

Bilateral Filters

Digital Visual Effects, Spring 2009

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2009/5/21

with slides by Fredo Durand, Ramesh Raskar, Sylvain Paris, Soonmin Bae

Bilateral filtering



[Ben Weiss, Siggraph 2006]

Image Denoising



noisy image



naïve denoising
Gaussian blur



better denoising
edge-preserving filter

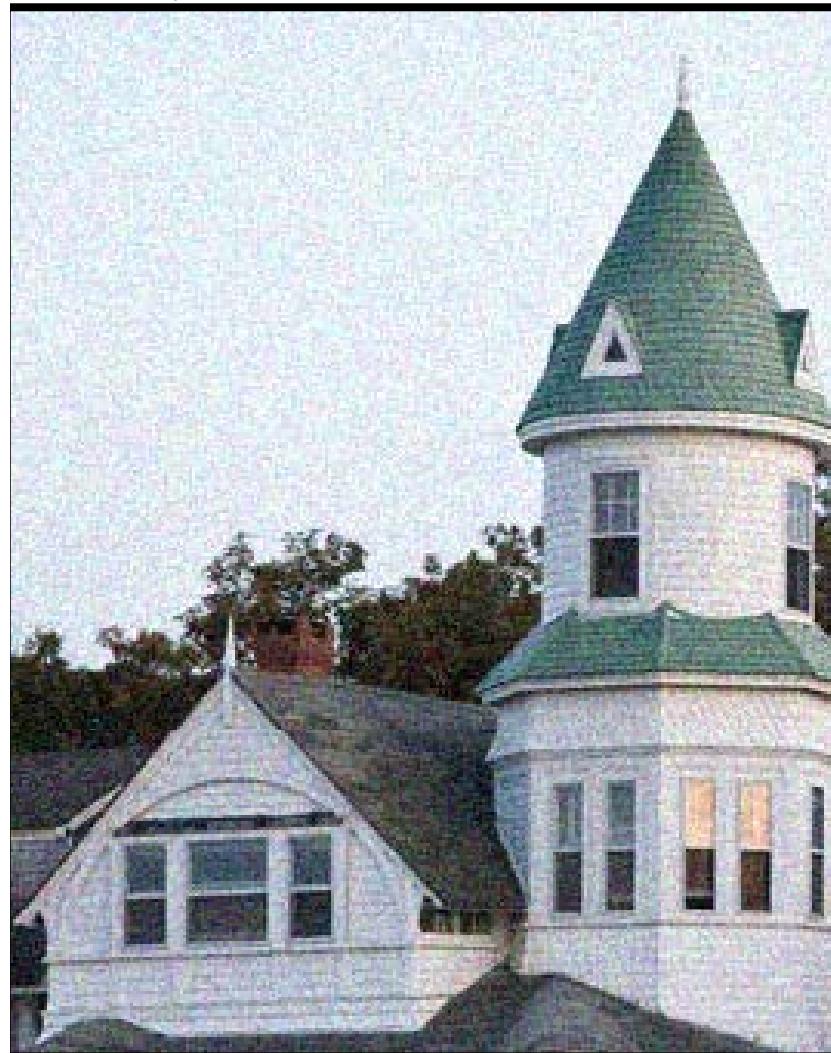
Smoothing an image without blurring its edges.

A Wide Range of Options

- Diffusion, Bayesian, Wavelets...
 - All have their pros and cons.
- Bilateral filter
 - not always the best result [Buades 05] but often good
 - easy to understand, adapt and set up

Basic denoising

Noisy input



Median 5x5



Basic denoising

Noisy input

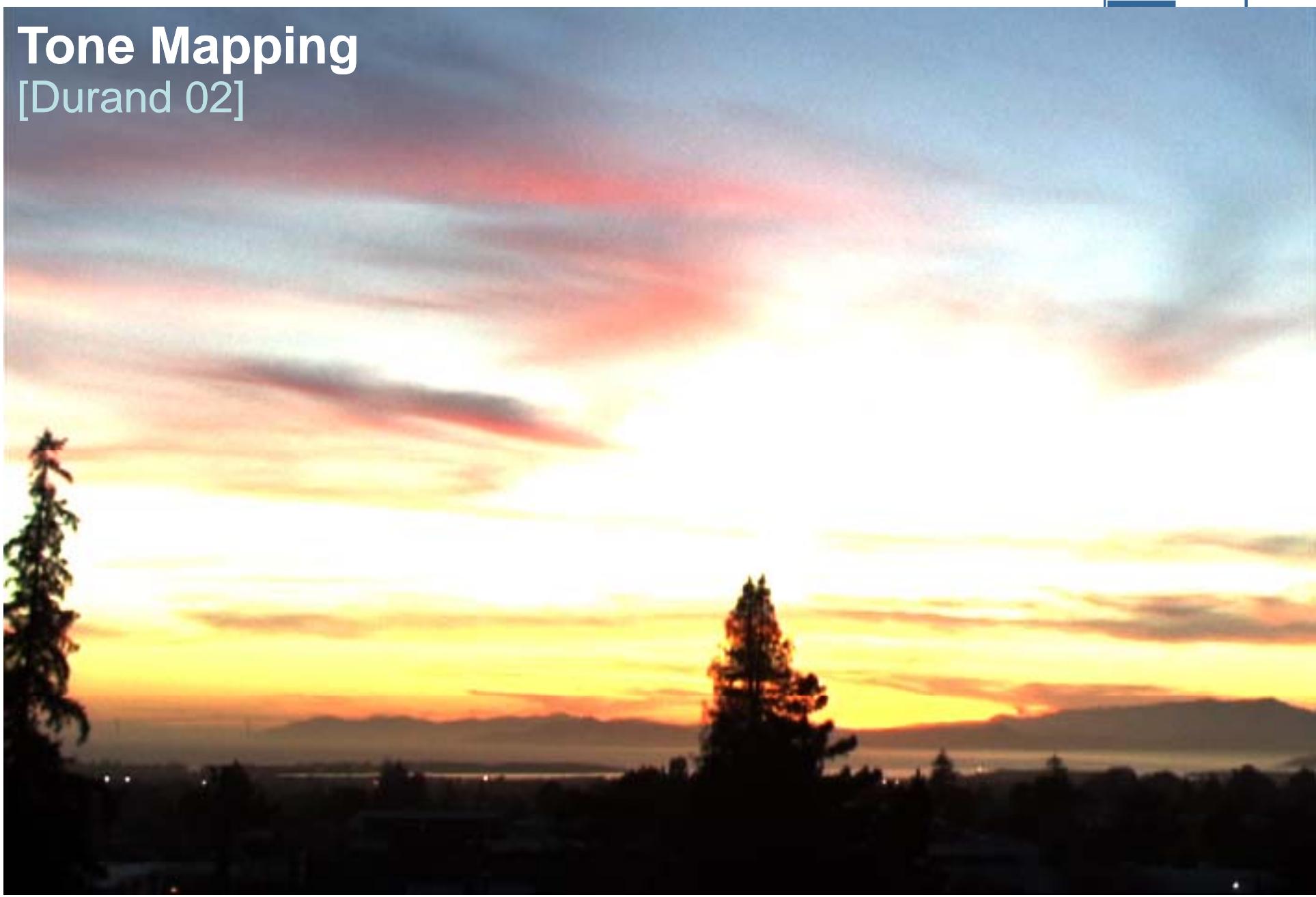


Bilateral filter 7x7 window



Tone Mapping

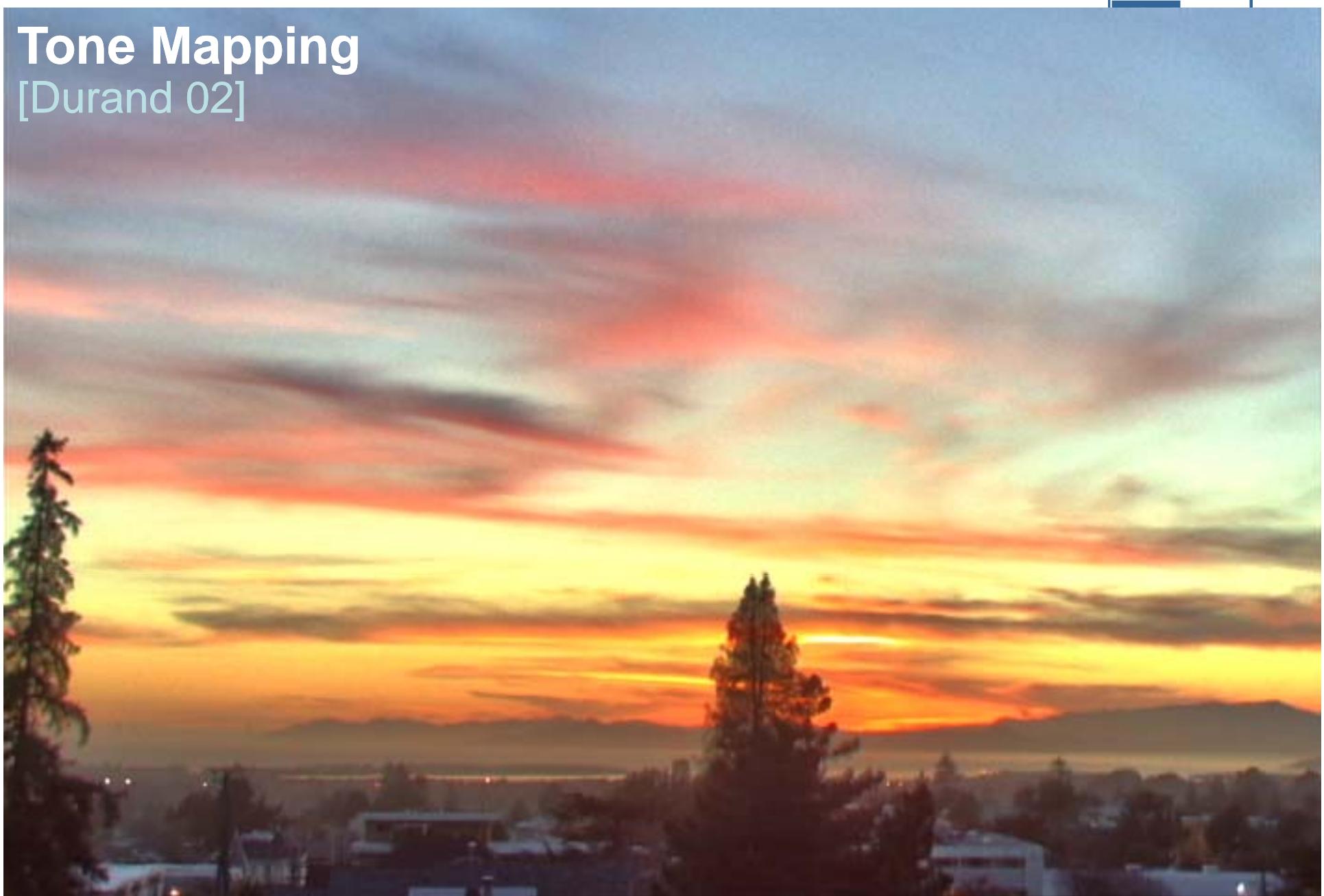
[Durand 02]



HDR input

Tone Mapping

[Durand 02]



output

Photographic Style Transfer

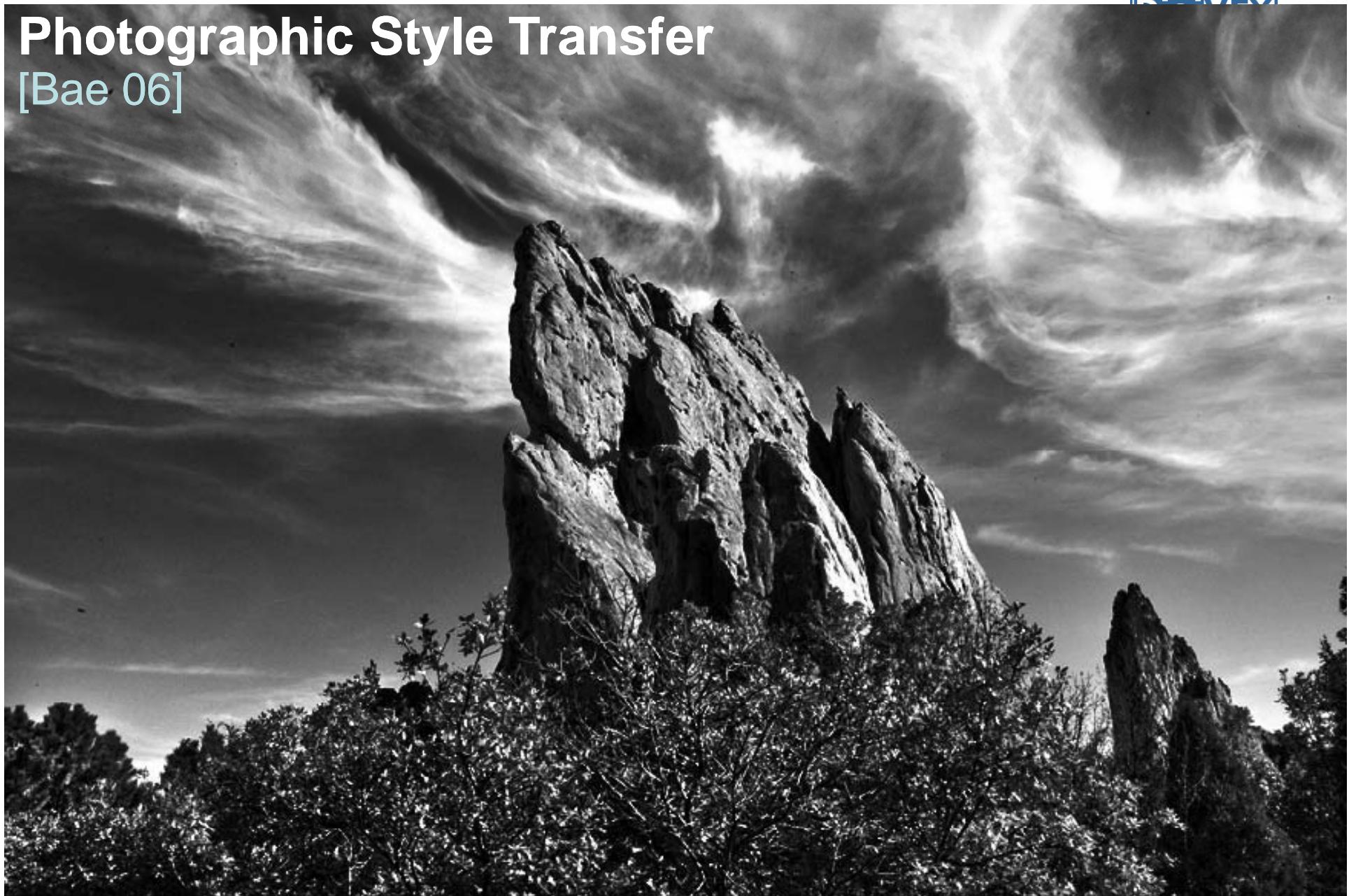
[Bae 06]



input

Photographic Style Transfer

[Bae 06]



output

Cartoon Rendition

[Winnemöller 06]



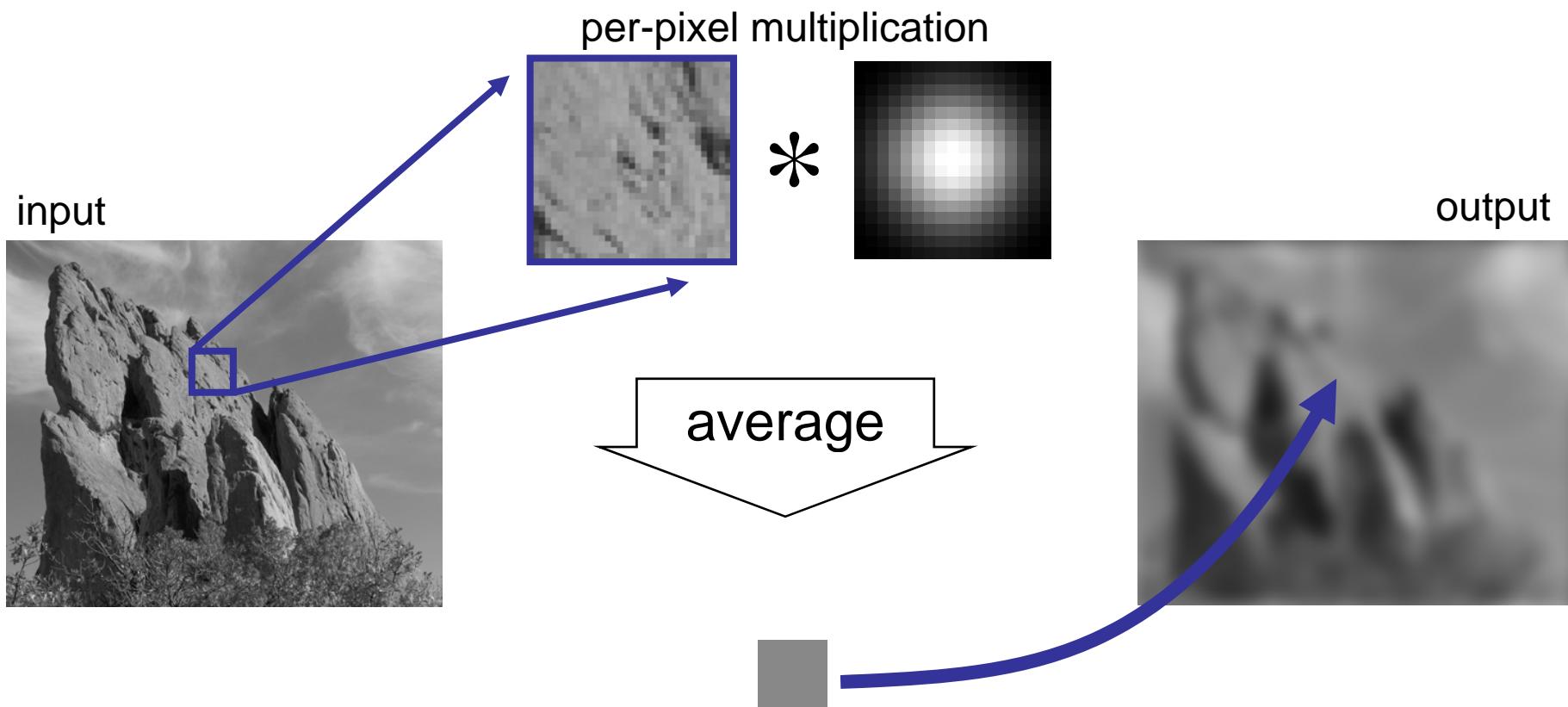
input

Cartoon Rendition
[Innemöller 06]

6 papers at
SIGGRAPH'07

output

Gaussian Blur



input



box average

Gaussian blur



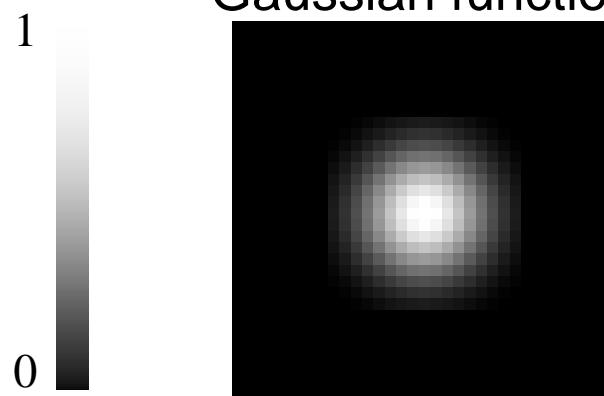
Equation of Gaussian Blur

Same idea: **weighted average of pixels.**

$$GB[I]_p = \sum_{q \in S} G_\sigma(||p-q||) I_q$$

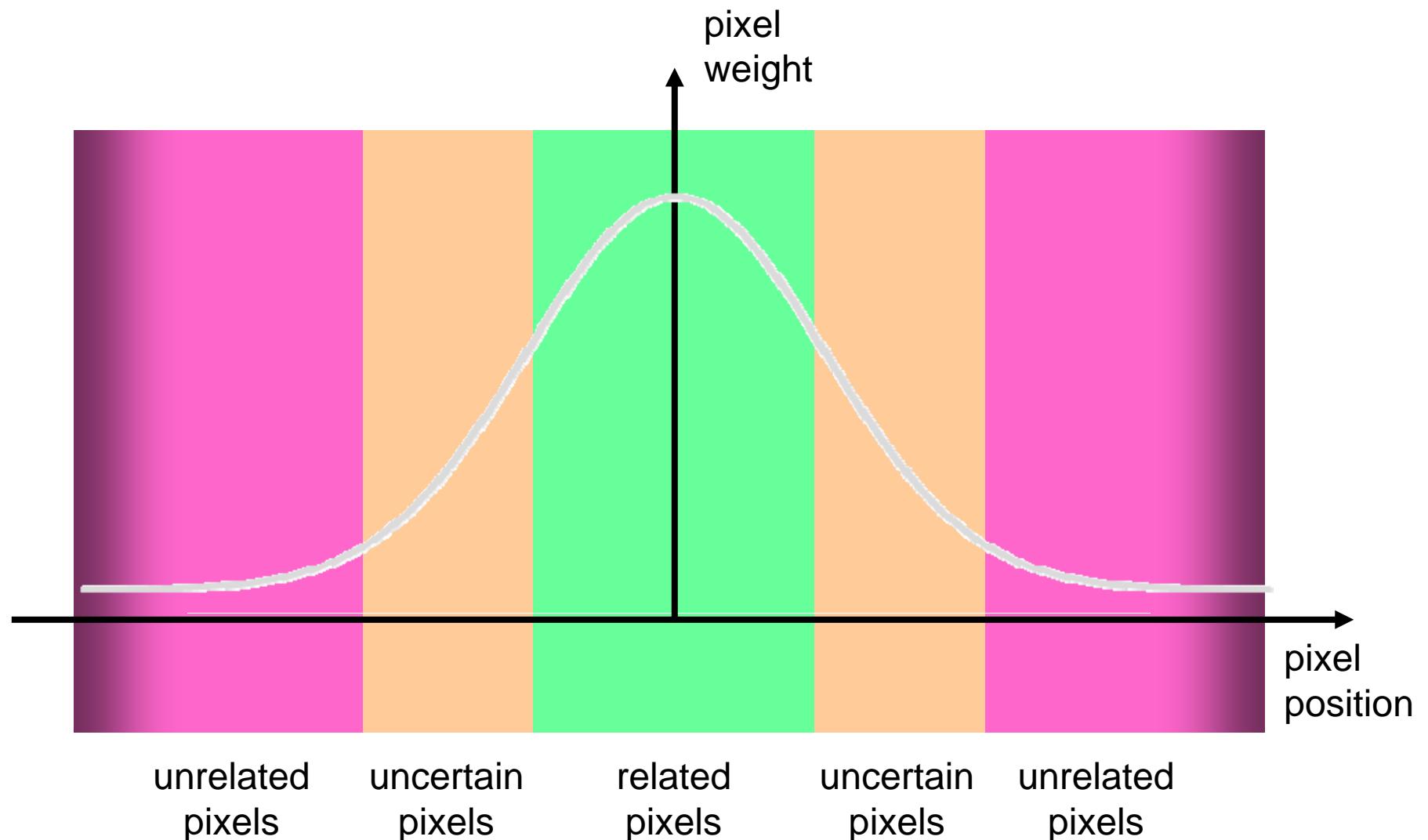


normalized
Gaussian function



Gaussian Profile

$$G_{\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$



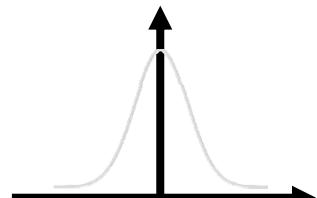
Spatial Parameter



input

$$GB[I]_p = \sum_{q \in S} G_{\sigma}(\| p - q \|) I_q$$

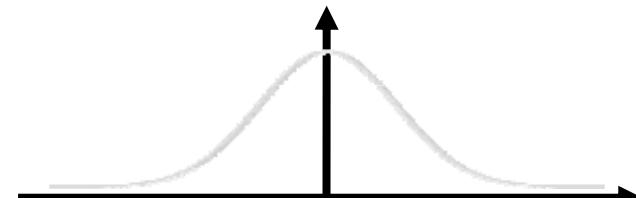
size of the window



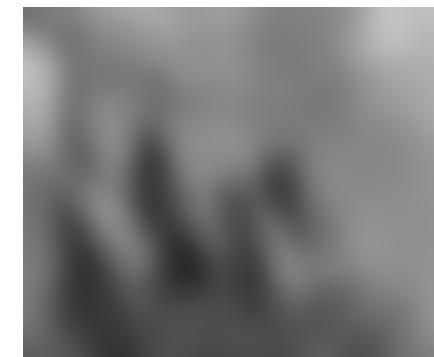
small σ



limited smoothing



large σ



strong smoothing

How to set σ

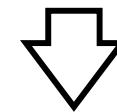
- Depends on the application.
- Common strategy: proportional to image size
 - e.g. 2% of the image diagonal
 - property: independent of image resolution

Properties of Gaussian Blur

- Weights independent of spatial location
 - linear convolution
 - well-known operation
 - efficient computation (recursive algorithm, FFT...)

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much:
edges are blurred.
 - Only spatial distance matters
 - No edge term



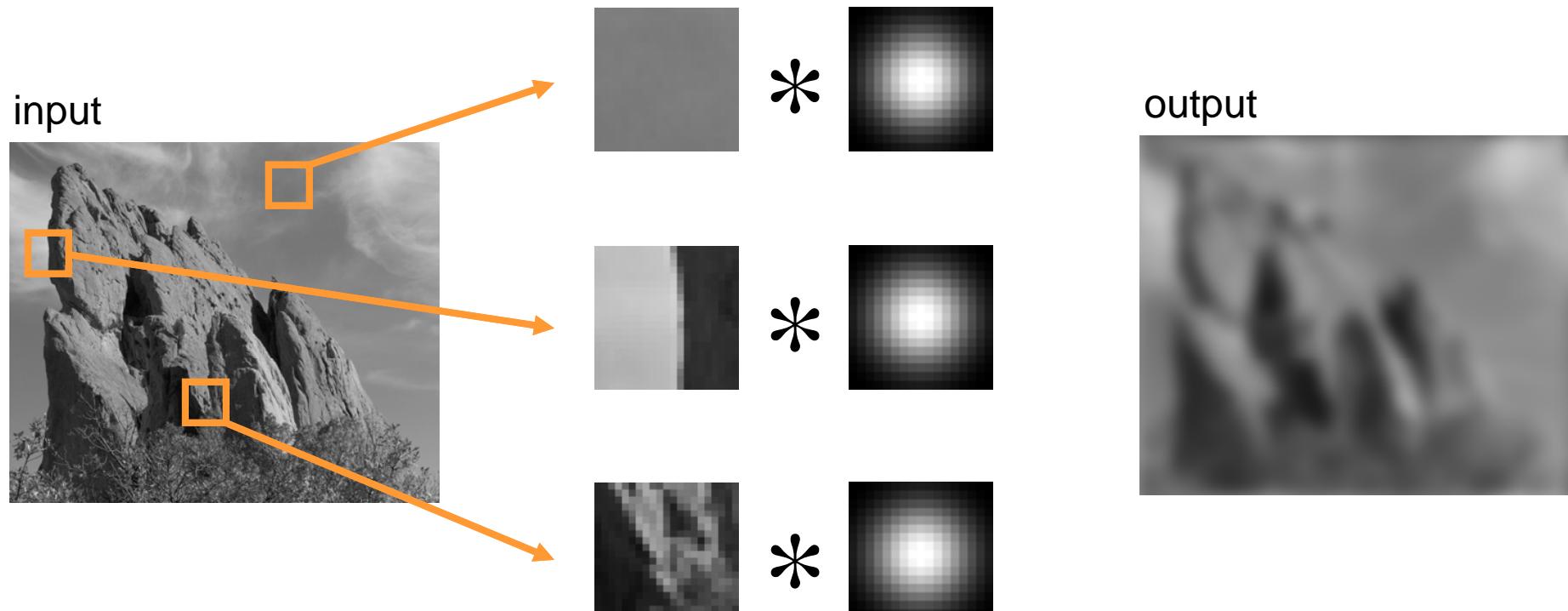
input



output

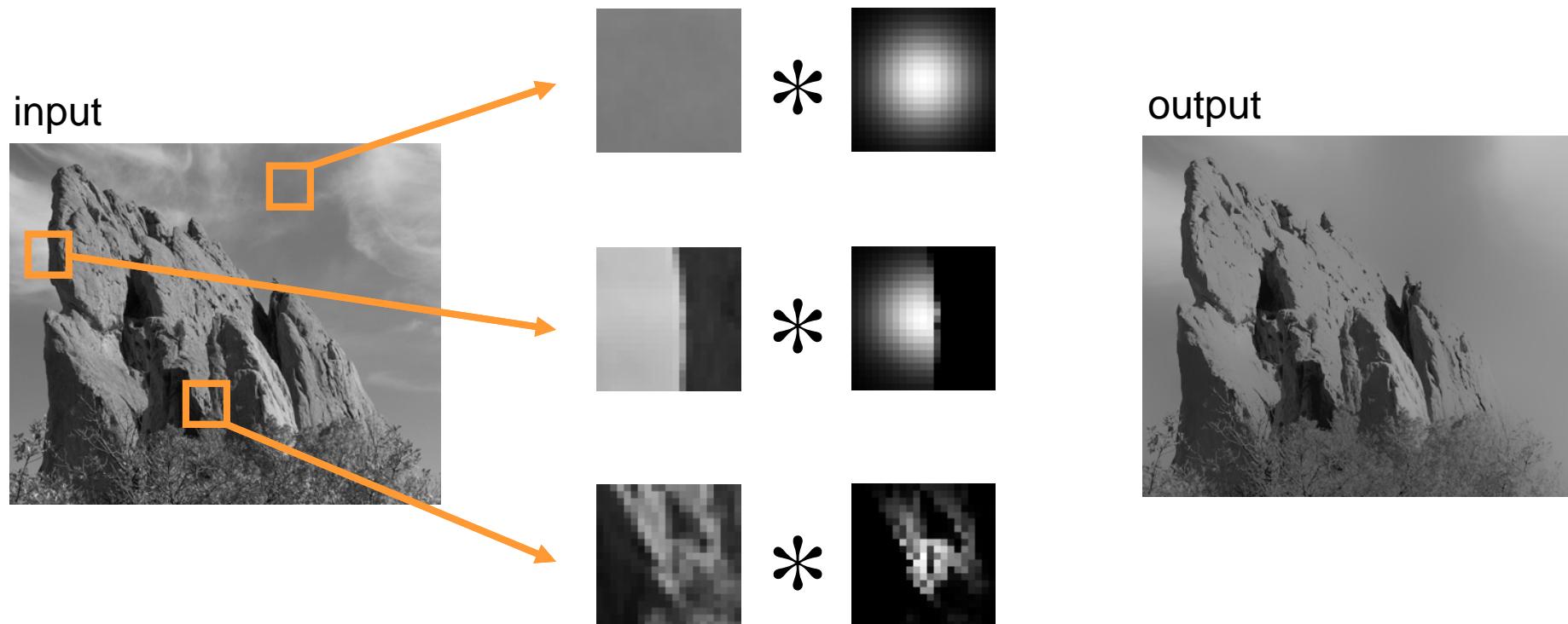
$$GB[I]_p = \sum_{q \in S} G_\sigma(\| p - q \|) I_q$$

Blur Comes from Averaging across Edges



Bilateral Filter No Averaging across Edges

[Aurich 95, Smith 97, Tomasi 98]



The kernel shape depends on the image content.

Bilateral Filter Definition

Same idea: **weighted average of pixels.**

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\| p - q \|) G_{\sigma_r}(|I_p - I_q|) I_q$$

new
not new
new

normalization factor
space weight
range weight

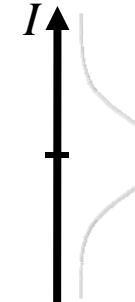
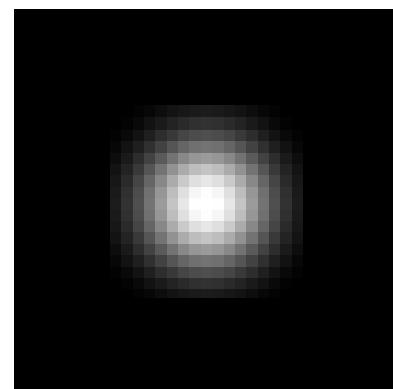
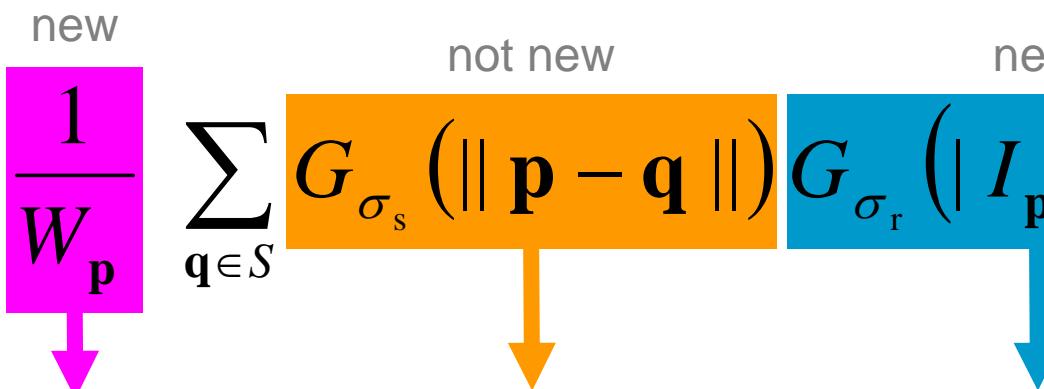
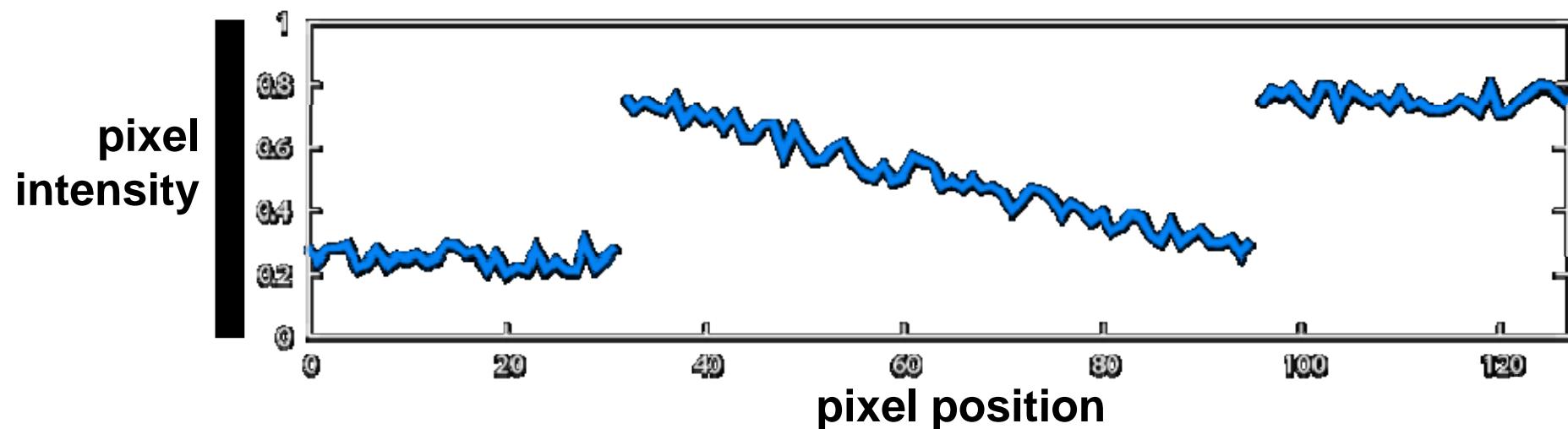


Illustration a 1D Image

- 1D image = line of pixels



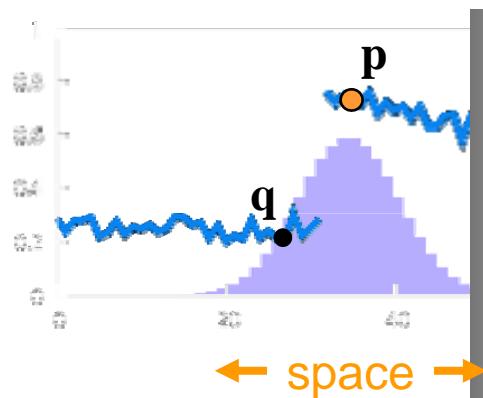
- Better visualized as a plot



Gaussian Blur and Bilateral Filter

DigiVFX

Gaussian blur

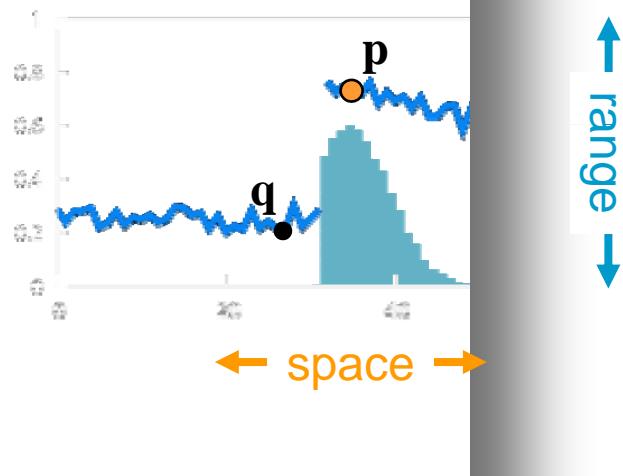


$$GB[I]_p = \sum_{q \in S} G_\sigma(\|p - q\|) I_q$$

space

Bilateral filter

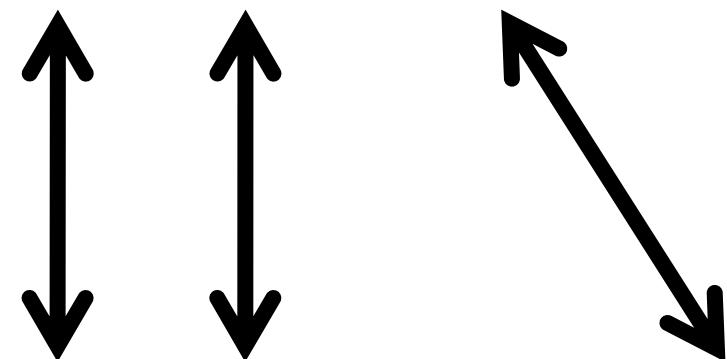
[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

normalization

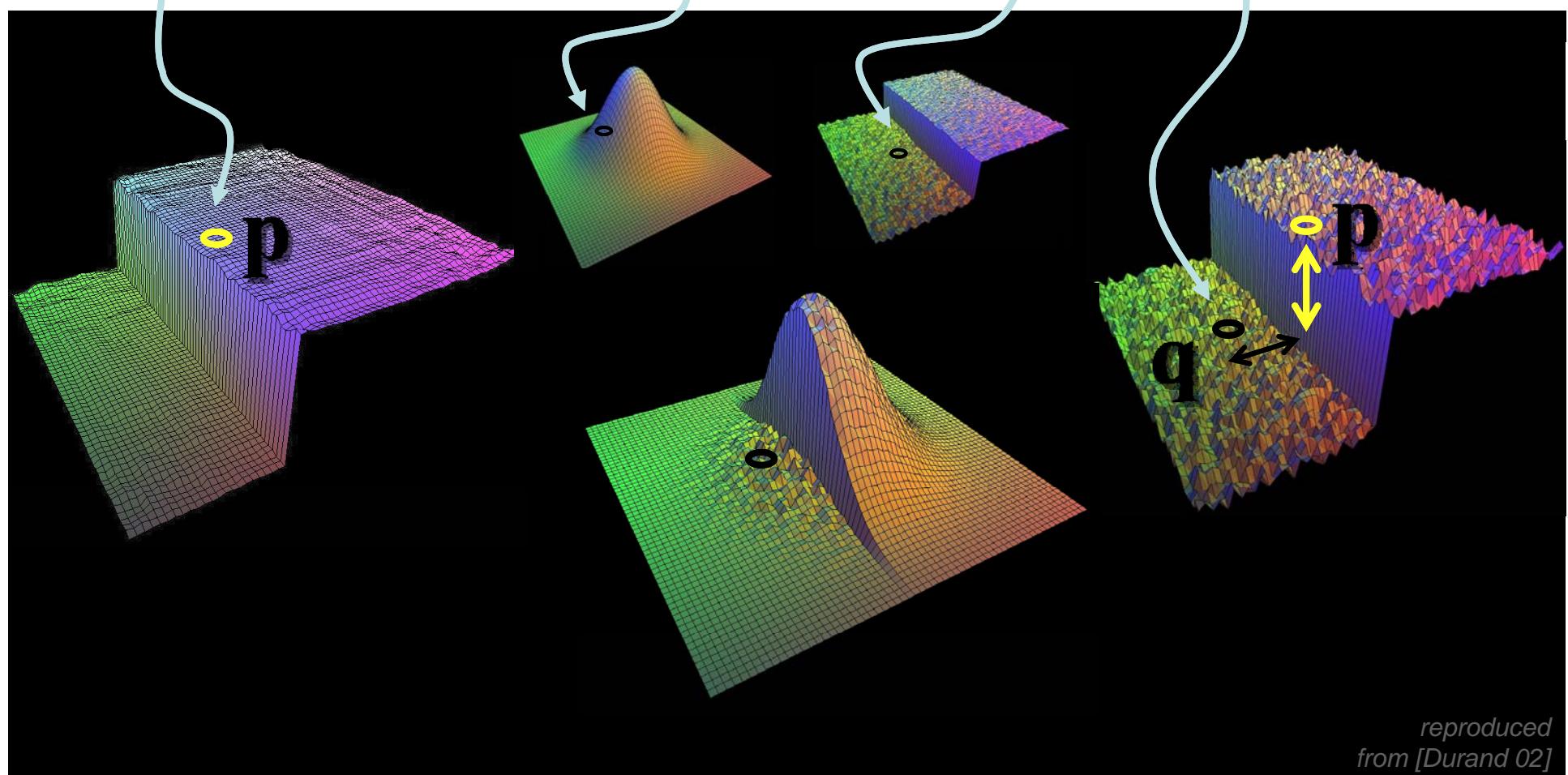
space range



Bilateral Filter on a Height Field

DigiVFX

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\| p - q \|) G_{\sigma_r}(|I_p - I_q|) I_q$$



reproduced
from [Durand 02]

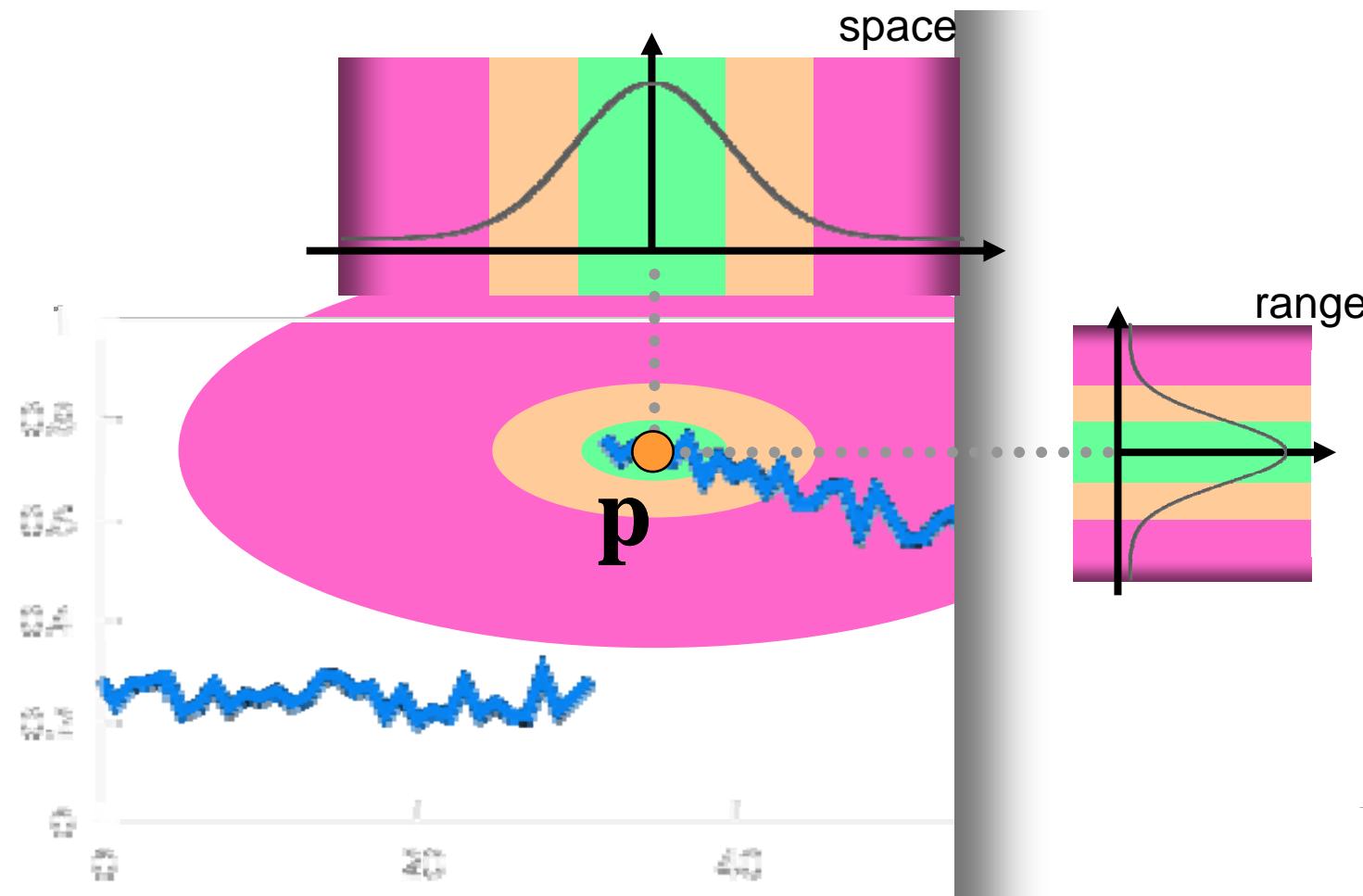
Space and Range Parameters

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\| p - q \|) G_{\sigma_r}(|I_p - I_q|) I_q$$


- space σ_s : spatial extent of the kernel, size of the considered neighborhood.
- range σ_r : “minimum” amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



Exploring the Parameter Space



input

$\sigma_r = 0.1$



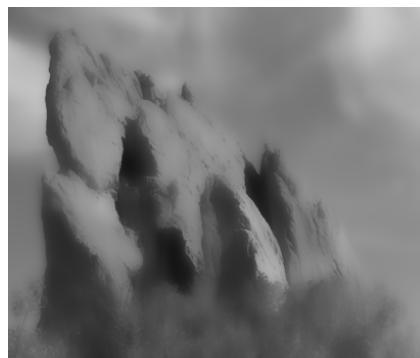
$\sigma_r = 0.25$



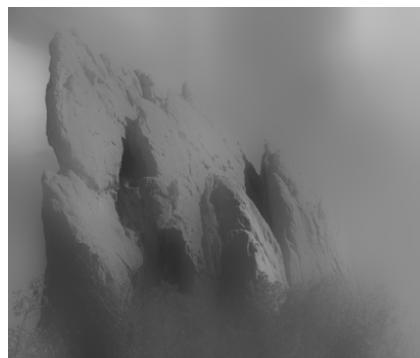
$\sigma_r = \infty$
(Gaussian blur)



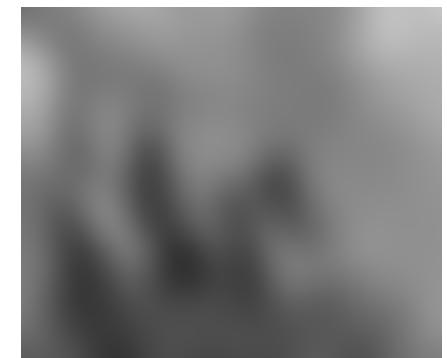
$\sigma_s = 2$



$\sigma_s = 6$



$\sigma_s = 18$





input

Varying the Range Parameter

$\sigma_r = 0.1$



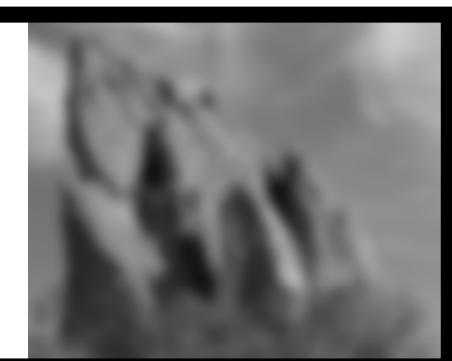
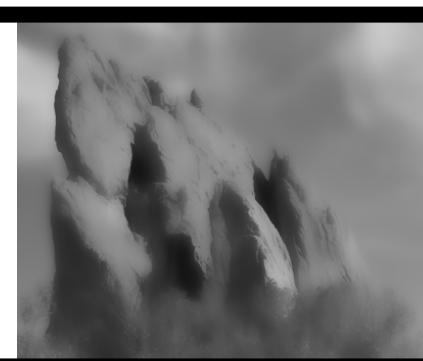
$\sigma_r = 0.25$



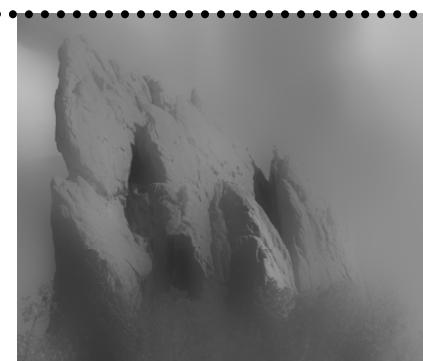
$\sigma_r = \infty$
(Gaussian blur)



$\sigma_s = 2$



$\sigma_s = 6$



$\sigma_s = 18$

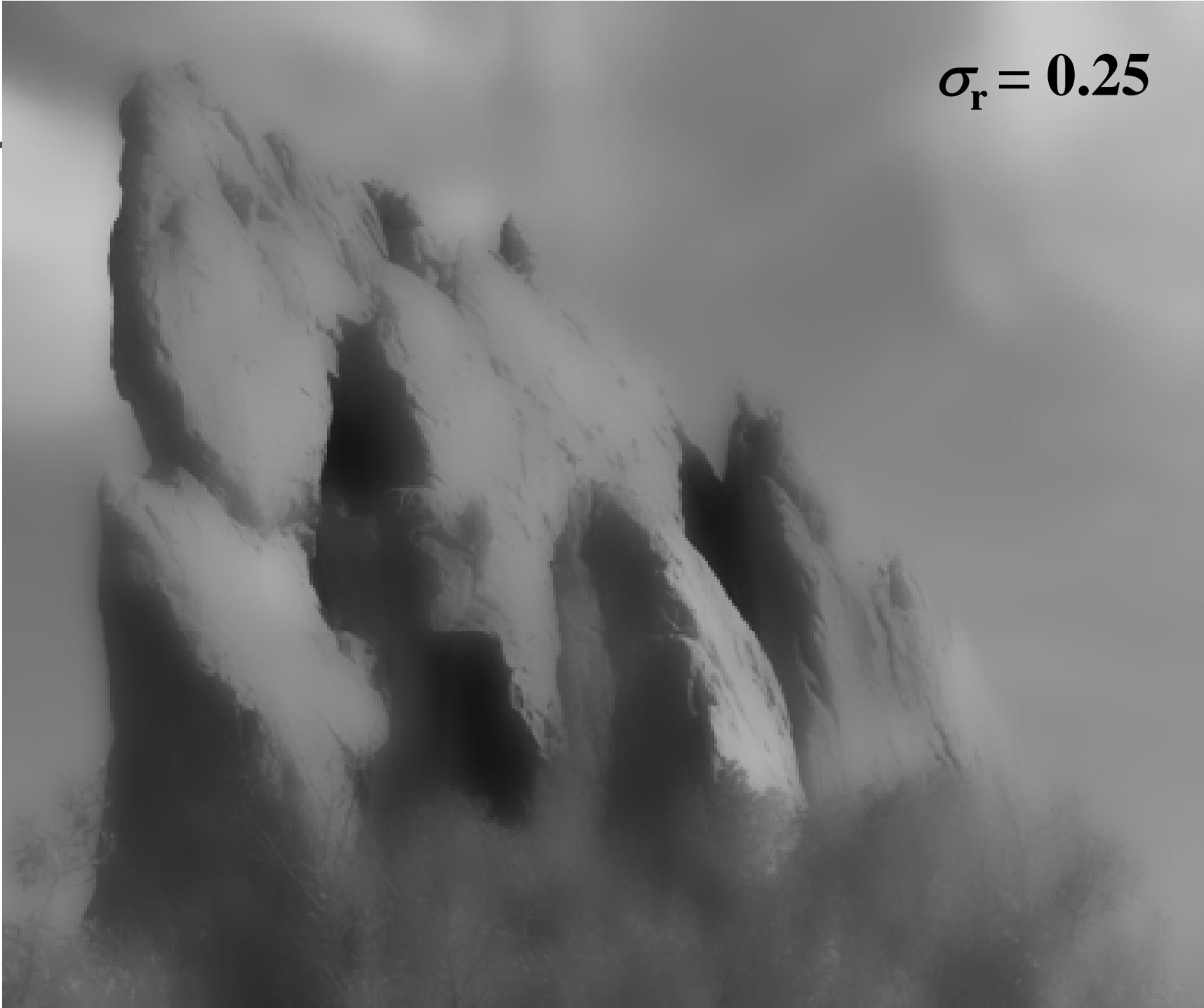
input



$\sigma_r = 0.1$



$\sigma_r = 0.25$



$\sigma_r = \infty$
(Gaussian blur)

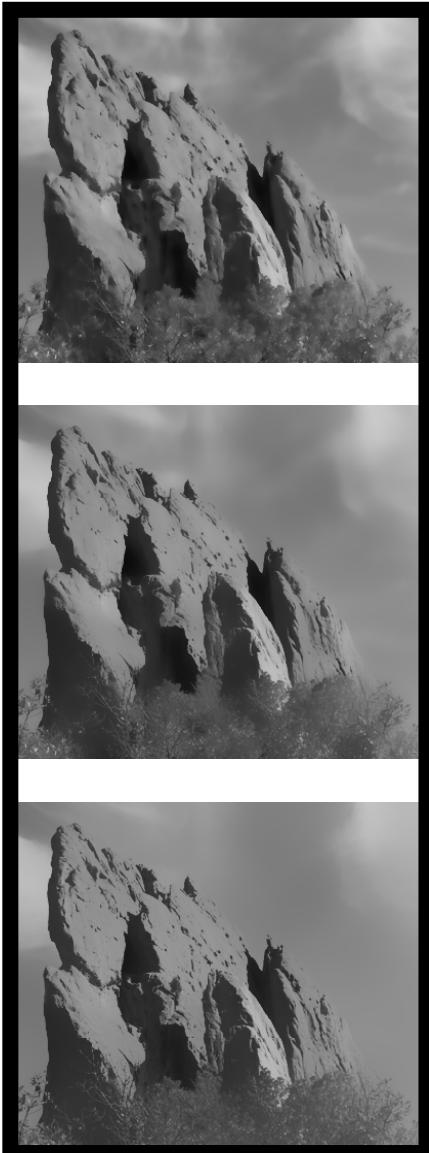


Varying the Space Parameter



input

$\sigma_s = 2$



$\sigma_s = 6$



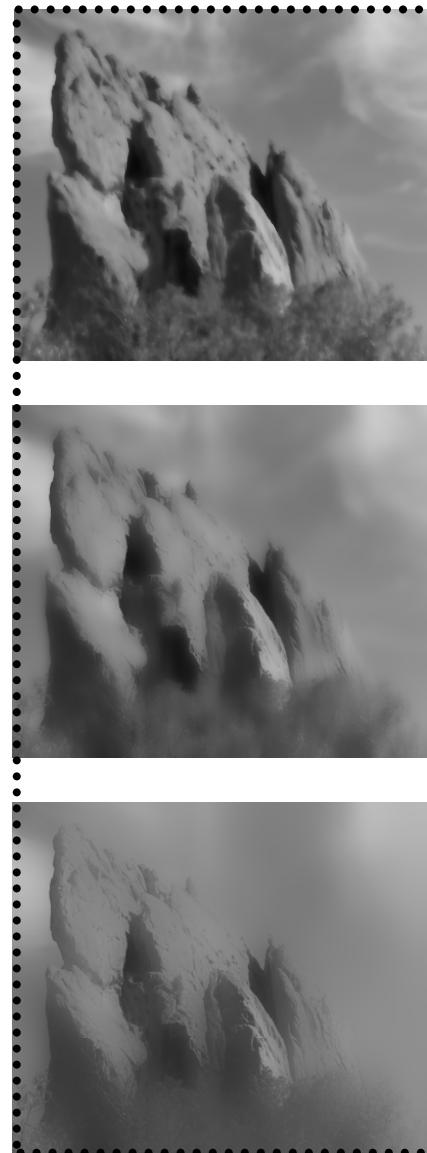
$\sigma_s = 18$



$\sigma_r = 0.1$

$\sigma_r = 0.25$

$\sigma_r = \infty$
(Gaussian blur)



input



$\sigma_s = 2$



$\sigma_s = 6$



$\sigma_s = 18$



How to Set the Parameters

Depends on the application. For instance:

- space parameter: proportional to image size
 - e.g., 2% of image diagonal
- range parameter: proportional to edge amplitude
 - e.g., mean or median of image gradients
- independent of resolution and exposure

Iterating the Bilateral Filter

$$I_{(n+1)} = BF[I_{(n)}]$$

- Generate more piecewise-flat images
- Often not needed in computational photo, but could be useful for applications such as NPR.

input



1 iteration



2 iterations



4 iterations



Advantages of Bilateral Filter

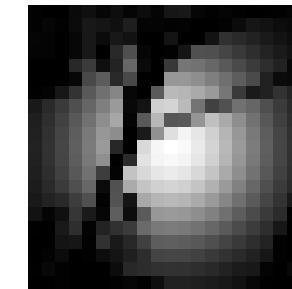
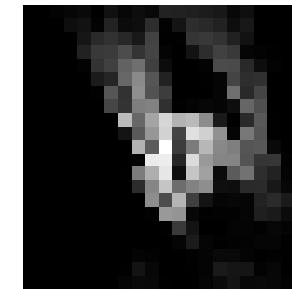
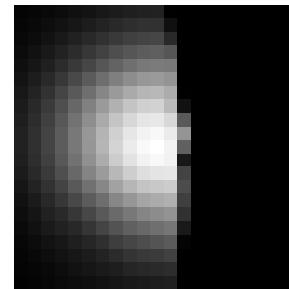
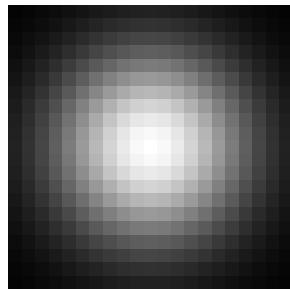
- Easy to understand
 - Weighted mean of nearby pixels
- Easy to adapt
 - Distance between pixel values
- Easy to set up
 - Non-iterative

Hard to Compute

- Nonlinear

$$BF [I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

- Complex, spatially varying kernels
 - Cannot be precomputed, no FFT...



- Brute-force implementation is slow > 10min

But Bilateral Filter is Nonlinear

- Slow but some accelerations exist:
 - [Elad 02]: Gauss-Seidel iterations
 - Only for many iterations
 - [Durand 02, Weiss 06]: fast approximation
 - No formal understanding of accuracy versus speed
 - [Weiss 06]: Only box function as spatial kernel

A Fast Approximation of the Bilateral Filter using a Signal Processing Approach

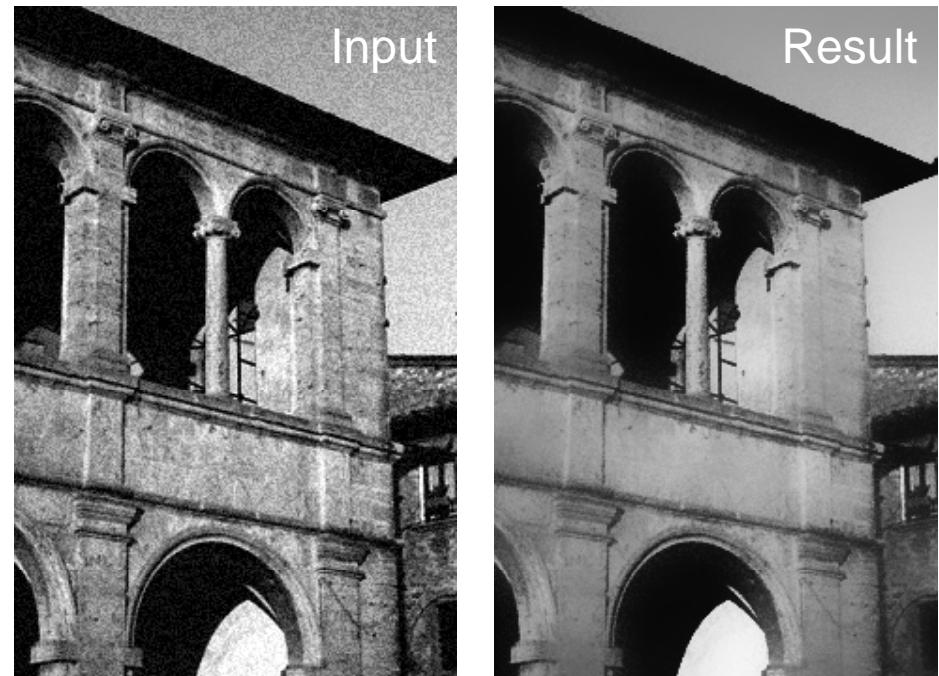
Sylvain Paris and Frédo Durand

Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology



Definition of Bilateral Filter

- [Smith 97, Tomasi 98]
- Smoothes an image and preserves edges
- Weighted average of neighbors
- Weights
 - Gaussian on *space* distance
 - Gaussian on *range* distance
 - sum to 1



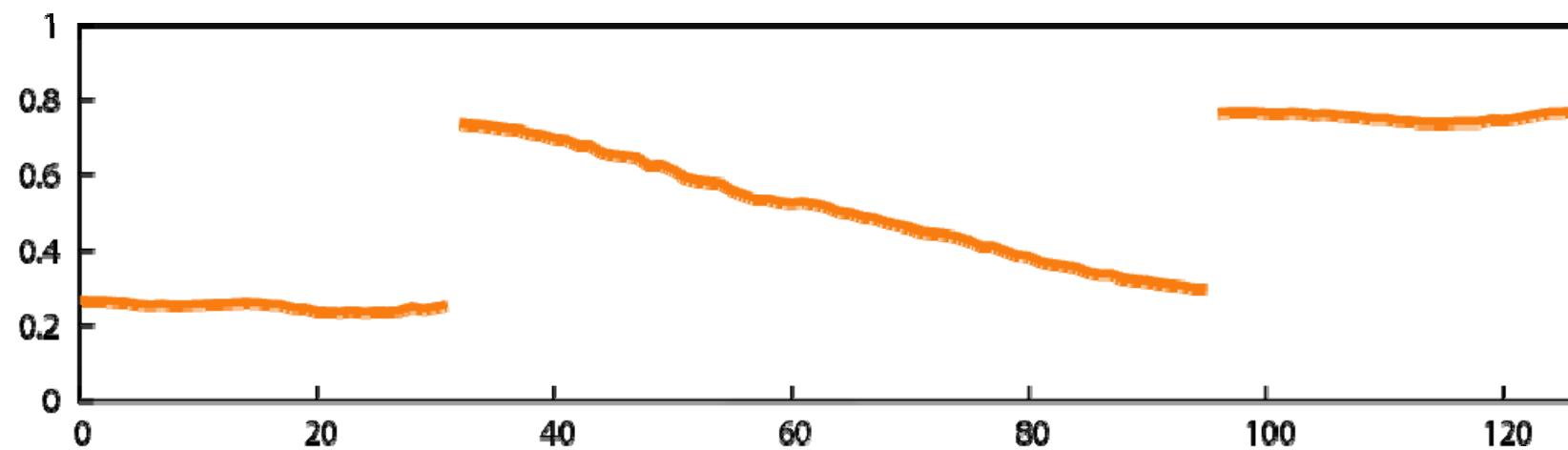
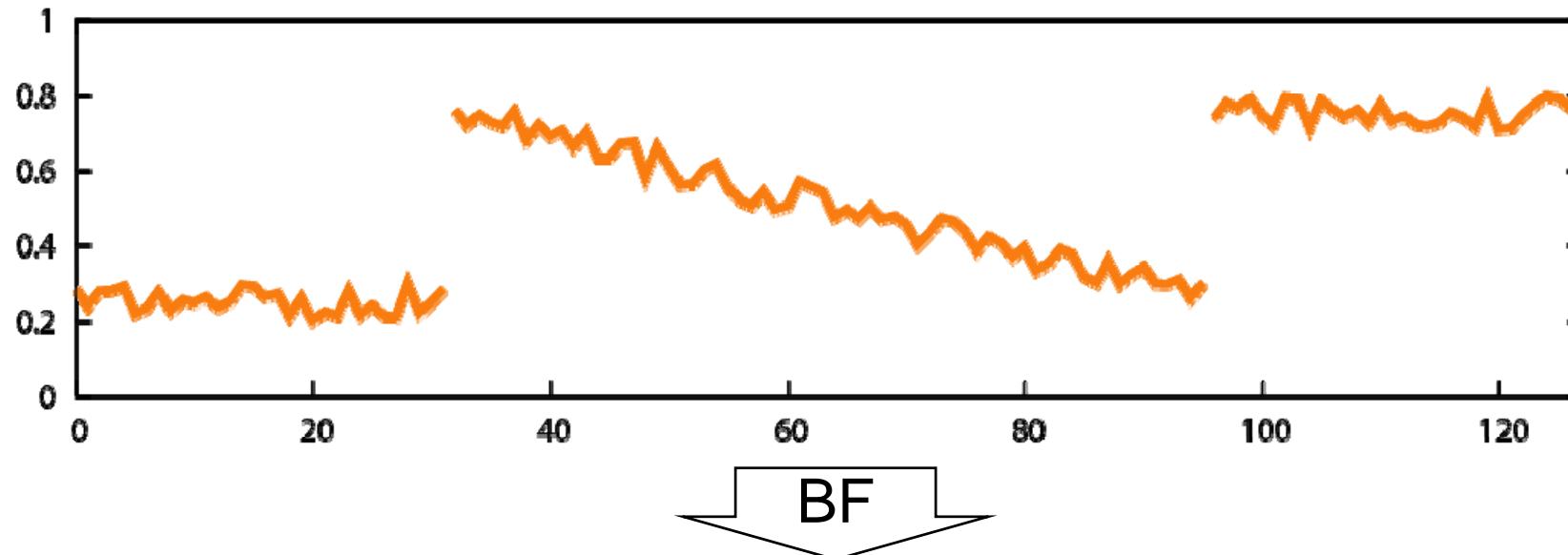
$$I_p^{bf} = \frac{1}{W_p^{bf}} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

space range

Contributions

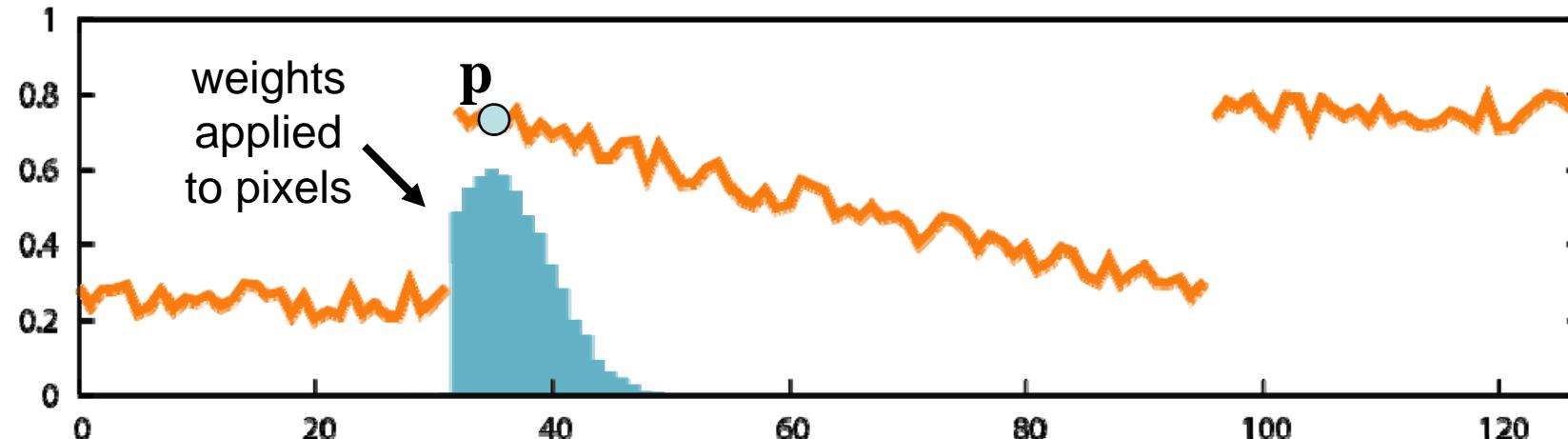
- Link with linear filtering
- Fast and accurate approximation

Intuition on 1D Signal



Intuition on 1D Signal

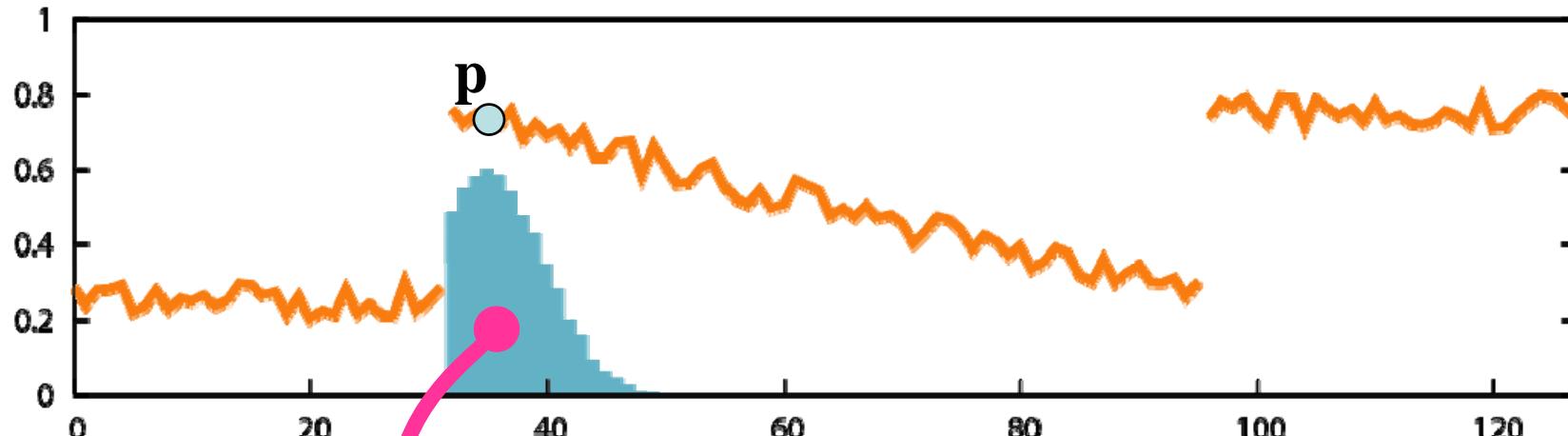
Weighted Average of Neighbors



- Near and similar pixels have influence.
- Far pixels have no influence.
- Pixels with different value have no influence.

Link with Linear Filtering

1. Handling the Division



sum of weights

$$I_p^{bf} = \frac{1}{W_p^{bf}} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

Handling the division with a projective space.

Formalization: Handling the Division

$$\begin{aligned} I_{\mathbf{p}}^{\text{bf}} &= \frac{1}{W_{\mathbf{p}}^{\text{bf}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}} \\ W_{\mathbf{p}}^{\text{bf}} &= \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \end{aligned}$$

- Normalizing factor as homogeneous coordinate
 - Multiply both sides by $W_{\mathbf{p}}^{\text{bf}}$

$$\begin{pmatrix} W_{\mathbf{p}}^{\text{bf}} & I_{\mathbf{p}}^{\text{bf}} \\ & W_{\mathbf{p}}^{\text{bf}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} I_{\mathbf{q}} \\ 1 \end{pmatrix}$$

Formalization: Handling the Division

$$\begin{pmatrix} W_p^{bf} & I_p^{bf} \\ & W_p^{bf} \end{pmatrix} = \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) \begin{pmatrix} W_q & I_q \\ & W_q \end{pmatrix} \text{ with } W_q = 1$$

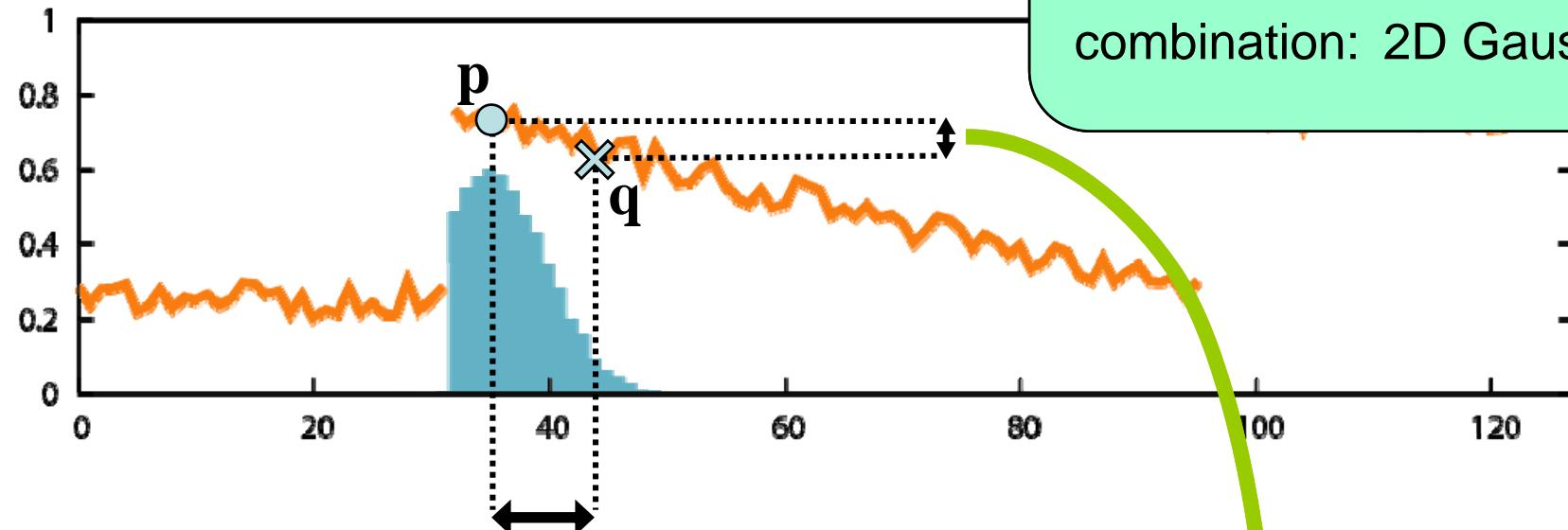
- Similar to homogeneous coordinates in projective space
- Division delayed until the end
- Next step: Adding a dimension to make a convolution appear

Link with Linear Filtering

2. Introducing a Convolution

space: 1D Gaussian
x range: 1D Gaussian

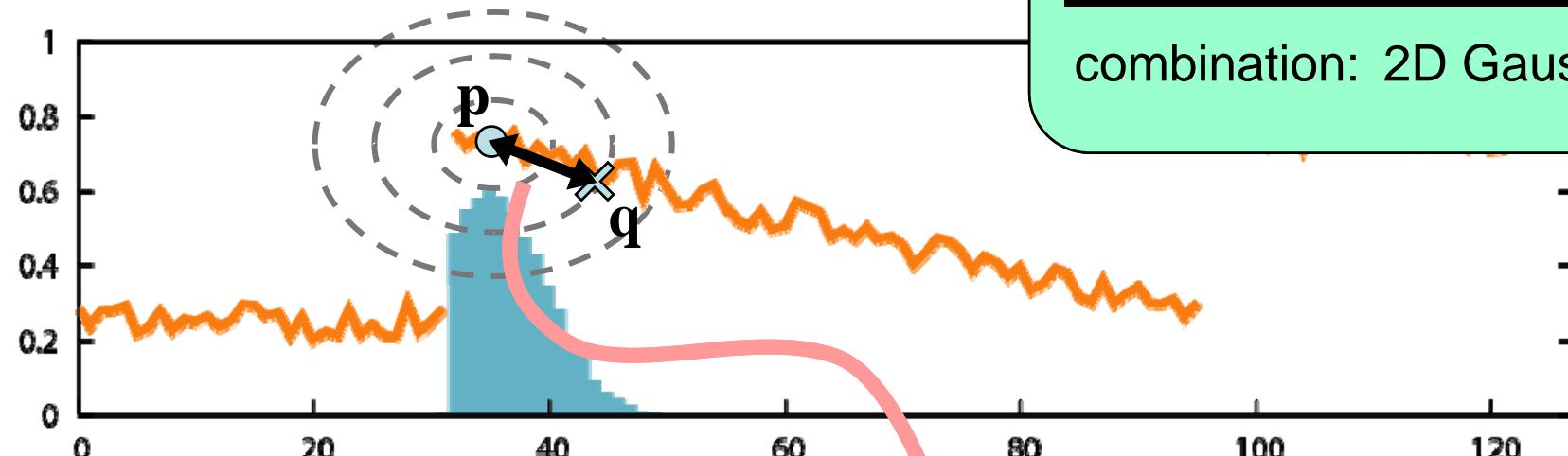
combination: 2D Gaussian



$$\begin{pmatrix} W_p^{\text{bf}} & I_p^{\text{bf}} \\ W_p^{\text{bf}} \end{pmatrix} = \sum_{q \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) \begin{pmatrix} W_q & I_q \\ W_q \end{pmatrix}$$

Link with Linear Filtering

2. Introducing a Convolution



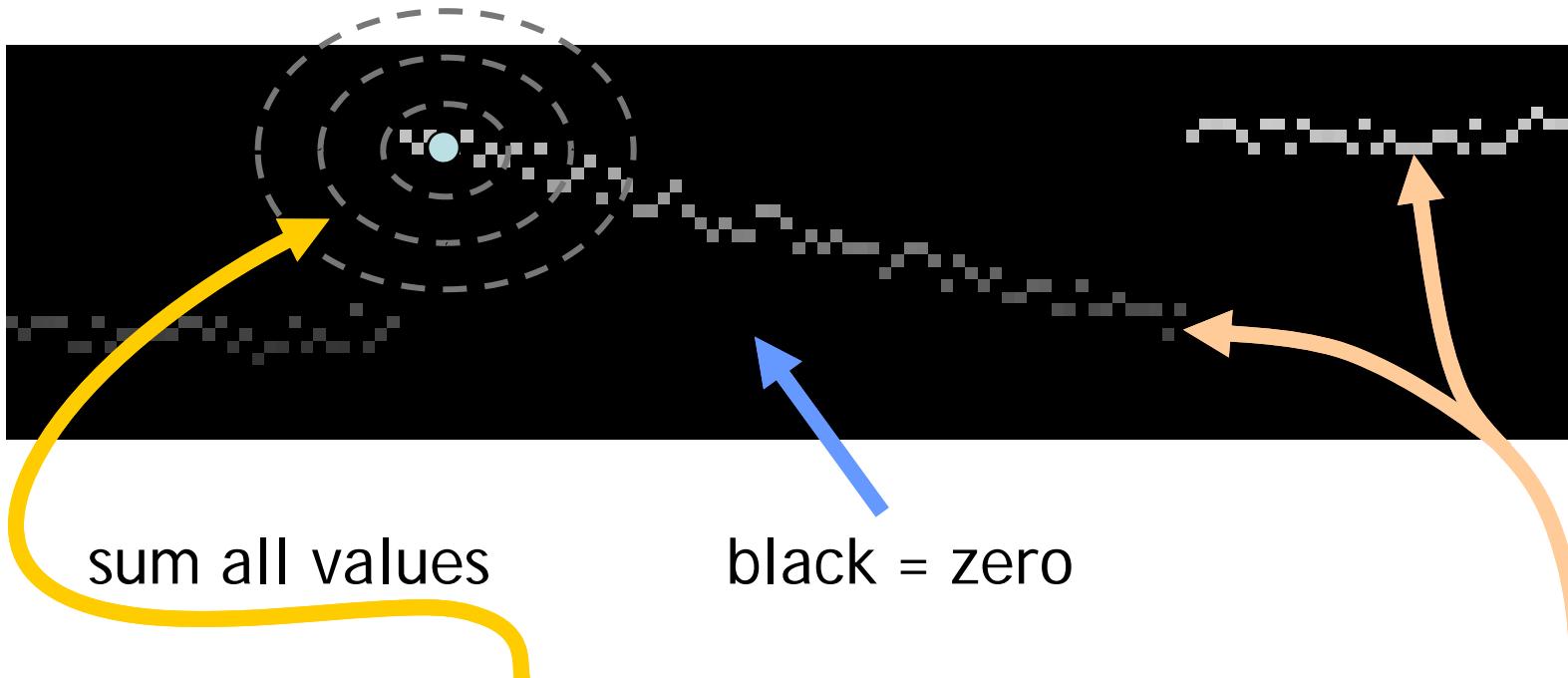
$$\begin{pmatrix} W_{\mathbf{p}}^{\text{bf}} & I_{\mathbf{p}}^{\text{bf}} \\ W_{\mathbf{p}} \end{pmatrix} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) \begin{pmatrix} W_{\mathbf{q}} & I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

space x range

Corresponds to a 3D Gaussian on a 2D image.

Link with Linear Filtering

2. Introducing a Convolution



$$\begin{pmatrix} W_{\mathbf{p}}^{\text{bf}} & I_{\mathbf{p}}^{\text{bf}} \\ W_{\mathbf{p}} \end{pmatrix} = \boxed{\sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}}}$$

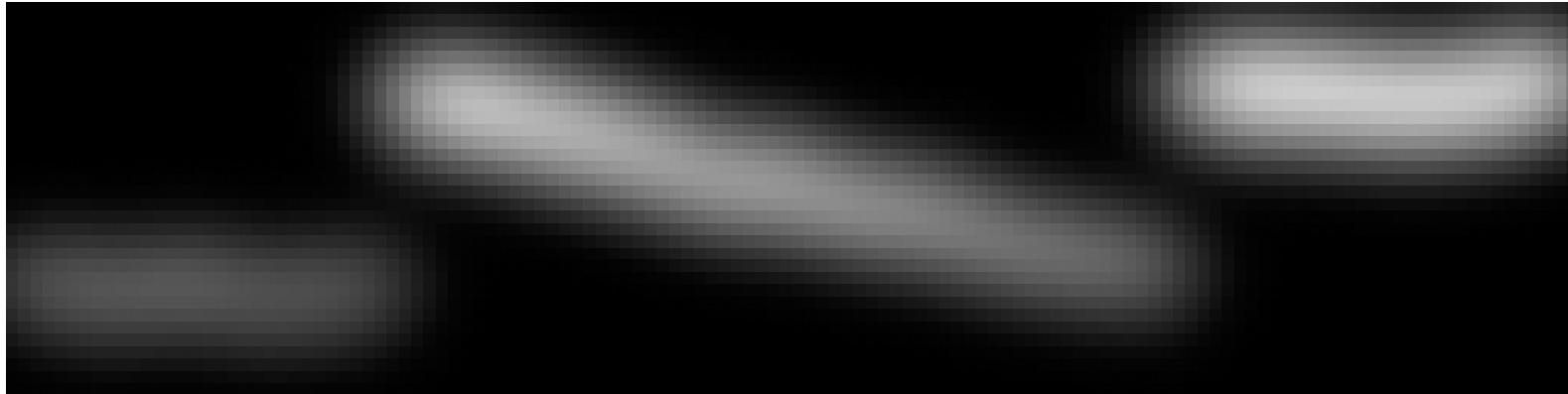
space-range Gaussian

$$\begin{pmatrix} W_{\mathbf{q}} & I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

sum all values multiplied by kernel \Rightarrow convolution

Link with Linear Filtering

2. Introducing a Convolution

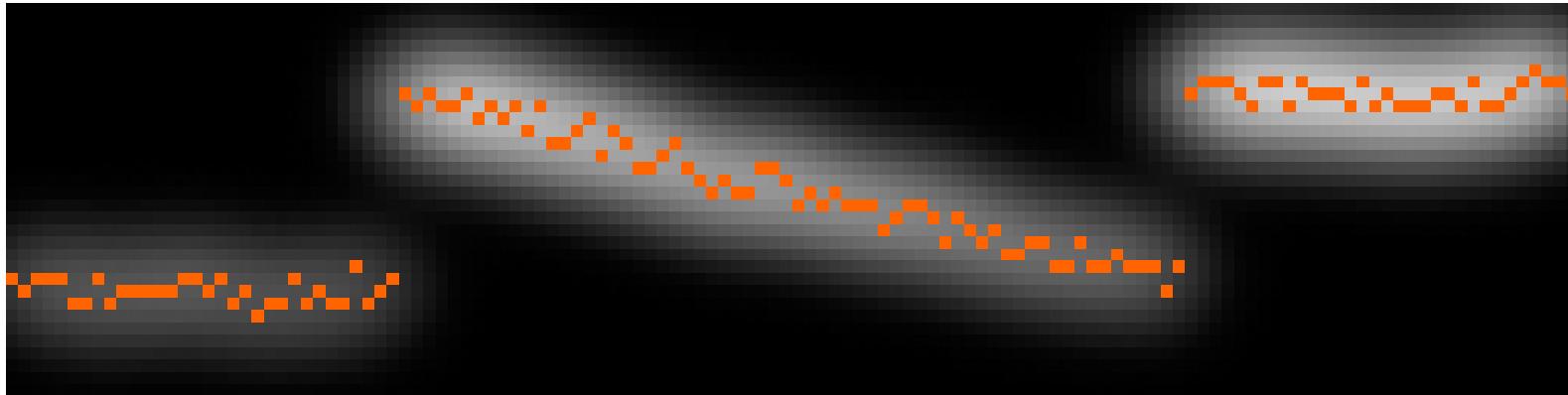


result of the convolution

$$\begin{pmatrix} W_{\mathbf{p}}^{\text{bf}} & I_{\mathbf{p}}^{\text{bf}} \\ W_{\mathbf{p}}^{\text{bf}} \end{pmatrix} = \sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_{\mathbf{q}} & I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$

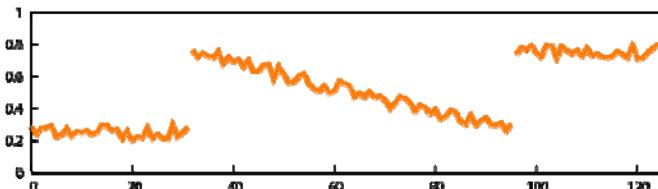
Link with Linear Filtering

2. Introducing a Convolution



result of the convolution

$$\begin{pmatrix} W_{\mathbf{p}}^{\text{bf}} & I_{\mathbf{p}}^{\text{bf}} \\ W_{\mathbf{p}}^{\text{bf}} \end{pmatrix} = \sum_{(\mathbf{q}, \zeta) \in \mathcal{S} \times \mathcal{R}} \text{space-range Gaussian} \begin{pmatrix} W_{\mathbf{q}} & I_{\mathbf{q}} \\ W_{\mathbf{q}} \end{pmatrix}$$



higher dimensional functions

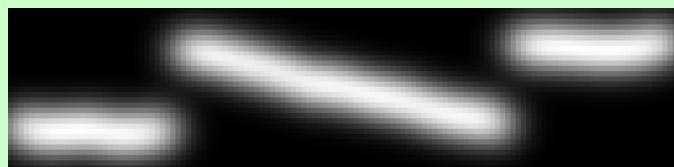
w_i



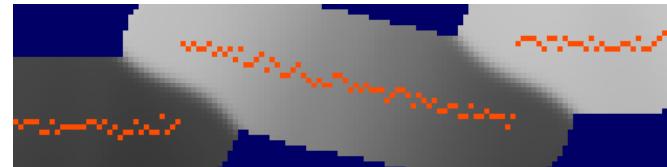
w



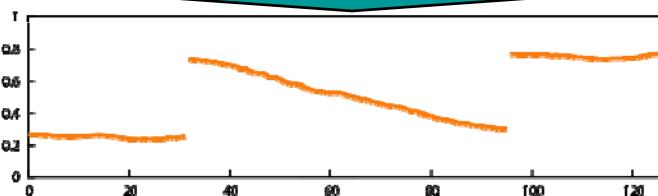
Gaussian convolution



division



slicing



Reformulation: Summary

linear: $(w^{\text{bf}} i^{\text{bf}}, w^{\text{bf}}) = g_{\sigma_s, \sigma_r} \otimes (wi, w)$

nonlinear: $I_p^{\text{bf}} = \frac{w^{\text{bf}}(\mathbf{p}, I_p) i^{\text{bf}}(\mathbf{p}, I_p)}{w^{\text{bf}}(\mathbf{p}, I_p)}$

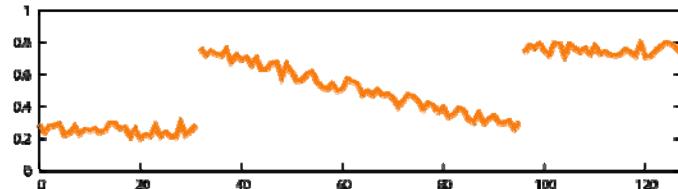
1. Convolution in higher dimension

- expensive but well understood (linear, FFT, etc)

2. Division and slicing

- nonlinear but simple and pixel-wise

Exact reformulation

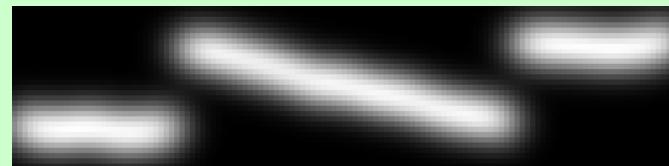


higher dimensional functions

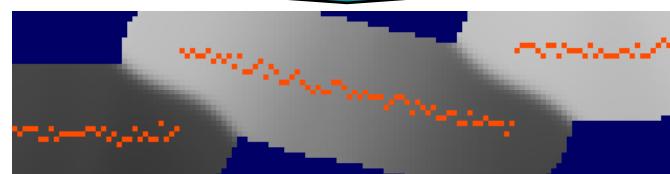


Low-pass filter

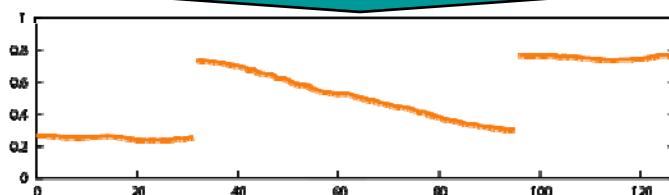
Gaussian convolution

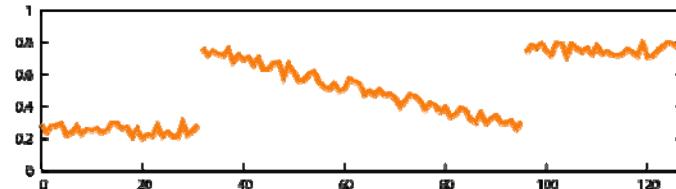


division



slicing





higher dimensional functions



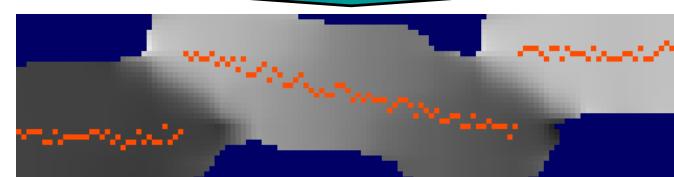
DOWNSAMPLE

Gaussian convolution

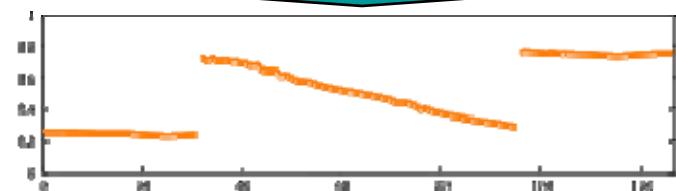


UPSAMPLE

division



slicing

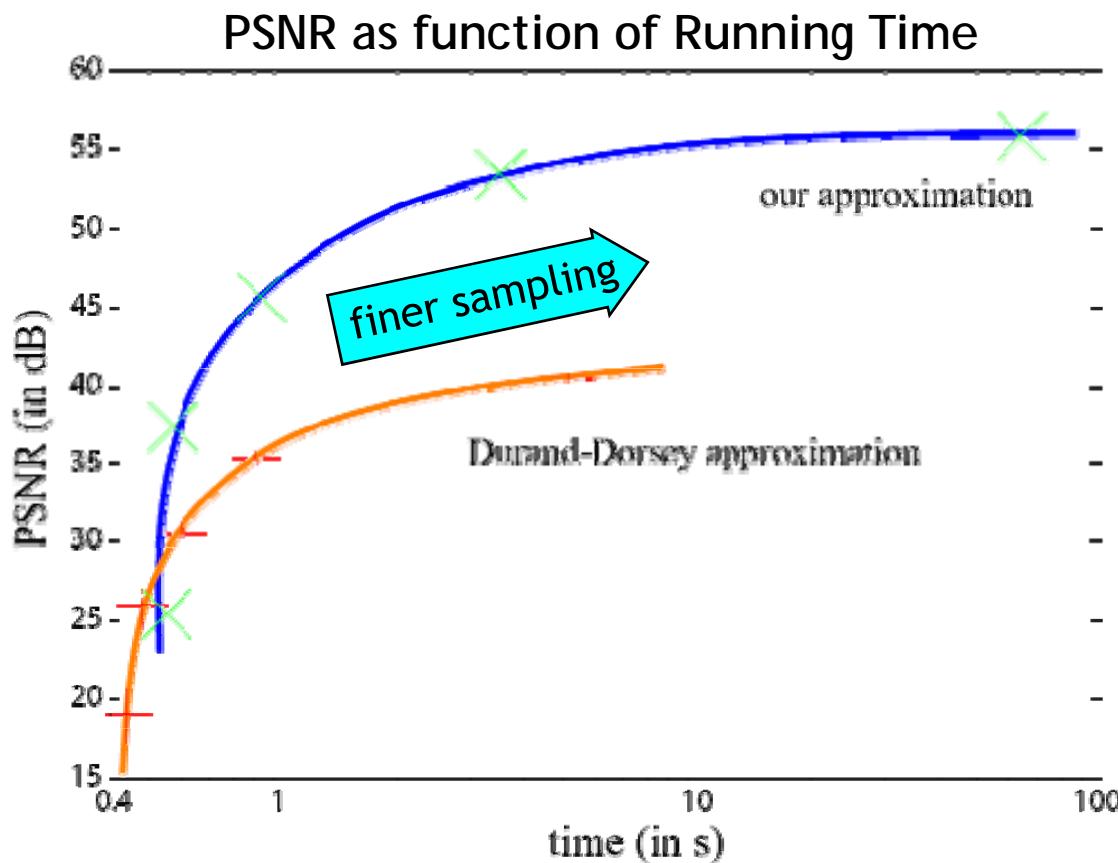


Fast Convolution by Downsampling

- Downsampling cuts frequencies above Nyquist limit
 - Less data to process
 - But induces error
- Evaluation of the approximation
 - Precision versus running time
 - Visual accuracy

Accuracy versus Running Time

- Finer sampling increases accuracy.
- More precise than previous work.



Digital
photograph
 1200×1600

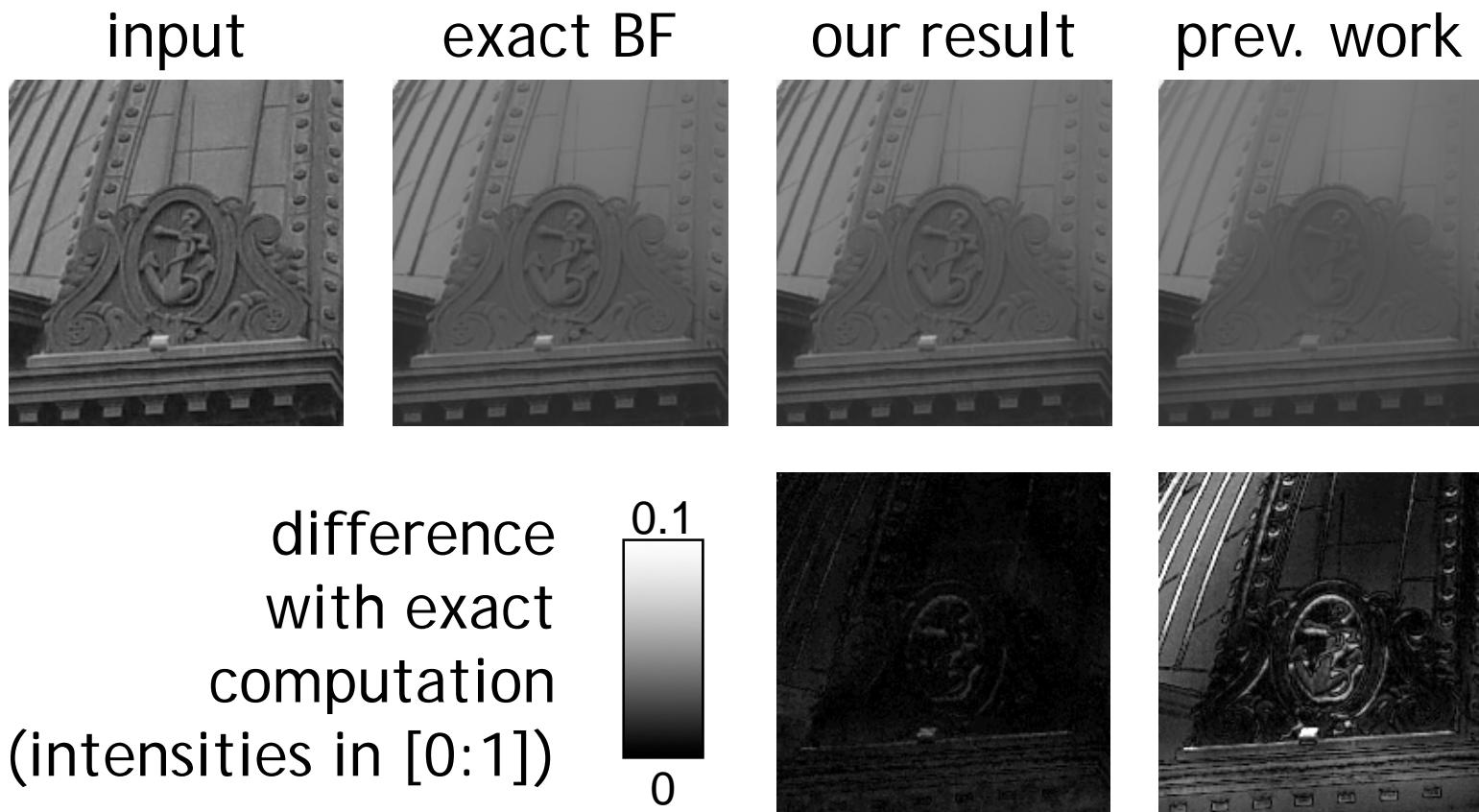
Straightforward
implementation is
over 10 minutes.

Visual Results

- Comparison with previous work [Durand 02]
 - running time = 1s for both techniques



1200 × 1600



Conclusions

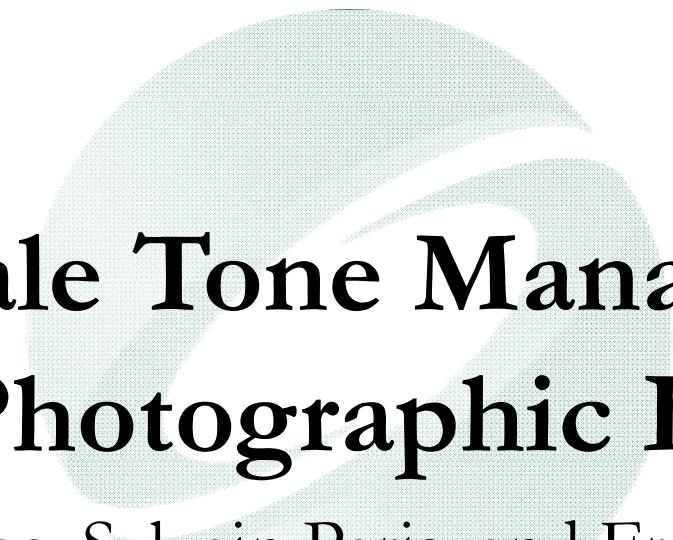
higher dimension \Rightarrow “better” computation

Practical gain

- Interactive running time
- Visually similar results
- Simple to code (100 lines)

Theoretical gain

- Link with linear filters
- Separation linear/nonlinear
- Signal processing framework



Two-scale Tone Management for Photographic Look

Soonmin Bae, Sylvain Paris, and Frédo Durand

MIT CSAIL

SIGGRAPH 2006

Ansel Adams



Ansel Adams, *Clearing Winter Storm*

An Amateur Photographer



A Variety of Looks



Goals

- Control over photographic look
- Transfer “look” from a model photo

For example,

we want



with the look of



Aspects of Photographic Look

- Subject choice
- Framing and composition
- ➔ Specified by input photos



Input

- Tone distribution and contrast
- ➔ Modified based on model photos



Model

Tonal Aspects of Look



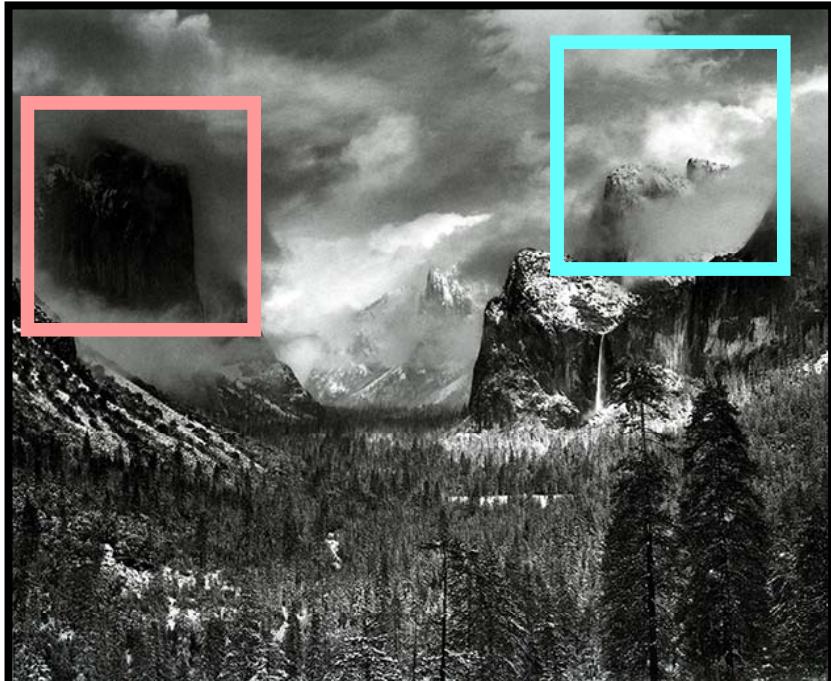
Ansel Adams



Kenro Izu

Tonal aspects of Look - Global Contrast

DigiVFX



Ansel Adams



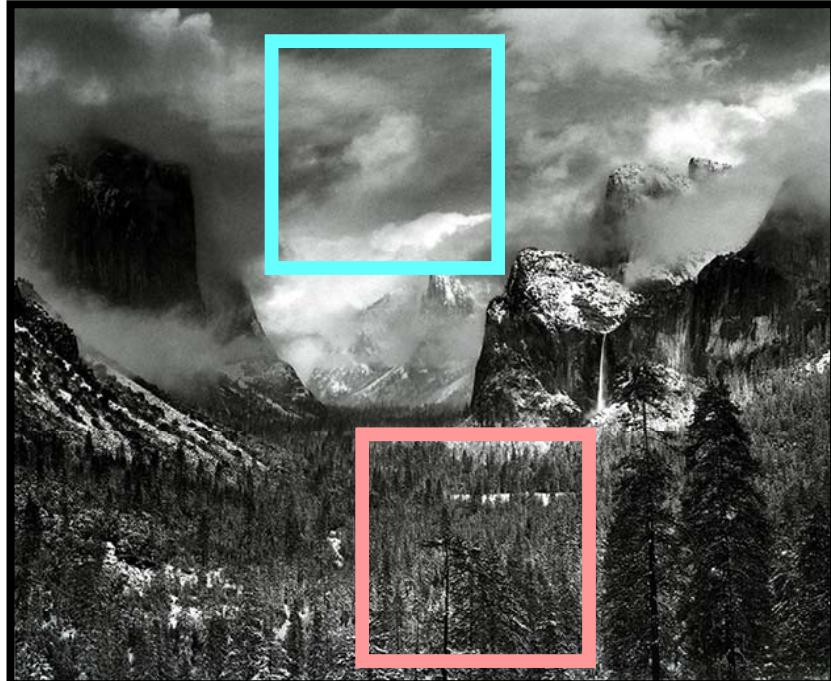
Kenro Izu

High Global Contrast

Low Global Contrast

Tonal aspects of Look - Local Contrast

DigiVFX



Ansel Adams



Kenro Izu

Variable amount of texture

Texture everywhere

Overview

DigiVFX



Input Image



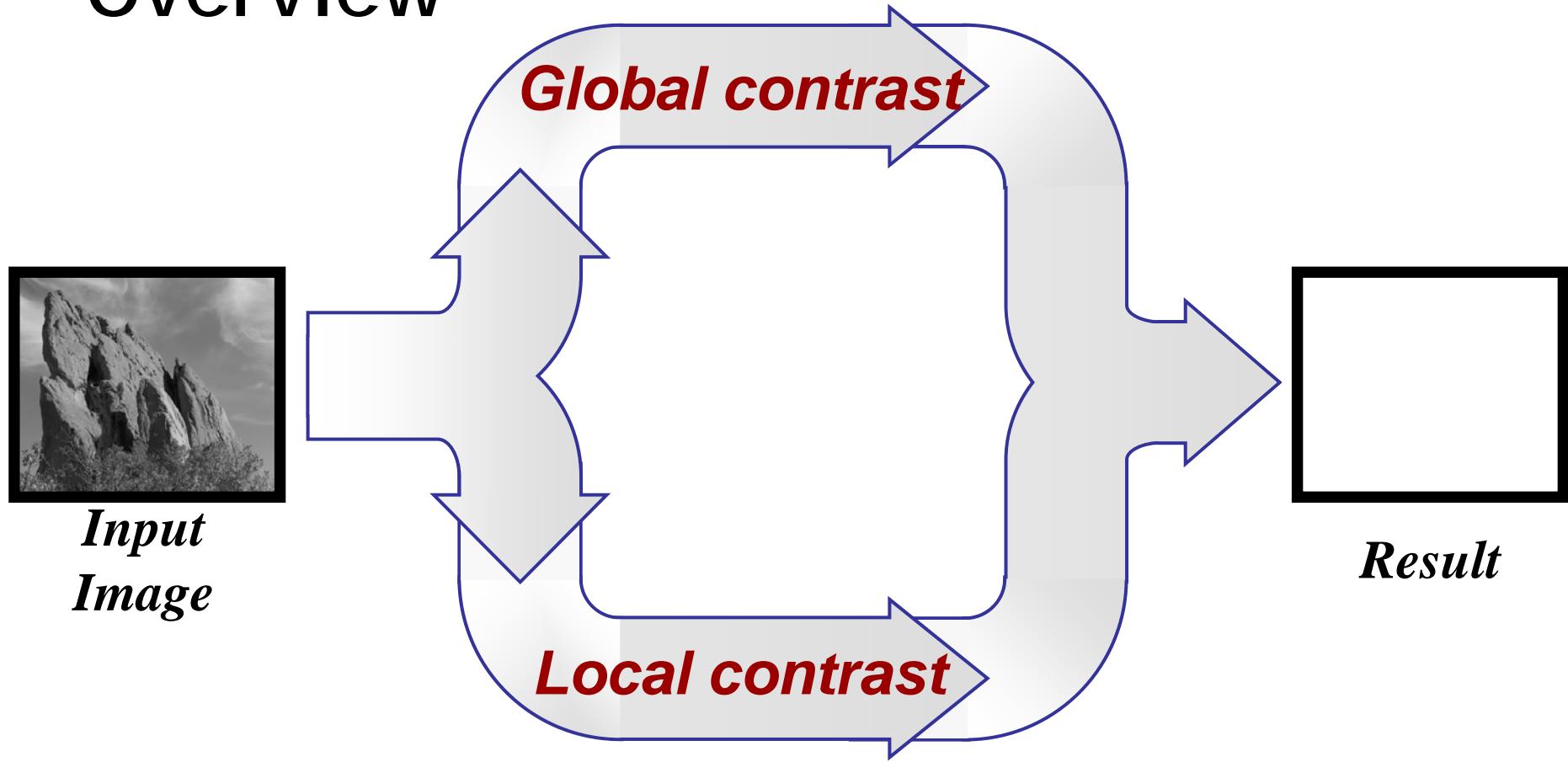
Model



Result

- Transfer look between photographs
 - Tonal aspects

Overview



- Separate global and local contrast

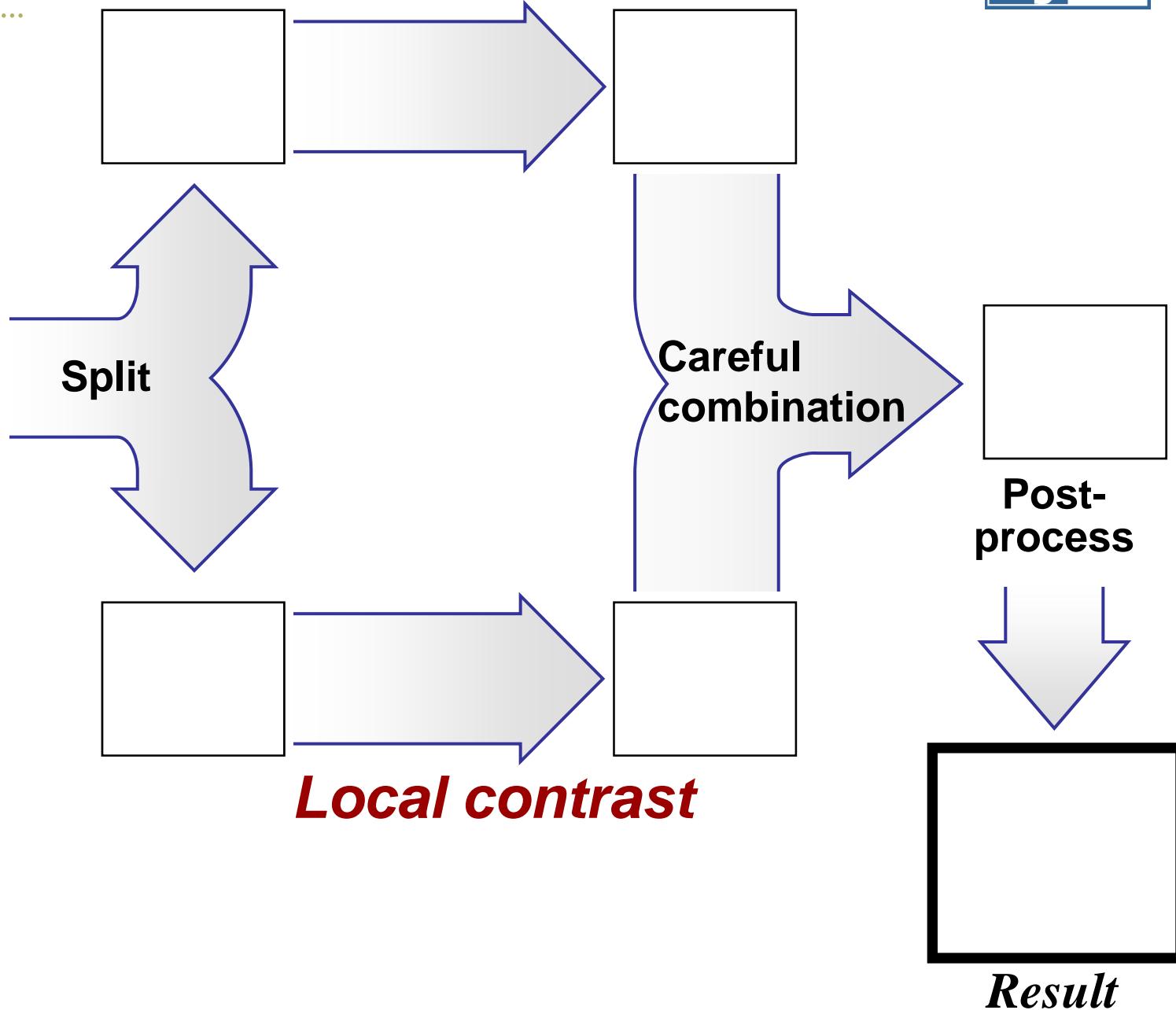
Overview

Global contrast

DigiVFX



*Input
Image*

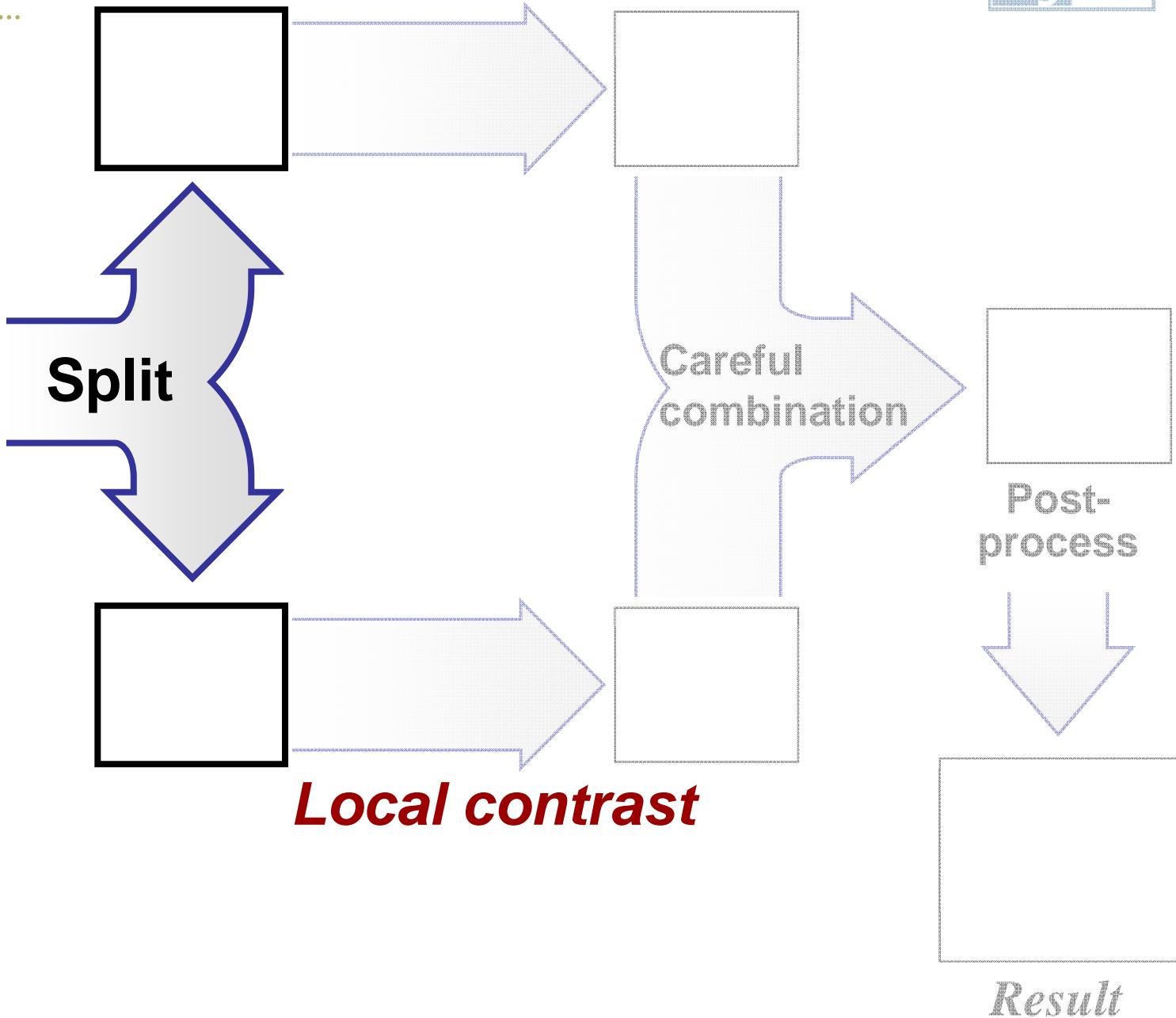


Overview

Global contrast



*Input
Image*

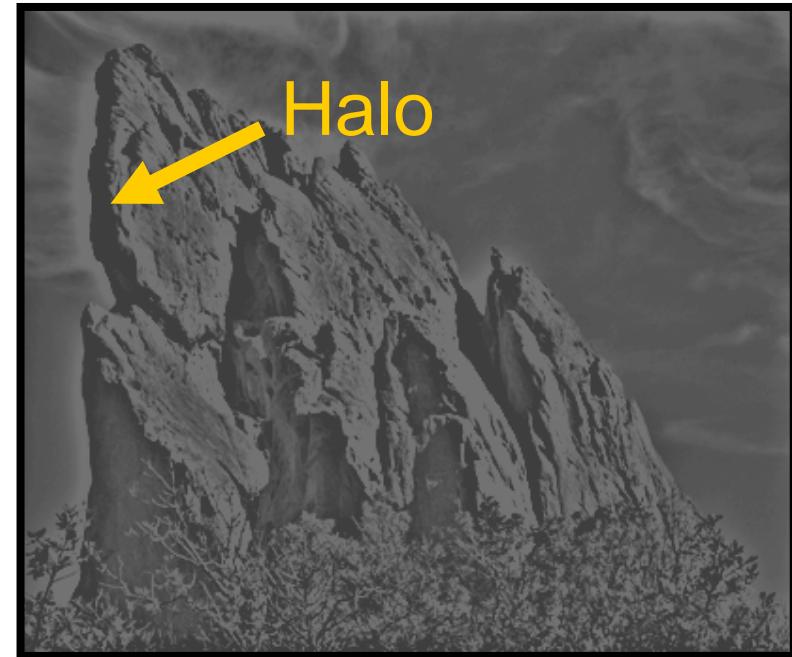


Split Global vs. Local Contrast

- Naïve decomposition: low vs. high frequency
 - Problem: introduce blur & halos



Low frequency
Global contrast



High frequency
Local contrast

Bilateral Filter

- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]



After bilateral filtering
Global contrast



Residual after filtering
Local contrast

Bilateral Filter

- Edge-preserving smoothing [Tomasi 98]
- We build upon tone mapping [Durand 02]

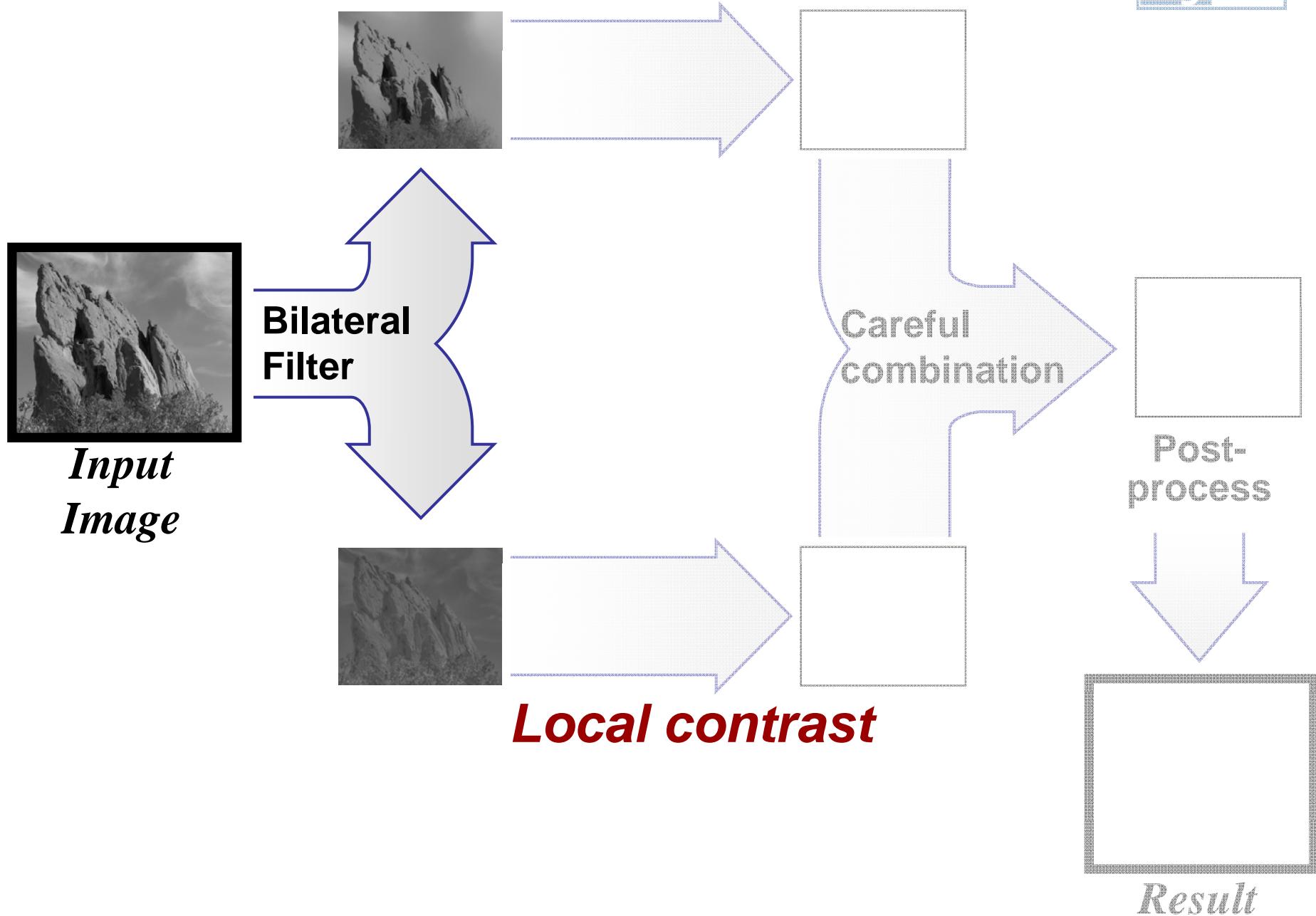


After bilateral filtering
Global contrast



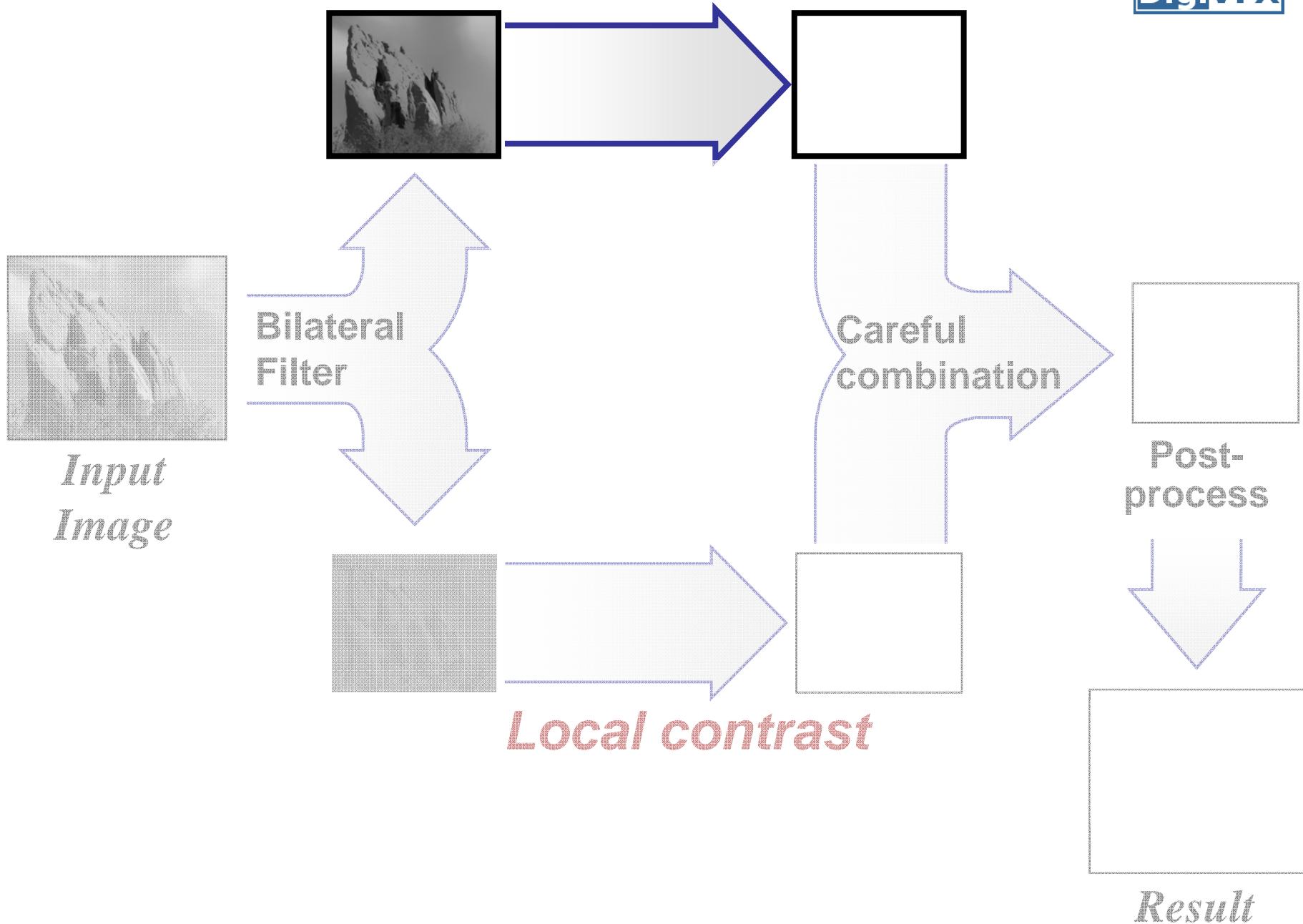
Residual after filtering
Local contrast

Global contrast



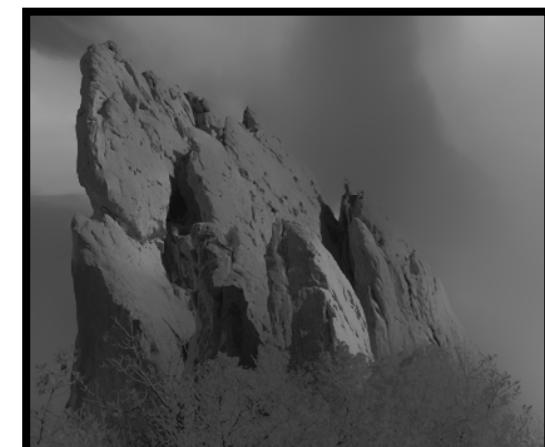
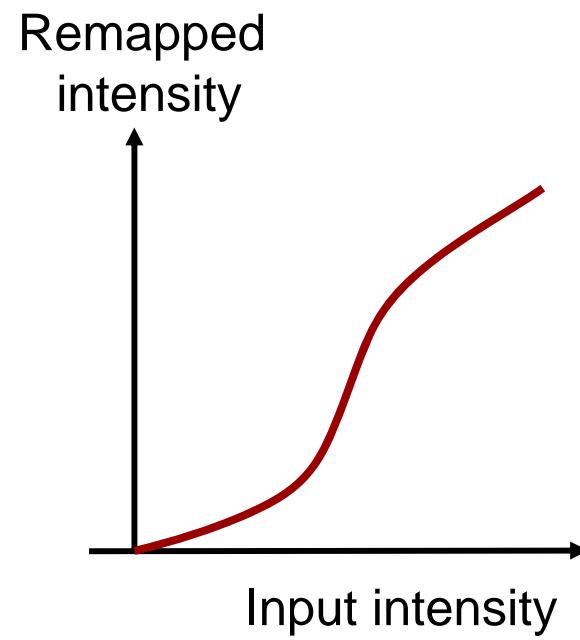
Global contrast

DigiVFX



Global Contrast

- Intensity remapping of base layer

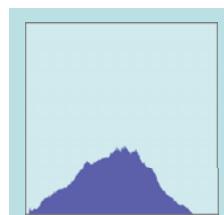


Global Contrast (Model Transfer)

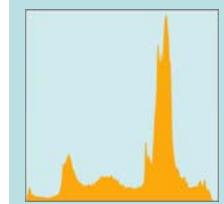
DigiVFX



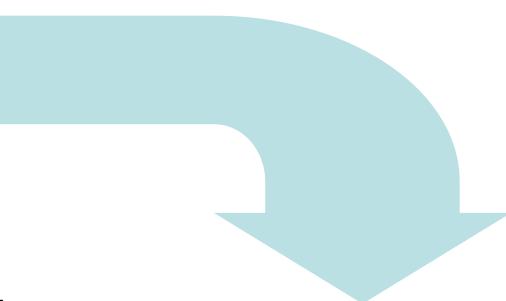
Model
base



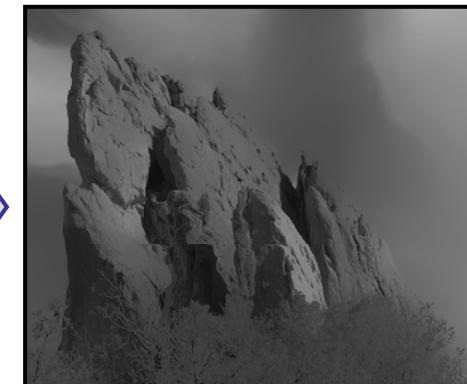
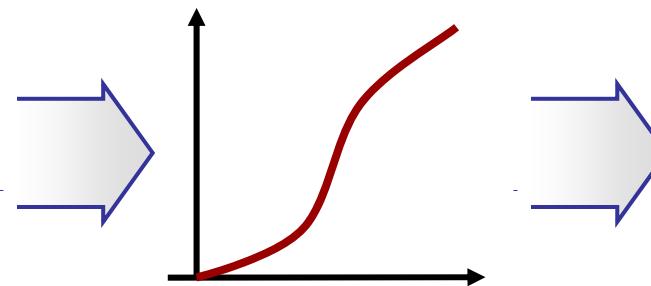
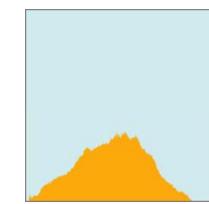
Input
base



- Histogram matching
 - Remapping function given input and model histogram

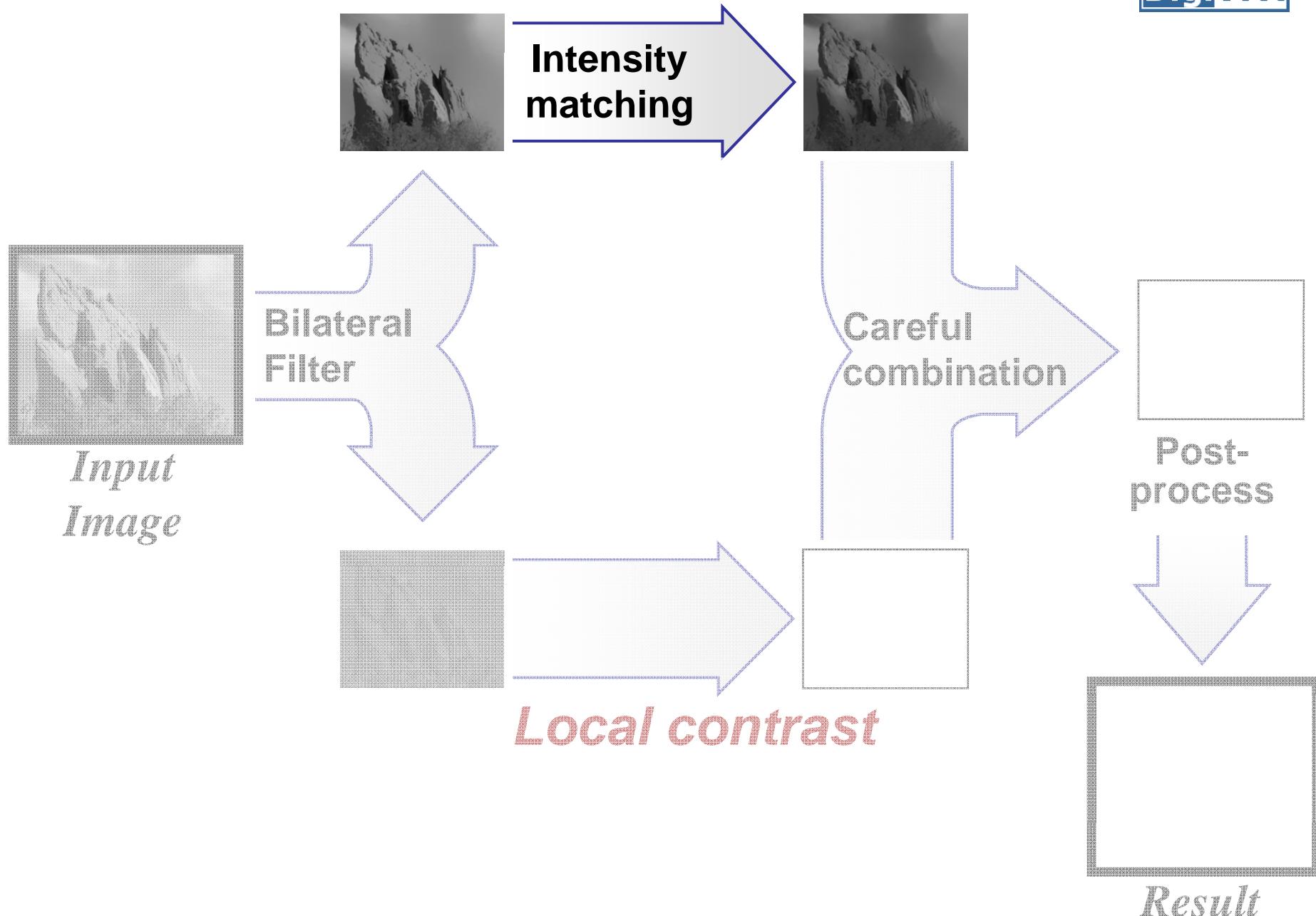


Output
base



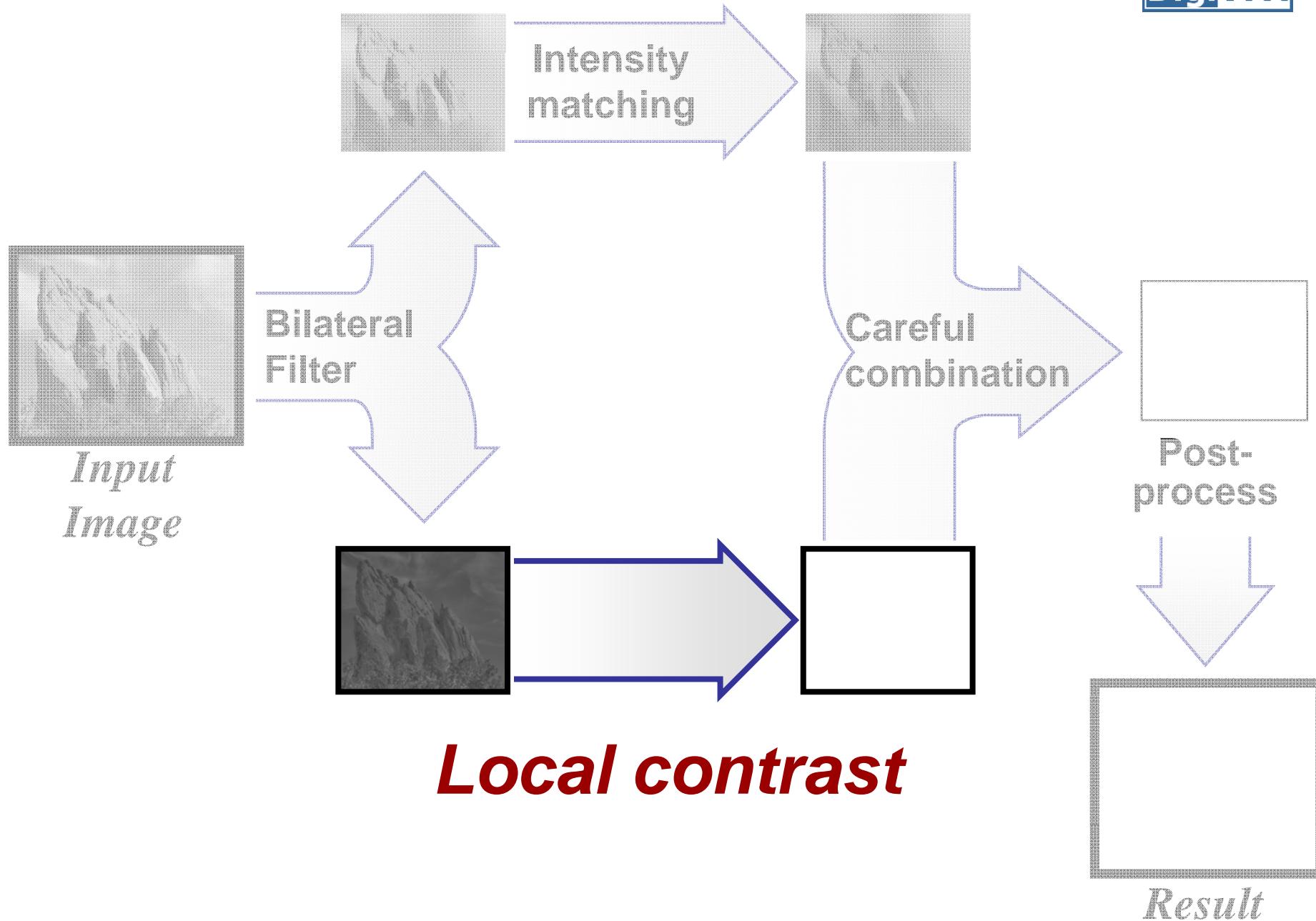
Global contrast

DigiVFX



Global contrast

DigiVFX

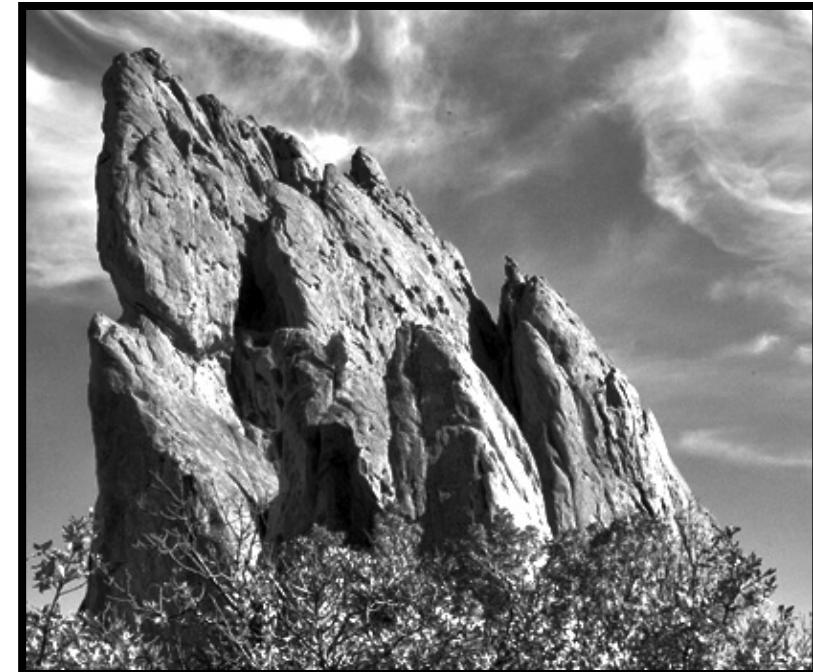


Local Contrast: Detail Layer

- Uniform control:
 - Multiply all values in the detail layer

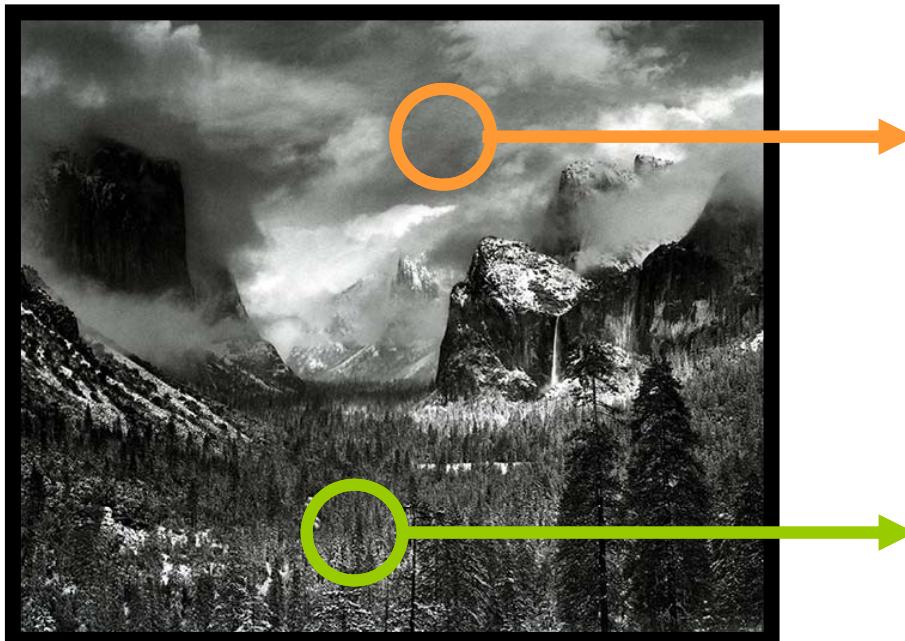


Input



Base + 3 × Detail

The amount of local contrast is not uniform

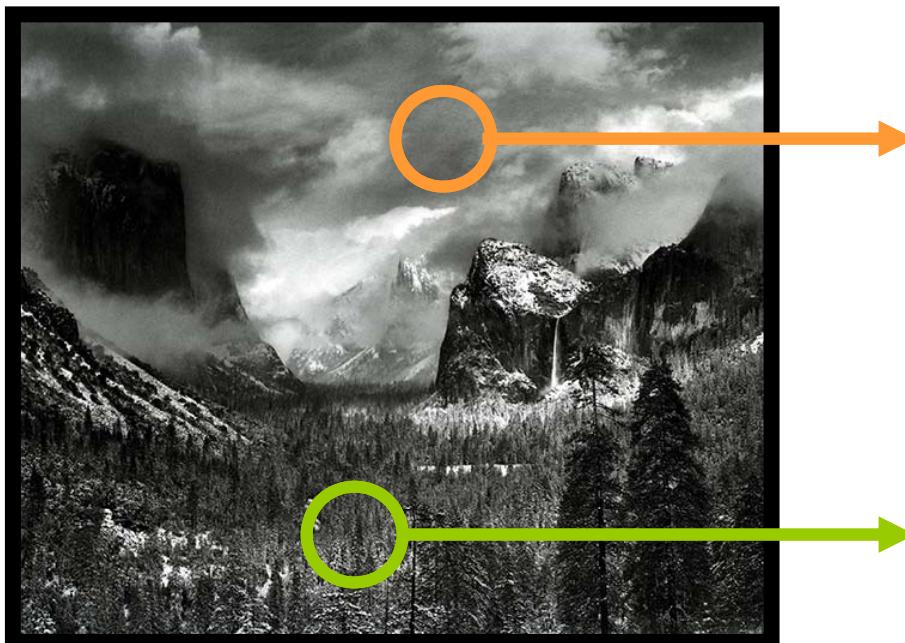


Smooth region

Textured region

Local Contrast Variation

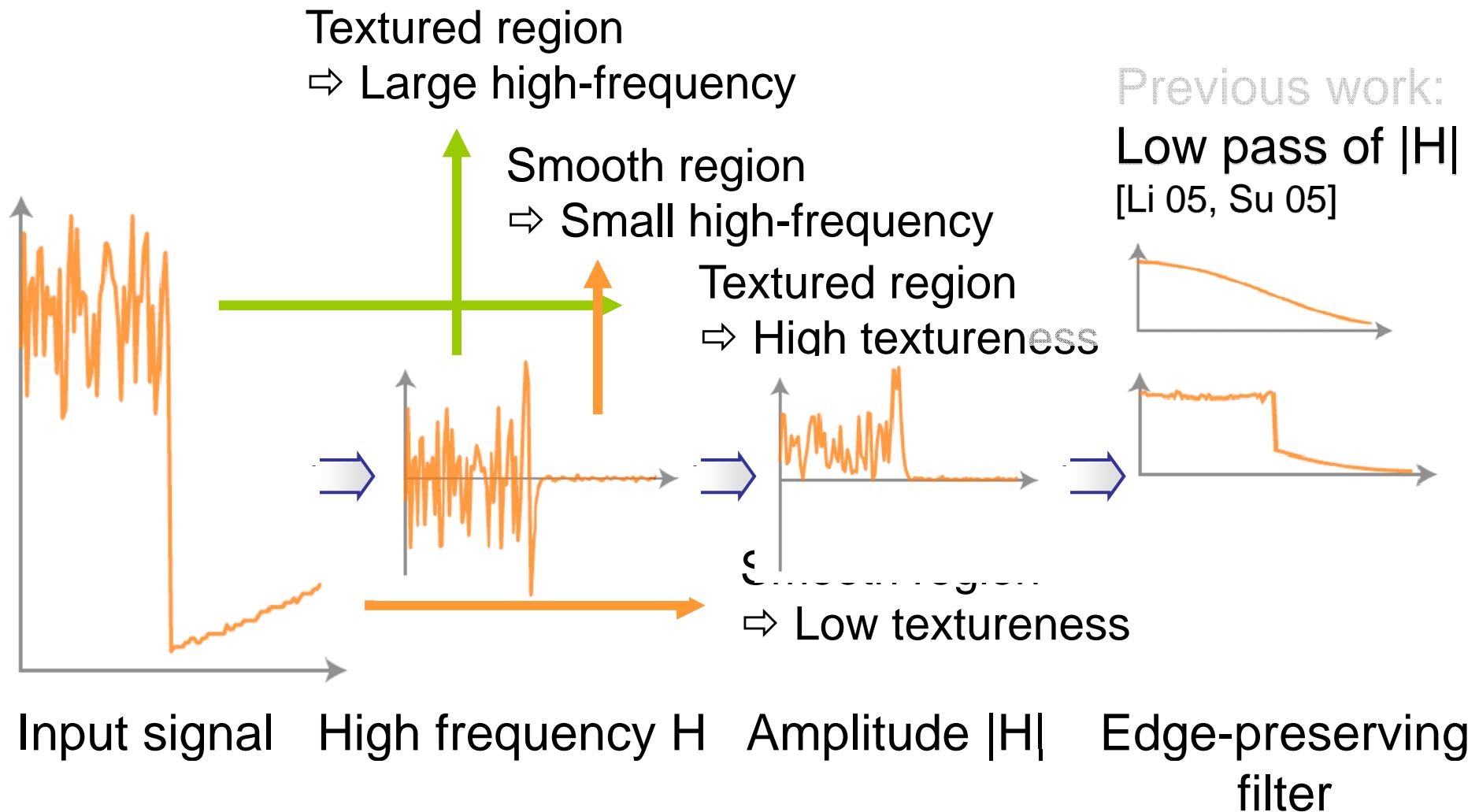
- We define “textureness”: amount of local contrast
 - at each pixel based on surrounding region



Smooth region
⇒ Low textureness

Textured region
⇒ High textureness

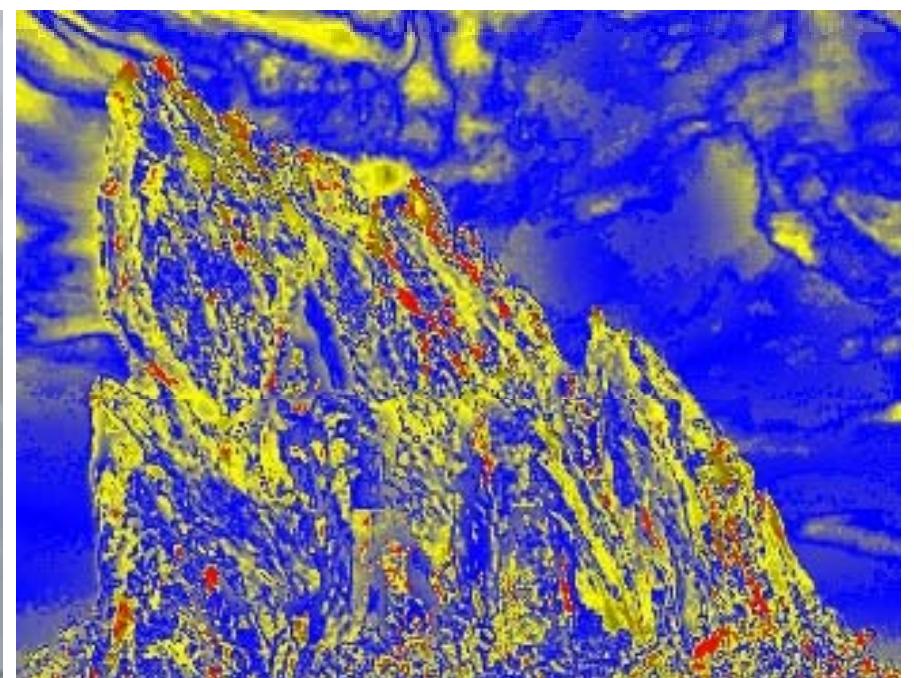
“Textureness”: 1D Example



Textureness



Input

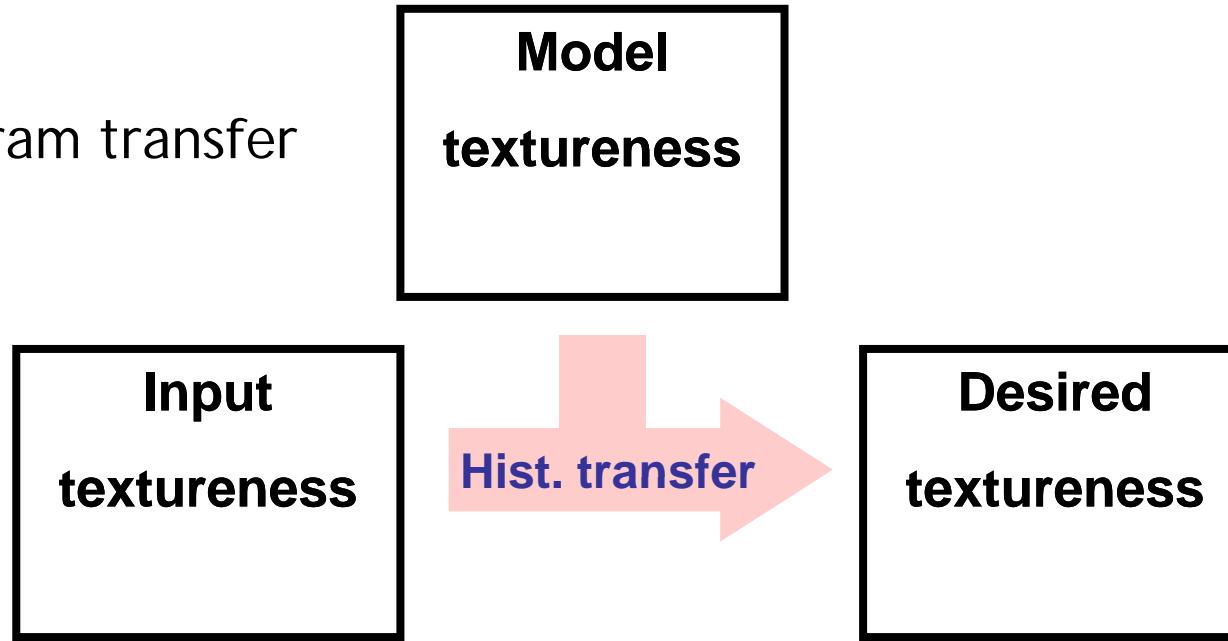


Textureness

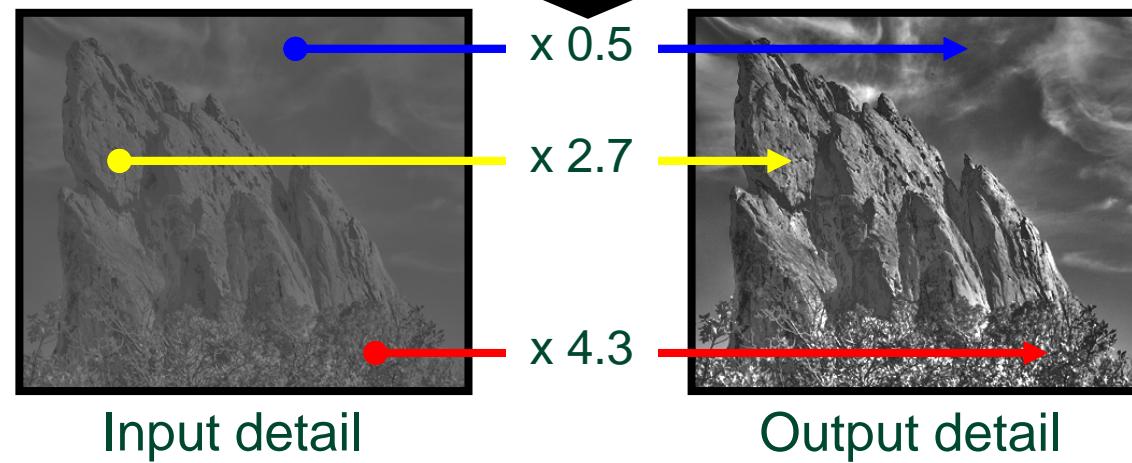
Textureness Transfer



Step 1:
Histogram transfer

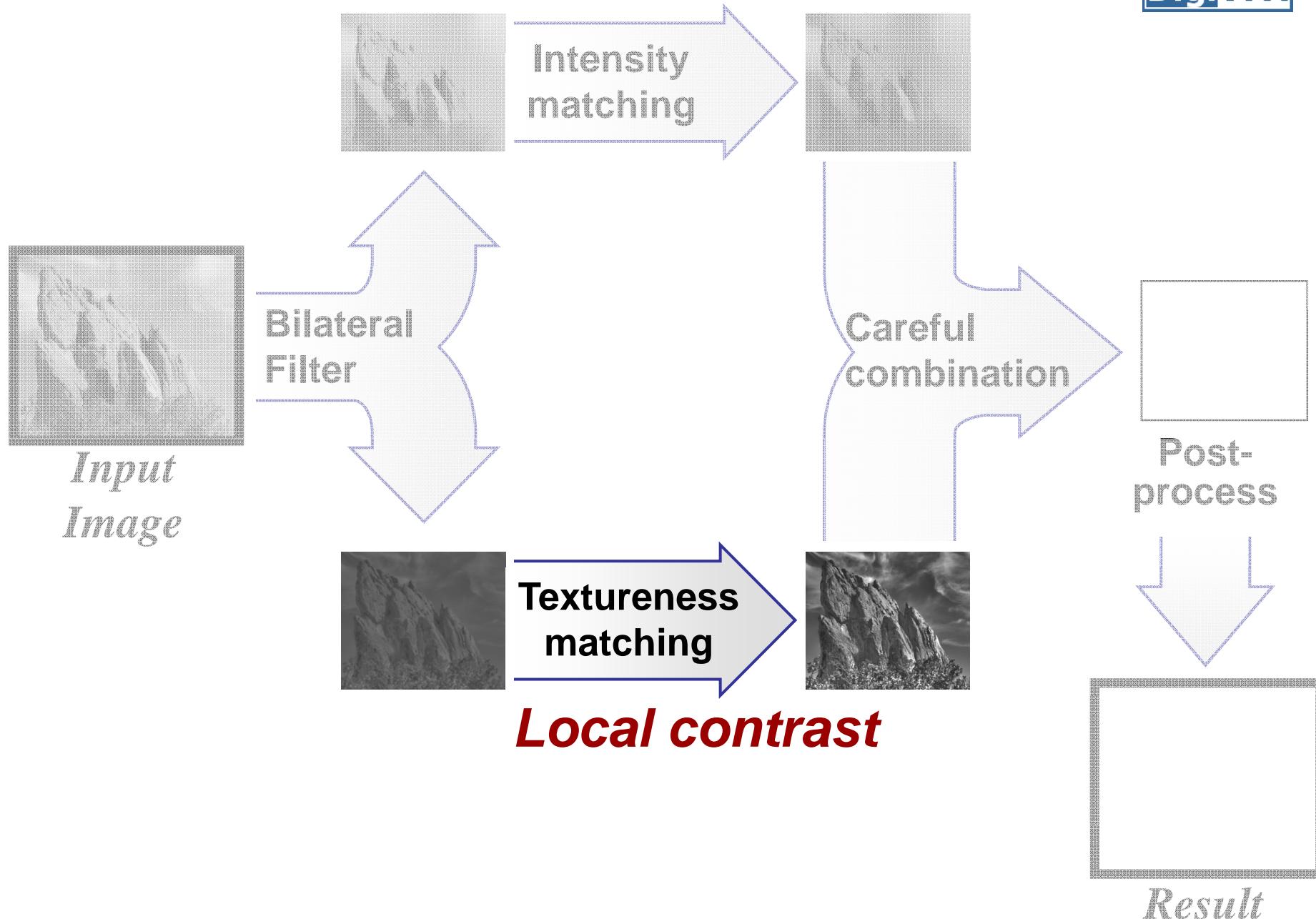


Step 2:
Scaling detail layer
(per pixel) to match
desired textureness



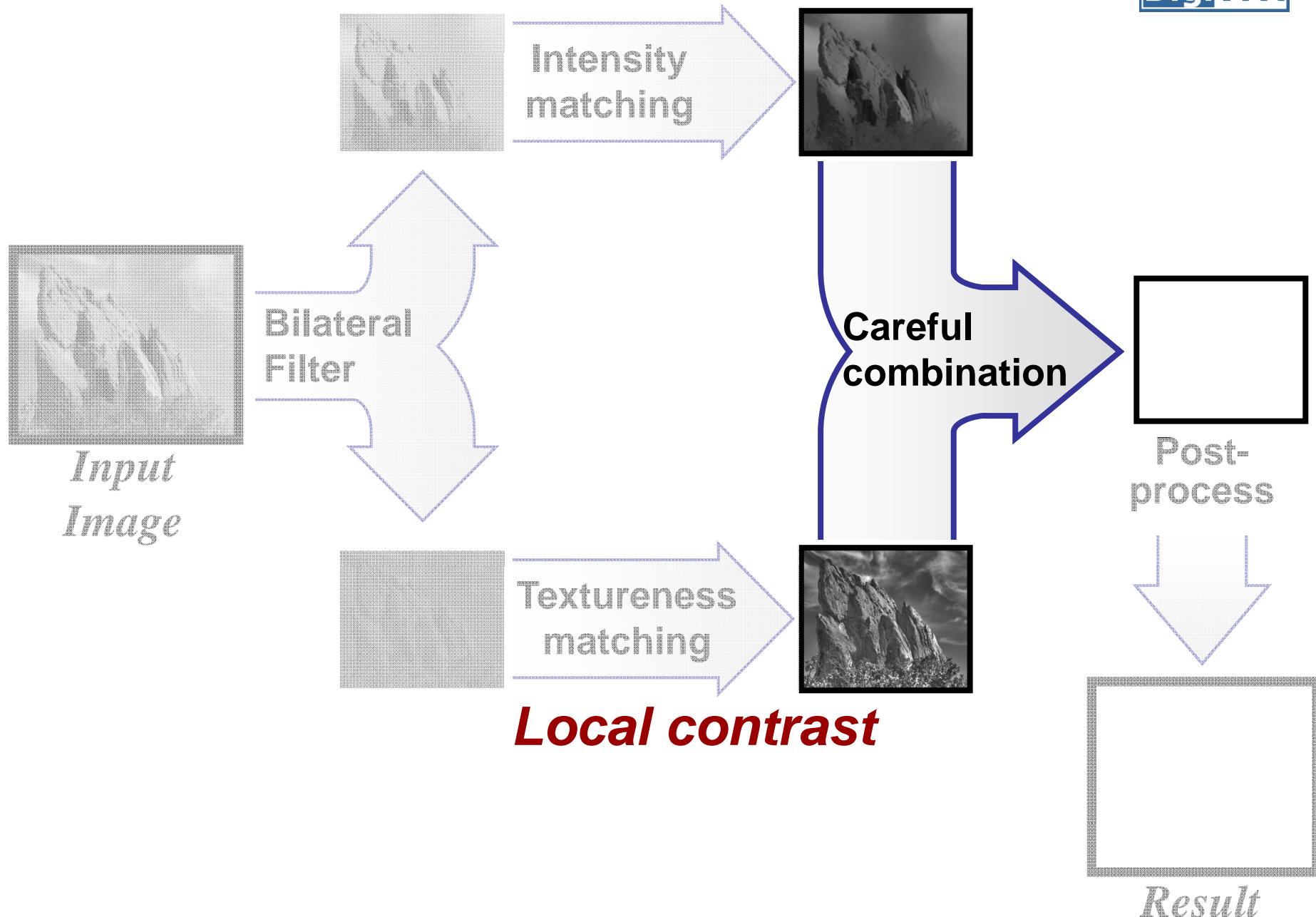
Global contrast

DigiVFX



Global contrast

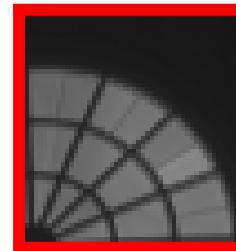
DigiVFX



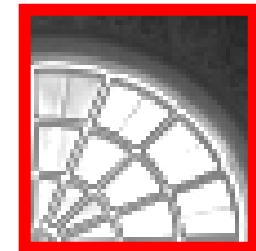
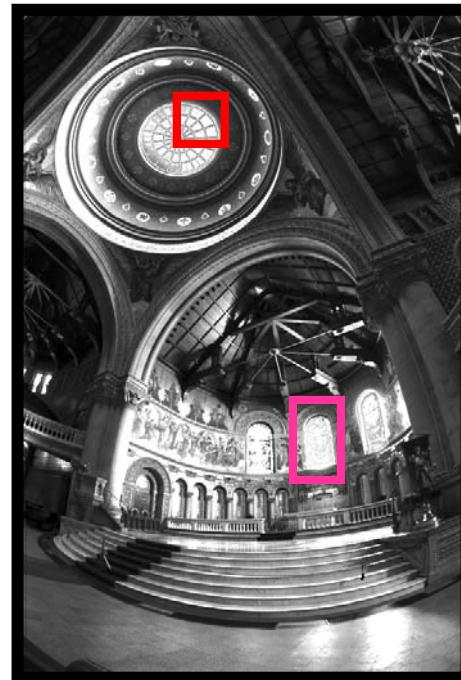
A Non Perfect Result

- Decoupled and large modifications (up to 6x)
→ Limited defects may appear

input (HDR)

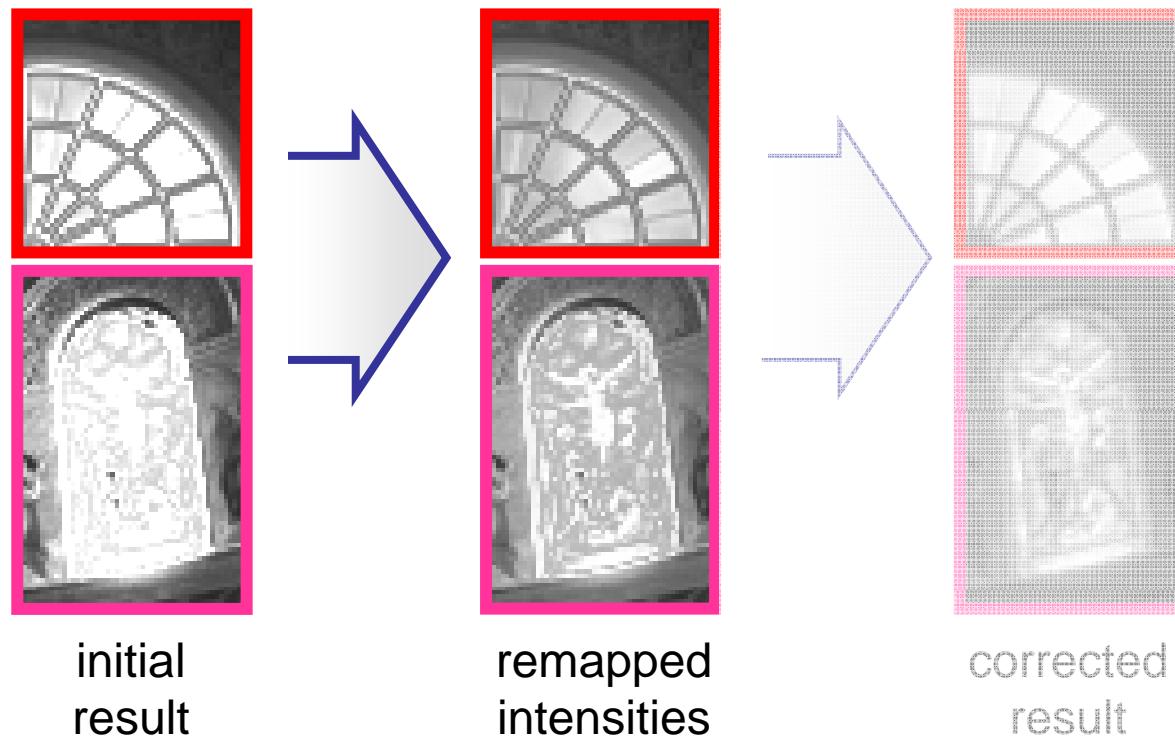


result after
global and local adjustments



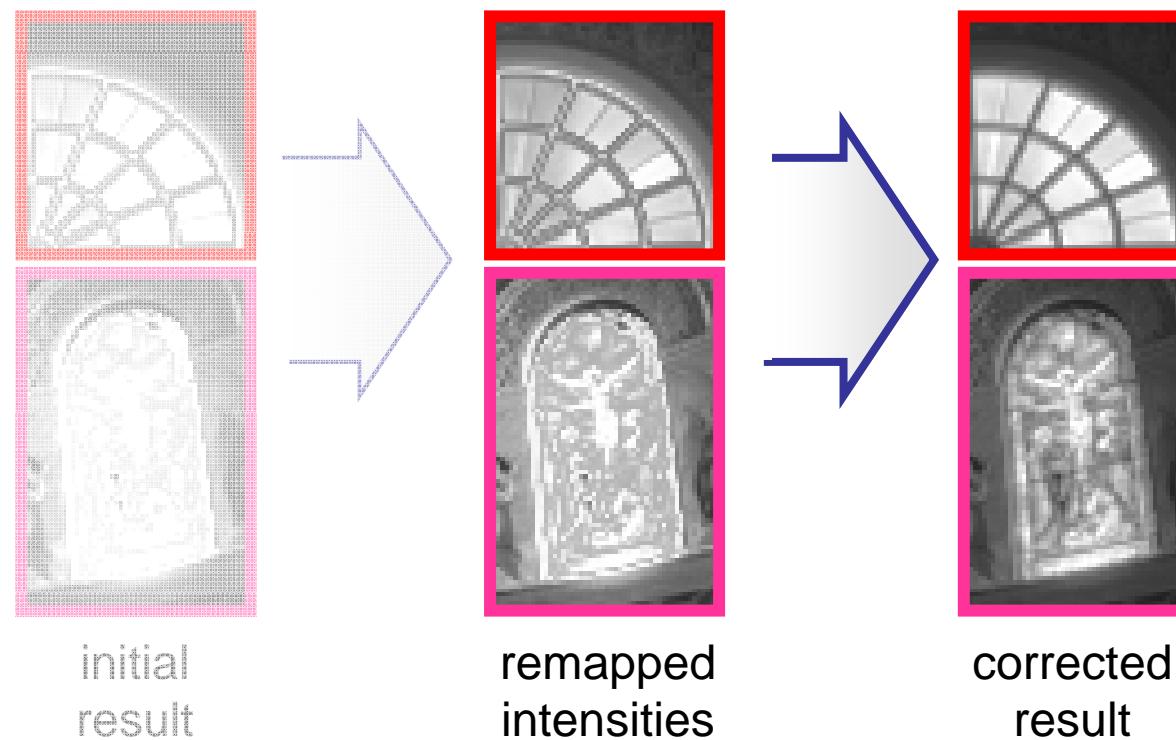
Intensity Remapping

- Some intensities may be outside displayable range.
→ Compress histogram to fit visible range.



Preserving Details

1. In the gradient domain:
 - Compare gradient amplitudes of input and current
 - Prevent extreme reduction & extreme increase
2. Solve the Poisson equation.



Effect of Detail Preservation

uncorrected result

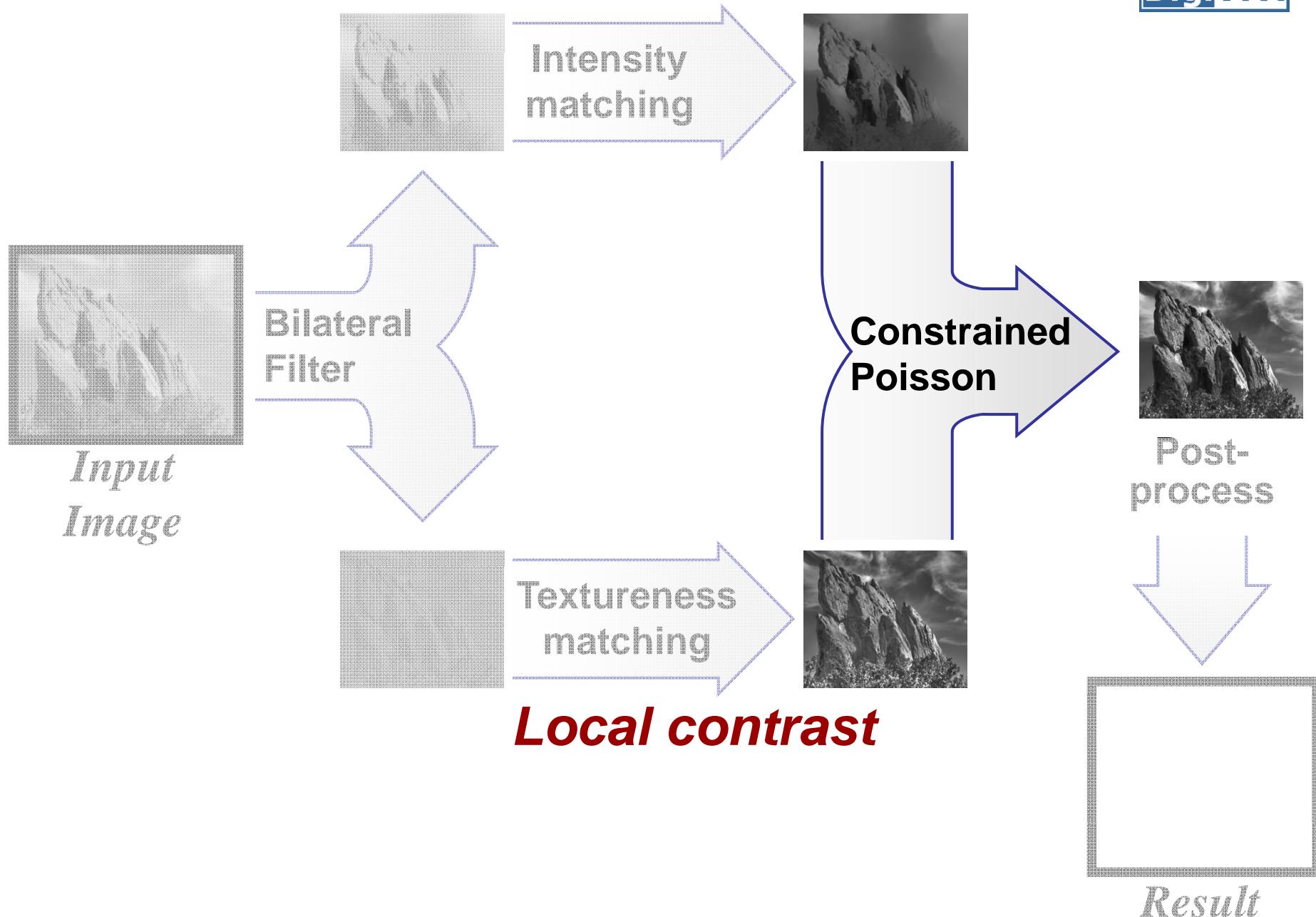


corrected result



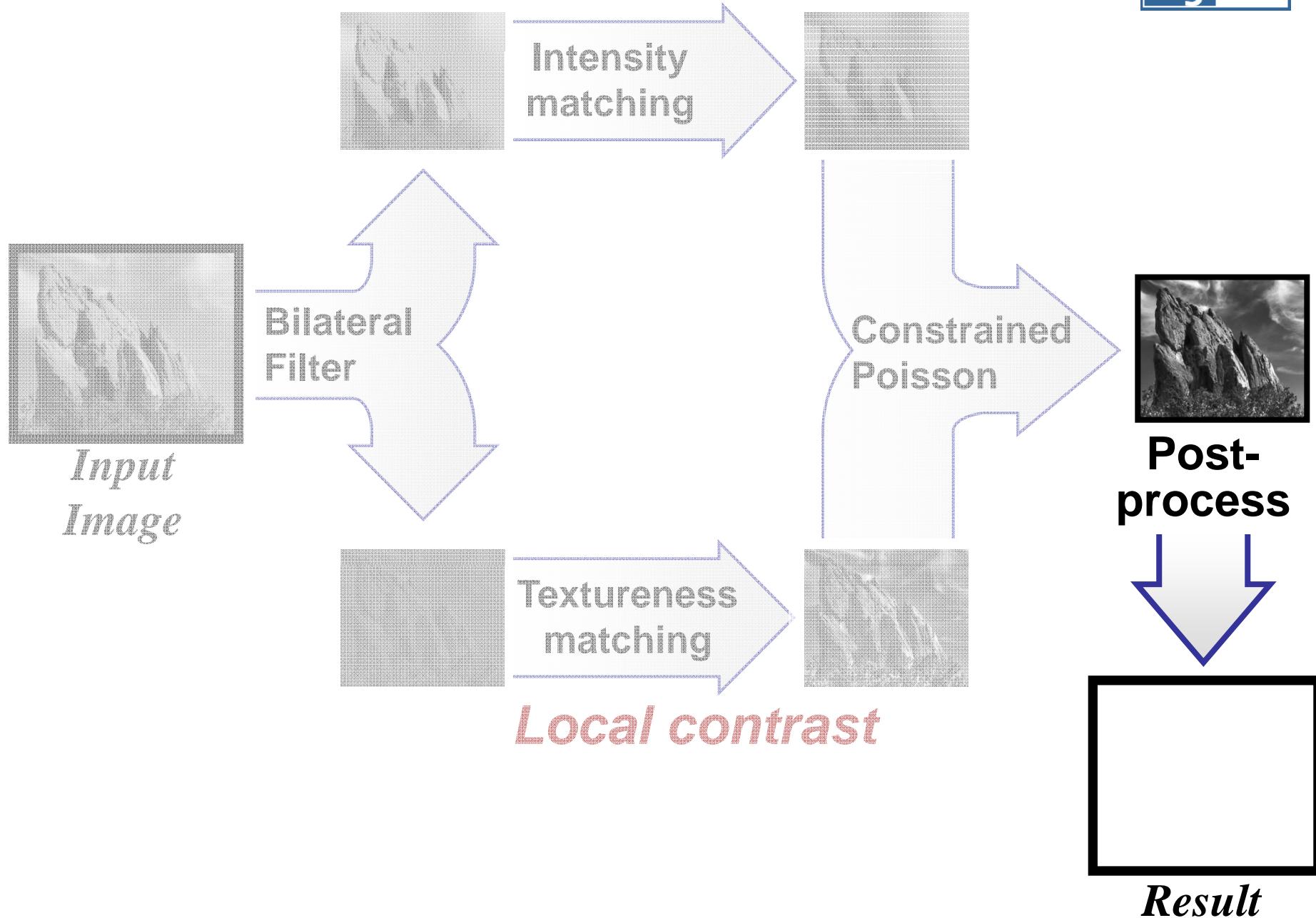
Global contrast

DigiVFX



Global contrast

DigiVFX



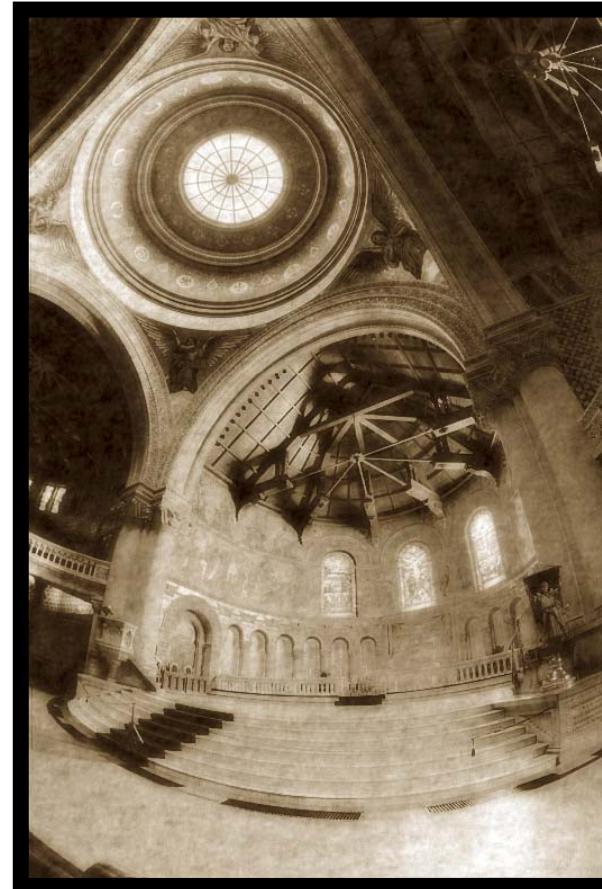
Additional Effects

model



- Soft focus (high frequency manipulation)
- Film grain (texture synthesis [Heeger 95])
- Color toning (chrominance = f (luminance))

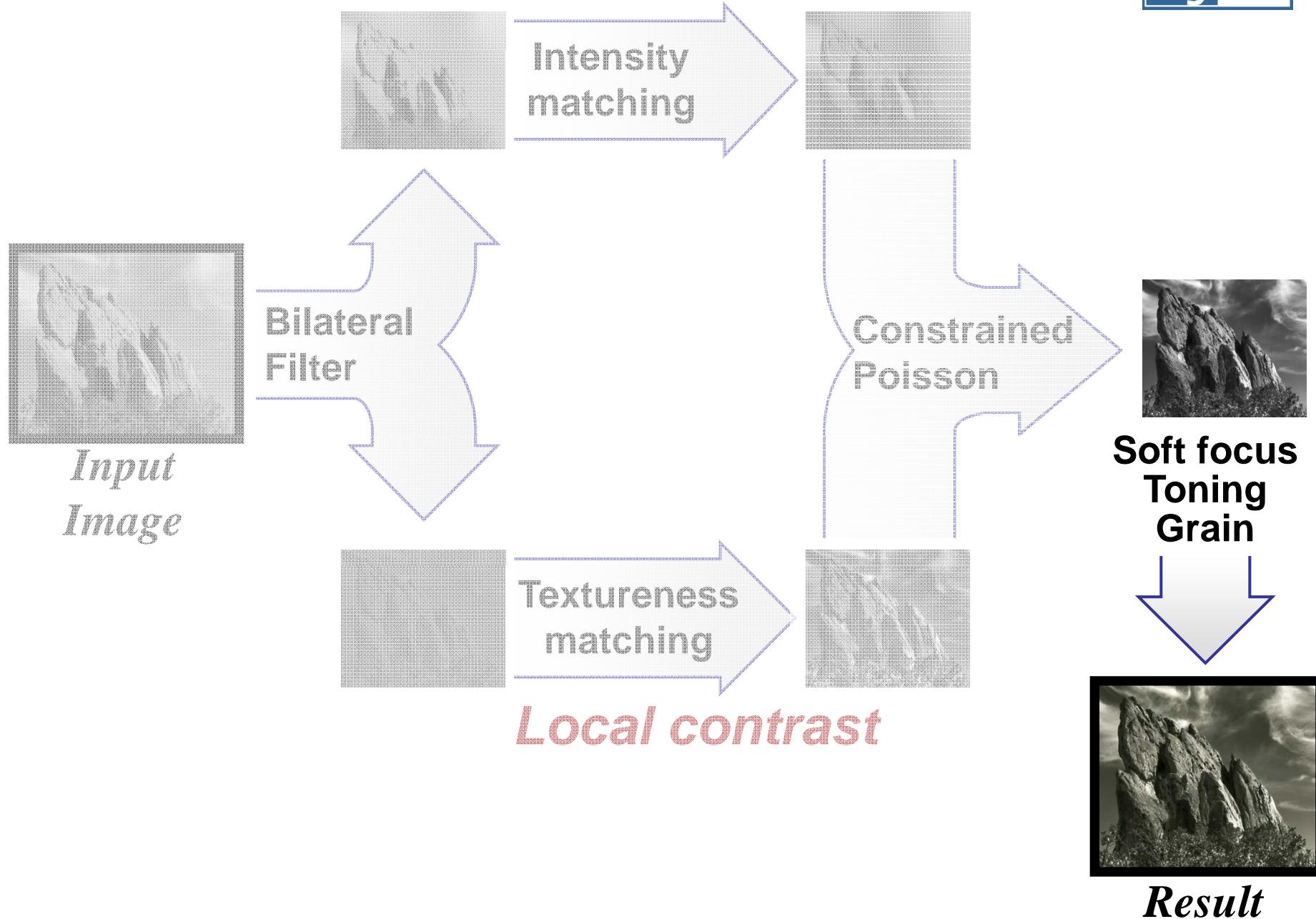
before
effects



after
effects

Global contrast

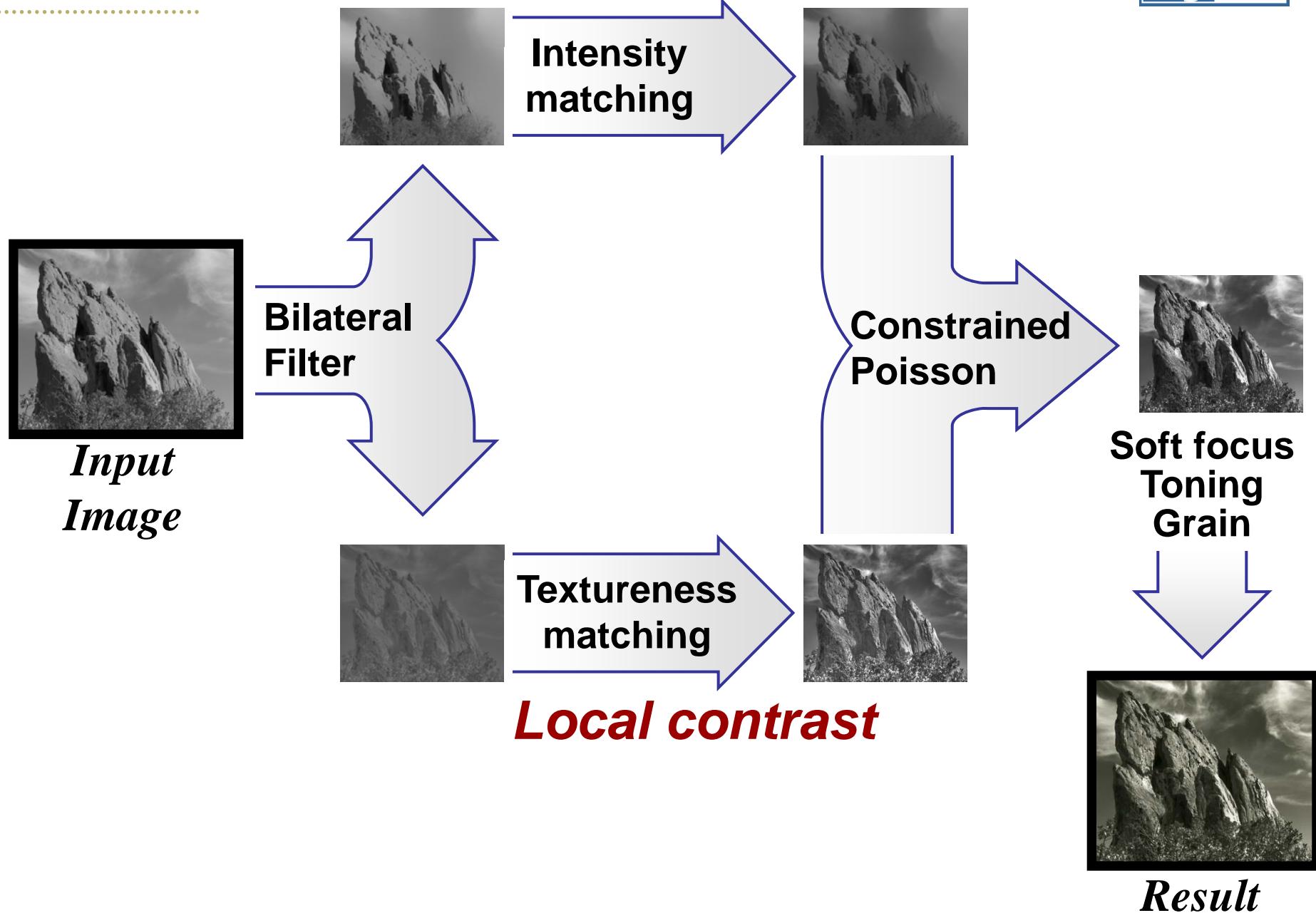
DigiVFX



Recap

Global contrast

DigiVFX



Results

User provides input and model photographs.

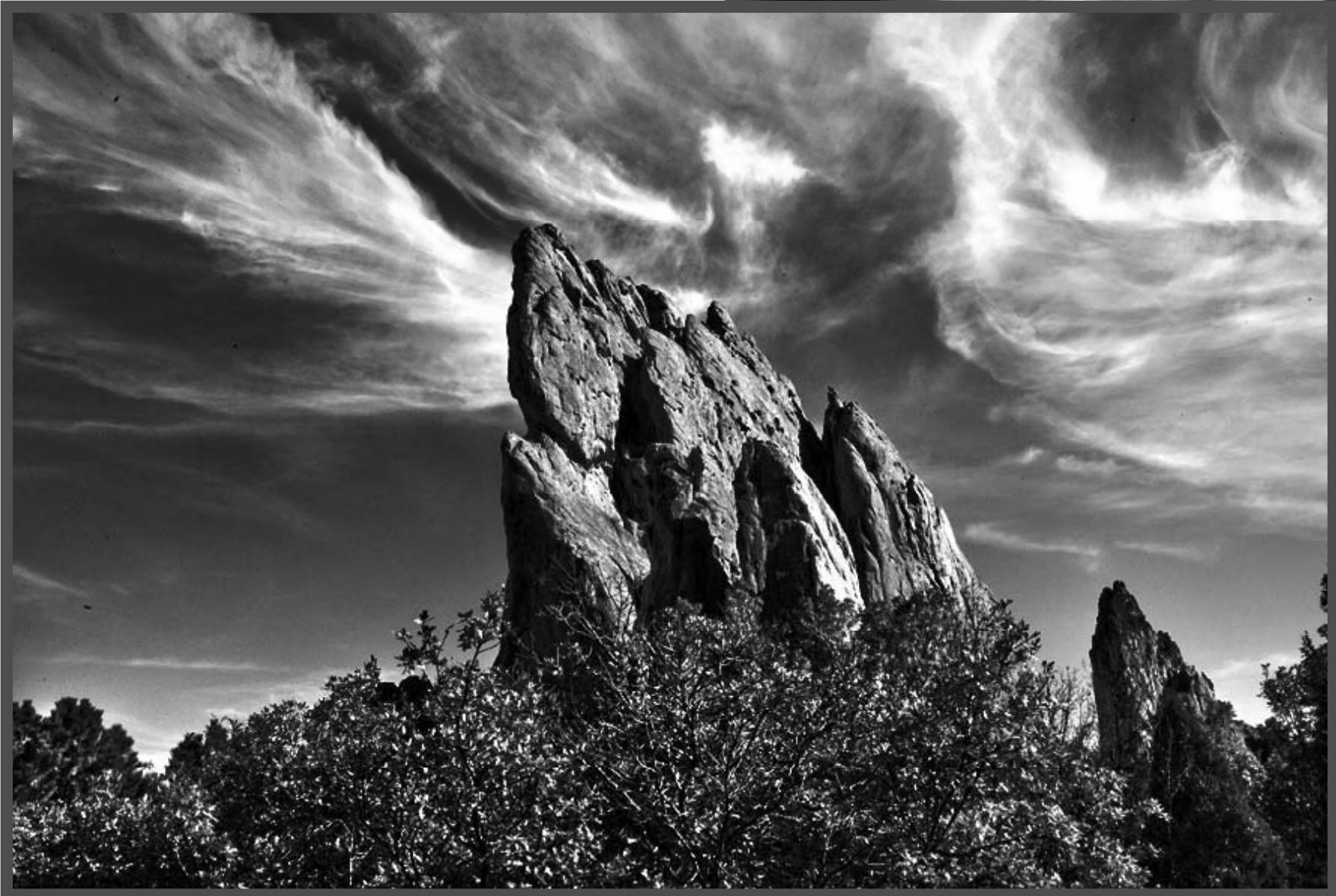
→ Our system automatically produces the result.

Running times:

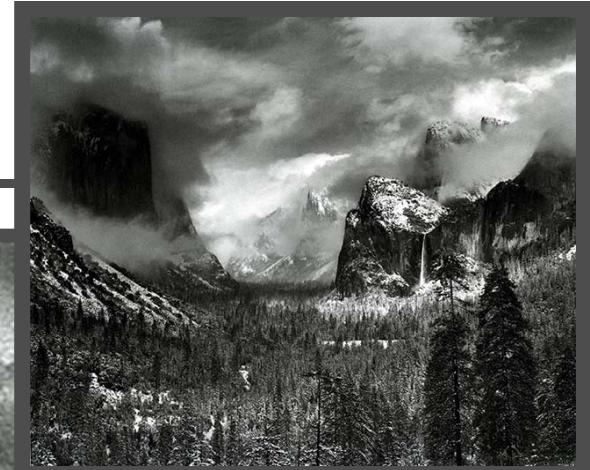
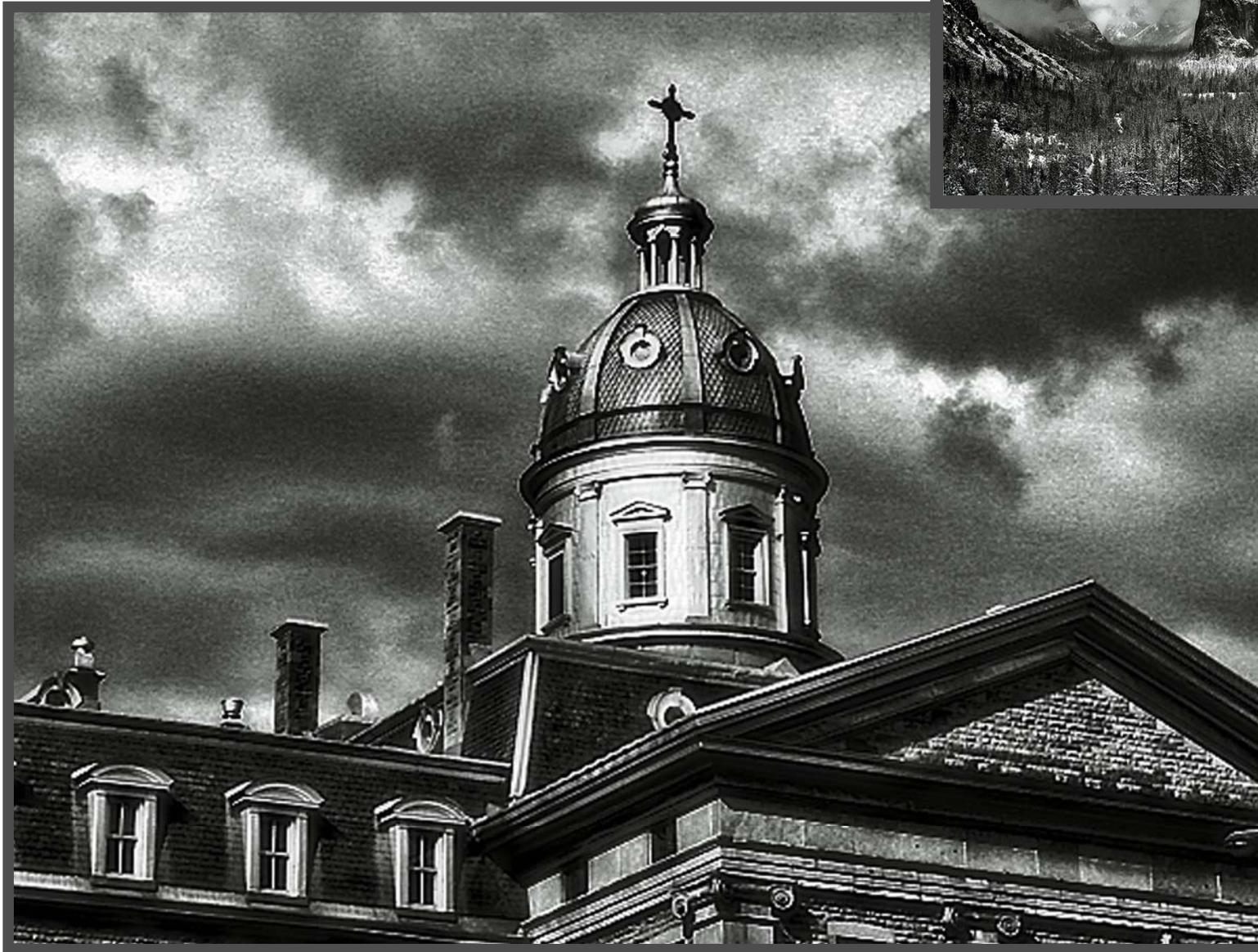
- 6 seconds for 1 MPixel or less
- 23 seconds for 4 MPixels
- multi-grid Poisson solver and fast bilateral filter [Paris 06]

Result

Model

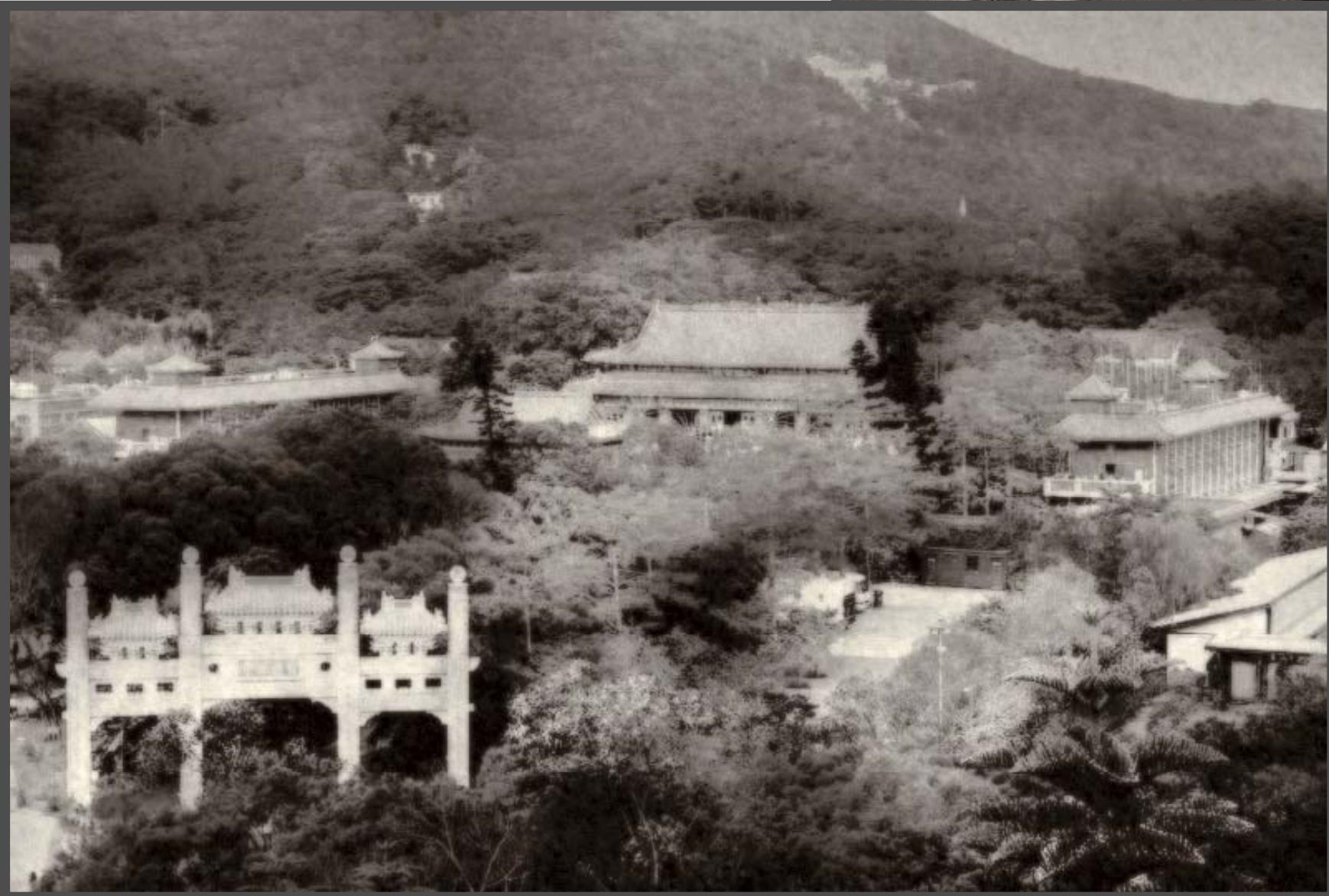


Result



Result

Model

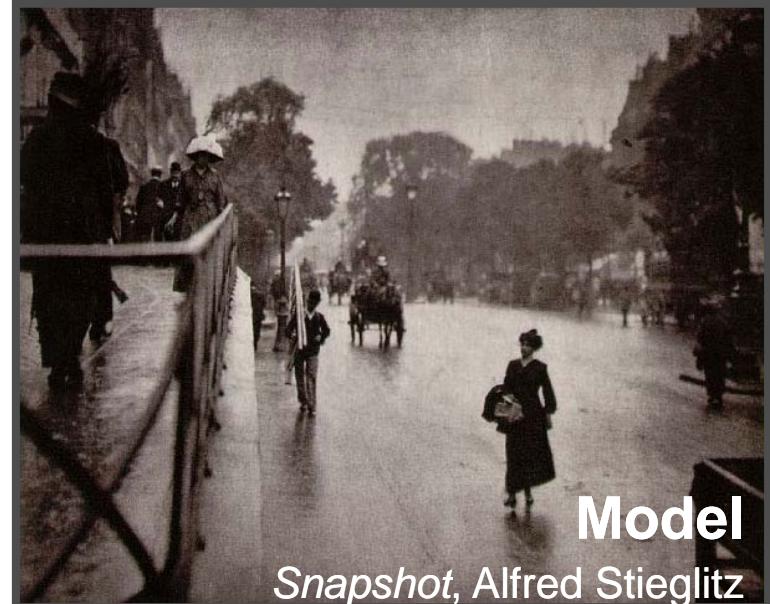


Comparison with Naïve Histogram Matching

Digital FX

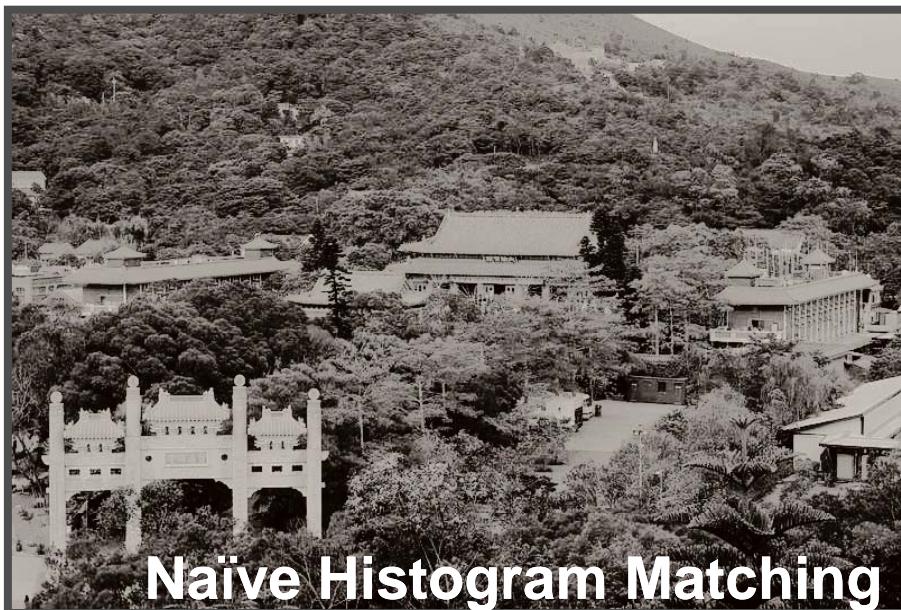


Input



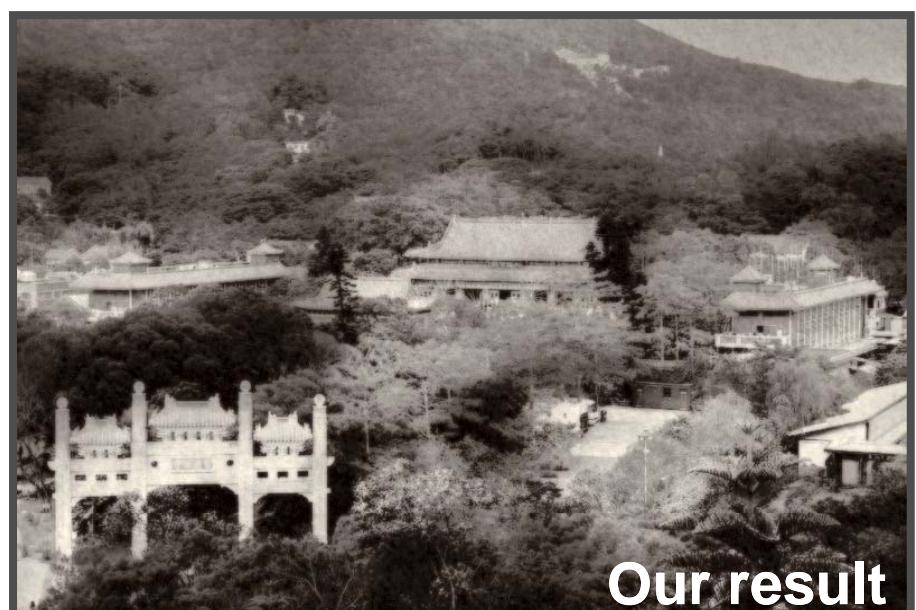
Model

Snapshot, Alfred Stieglitz



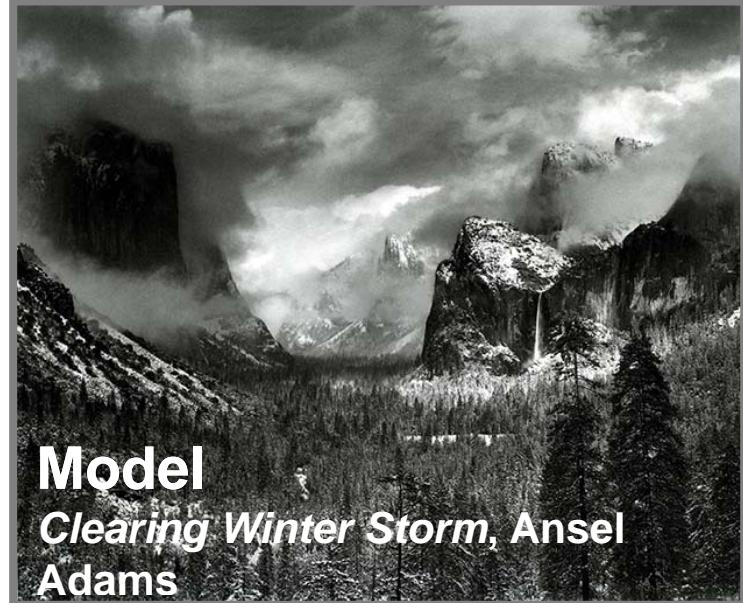
Naïve Histogram Matching

Local contrast, sharpness unfaithful



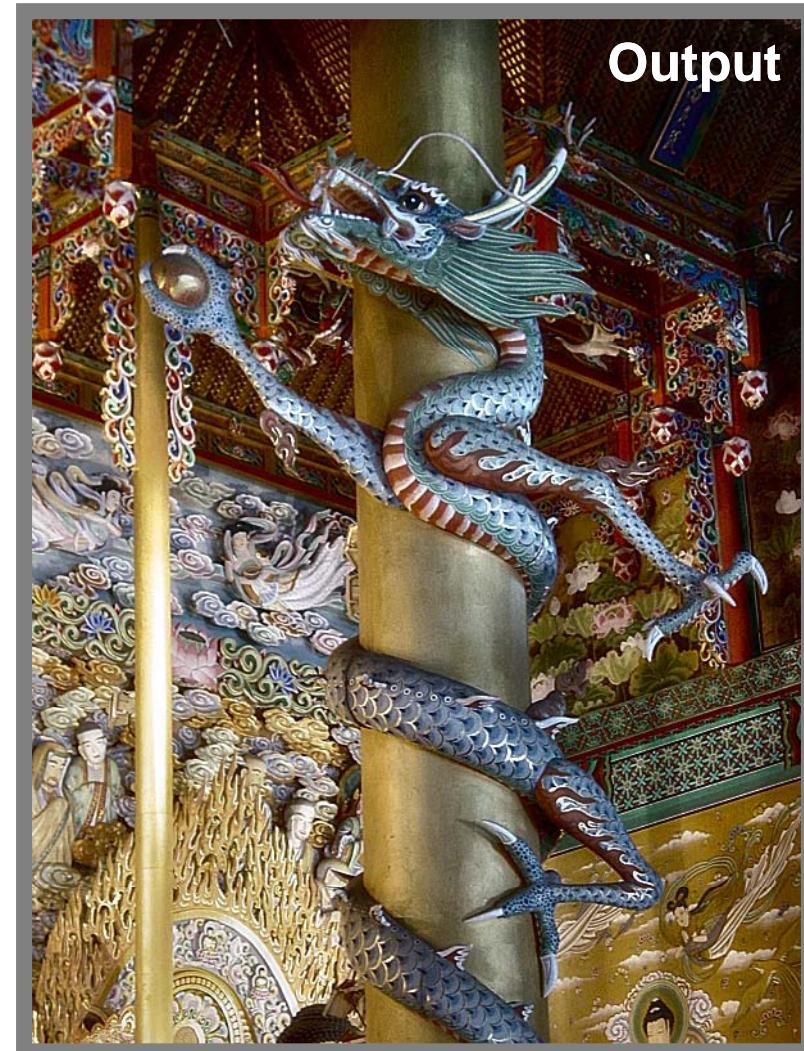
Our result

Comparison with Naïve Histogram Matching



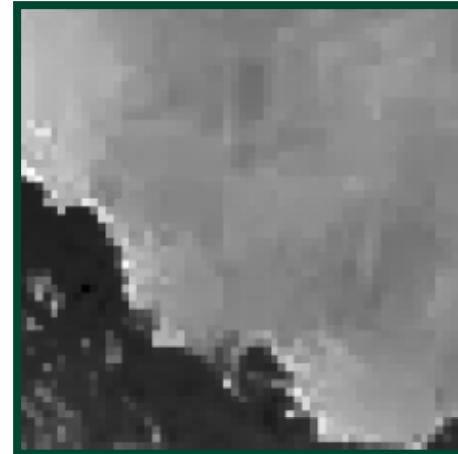
Color Images

- Lab color space: modify only luminance



Limitations

- Noise and JPEG artifacts
 - amplified defects
- Can lead to unexpected results if the image content is too different from the model
 - Portraits, in particular, can suffer

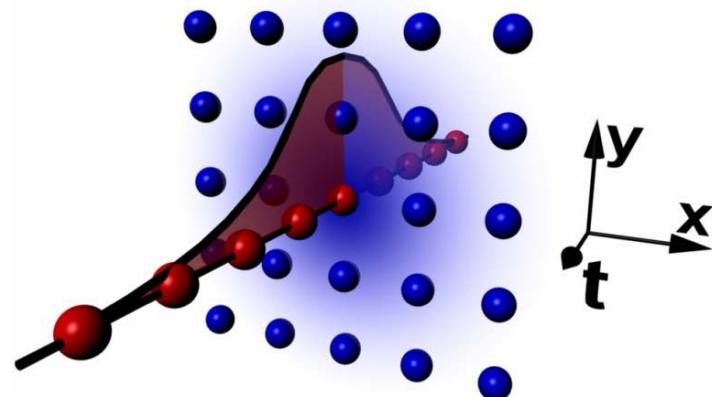


Video Enhancement Using Per Pixel Exposures (Bennett, 06)

DigiVFX

From this video:

ASTA: Adaptive
Spatio-
Temporal
Accumulation Fi



Joint bilateral filtering

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||I_p - I_q||)$$

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||\tilde{I}_p - \tilde{I}_q||)$$

Flash / No-Flash Photo Improvement (Petschnigg04) (Eisemann04)

DigiVFX

Merge best features: warm, cozy candle light (no-flash)
low-noise, detailed flash image



Overview

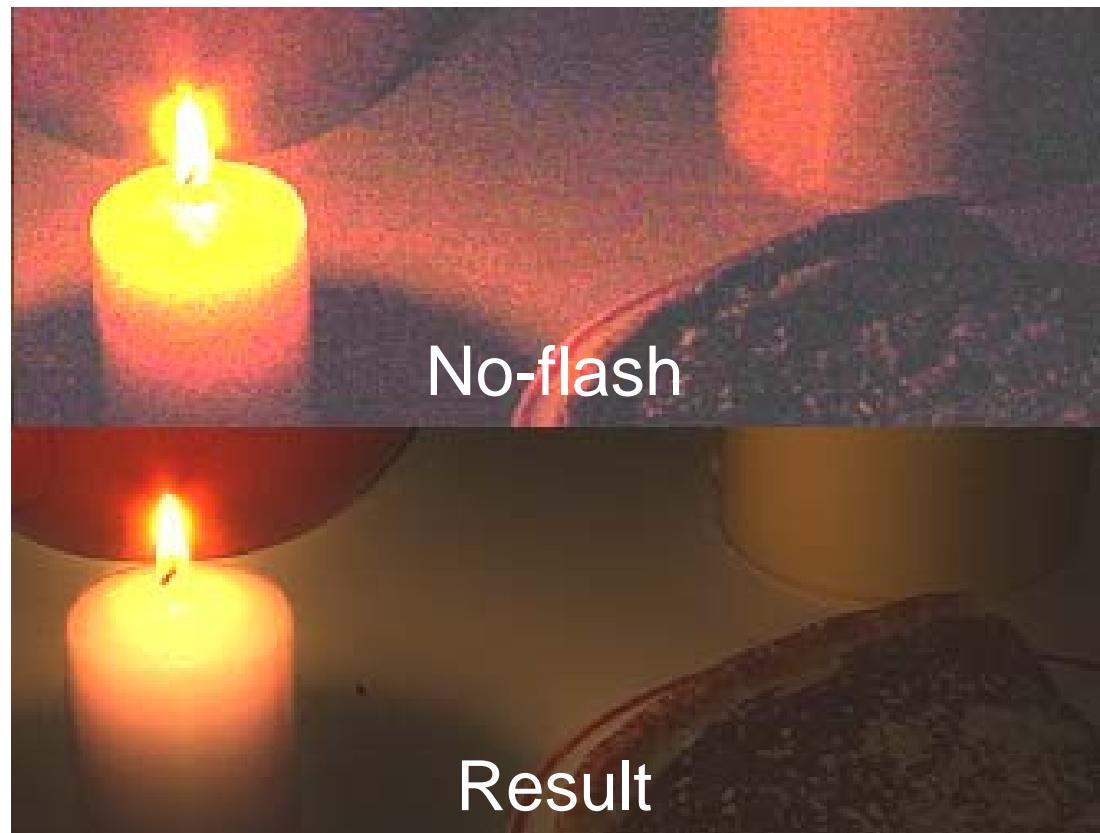
Basic approach of both flash/noflash papers

Remove noise + details
from image A,

Keep as image A Lighting

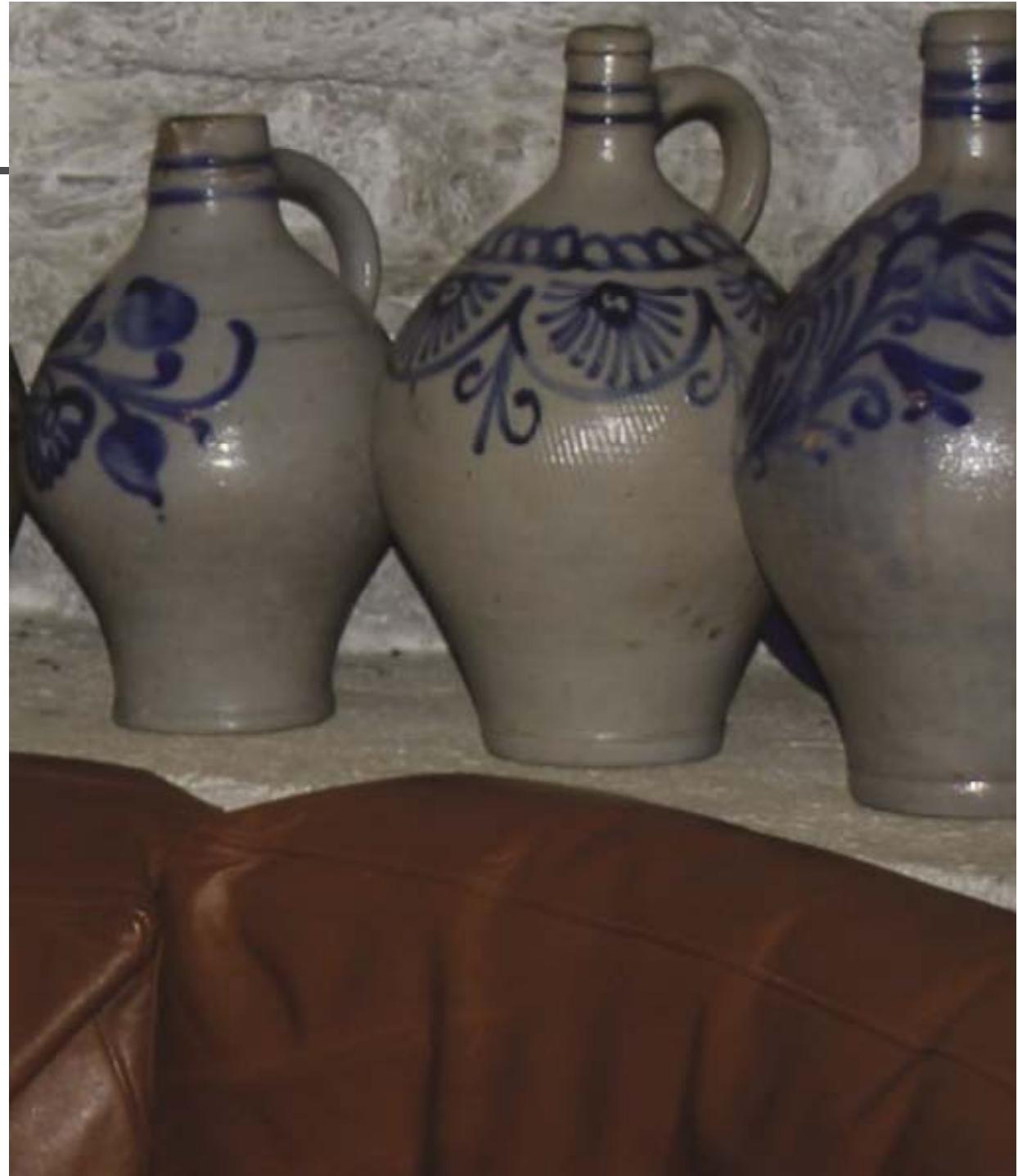
Obtain noise-free details
from image B,

Discard Image B Lighting



Petschnigg:

- Flash



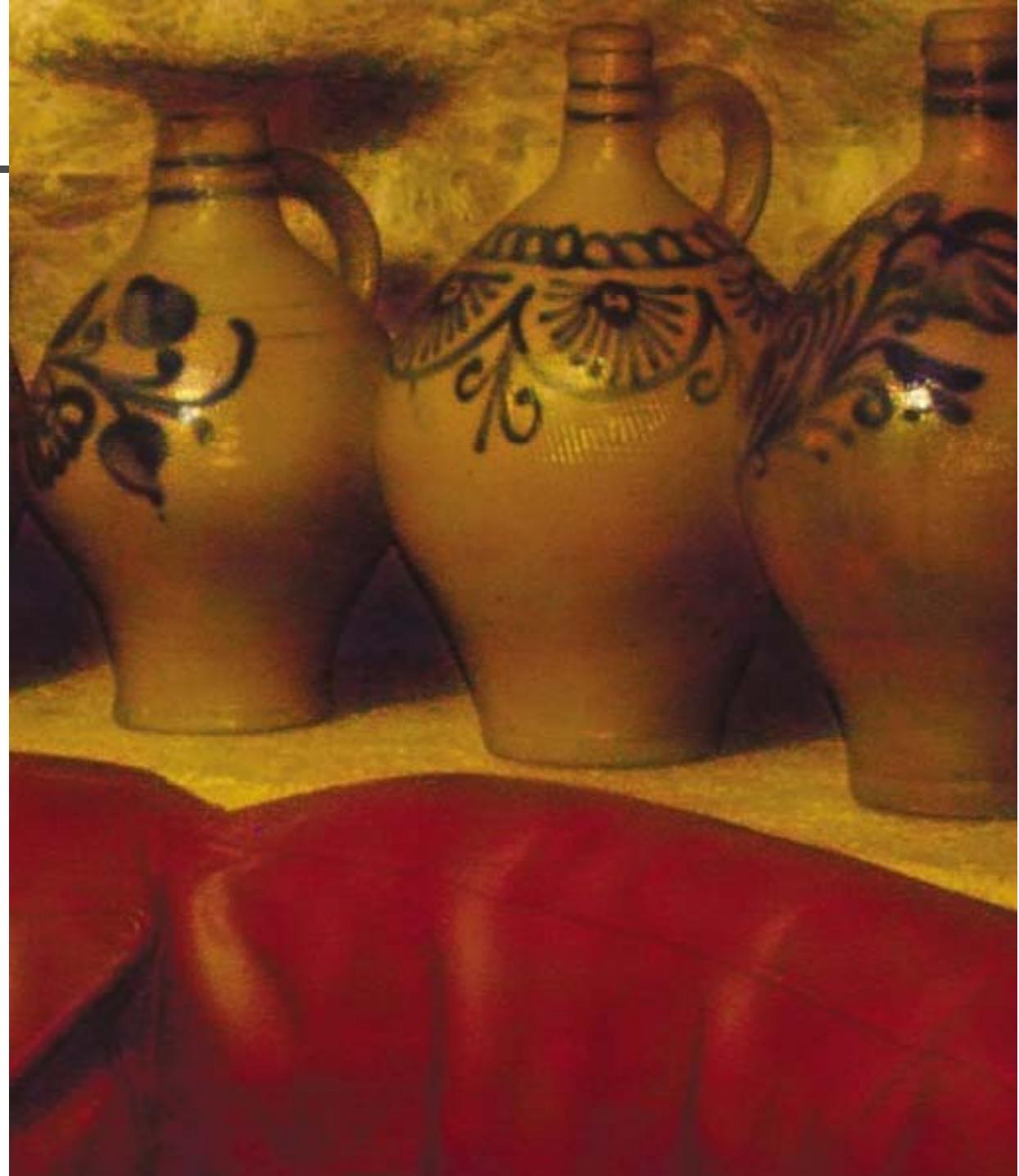
Petschnigg:

- No Flash,



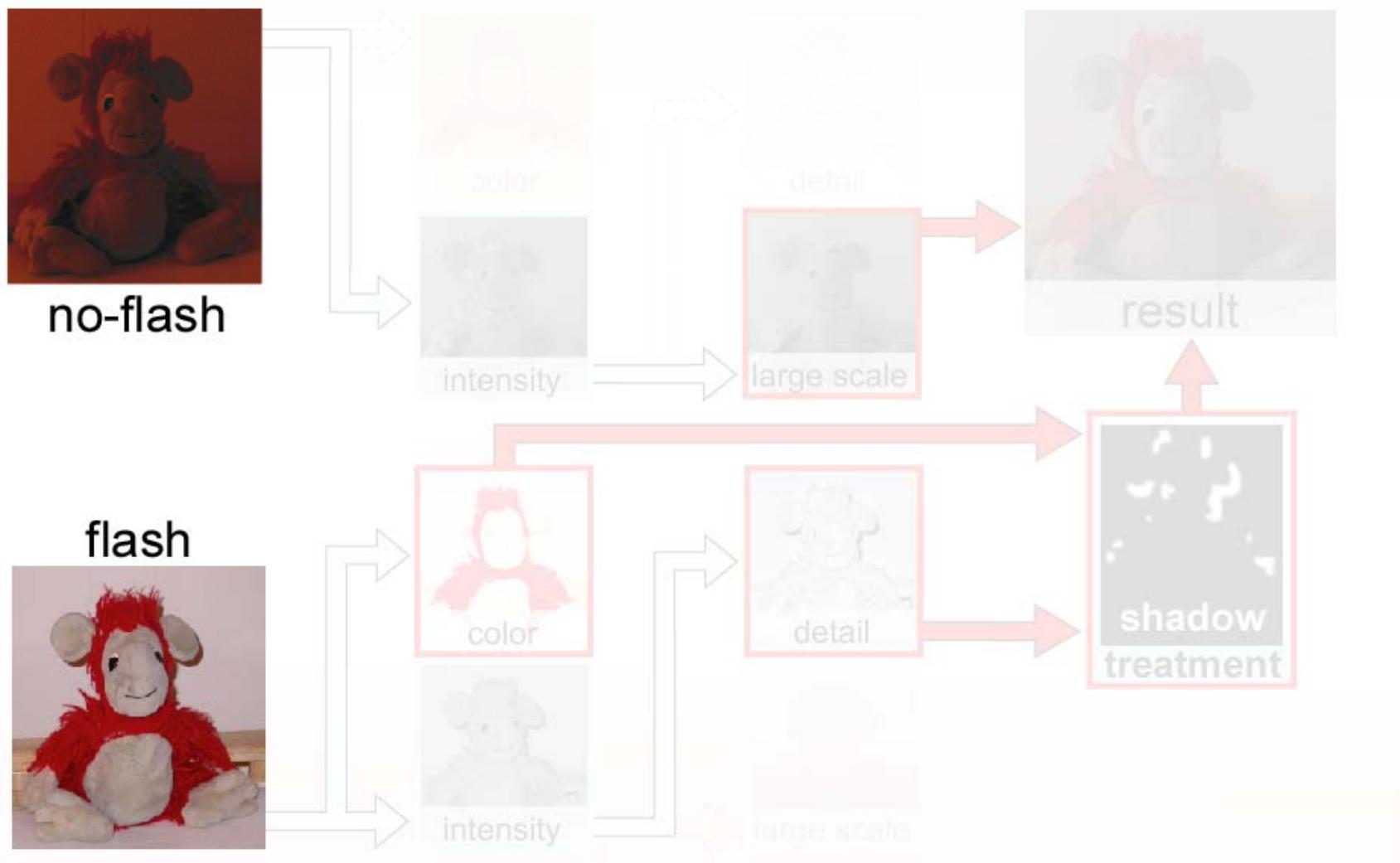
Petschnigg:

- Result



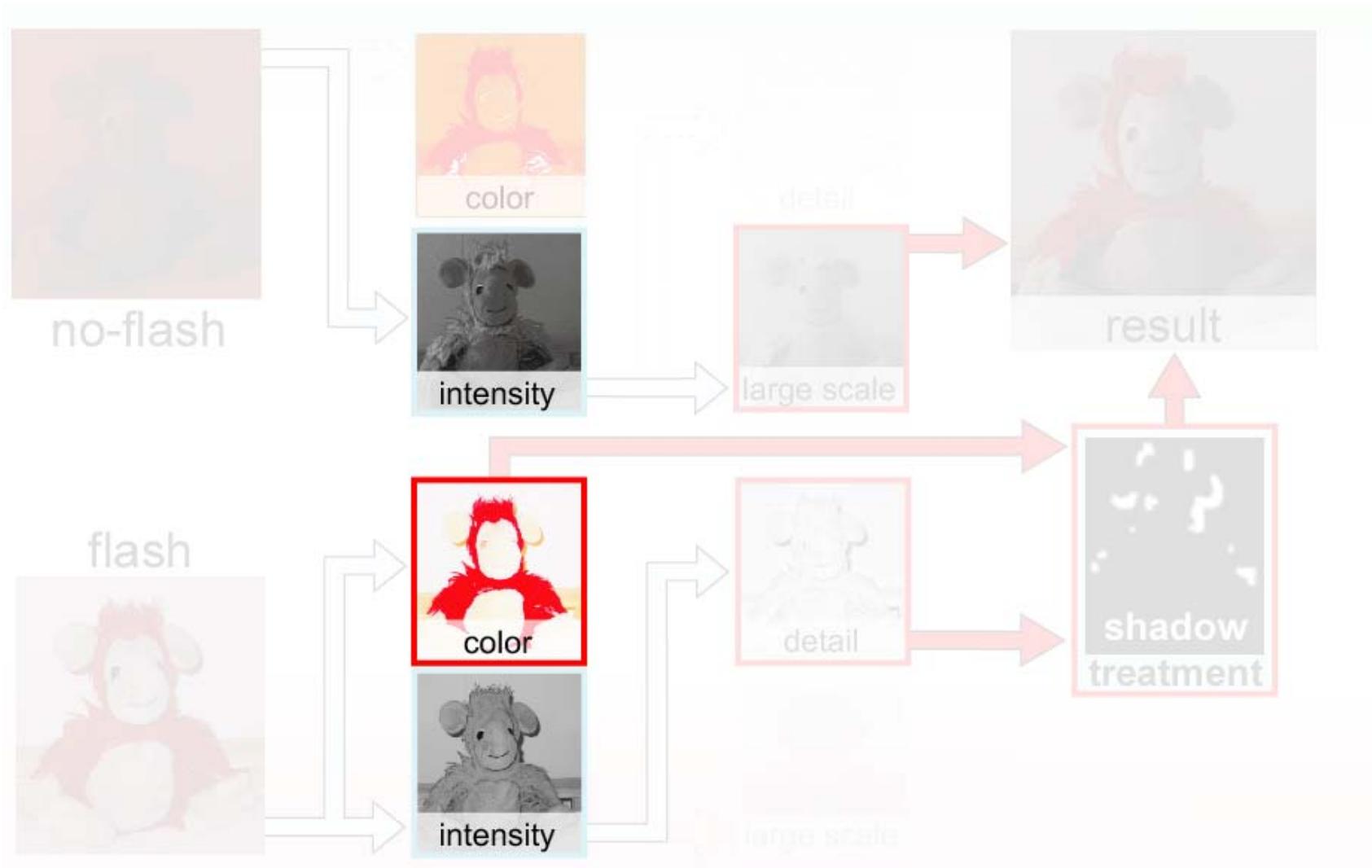
Our Approach

Registration



Our Approach

Decomposition



Decomposition

Color / Intensity:



original

=



intensity

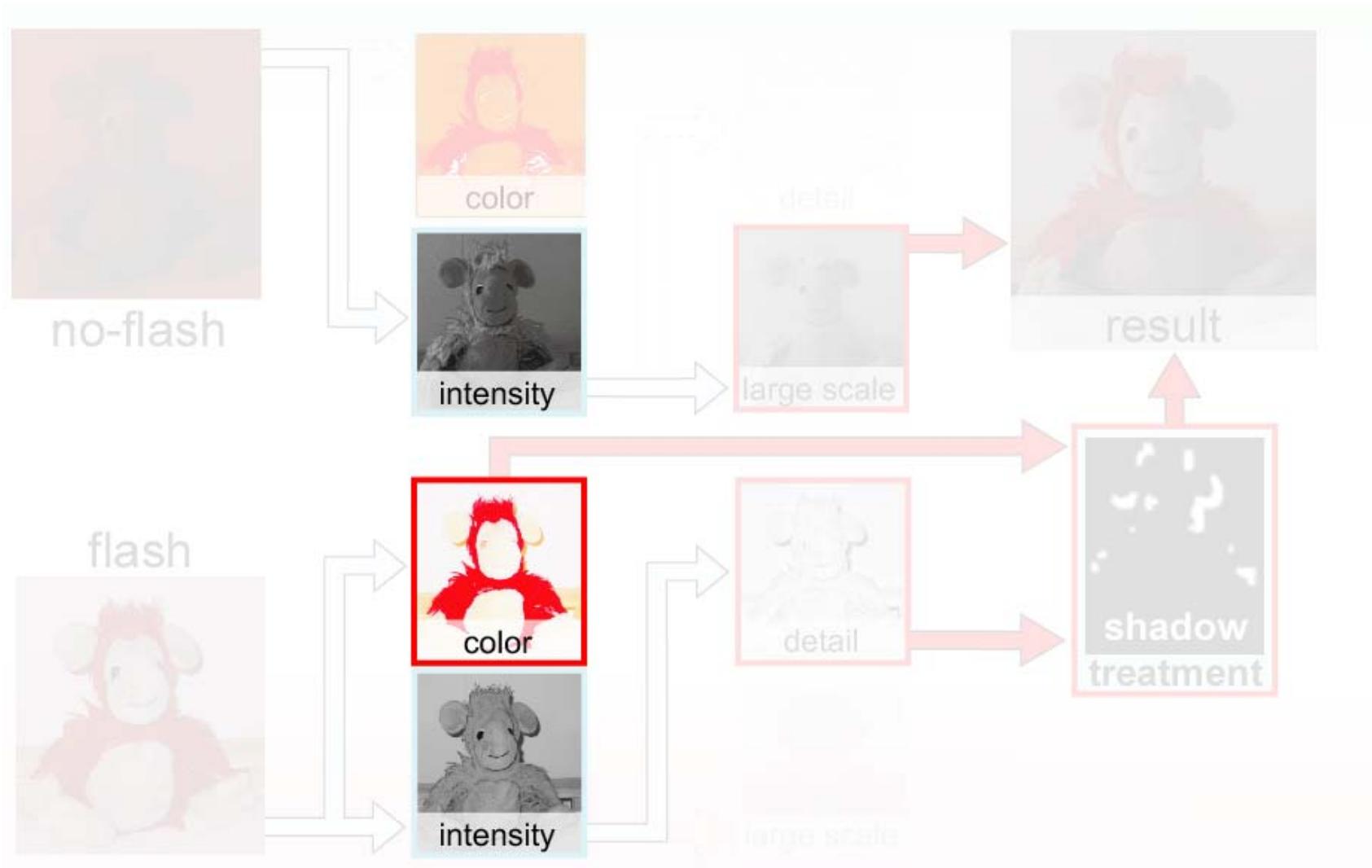
*



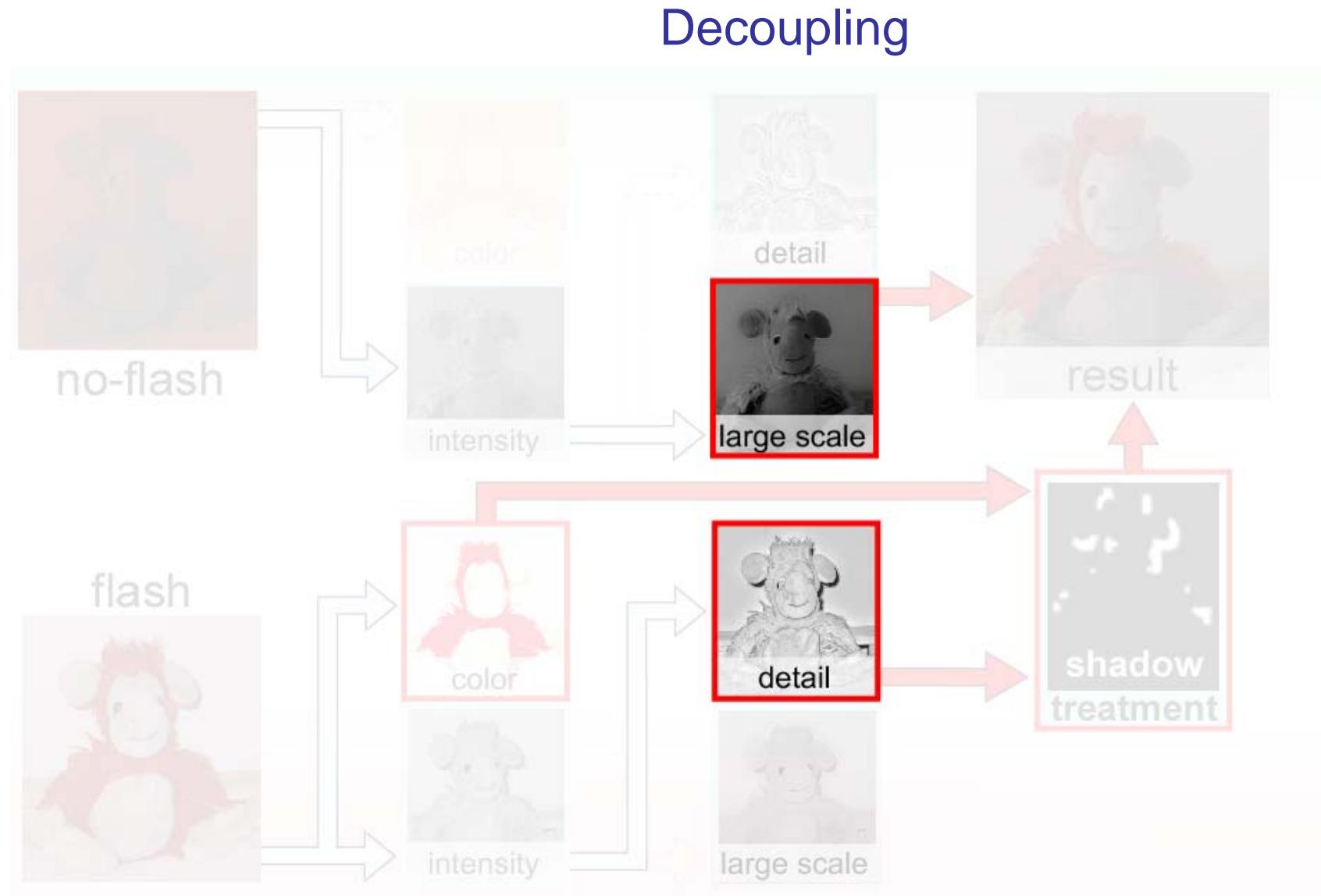
color

Our Approach

Decomposition



Our Approach



Decoupling

- Lighting : Large-scale variation
- Texture : Small-scale variation
- **Lighting** : Large-scale variation
- Texture : Small-scale variation



Lighting

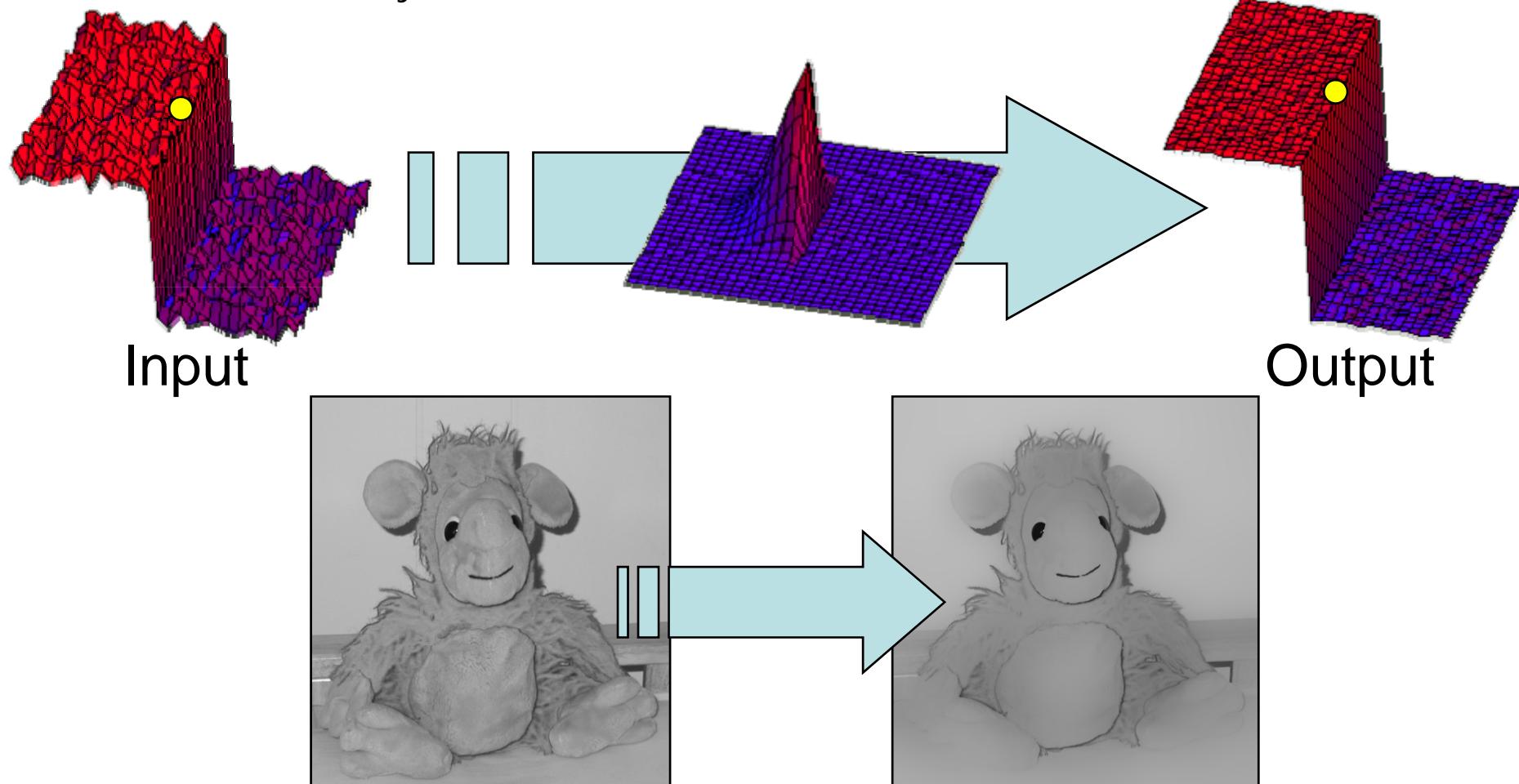


Texture

Large-scale Layer

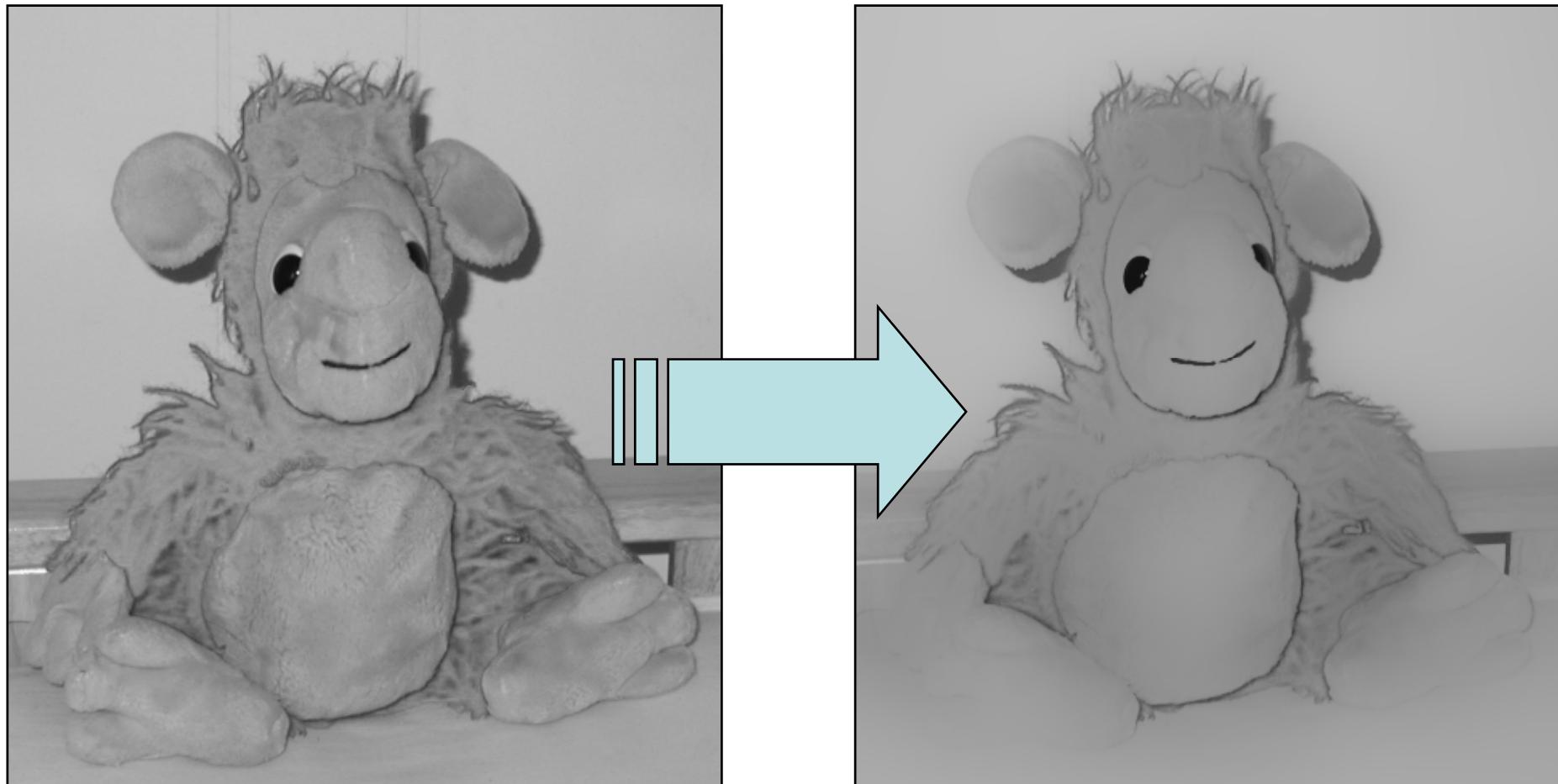
- **Bilateral filter – edge preserving filter**

Smith and Brady 1997; Tomasi and Manducci 1998; Durand et al. 2002



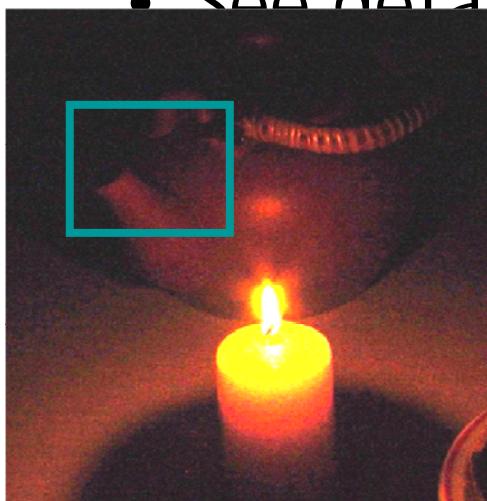
Large-scale Layer

- Bilateral filter



Cross Bilateral Filter

- Similar to joint bilateral filter by Petschnigg et al.
- When no-flash image is too noisy
- Borrow similarity from flash image
 - edge stopping from flash image
- See detail in paper



Bilateral



Cross Bilateral

Detail Layer



Intensity



Large-scale

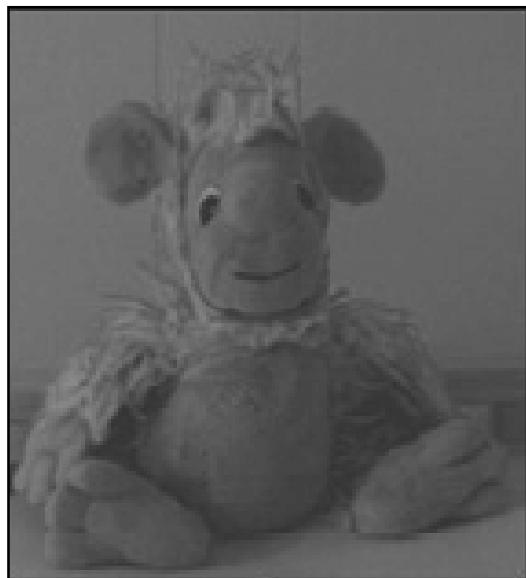


Detail



Recombination: Large scale * Detail = Intensity

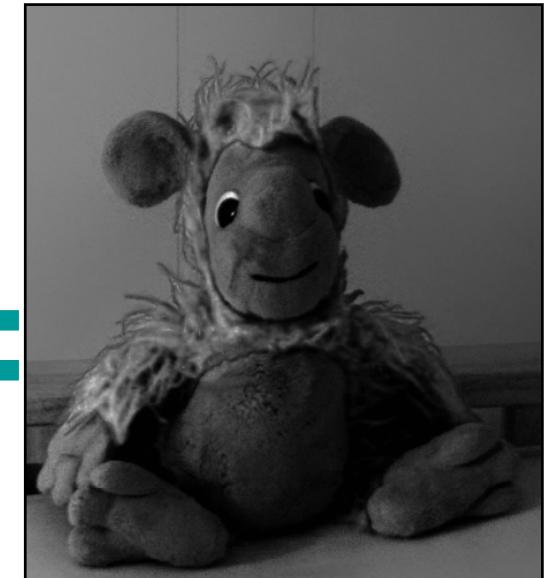
Recombination



Large-scale
No-flash



Detail
Flash

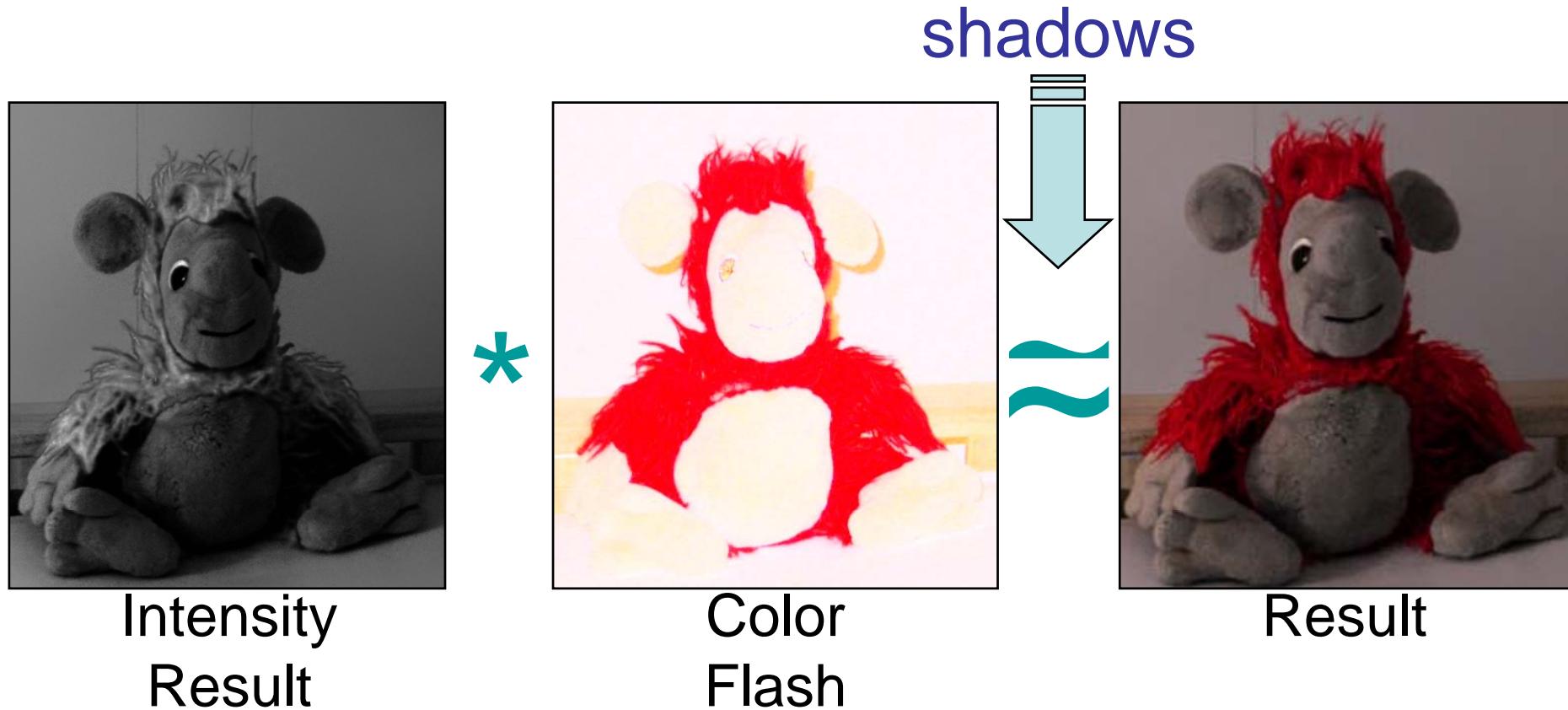


Intensity
Result

Recombination: Large scale * Detail = Intensity

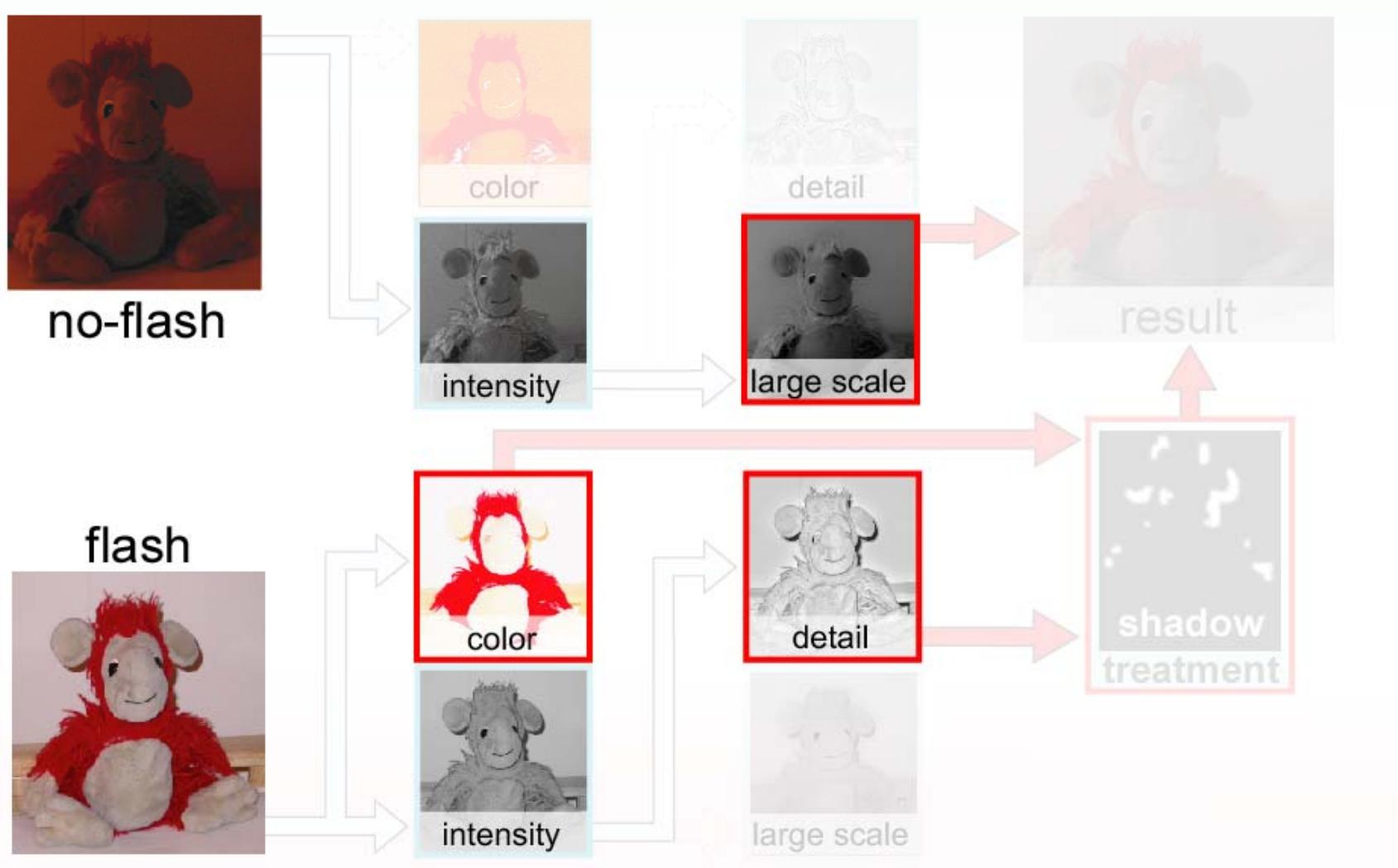
Recombination

DigiVFX

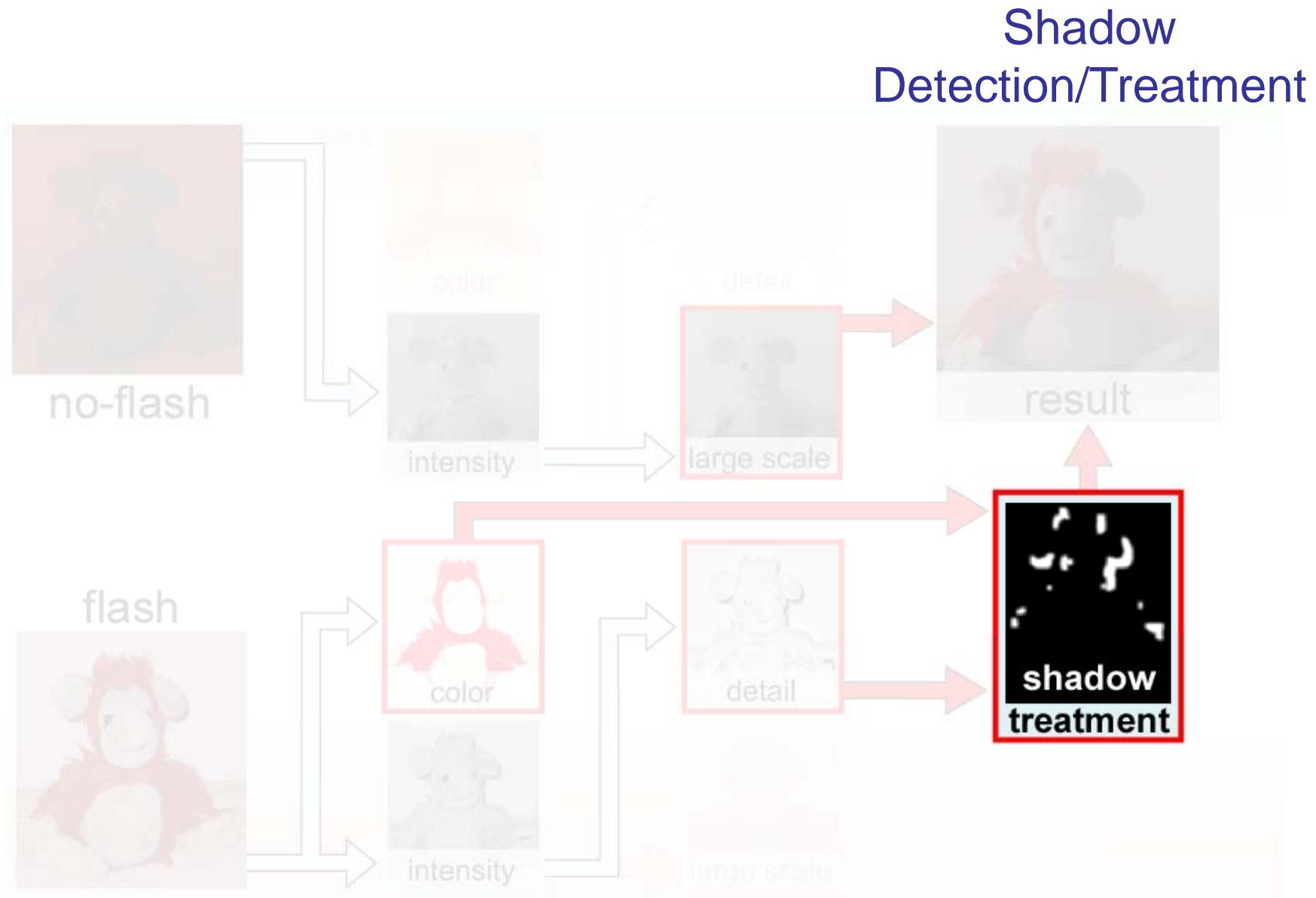


Recombination: Intensity * Color = Original

Our Approach



Our Approach



Results



No-flash



Flash

Joint bilateral upsampling

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||I_p - I_q||)$$

$$J_p = \frac{1}{k_p} \sum_{q \in \Omega} I_q f(||p - q||) g(||\tilde{I}_p - \tilde{I}_q||)$$

$$\tilde{S}_p = \frac{1}{k_p} \sum_{q_\downarrow \in \Omega} S_{q_\downarrow} f(||p_\downarrow - q_\downarrow||) g(||\tilde{I}_p - \tilde{I}_q||)$$

Joint bilateral upsampling

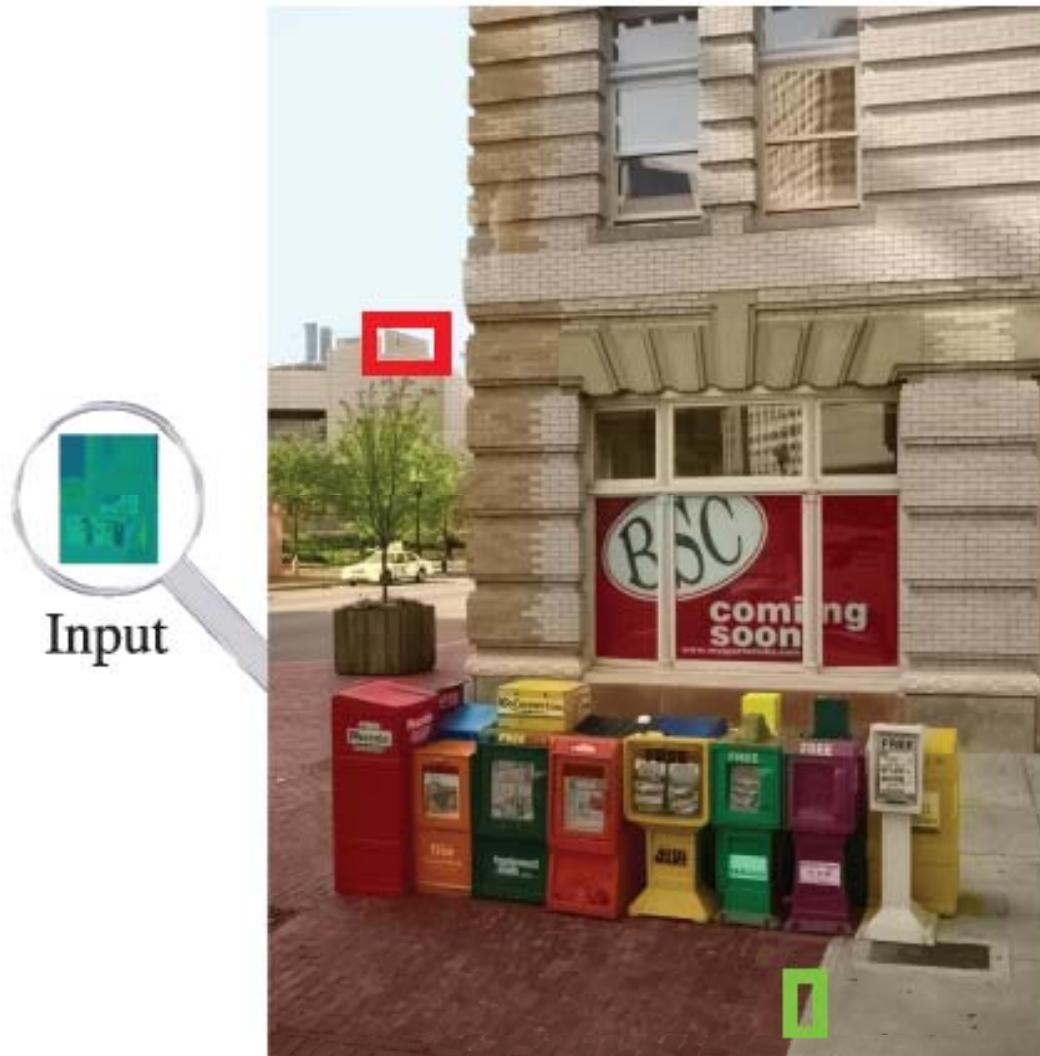


Upsampled Result

Joint bilateral upsampling

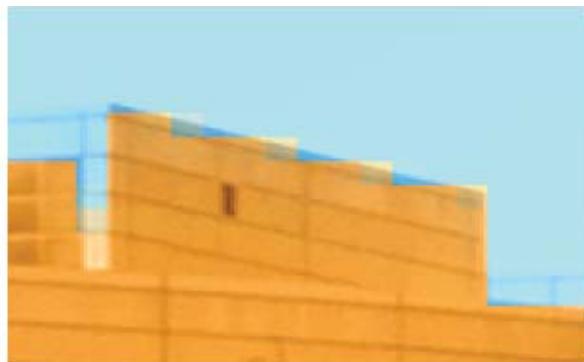


Joint bilateral upsampling

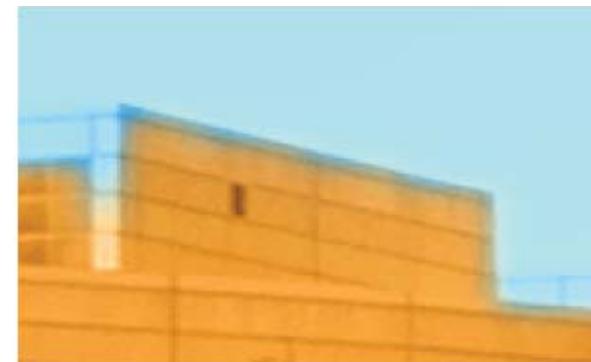
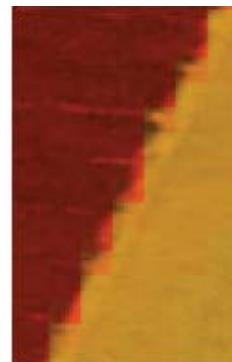


Upsampled Result

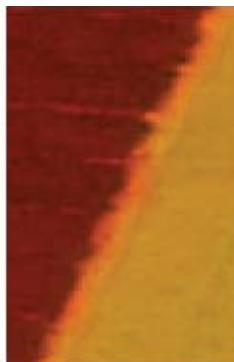
Joint bilateral upsampling



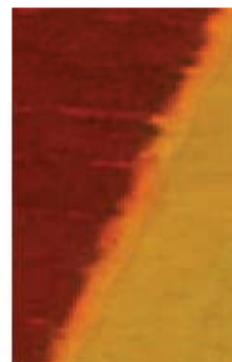
Nearest Neighbor Upsampling



Bicubic Upsampling



Gaussian Upsampling



Joint Bilateral Upsampling



Joint bilateral upsampling



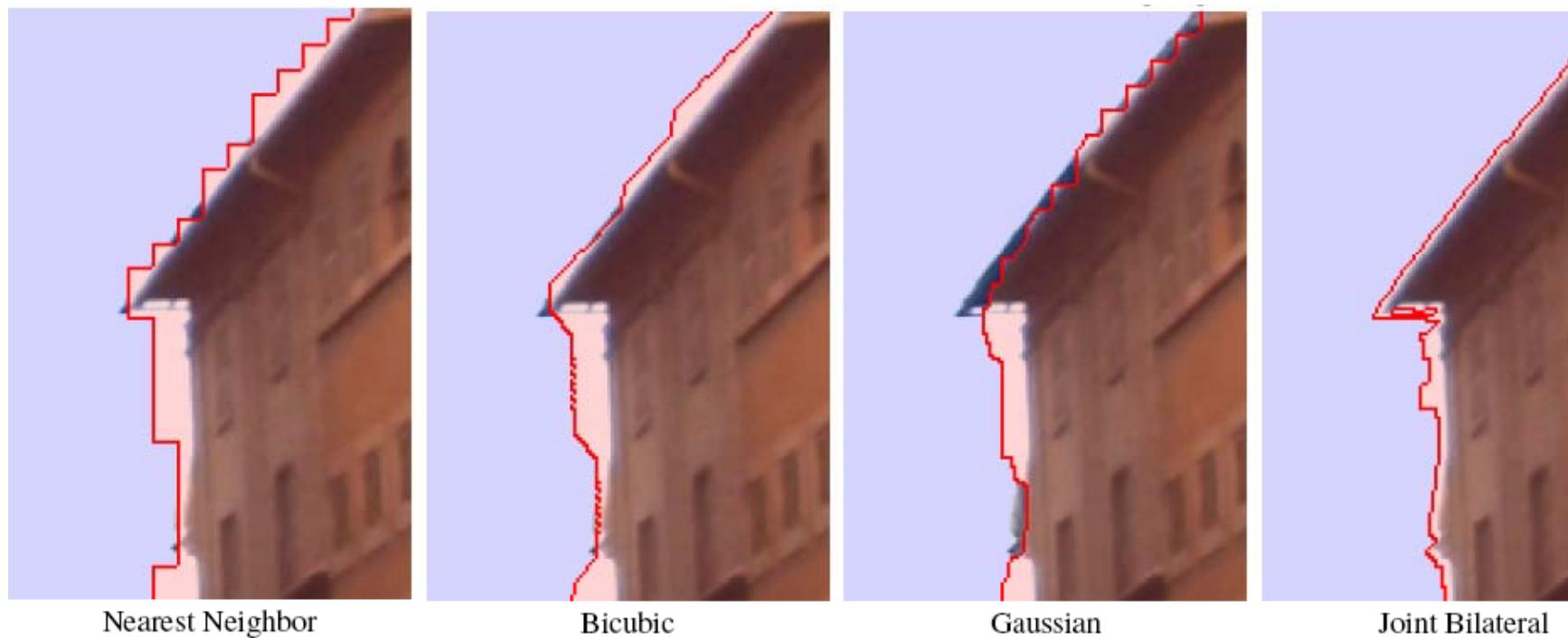
Downsampled



Input Solution

Input Images

Joint bilateral upsampling



Joint bilateral upsampling



Upsampled Result