sy$$



$$x = Sy$$

Performance

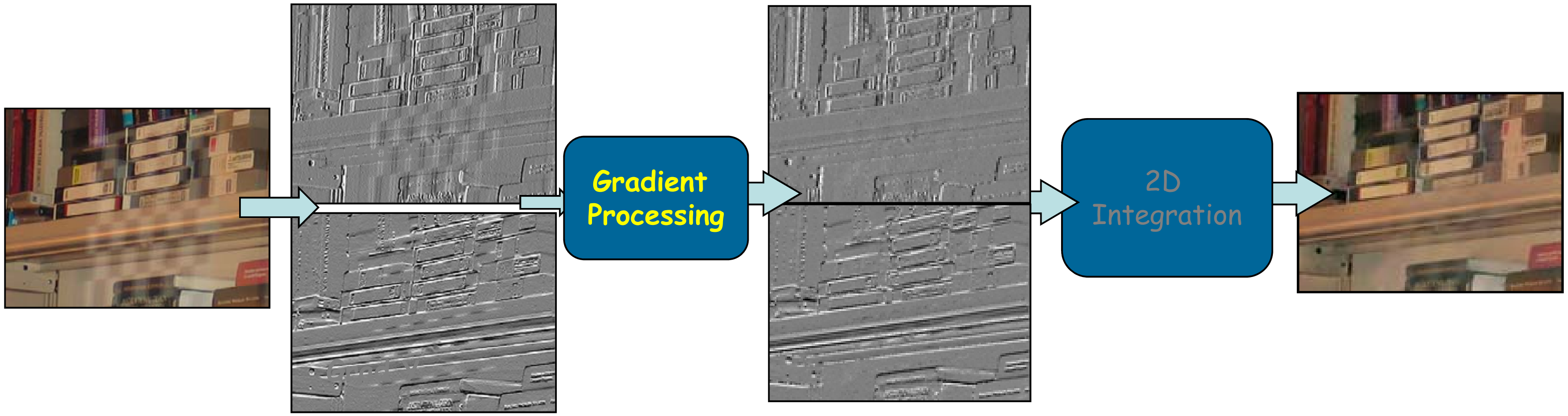


Cut-and-paste

Cut-and-paste

Intensity Gradient Manipulation

A Common Pipeline



Gradient Domain Manipulations: Overview

(A) Per pixel

(B) Corresponding gradients in two images

(C) Corresponding gradients in multiple images

(D) Combining gradients along seams

Gradient Domain Manipulations: Overview

(A) Per pixel
- Non-linear operations (HDR compression, local illumination change)
- Set to zero (shadow removal, intrinsic images, texture de-emphasis)
- Poisson Matting

(B) Corresponding gradients in two images
- Vector operations (gradient projection)
  - Combining flash/no-flash images, Reflection removal
- Projection Tensors
  - Reflection removal, Shadow removal
- Max operator
  - Day/Night fusion, Visible/IR fusion, Extending DoF
- Binary, choose from first or second, copying
  - Image editing, seamless cloning

Gradient Domain Manipulations

(C) Corresponding gradients in multiple images

- Median operator
  - Specularity reduction
  - Intrinsic images
- Max operation
  - Extended DOF

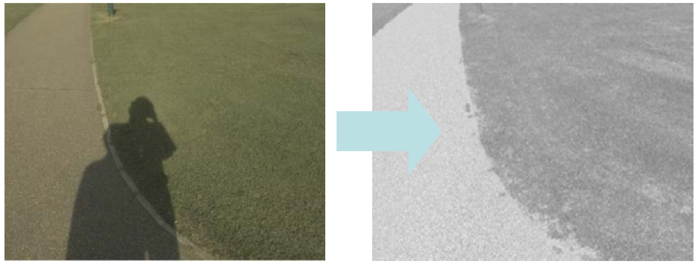
(D) Combining gradients along seams

- Weighted averaging
- Optimal seam using graph cut
  - Image stitching, Mosaics, Panoramas, Image fusion
  - A usual pipeline: Graph cut to find seams + gradient domain fusion

A. Per Pixel Manipulations

- Non-linear operations
  - HDR compression, local illumination change

- Set to zero
  - Shadow removal, intrinsic images, texture de-emphasis



- Poisson Matting

High Dynamic Range Imaging

Images from Raanan Fattal

Gradient Domain Compression

HDR Image L

Log L

Gradients L_x, L_y

Multiply

2D Integration

Gradient Attenuation Function G

Local Illumination Change

Original Image: f

$$\mathbf{v} = \alpha^{\beta} |\nabla f^*|^{-\beta} \nabla f^*,$$

Original gradient field: ∇f^*

Modified gradient field: v

Perez et al. Poisson Image editing, SIGGRAPH 2003

Illumination Invariant Image

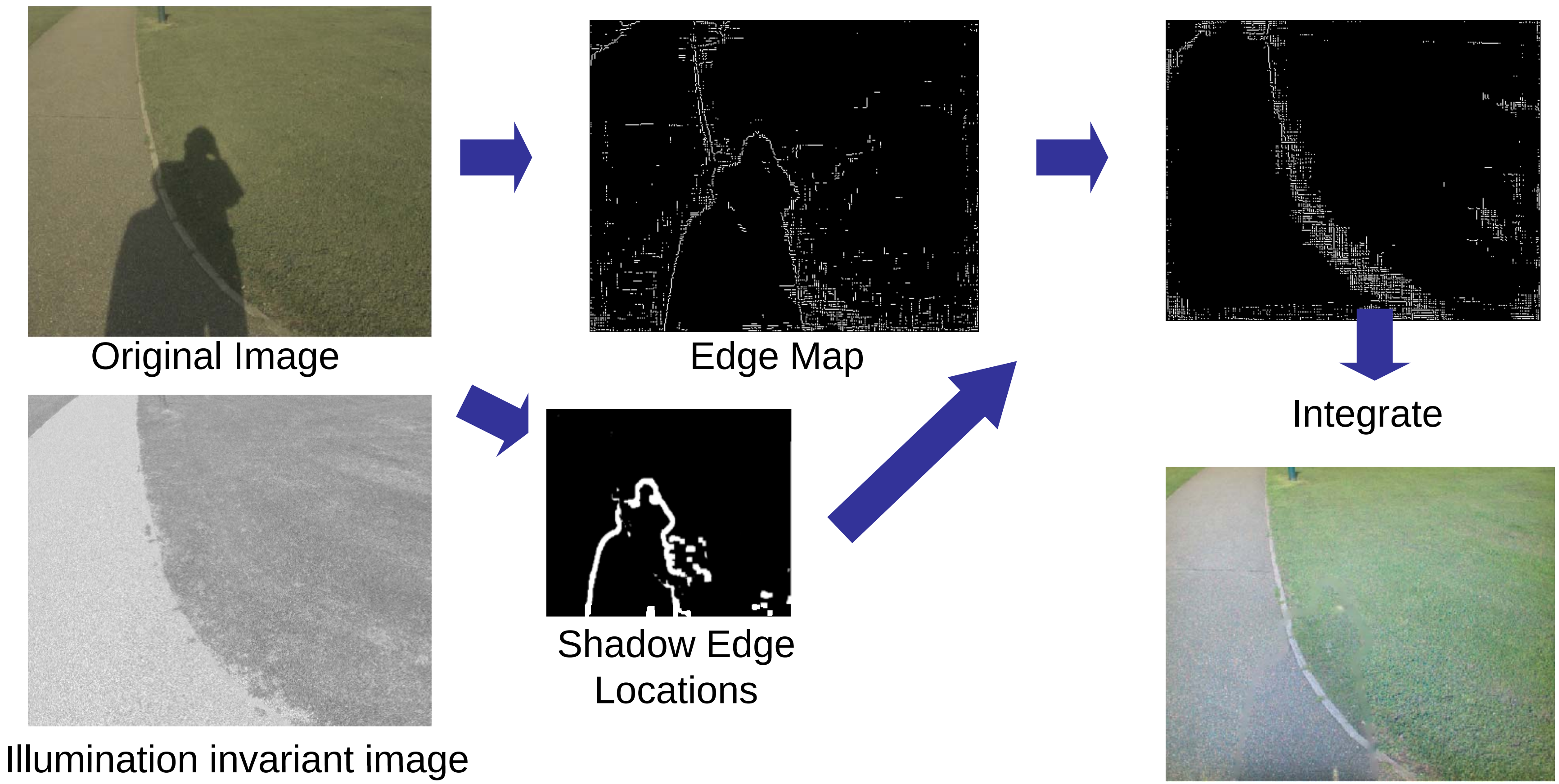
Original Image

Illumination invariant image

- Assumptions
  - Sensor response = delta functions R, G, B in wavelength spectrum
  - Illumination restricted to Outdoor Illumination

G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

Shadow Removal Using Illumination Invariant Image



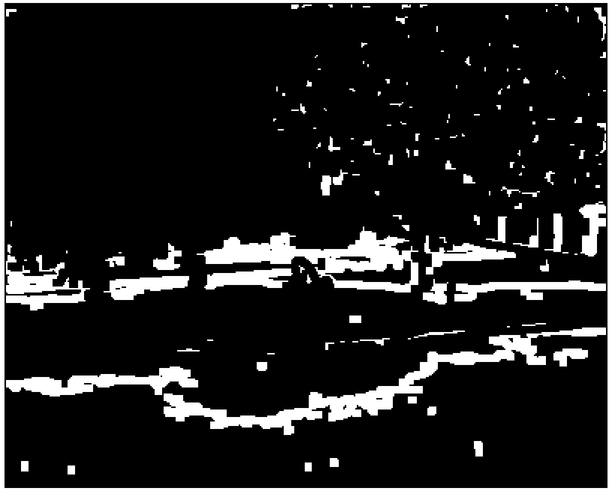
G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

Illumination invariant image

Original Image

Invariant Image

Detected Shadow Edges



Shadow Removed

G. D. Finlayson, S.D. Hordley & M.S. Drew, Removing Shadows From Images, ECCV 2002

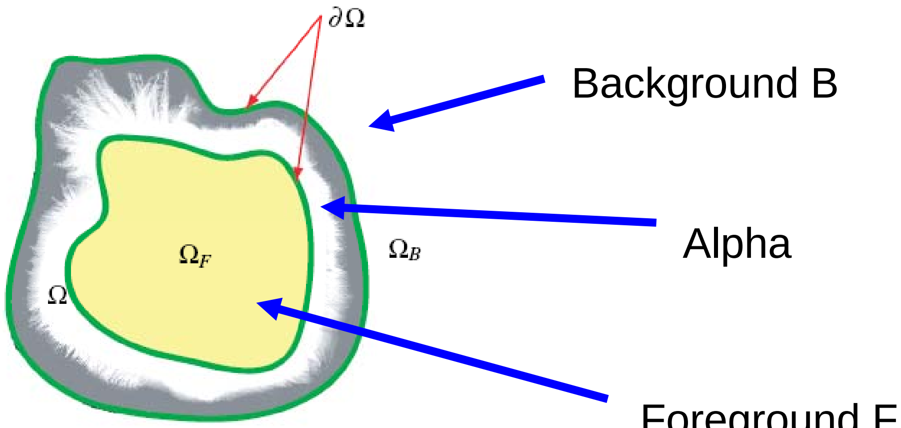
Intrinsic Image

- Photo = Illumination Image * Intrinsic Image
- Retinex [Land & McCann 1971, Horn 1974]
  - Illumination is smoothly varying
  - Reflectance, piece-wise constant, has strong edges
  - Keep strong image gradients, integrate to obtain reflectance

low-frequency attenuate more

high-frequency attenuate less

Poisson Matting



Trimap: User specified

Jian Sun, Jiaya Jia, Chi-Keung Tang, Heung-Yeung Shum, Poisson Matting, SIGGRAPH 2004

Poisson Matting

DigiVFX

$$I = \alpha F + (1-\alpha)B$$

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

Approximate: Assume F and B are smooth

$$\nabla I = (F-B)\nabla\alpha$$

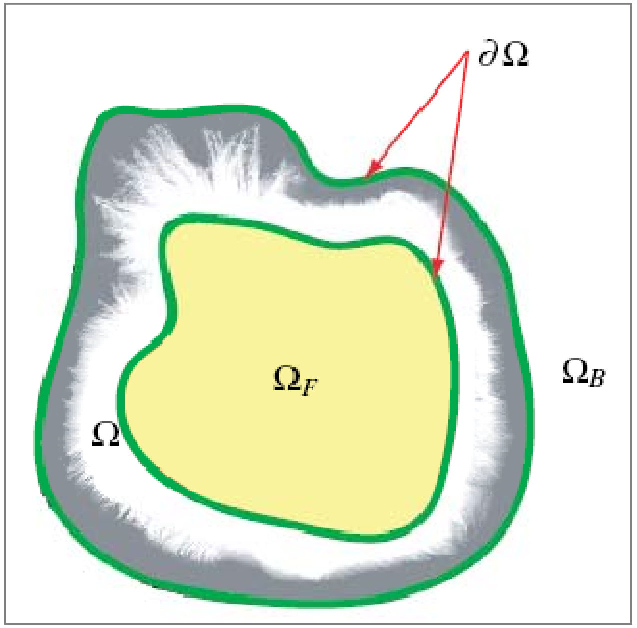
$$\nabla\alpha \approx \frac{1}{F-B}\nabla I \quad \Rightarrow \quad \Delta\alpha = div\left(\frac{\nabla I}{F-B}\right)$$

F and B in tri-map using nearest pixels

Poisson Equation

Poisson Matting

- Steps
  - Approximate F and B in trimap Ω.
  - Solve for α $\Delta\alpha = div(\frac{\nabla I}{F-B})$
  - Refine F and B using α
  - Iterate



Gradient Domain Manipulations: Overview

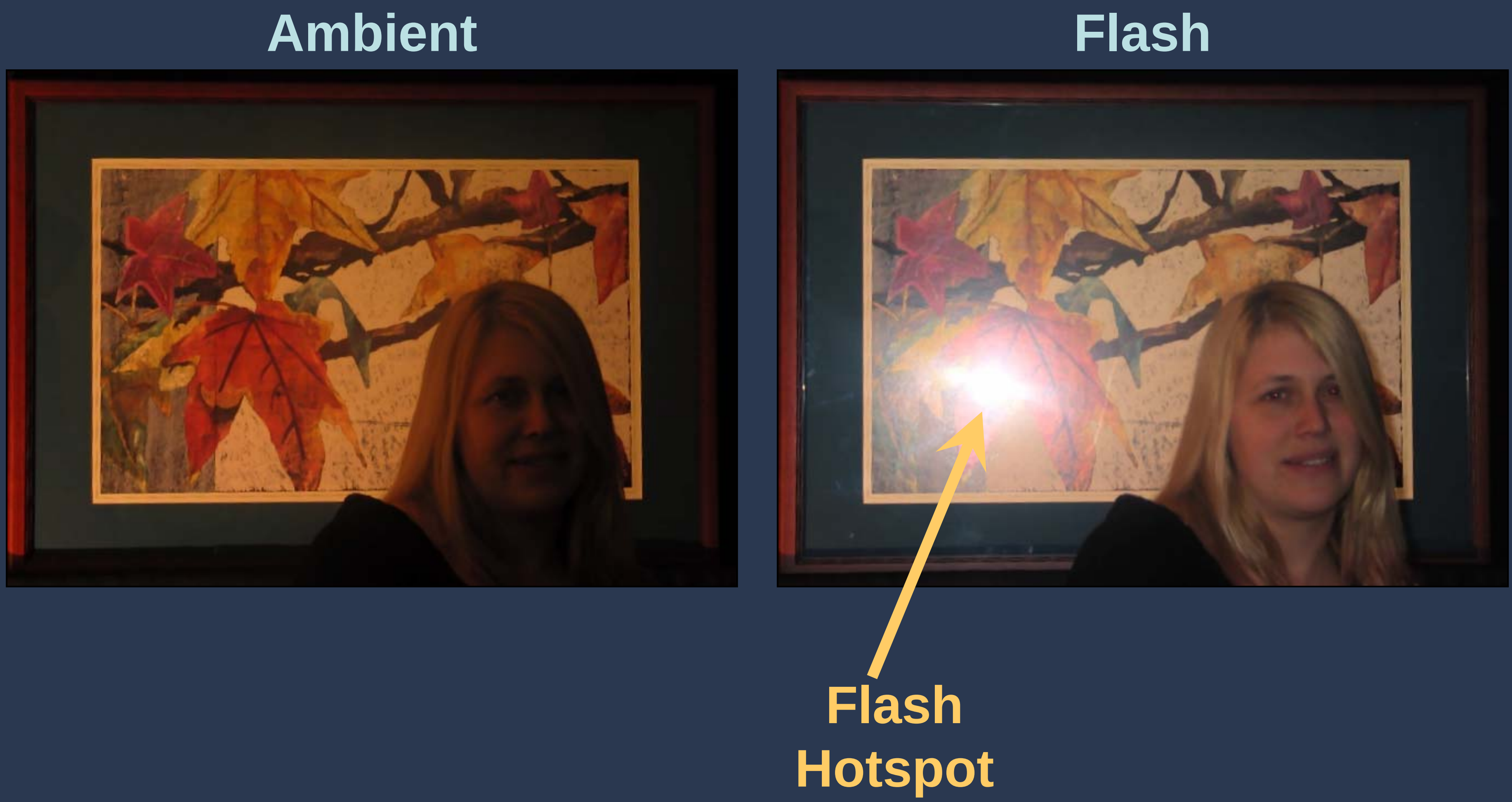
(A) Per pixel

(B) Corresponding gradients in <u>two images</u>

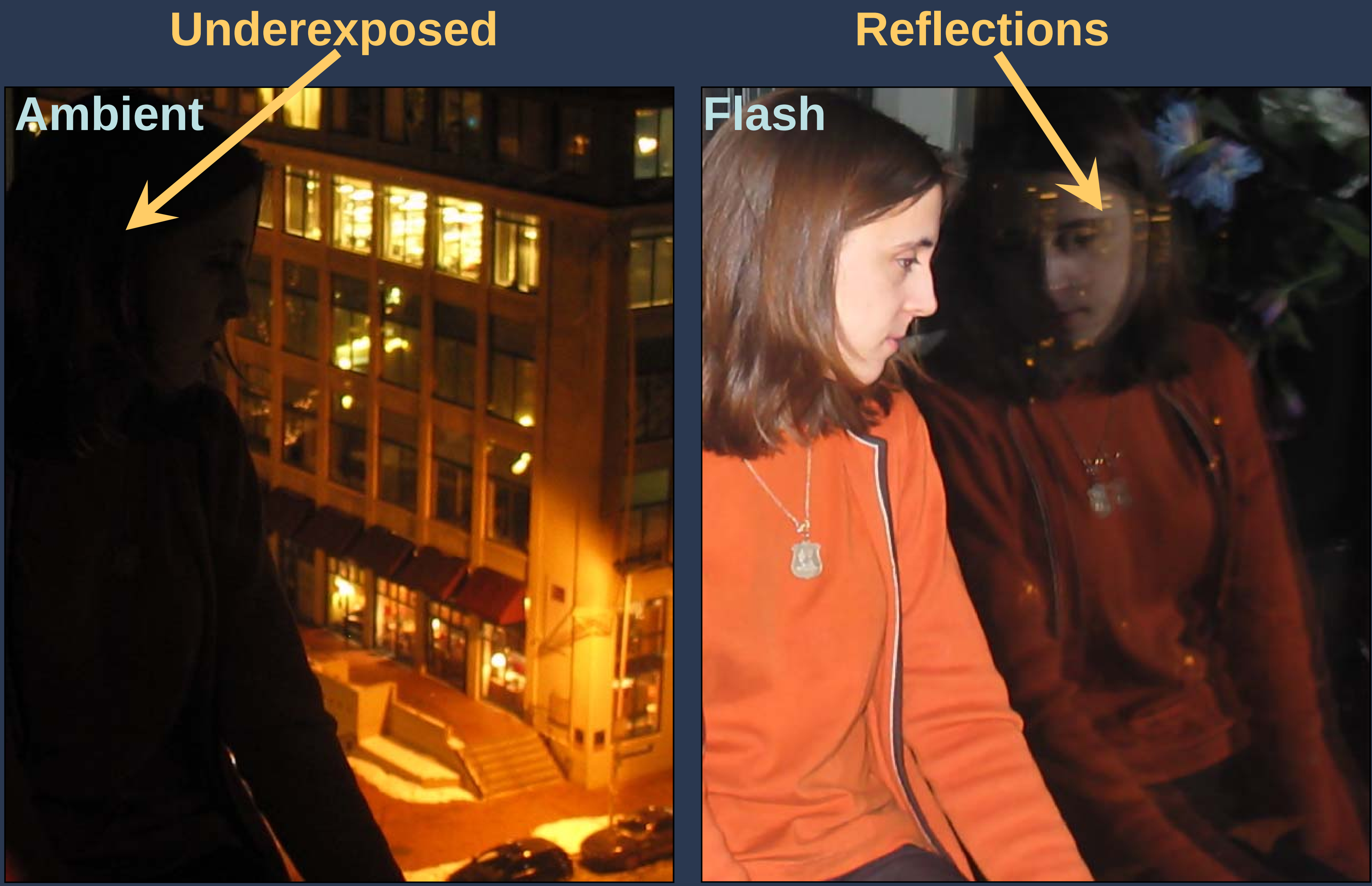
(C) Corresponding gradients in <u>multiple images</u>

(D) Combining gradients <u>along seams</u>

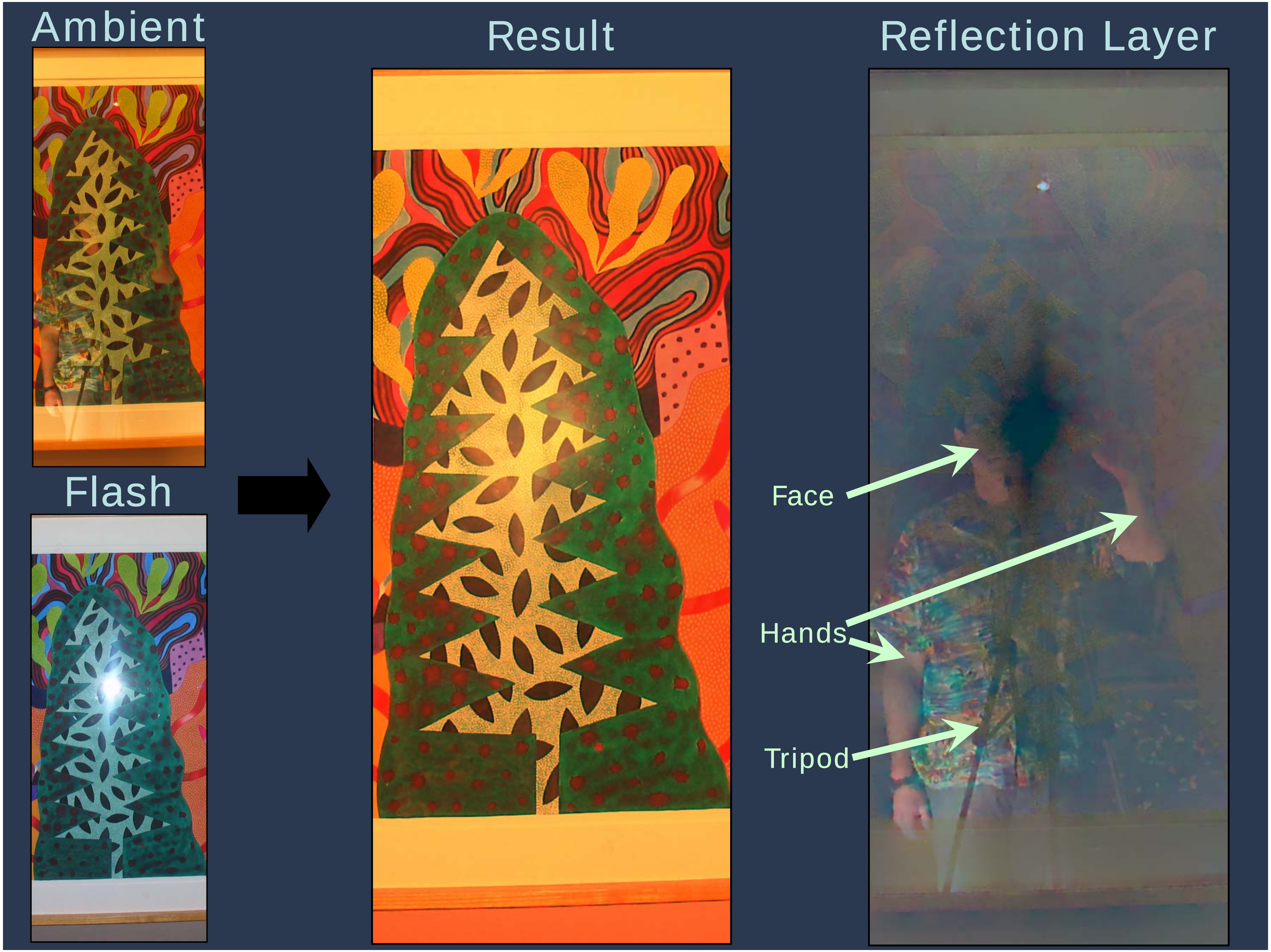
Photography Artifacts: Flash Hotspot



Reflections due to Flash

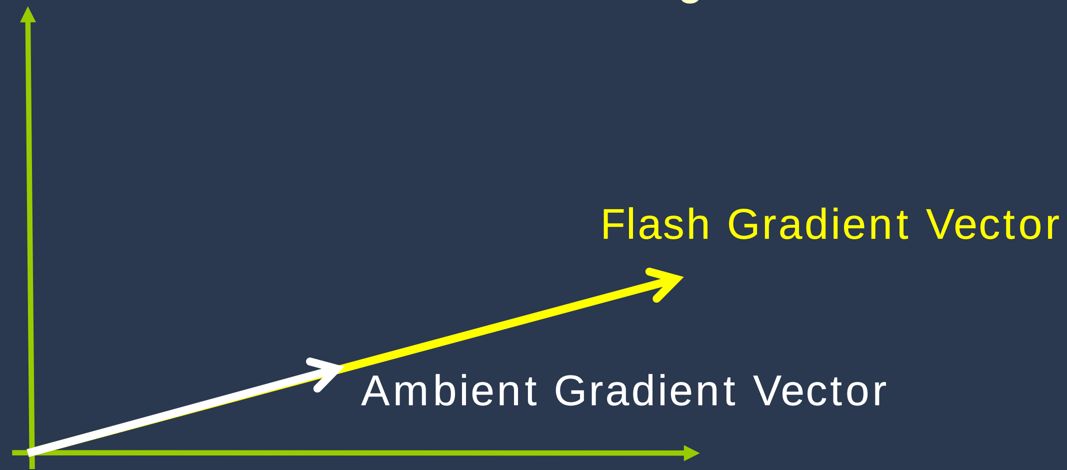


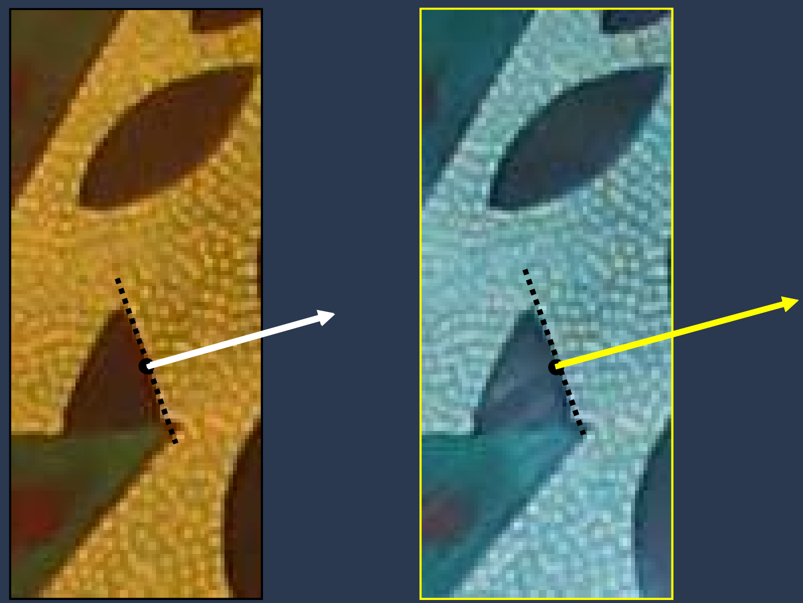
Self-Reflections and Flash Hotspot


Ambient
Flash
Result
Reflection Layer
Face
Hands
Tripod

Intensity Gradient Vectors in Flash and Ambient Images

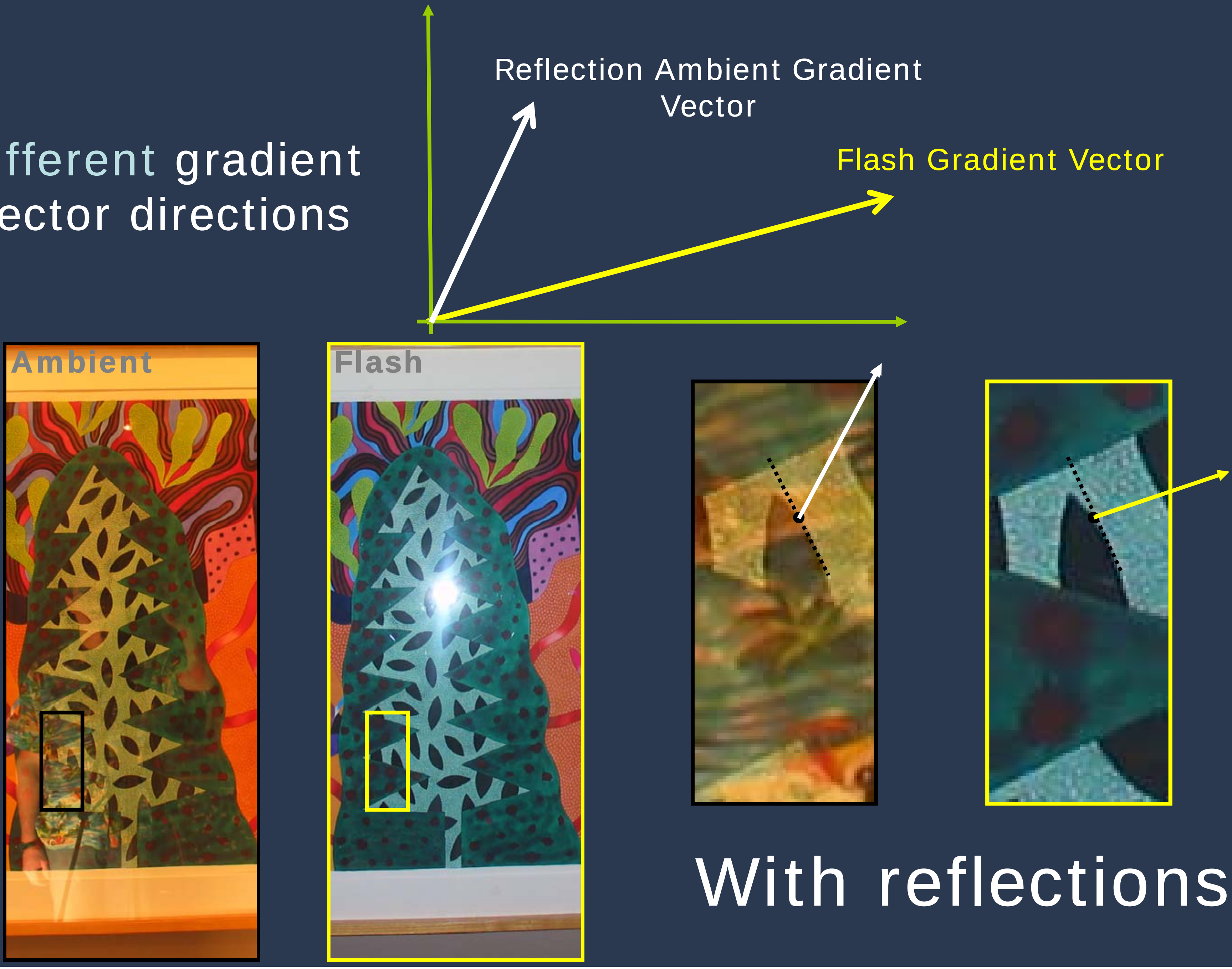
Same gradient vector direction





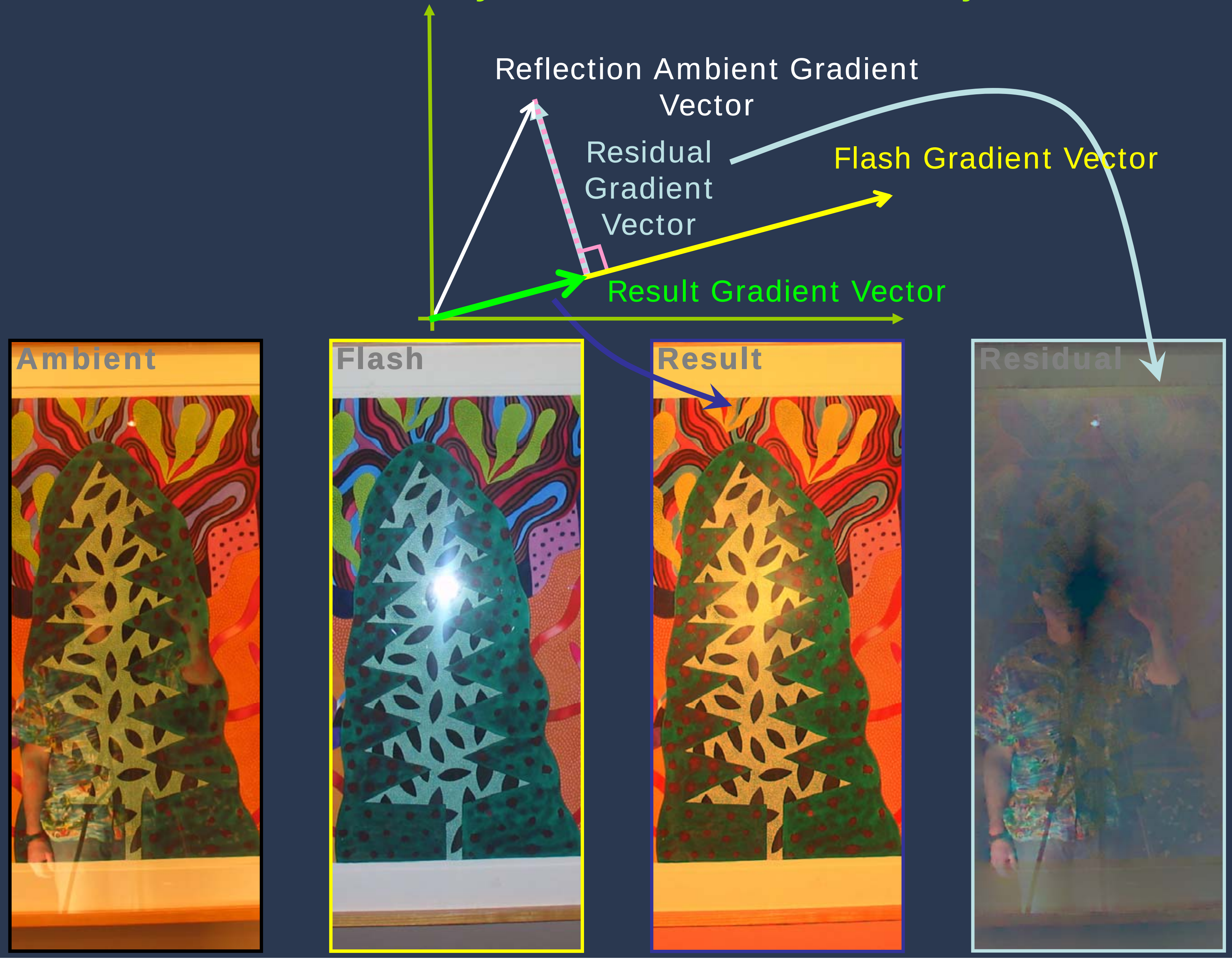
No reflections

Different gradient vector directions



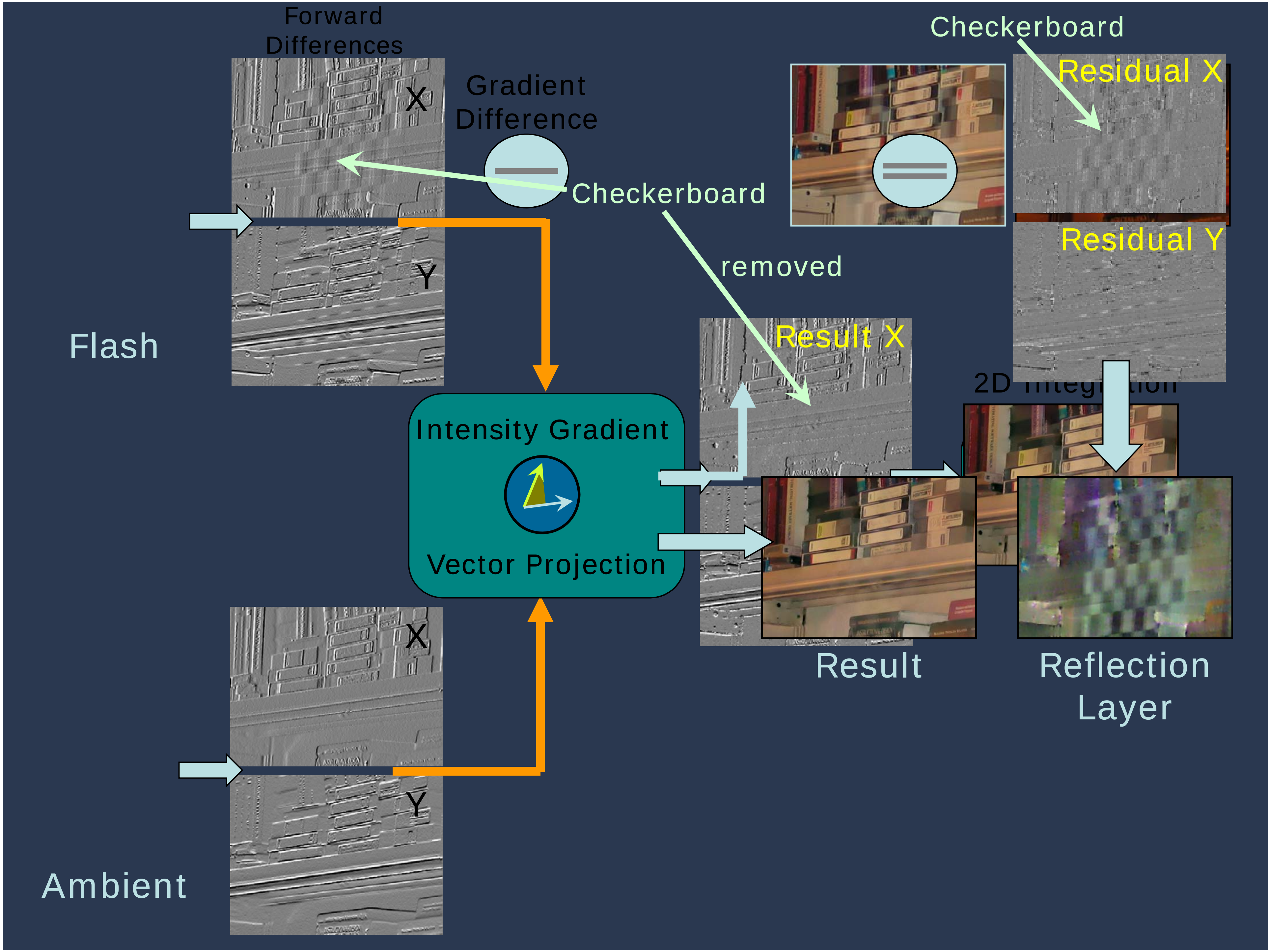
With reflections

Intensity Gradient Vector Projection



Ambient | Flash | Projection = Result | Residual = Reflection Layer


Flash
Ambient
Checkerboard outside glass window
Reflections on glass window



Forward Differences
Gradient Difference
Checkerboard
Checkerboard
Residual X
Residual Y
removed
Result X
2D Integration
Flash
Intensity Gradient
Vector Projection
X
Y
X
Y
Ambient
Result
Reflection Layer

Image Fusion for Context Enhancement and Video Surrealism

Ramesh Raskar

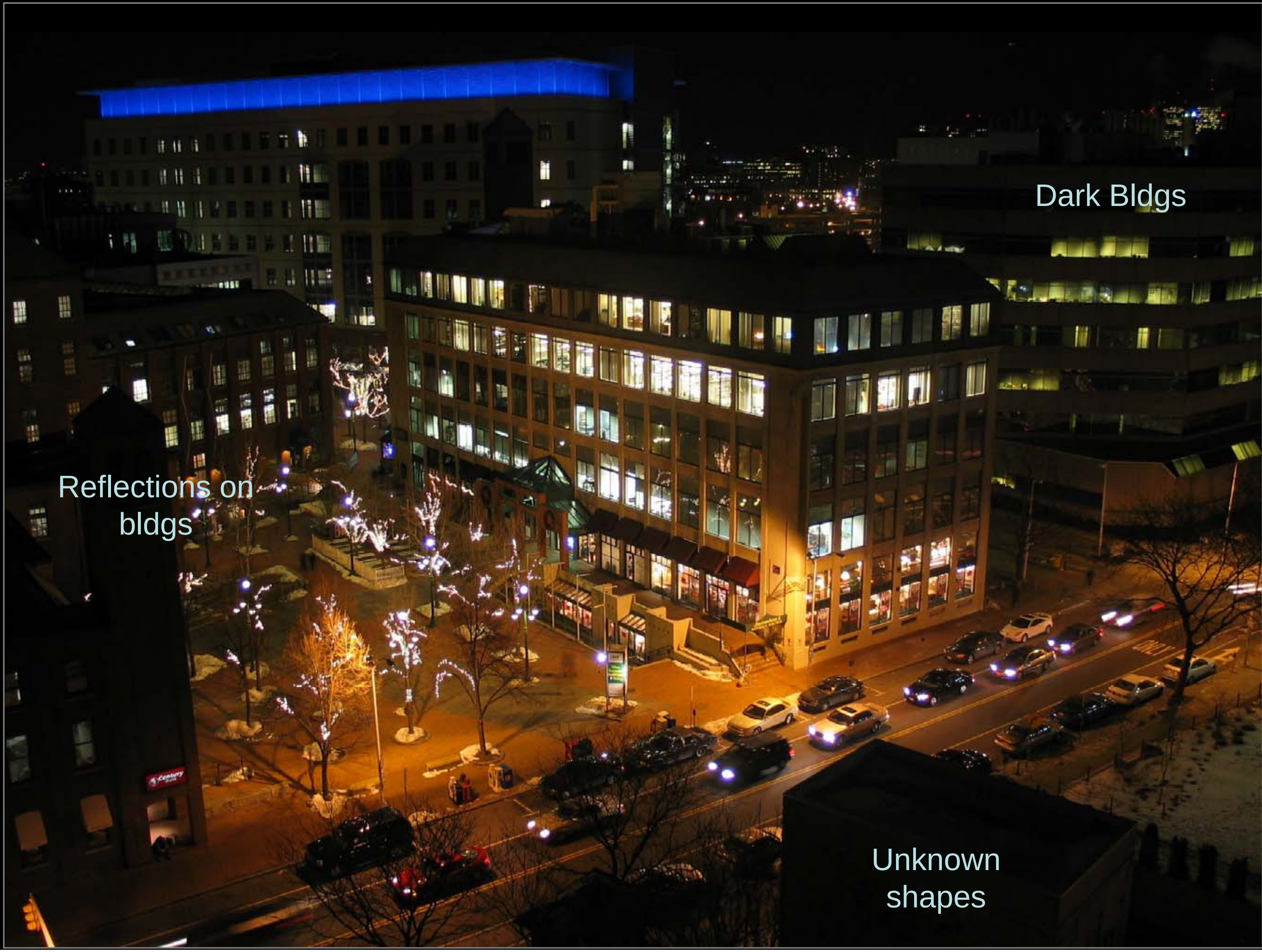
Mitsubishi Electric Research Labs, (MERL)

Adrian Ilie

UNC Chapel Hill

Jingyi Yu

MIT


Dark Bldgs
Reflections on bldgs
Unknown shapes

Night Image

Background is captured from day-time scene using the same fixed camera

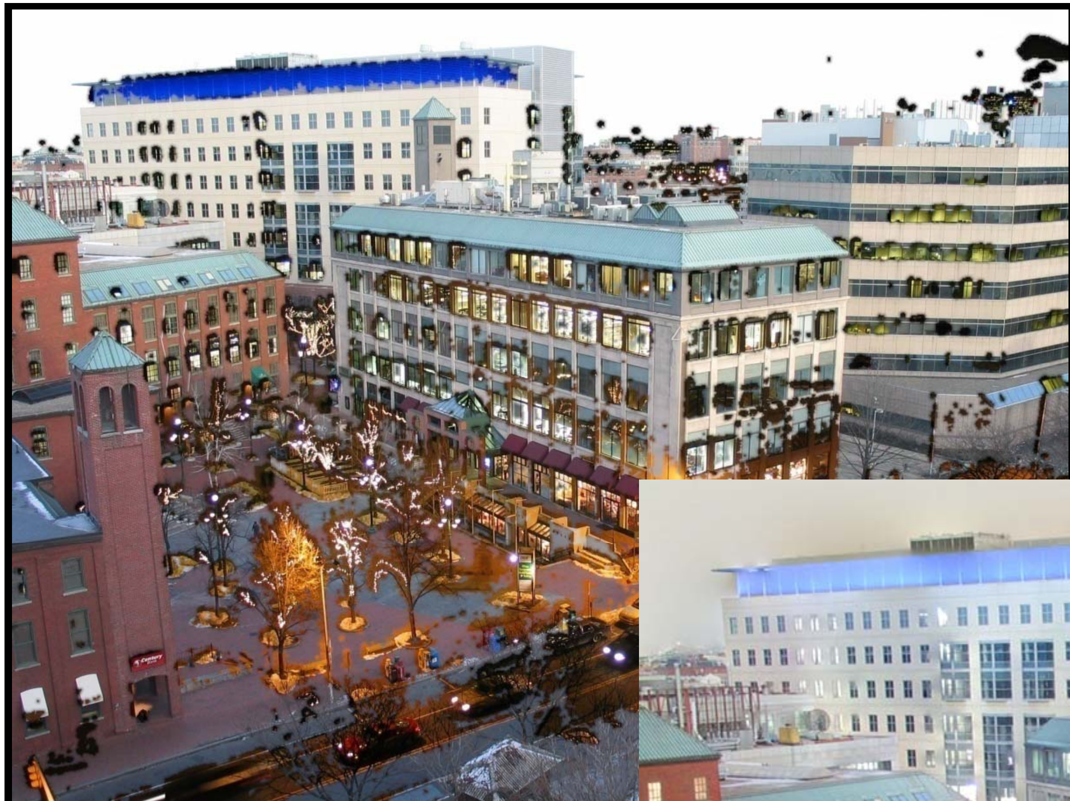
Context Enhanced Image

Day Image

Mask is automatically computed from scene contrast

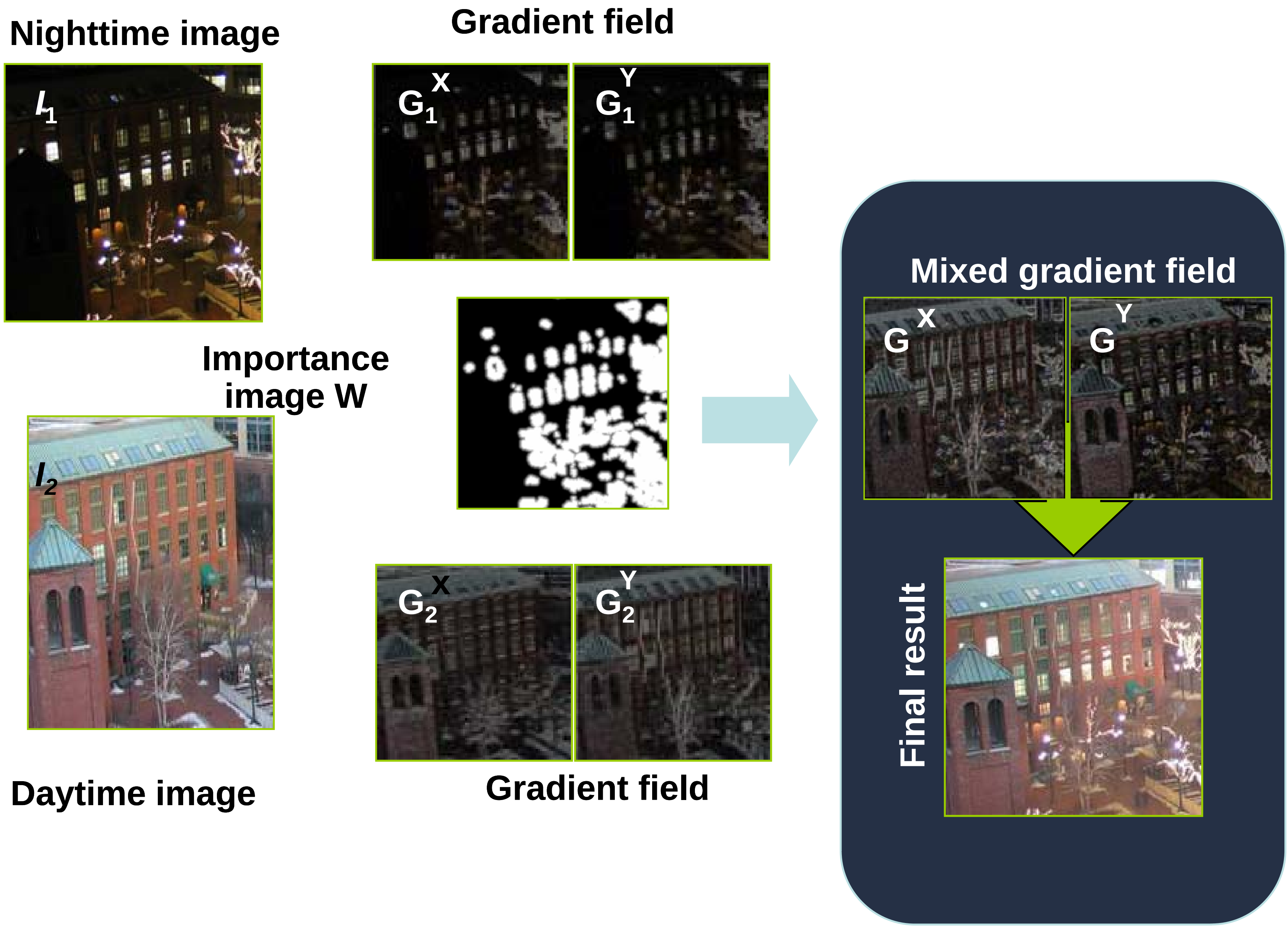
But, Simple Pixel Blending Creates Ugly Artifacts


Century
Bank



Pixel Blending

solution:
Integration of
blended Gradients


Nighttime image
I1
Gradient field
G1 X
G1 Y
Importance image W
I2
Daytime image
G2 X
G2 Y
Gradient field
Mixed gradient field
G X
G Y
Final result

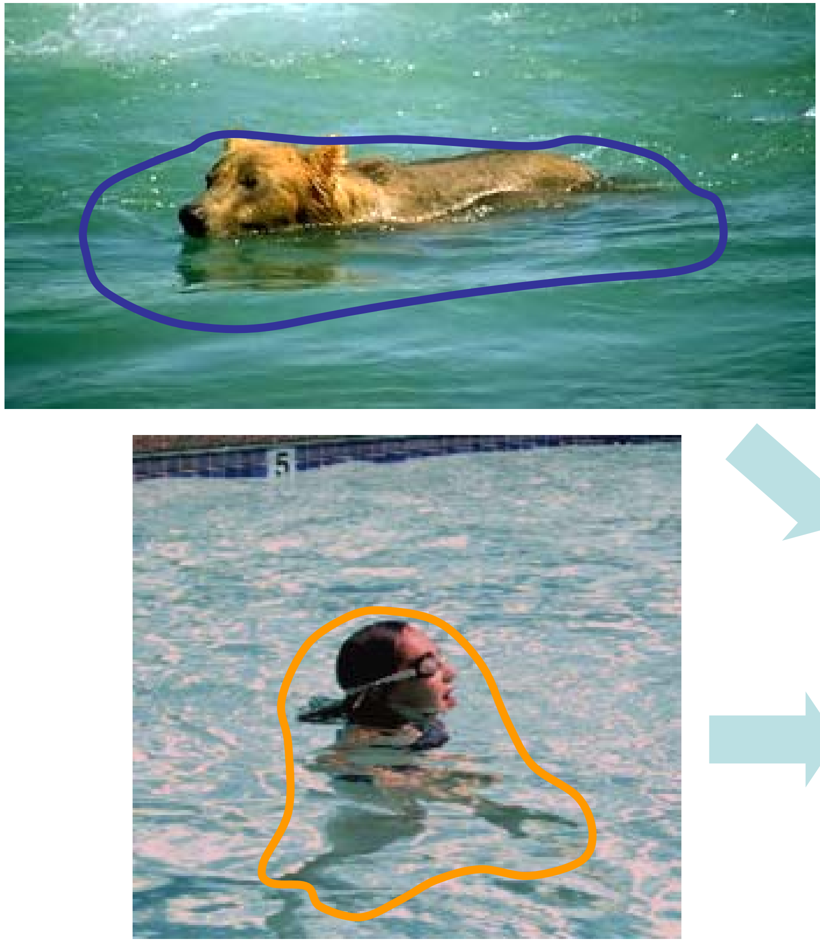
Poisson Image Editing

- Precise selection: tedious and unsatisfactory
- Alpha-Matting: powerful but involved
- Seamless cloning: loose selection but no seams?

Conceal

Copy Background gradients (user strokes)

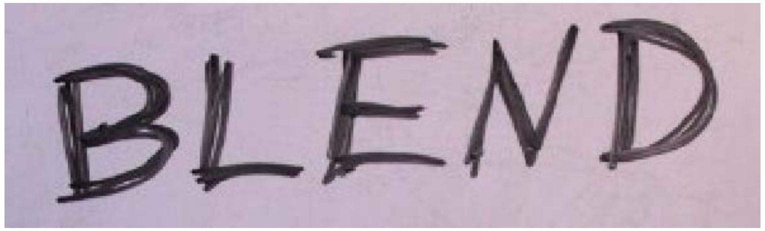
Compose

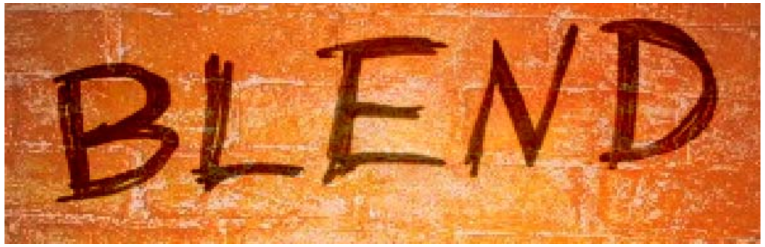


Source Images

Target Image

Transparent Cloning





$$\mathbf{v} = \nabla g$$

$$\mathbf{v} = \frac{\nabla f^* + \nabla g}{2}$$

max

Largest variation from source and destination at each point

Gradient Domain Manipulations: Overview

(A) Per pixel

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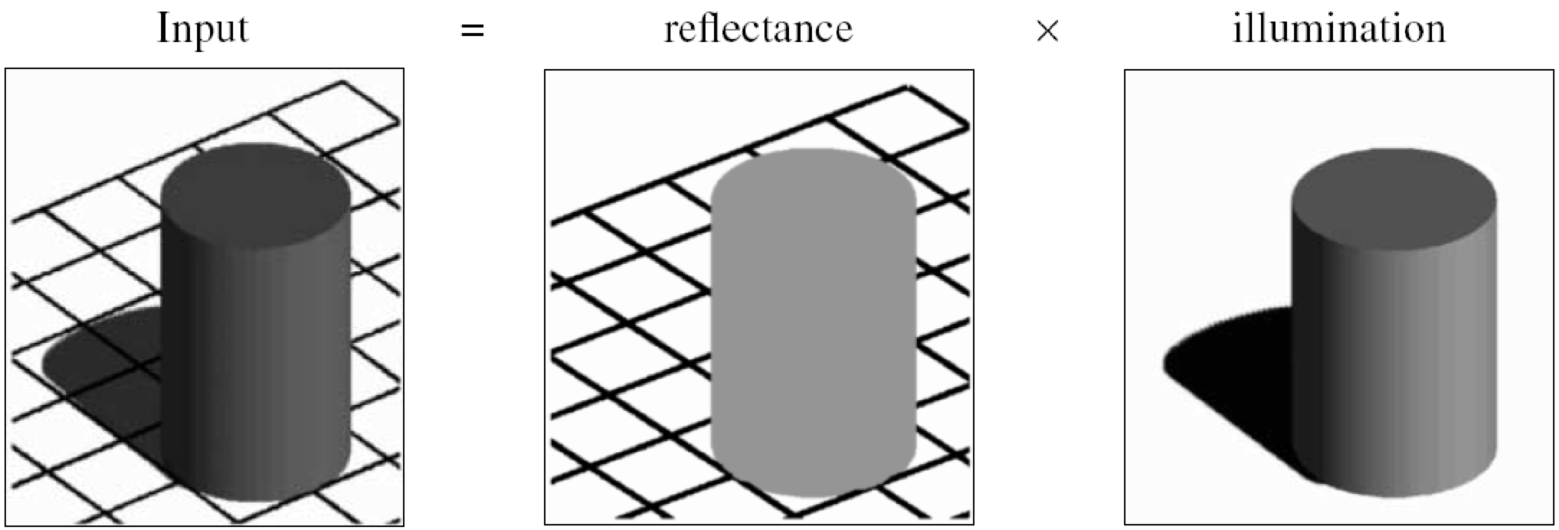
(D) Combining gradients along seams

Intrinsic images

- I = L * R
- L = illumination image
- R = reflectance image



Intrinsic images

- Use multiple images under different illumination
- Assumption
  - Illumination image gradients = Laplacian PDF
  - Under Laplacian PDF, Median = ML estimator
- At each pixel, take Median of gradients across images
- Integrate to remove shadows

Yair Weiss, "Deriving intrinsic images from image sequences", ICCV 2001

frame 1

frame 11

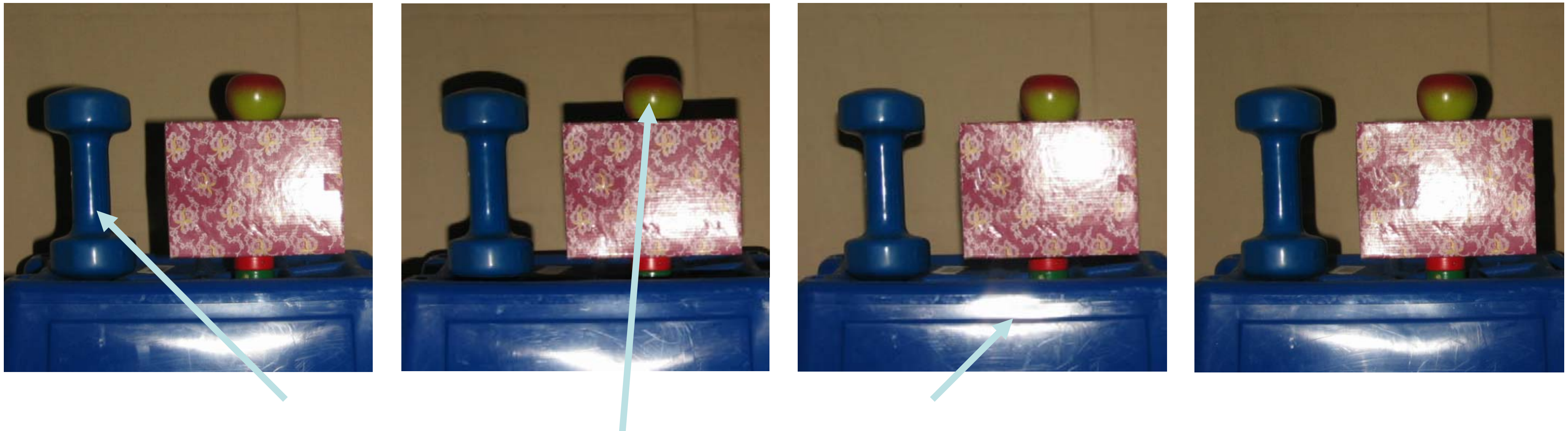
ML reflectance

Shadow free

Intrinsic Image

Result = Illumination Image * (Label in Intrinsic Image)

Specularity Reduction in Active Illumination



Line Specularity Point Specularity Area Specularity

Multiple images with same viewpoint, varying illumination

How do we remove highlights?






Specularity Reduced
Image

Gradient Domain Manipulations: Overview

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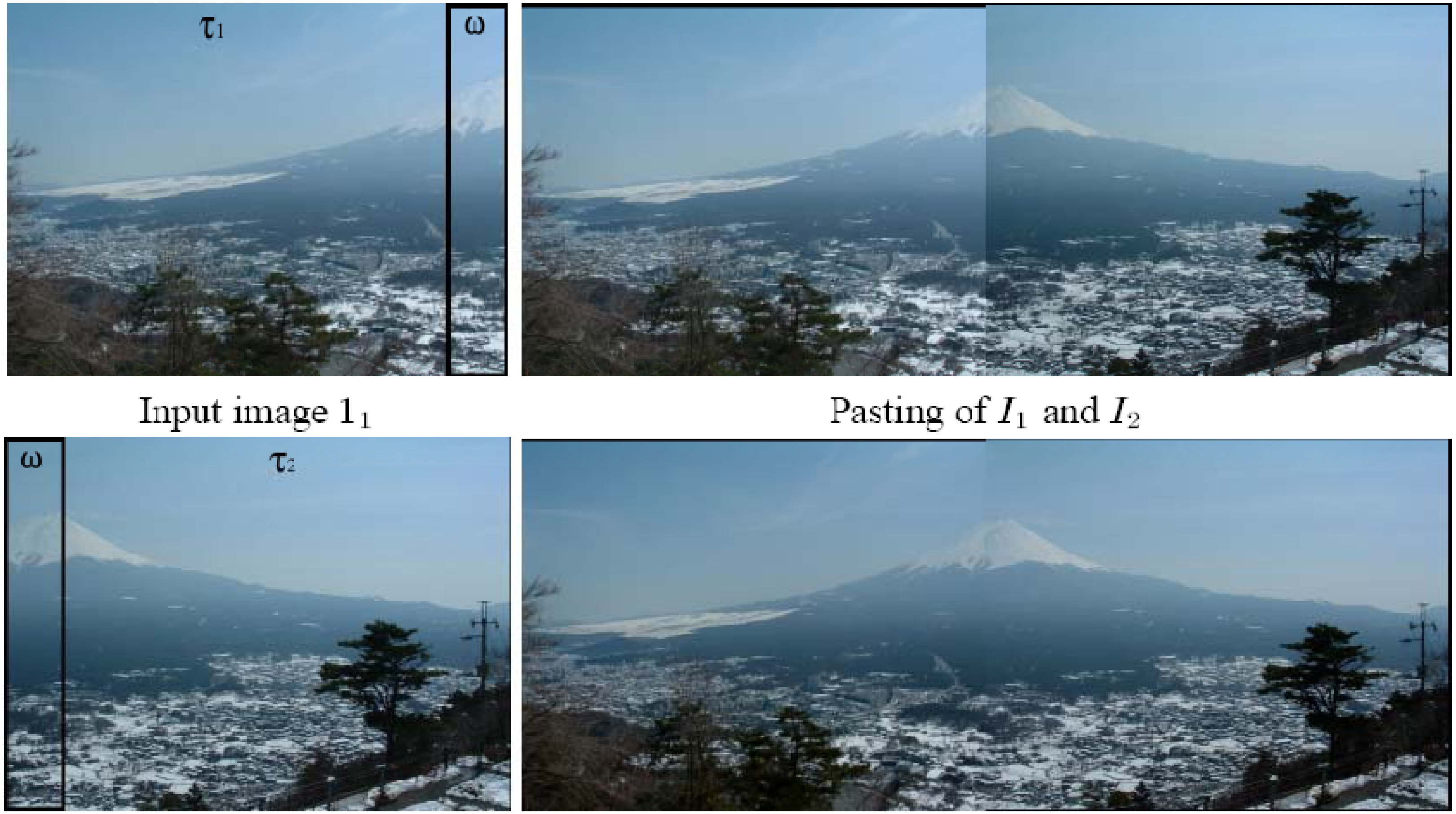
Seamless Image Stitching

DigiVFX



Input image 1_1 — Pasting of I_1 and I_2

Input image I_2 — Stitching result

Anat Levin, Assaf Zomet, Shmuel Peleg and Yair Weiss, "Seamless Image Stitching in the Gradient Domain", ECCV 2004