Image stitching

Digital Visual Effects, Spring 2008

Yung-Yu Chuang

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with slides by Richard Szeliski, Steve Seitz, Matthew Brown and Vaclav Hlavac

Image stitching

- Stitching = alignment + blending

geometrical registration (alignment)

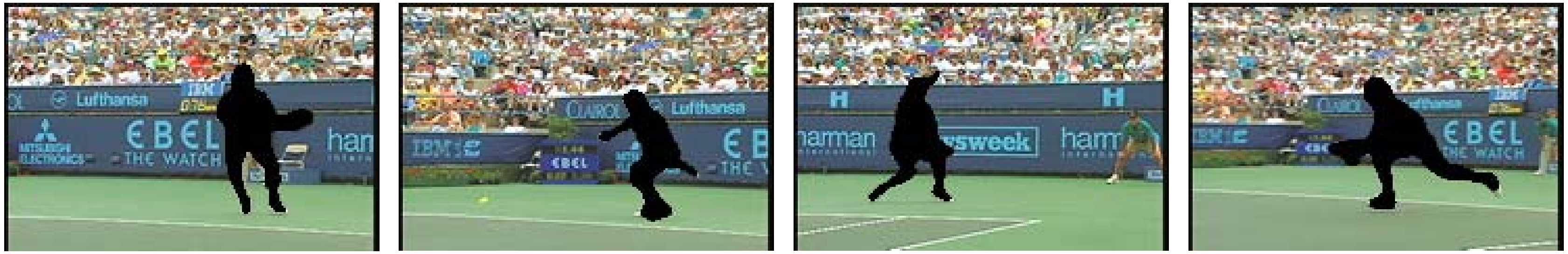
photometric registration (blending)

Applications of image stitching

- Video stabilization
- Video summarization
- Video compression
- Video matting
- Panorama creation

Video summarization

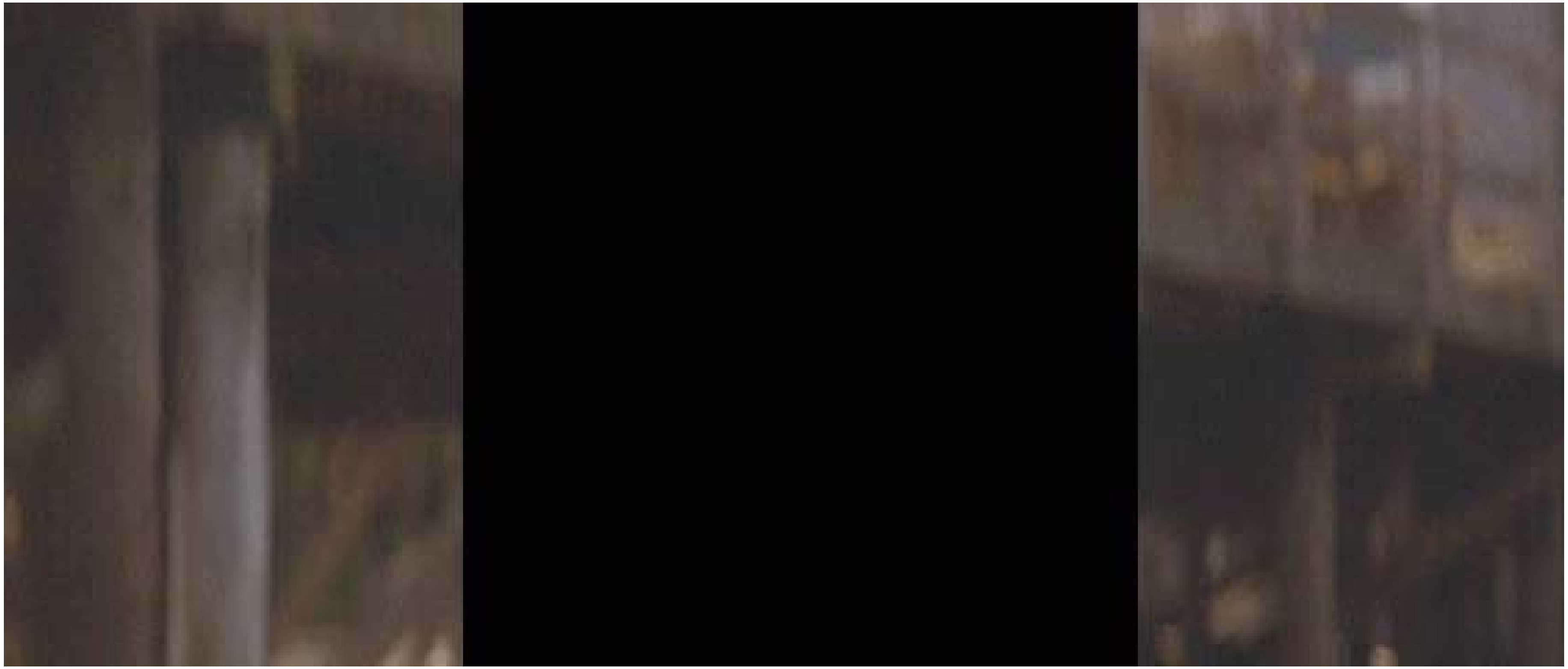
Video compression



Object removal

input video

Object removal



remove foreground

Object removal

estimate background

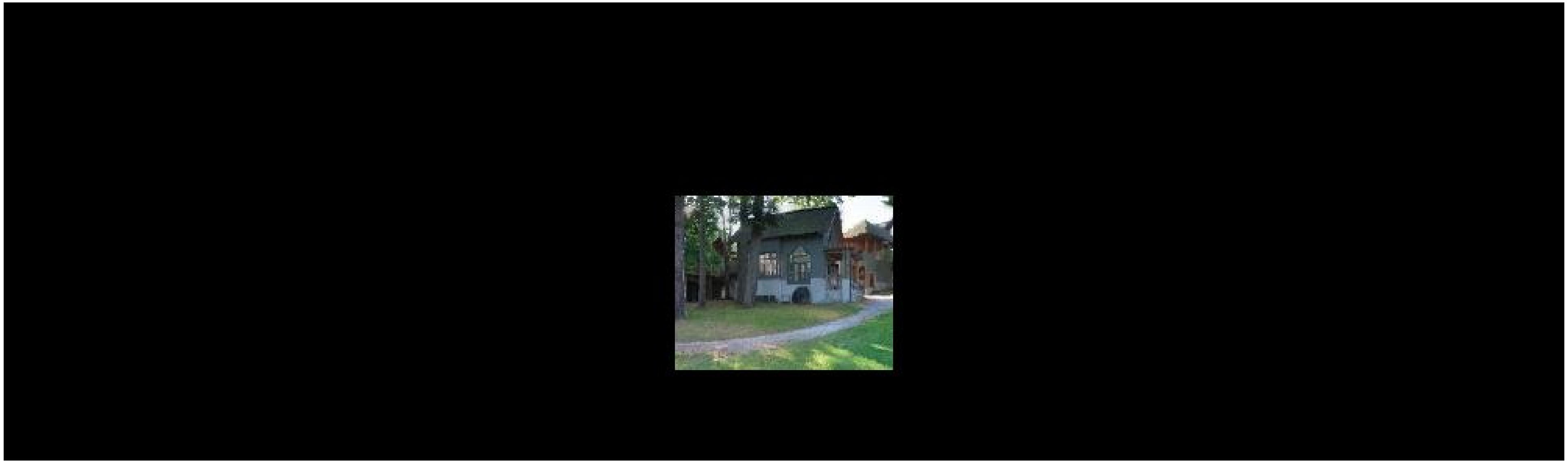
Object removal

background estimation

Panorama creation

Why panorama?

- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°



Why panorama?

- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°
  - Human FOV = 200 x 135°

Why panorama?

- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°
  - Human FOV = 200 x 135°
  - Panoramic Mosaic = 360 x 180°

Panorama examples

- Like HDR, it is a topic of computational photography, seeking ways to build a better camera mostly in software.
- Most consumer cameras have a panorama mode
- Mars:

http://www.panoramas.dk/fullscreen3/f2_mars97.html

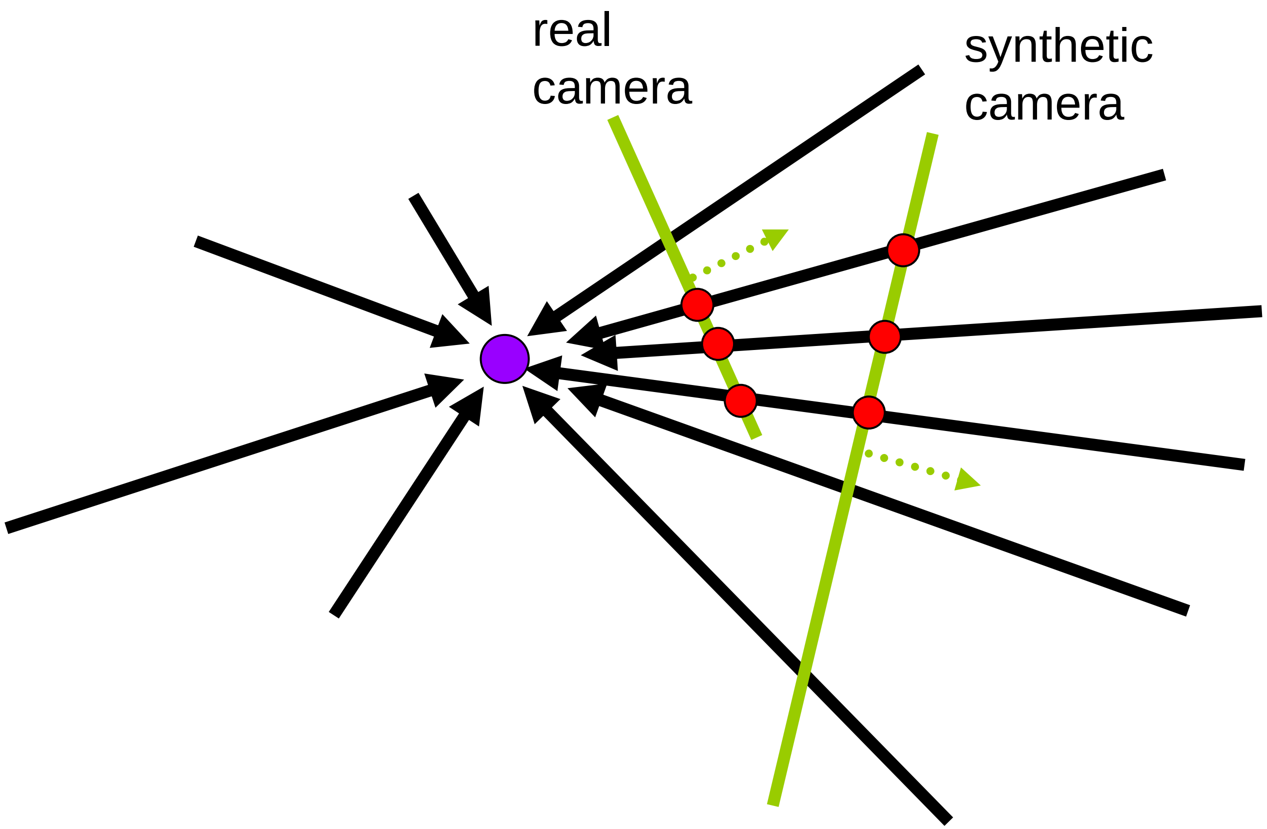
- Earth:

http://www.panoramas.dk/new-year-2006/taipei.html

What can be globally aligned?

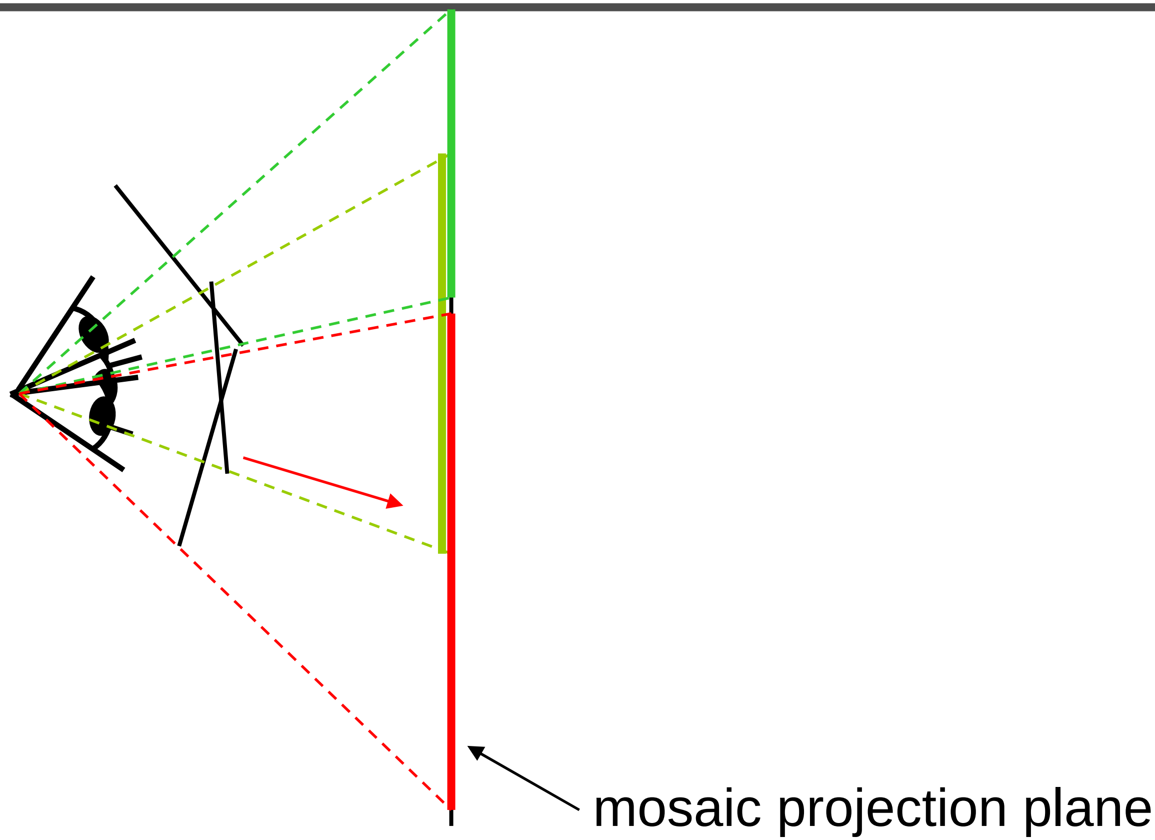
- In image stitching, we seek for a matrix to globally warp one image into another. Are any two images of the same scene can be aligned this way?
  - Images captured with the same center of projection
  - A planar scene or far-away scene

A pencil of rays contains all views



Can generate any synthetic camera view
as long as it has **the same center of projection**!

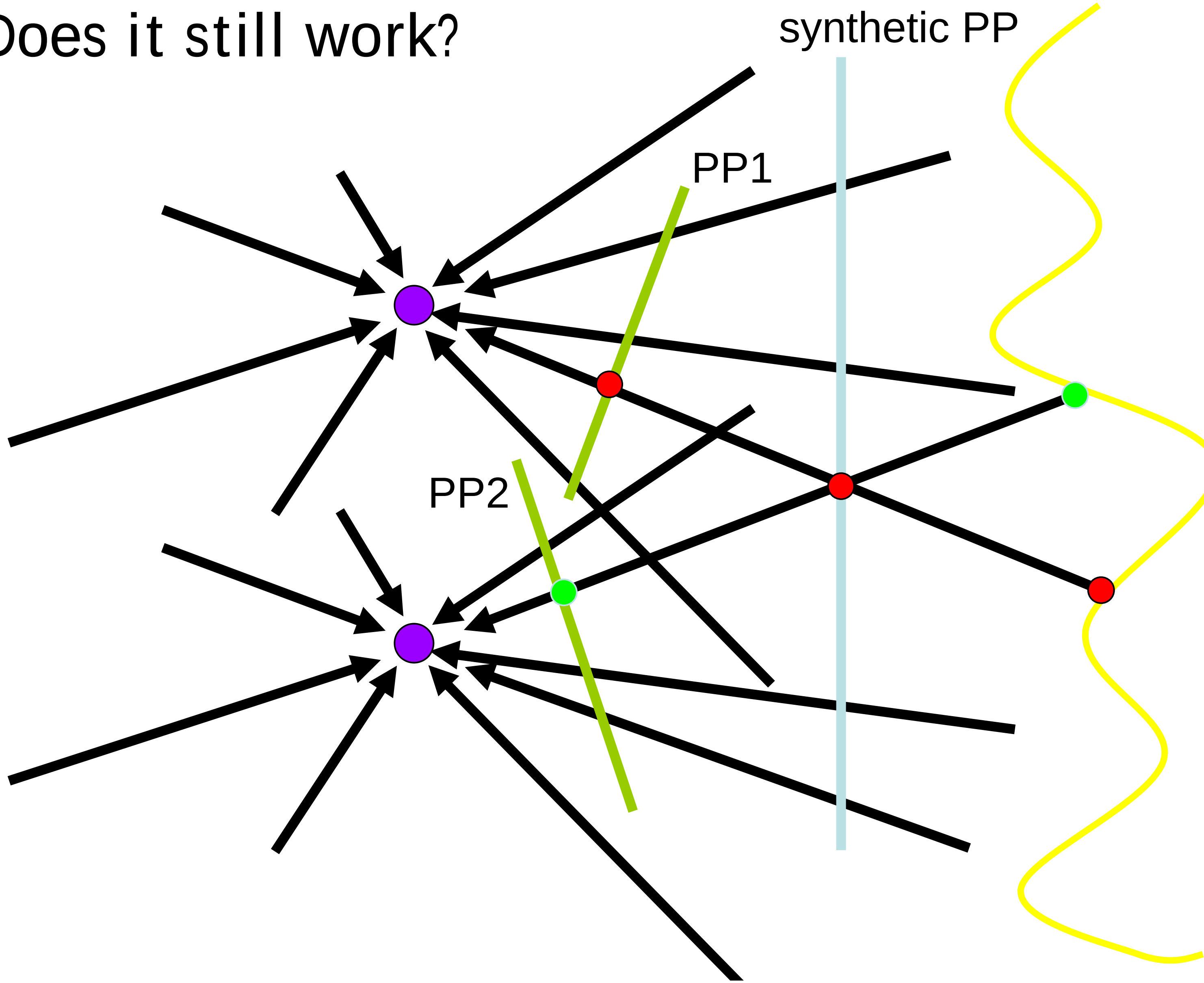
Mosaic as an image reprojection



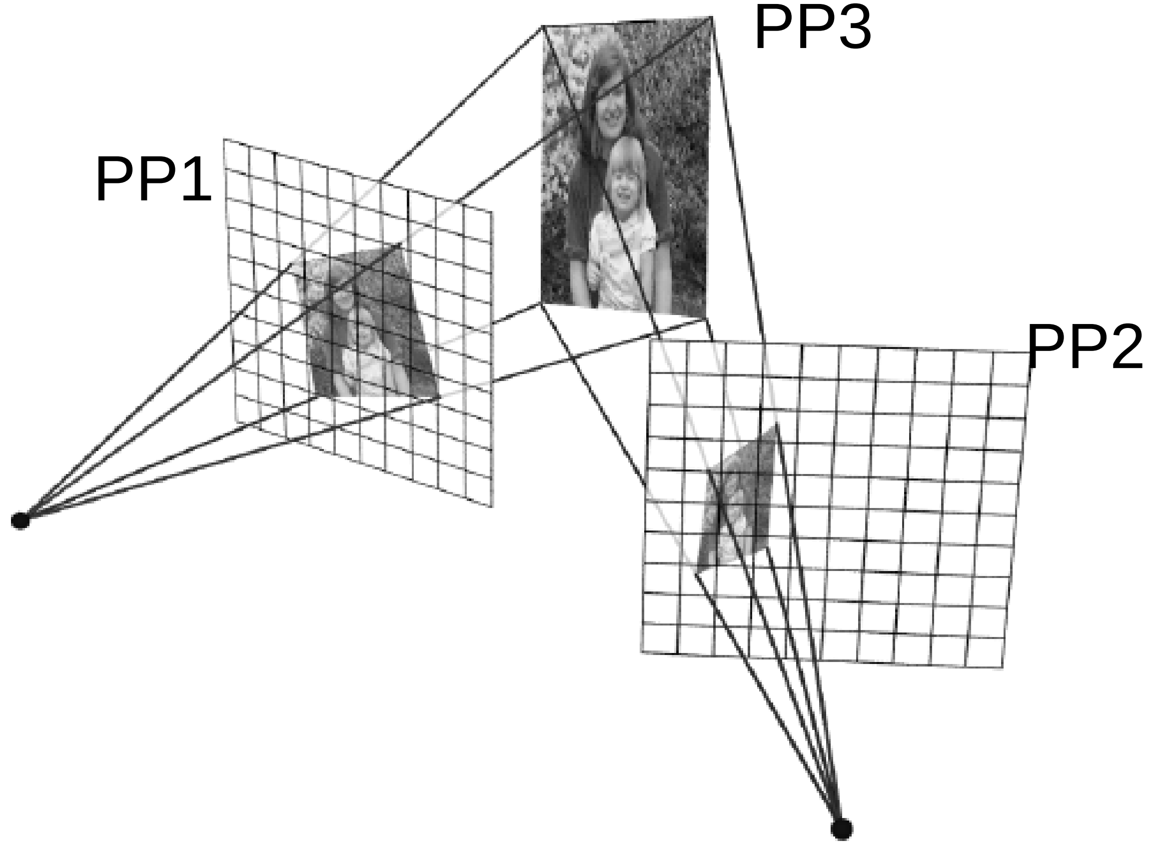
- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Changing camera center

- Does it still work?



Planar scene (or far away)

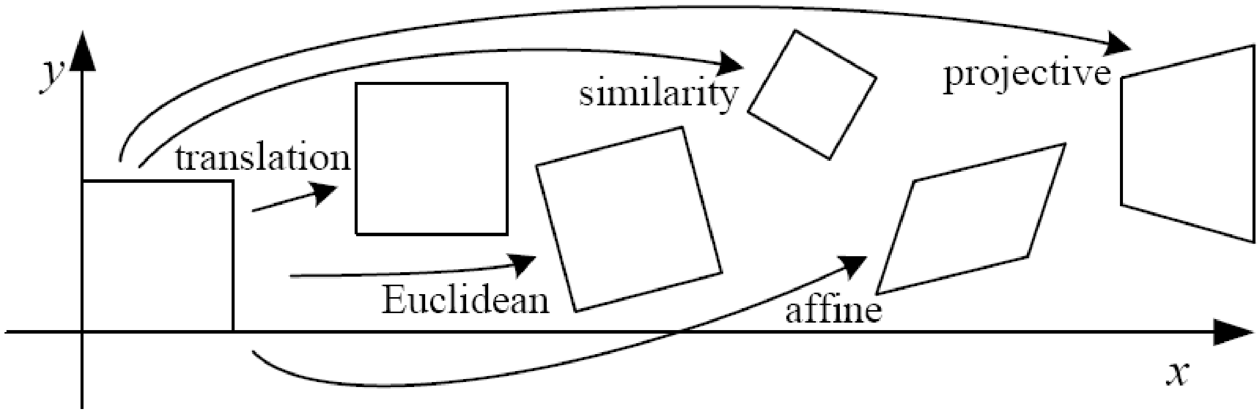


- PP3 is a projection plane of both centers of projection, so we are OK!
- This is how big aerial photographs are made

Motion models

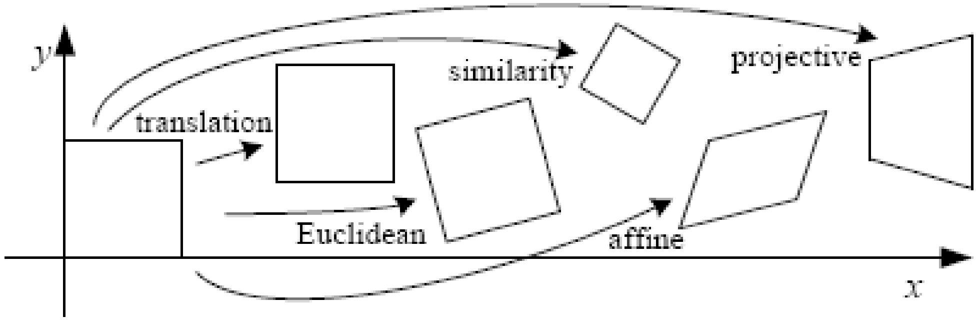
- Parametric models as the assumptions on the relation between two images.

2D Motion models

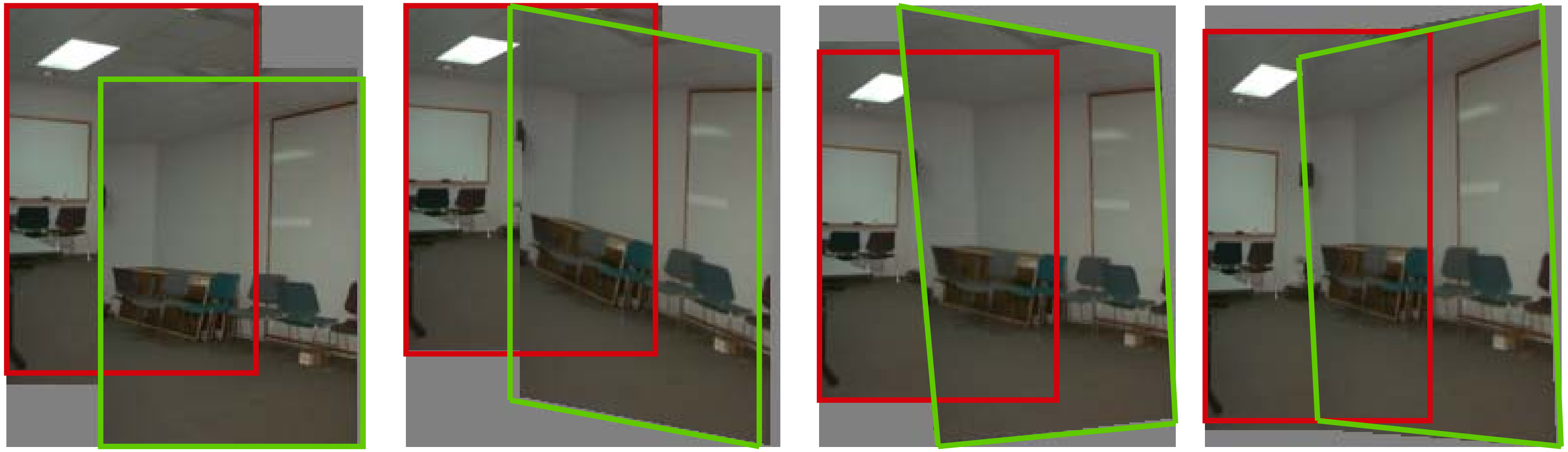


Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

Motion models



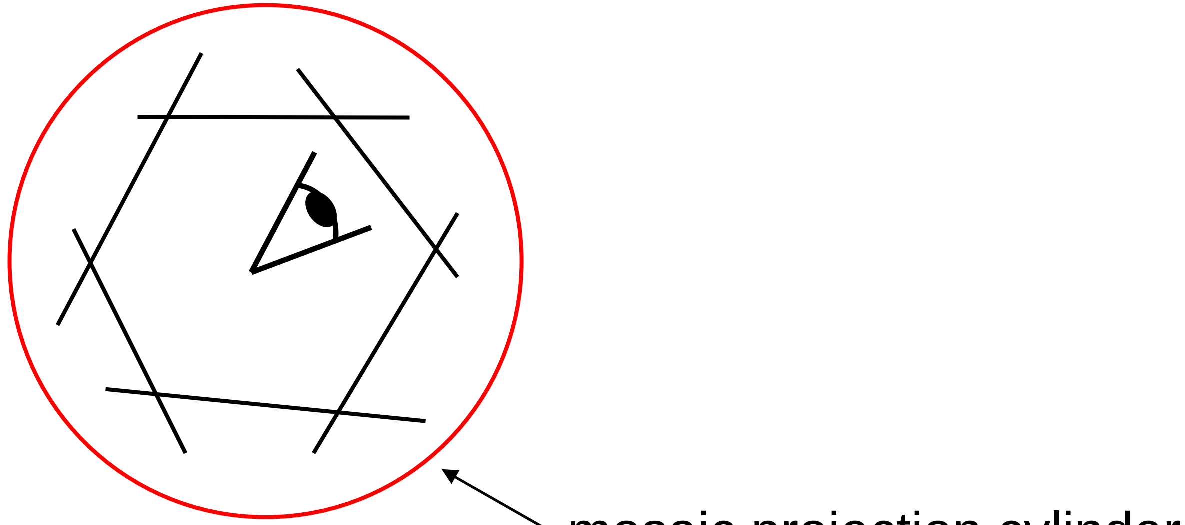
Translation | **Affine** | **Perspective** | **3D rotation**



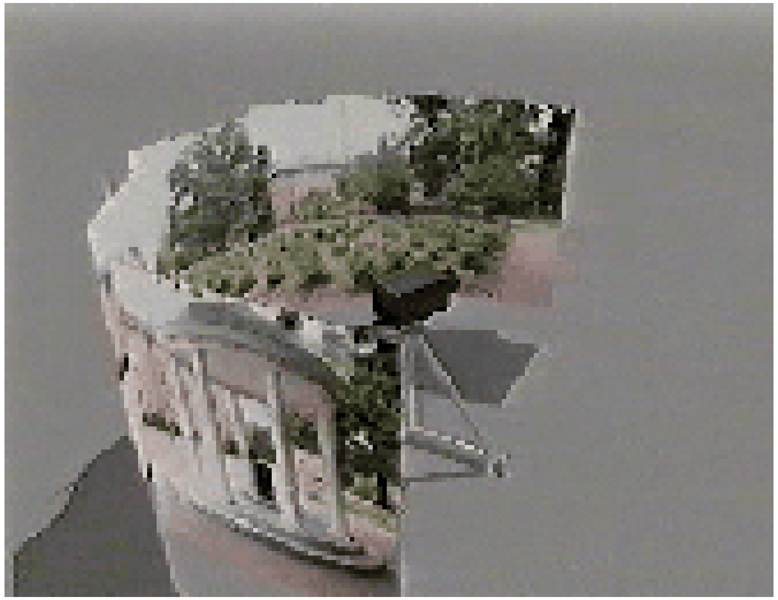
2 unknowns | **6 unknowns** | **8 unknowns** | **3 unknowns**

A case study: cylindrical panorama

- What if you want a 360° field of view?



Cylindrical panoramas



- Steps
  - Reproject each image onto a cylinder
  - Blend
  - Output the resulting mosaic

Cylindrical panorama

1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinate
3. Compute pairwise alignments
4. Fix up the end-to-end alignment
5. Blending
6. Crop the result and import into a viewer

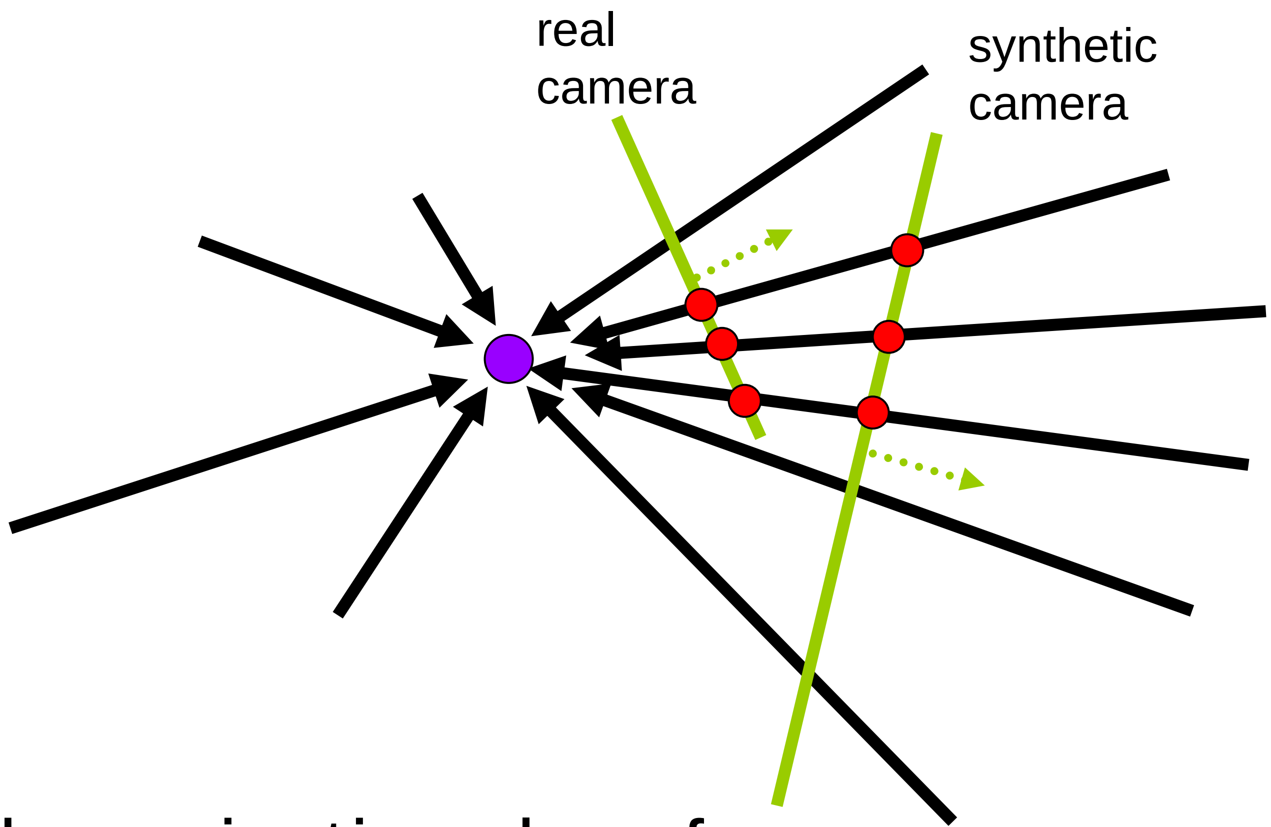
Taking pictures

Kaidan panoramic tripod head

Translation model

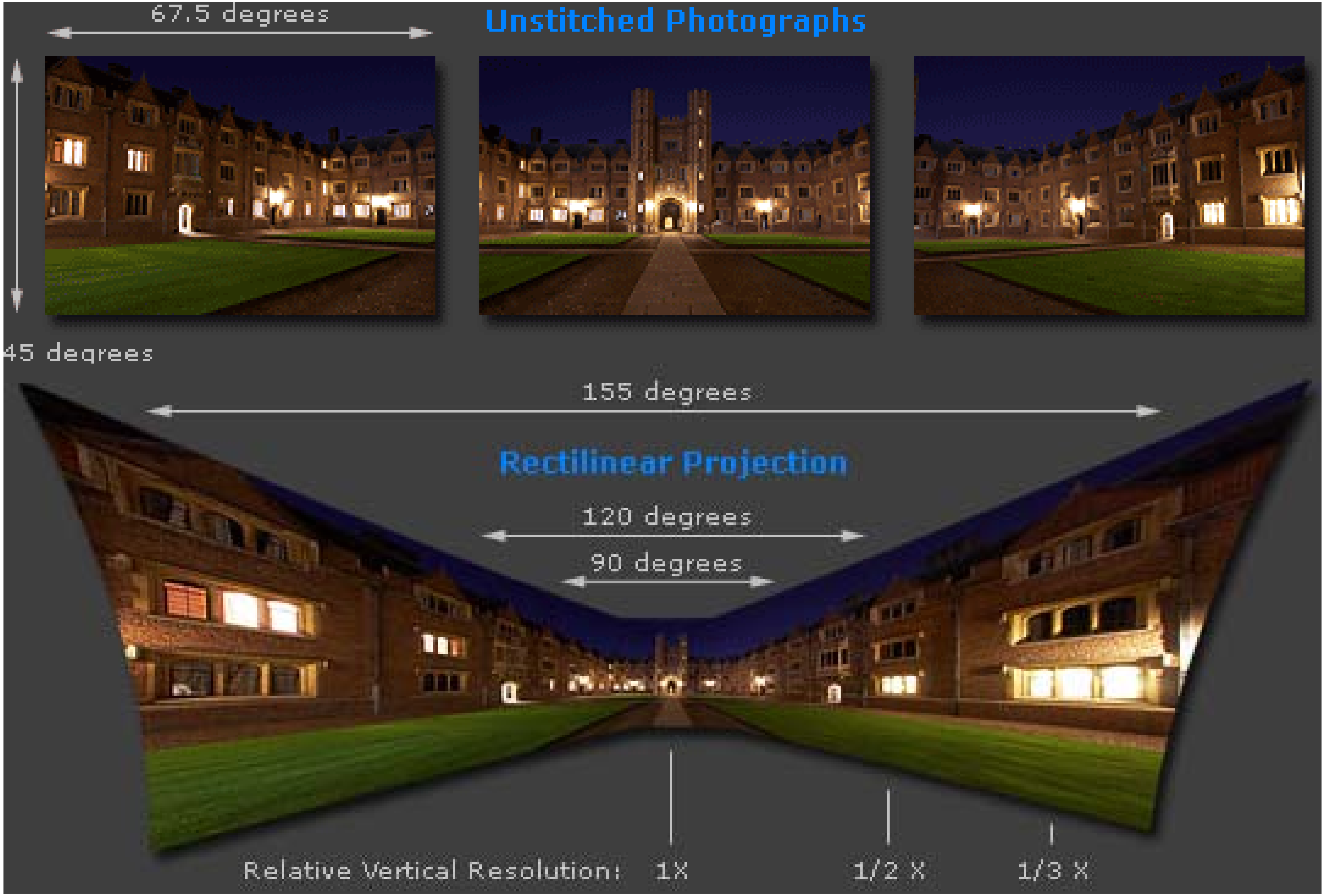
Try to align this in PaintShop Pro

Where should the synthetic camera be



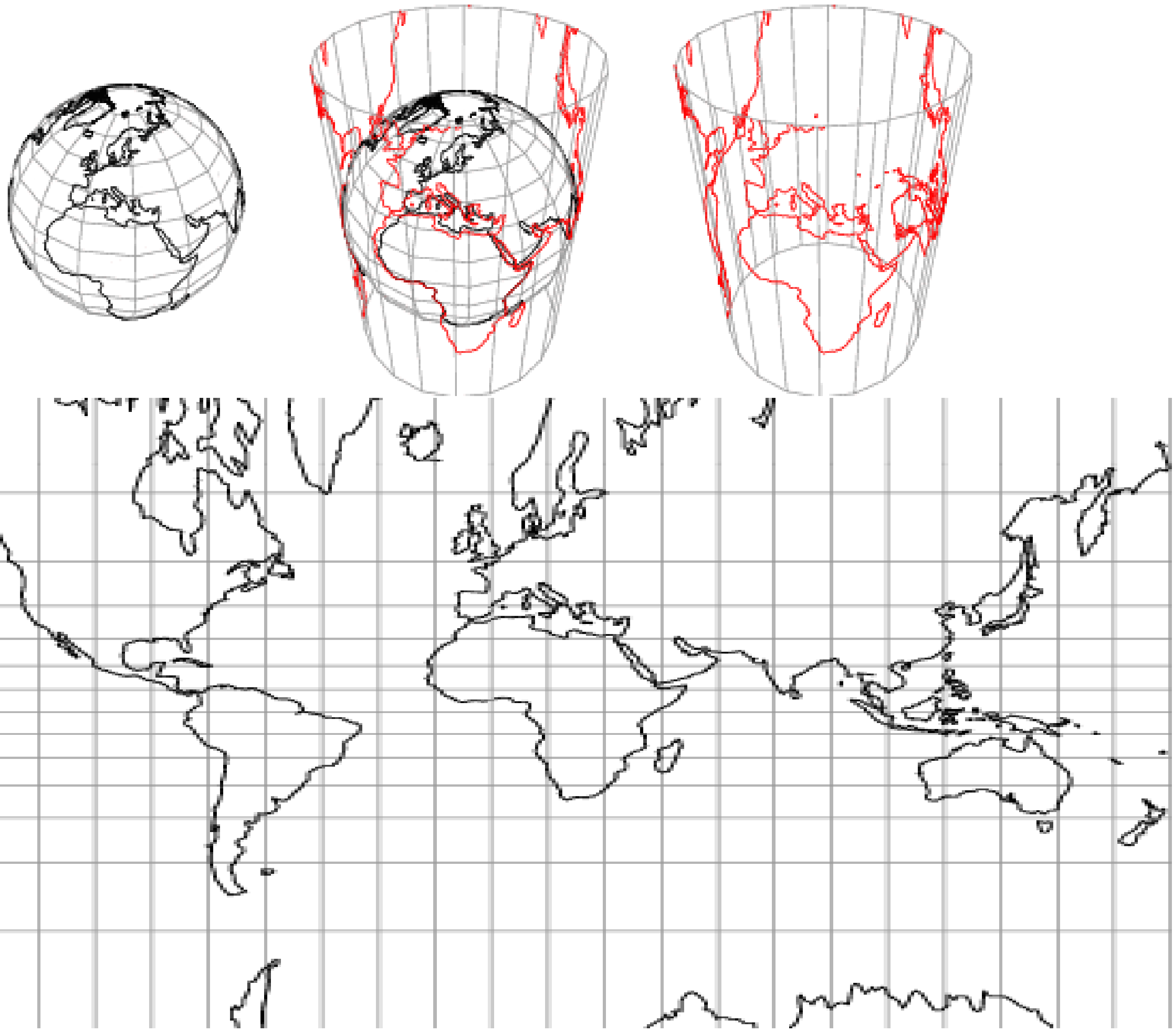
- The projection plan of some camera
- Onto a cylinder

Cylindrical projection

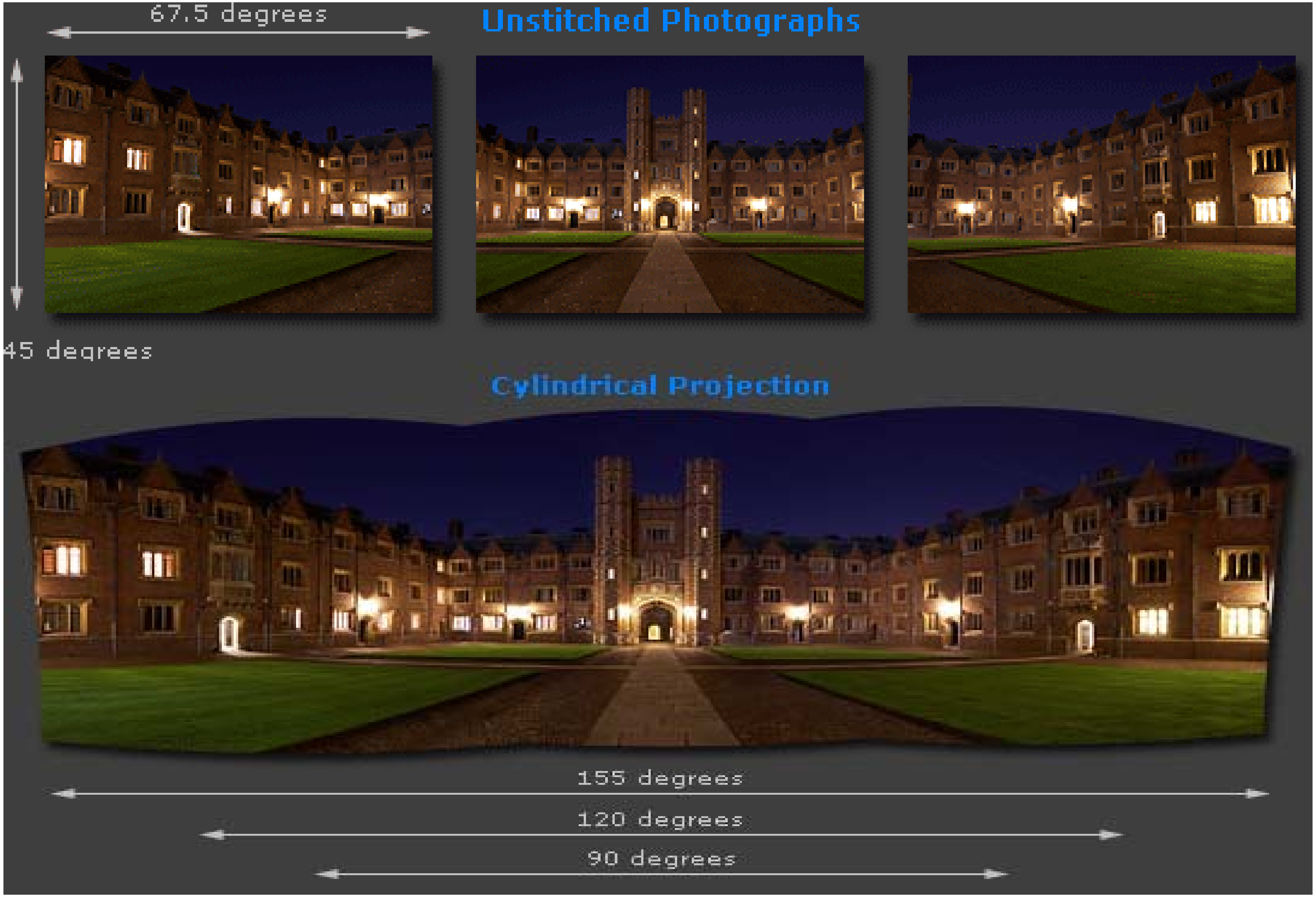


Adopted from http://www.cambridgeincolour.com/tutorials/image-projections.htm

Cylindrical projection



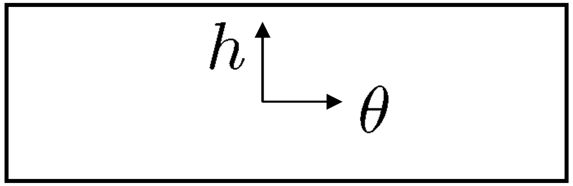
Cylindrical projection



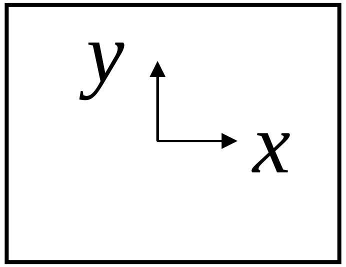
Adopted from http://www.cambridgeincolour.com/tutorials/image-projections.htm

Cylindrical projection

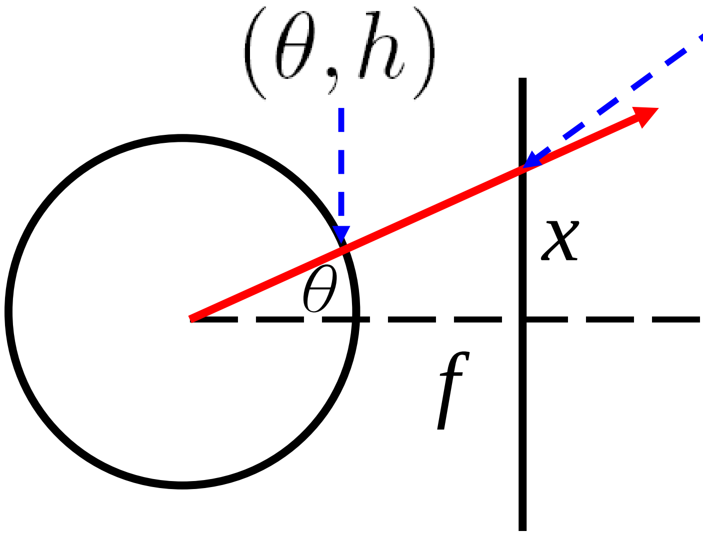
DigiVFX



unwrapped cylinder

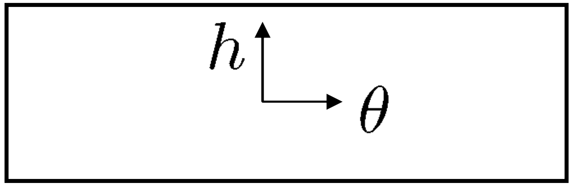


$$(\sin\theta, h, \cos\theta) \propto (x, y, f)$$

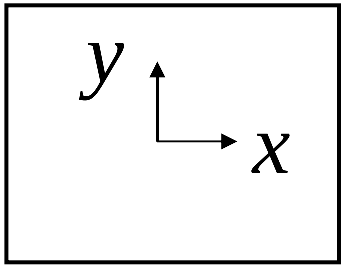


$$\theta = \tan^{-1}\frac{x}{f}$$

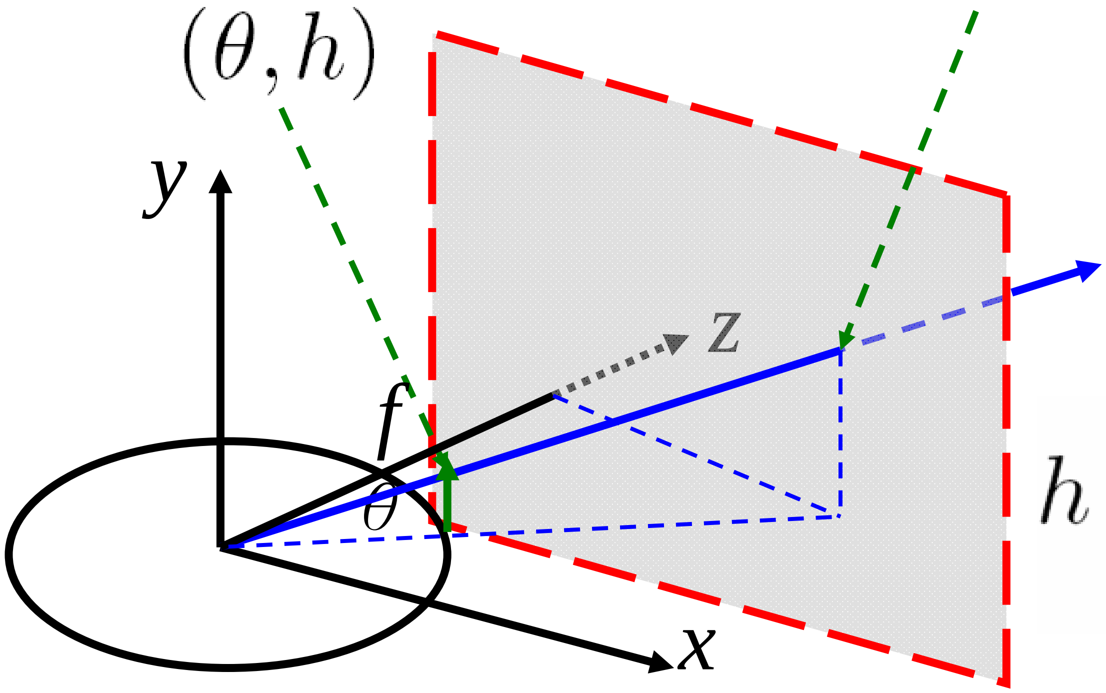
Cylindrical projection



unwrapped cylinder

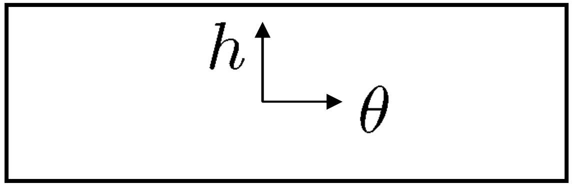


$$(\sin\theta, h, \cos\theta) \propto (x, y, f)$$

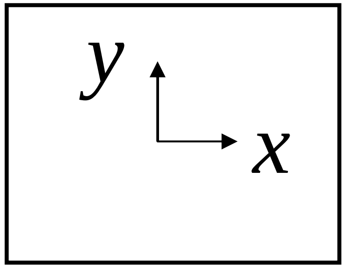


$$h = \frac{y}{\sqrt{x^2 + f^2}}$$

Cylindrical projection

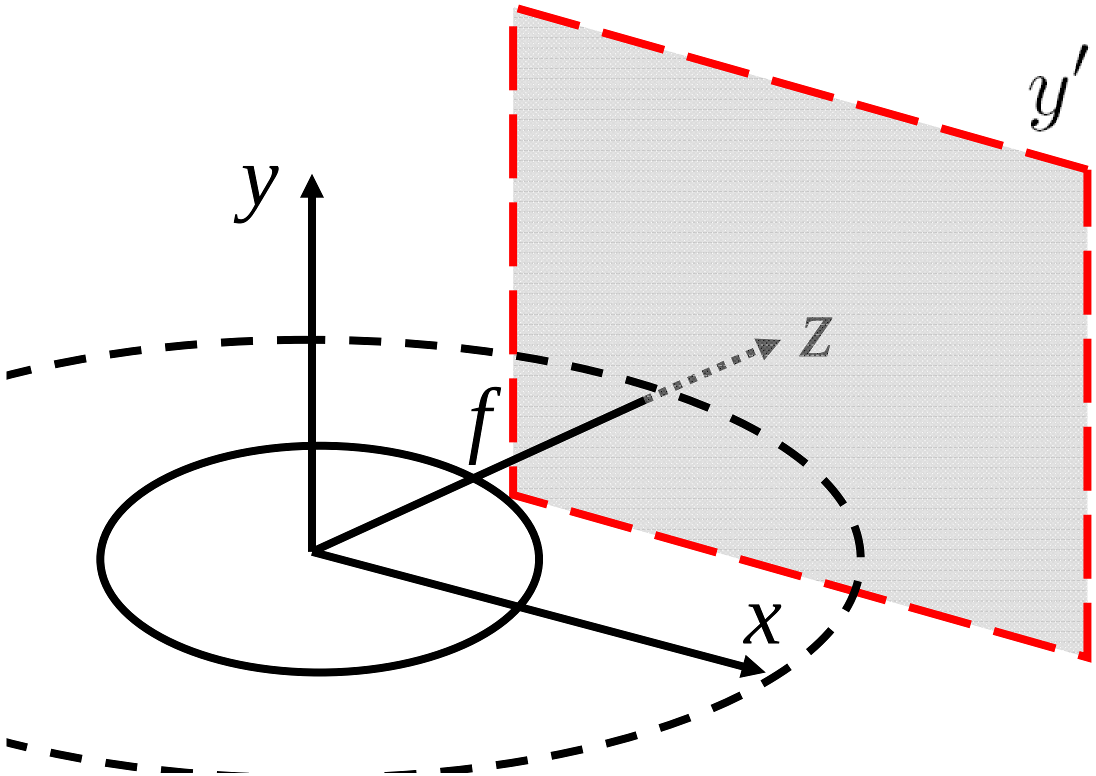


unwrapped cylinder



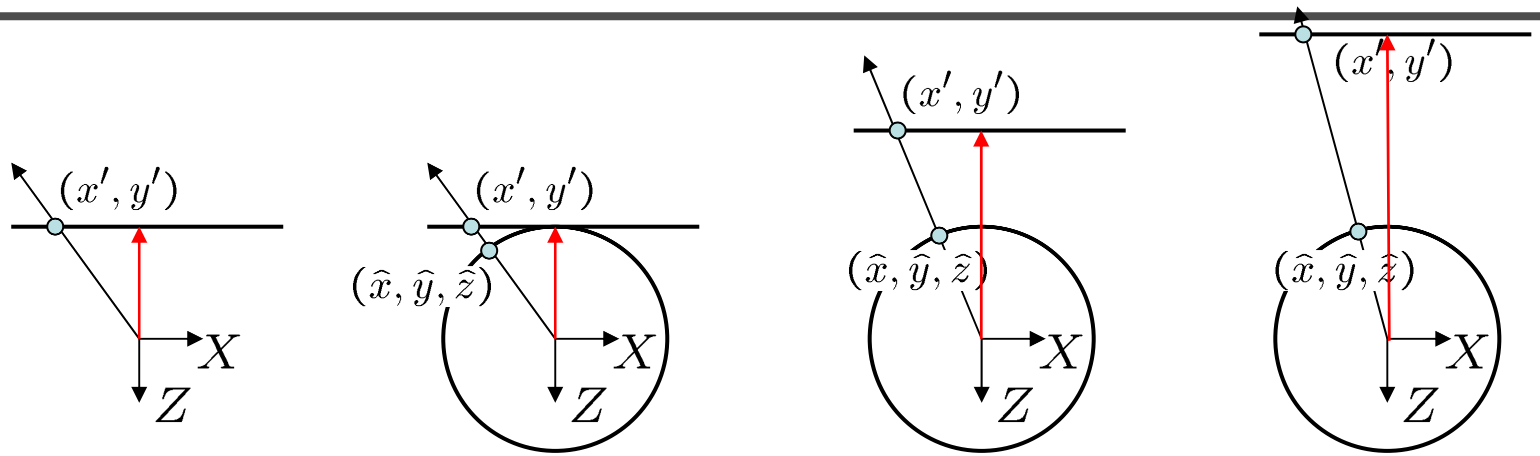
$$x' = s\theta = s\tan^{-1}\frac{x}{f}$$

$$y' = sh = s\frac{y}{\sqrt{x^2+f^2}}$$



$s=f$ gives less distortion

Cylindrical reprojection



top-down view

Focal length – the dirty secret…

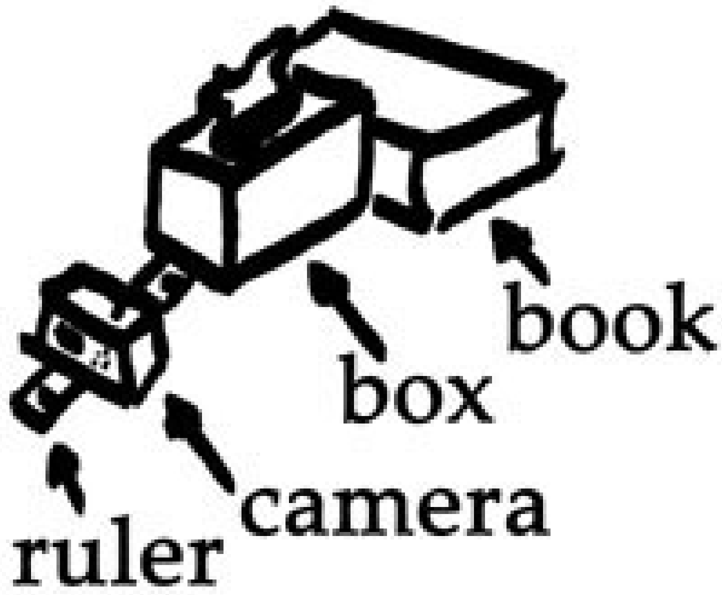
Image 384x300

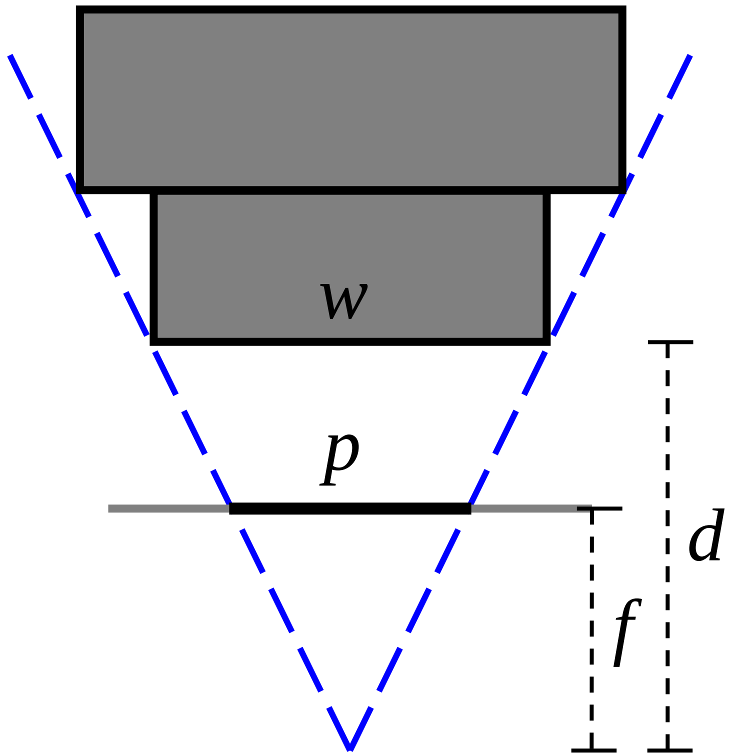
f = 180 (pixels)

f = 280

f = 380

A simple method for estimating f





Or, you can use other software, such as AutoStich, to help.

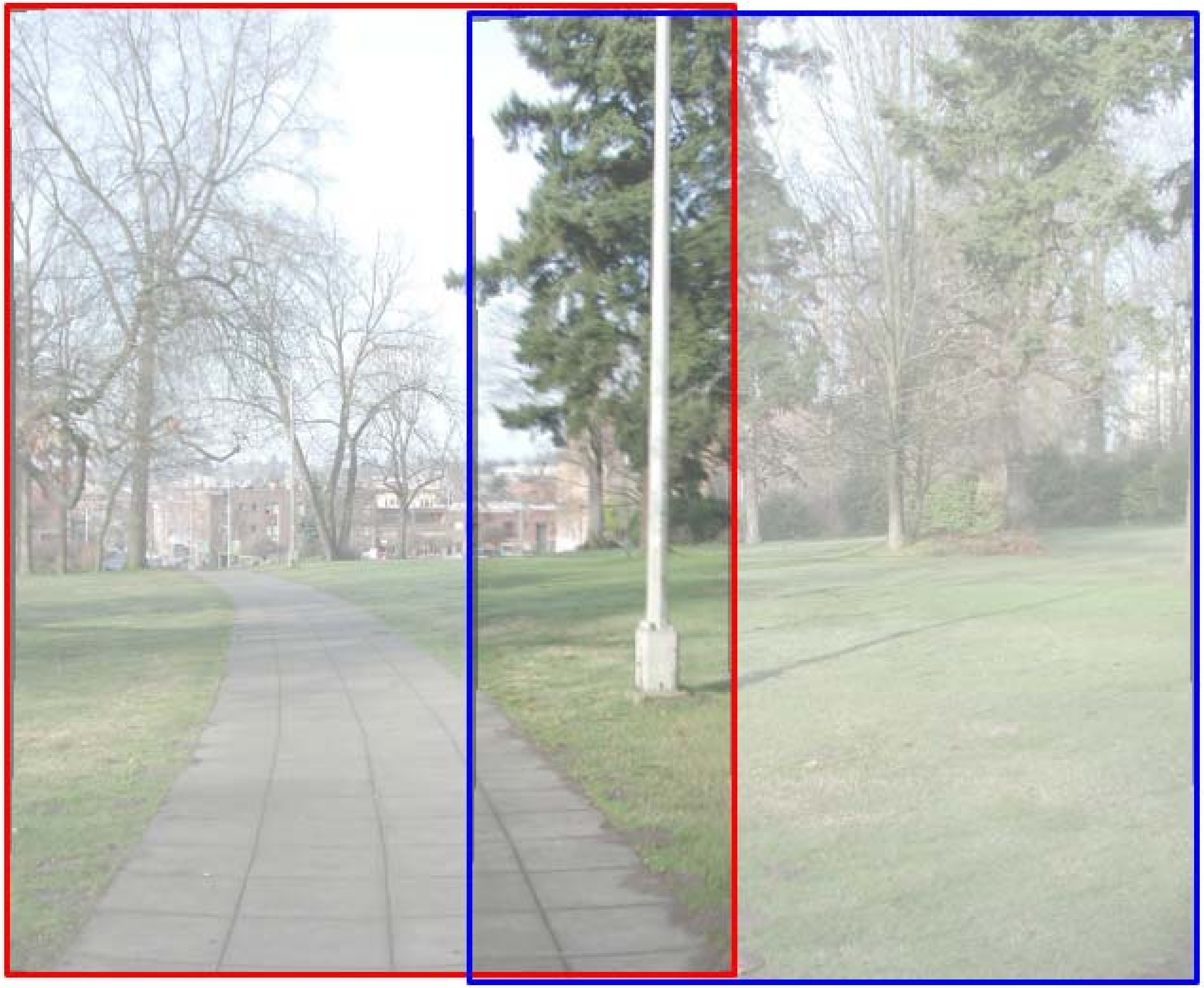
Input images


Cylindrical warping

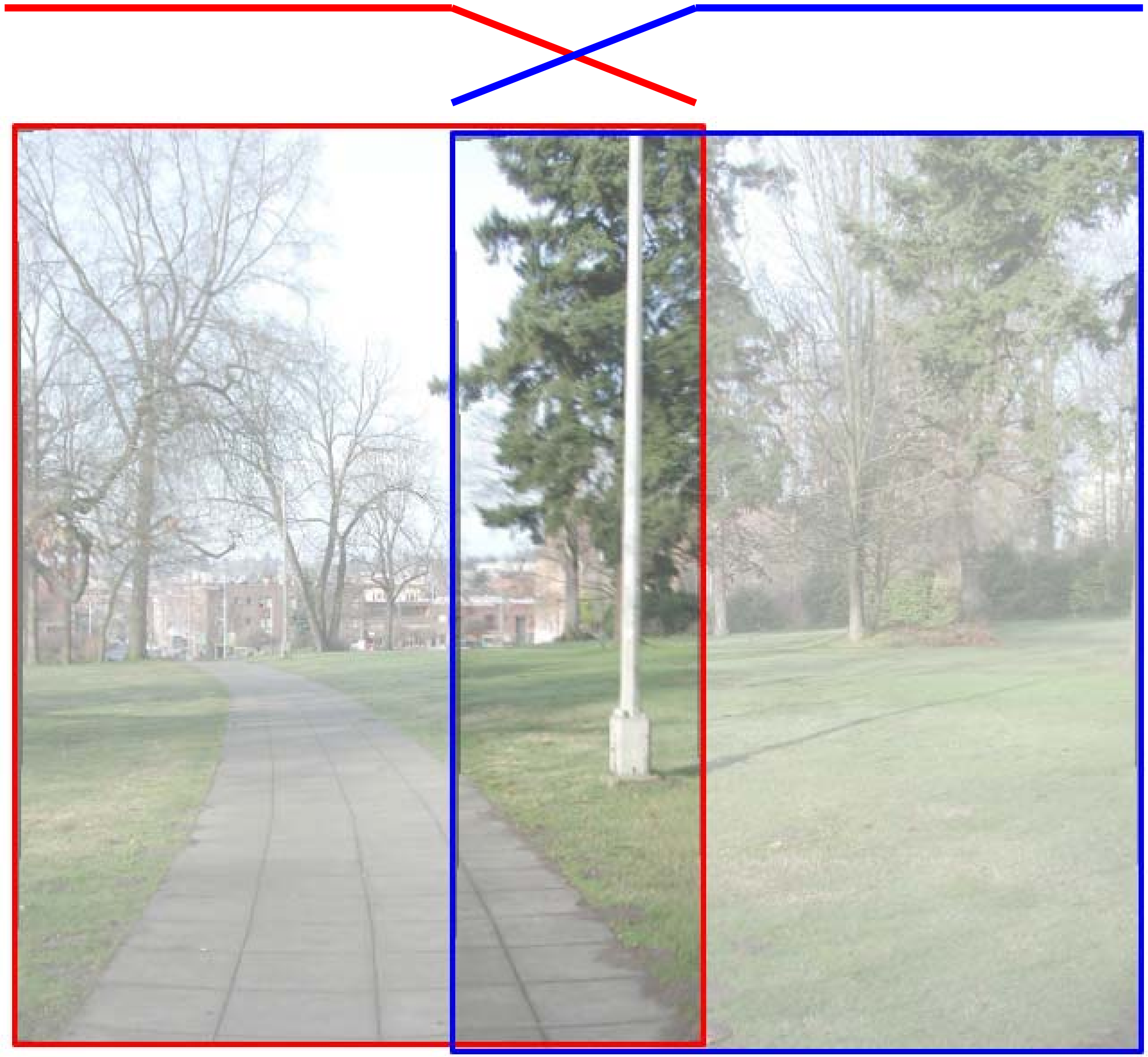
Blending

- Why blending: parallax, lens distortion, scene motion, exposure difference

Blending

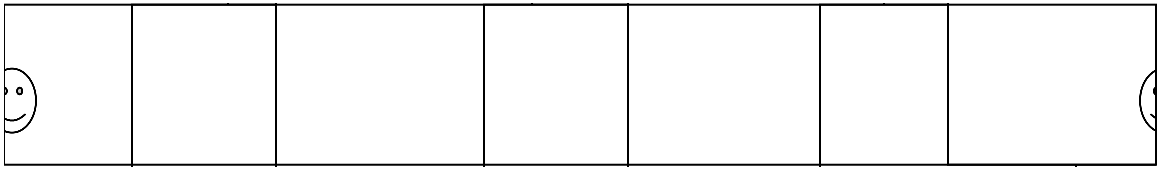


Blending



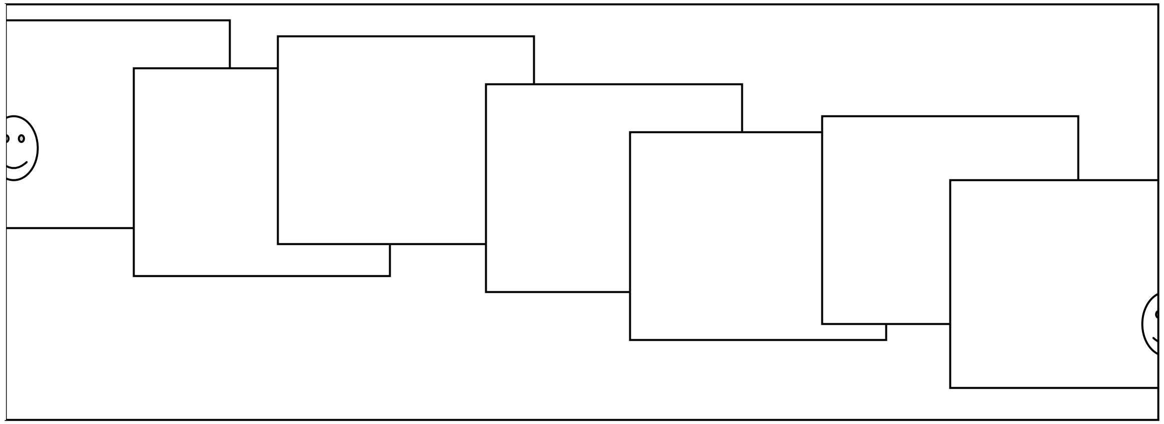
Blending

Assembling the panorama



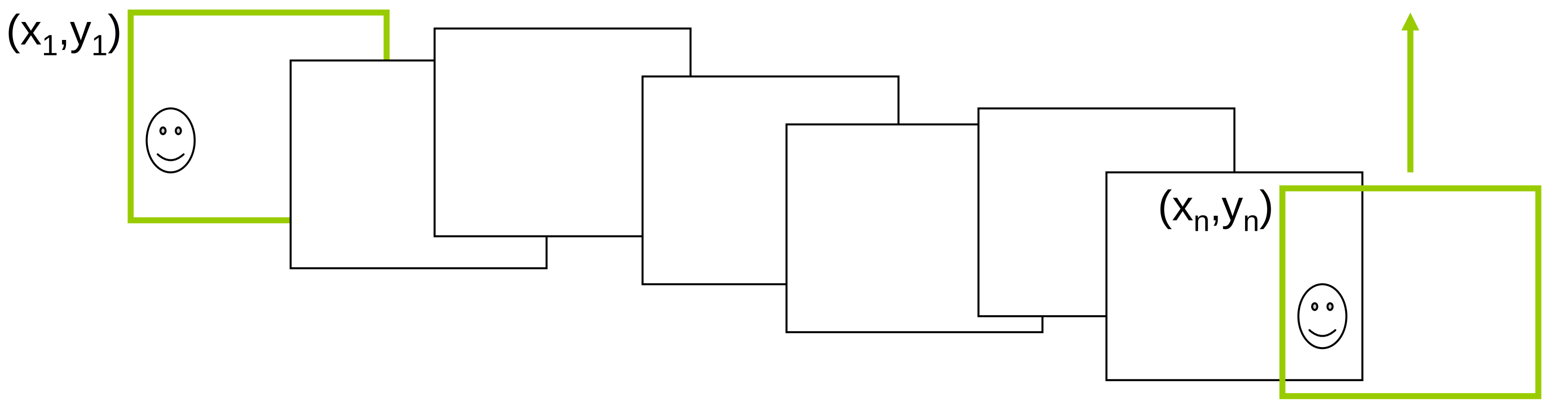
- Stitch pairs together, blend, then crop

Problem: Drift



- Error accumulation
  - small errors accumulate over time

Problem: Drift



- copy of first image

- Solution
  - add another copy of first image at the end
  - there are a bunch of ways to solve this problem
    - add displacement of $(y_1 – y_n)/(n -1)$ to each image after the first
    - compute a global warp: $y' = y + ax$
    - run a big optimization problem, incorporating this constraint
      - best solution, but more complicated
      - known as “bundle adjustment”

End-to-end alignment and crop

Viewer: panorama

example: http://www.cs.washington.edu/education/courses/cse590ss/01wi/projects/project1/students/dougz/index.html

Viewer: texture mapped model

example: http://www.panoramas.dk/

Cylindrical panorama

1. Take pictures on a tripod (or handheld)
2. Warp to cylindrical coordinate
3. Compute pairwise alignments
4. Fix up the end-to-end alignment
5. Blending
6. Crop the result and import into a viewer

Determine pairwise alignment?

- Feature-based methods: only use feature points to estimate parameters
- We will study the “Recognising panorama” paper published in ICCV 2003
- Run SIFT for each image, find feature matches.

Determine pairwise alignment

- p'=Mp, where M is a transformation matrix, p and p' are feature matches
- It is possible to use more complicated models such as affine or perspective
- For example, assume M is a 2x2 matrix

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- Find M with the least square error

$$\sum_{i=1}^{n} \left(Mp - p' \right)^2$$

Determine pairwise alignment

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x_1 m_{11} + y_1 m_{12} = x_1^{'}$$
$$x_1 m_{21} + y_1 m_{22} = y_1^{'}$$

- Overdetermined system

$$\begin{pmatrix} x_1 & y_1 & 0 & 0 \\ 0 & 0 & x_1 & y_1 \\ x_2 & y_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 0 & 0 \\ 0 & 0 & x_n & y_n \end{pmatrix} \begin{pmatrix} m_{11} \\ m_{12} \\ m_{21} \\ m_{22} \end{pmatrix} = \begin{pmatrix} x_1^{'} \\ y_1^{'} \\ x_2^{'} \\ \vdots \\ x_n^{'} \\ y_n^{'} \end{pmatrix}$$

Normal equation

Given an overdetermined system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

the normal equation is that which minimizes the sum of the square differences between left and right sides

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Why?

Normal equation

$$E(\mathbf{x}) = (\mathbf{A}\mathbf{x} - \mathbf{b})^2$$

$$\begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ \vdots \\ \vdots \\ b_n \end{bmatrix}$$

nxm, n equations, m variables

Normal equation

$$\mathbf{Ax}-\mathbf{b}=\begin{bmatrix}\sum_{j=1}^{m}a_{1j}x_j\\ \vdots\\ \sum_{j=1}^{m}a_{ij}x_j\\ \vdots\\ \sum_{j=1}^{m}a_{nj}x_j\end{bmatrix}-\begin{bmatrix}b_1\\ \vdots\\ b_i\\ \vdots\\ b_n\end{bmatrix}=\begin{bmatrix}\left(\sum_{j=1}^{m}a_{1j}x_j\right)-b_1\\ \vdots\\ \left(\sum_{j=1}^{m}a_{ij}x_j\right)-b_i\\ \vdots\\ \left(\sum_{j=1}^{m}a_{nj}x_j\right)-b_n\end{bmatrix}$$

$$E(\mathbf{x})=(\mathbf{Ax}-\mathbf{b})^2=\sum_{i=1}^{n}\left[\left(\sum_{j=1}^{m}a_{ij}x_j\right)-b_i\right]^2$$

Normal equation

$$E(\mathbf{x}) = (\mathbf{A}\mathbf{x}-\mathbf{b})^2 = \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{m} a_{ij}x_j\right)-b_i\right]^2$$

$$0 = \frac{\partial E}{\partial x_1} = \sum_{i=1}^{n} 2\left[\left(\sum_{j=1}^{m} a_{ij}x_j\right)-b_i\right]a_{i1}$$

$$= 2\sum_{i=1}^{n} a_{i1}\sum_{j=1}^{m} a_{ij}x_j - 2\sum_{i=1}^{n} a_{i1}b_i$$

$$0 = \frac{\partial E}{\partial \mathbf{x}} = 2(\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x}-\mathbf{A}^{\mathrm{T}}\mathbf{b}) \rightarrow \mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Normal equation

$$(\mathbf{Ax}-\mathbf{b})^2$$

$$=(\mathbf{Ax}-\mathbf{b})^T(\mathbf{Ax}-\mathbf{b})$$

$$=\left((\mathbf{Ax})^T-\mathbf{b}^T\right)(\mathbf{Ax}-\mathbf{b})$$

$$=\left(\mathbf{x}^\mathbf{T}\mathbf{A}^\mathbf{T}-\mathbf{b}^\mathbf{T}\right)(\mathbf{Ax}-\mathbf{b})$$

$$=\mathbf{x}^\mathbf{T}\mathbf{A}^\mathbf{T}\mathbf{Ax}-\mathbf{b}^\mathbf{T}\mathbf{Ax}-\mathbf{x}^\mathbf{T}\mathbf{A}^\mathbf{T}\mathbf{b}+\mathbf{b}^\mathbf{T}\mathbf{b}$$

$$=\mathbf{x}^\mathbf{T}\mathbf{A}^\mathbf{T}\mathbf{Ax}-\left(\mathbf{A}^\mathbf{T}\mathbf{b}\right)^\mathbf{T}\mathbf{x}-\left(\mathbf{A}^\mathbf{T}\mathbf{b}\right)^\mathbf{T}\mathbf{x}+\mathbf{b}^\mathbf{T}\mathbf{b}$$

$$\frac{\partial E}{\partial \mathbf{x}}=2\mathbf{A}^\mathbf{T}\mathbf{Ax}-2\mathbf{A}^\mathbf{T}\mathbf{b}$$

Determine pairwise alignment?

- p'=Mp, where M is a transformation matrix, p and p' are feature matches
- For translation model, it is easier.

$$E=\sum_{i=1}^{n}\left[\left(m_1+x_i-x_i^{'}\right)^2+\left(m_2+y_i-y_i^{'}\right)^2\right]$$

$$0=\frac{\partial E}{\partial m_1}$$

- What if the match is false? Avoid impact of outliers.

RANSAC

- RANSAC = Random Sample Consensus
- An algorithm for robust fitting of models in the presence of many data outliers
- Compare to robust statistics

- Given N data points x_i, assume that mjority of them are generated from a model with parameters Θ, try to recover Θ.

RANSAC algorithm

Run k times: ← How many times?

(1) draw n samples randomly ← How big? Smaller is better

(2) fit parameters Θ with these n samples

(3) for each of other N-n points, calculate its distance to the fitted model, count the number of inlier points, c

Output Θ with the largest c

How to define? Depends on the problem.

How to determine k

p: probability of real inliers

P: probability of success after k trials

$$P = 1 - (1 - p^n)^k$$

p^n: n samples are all inliers

$1 - p^n$: a failure

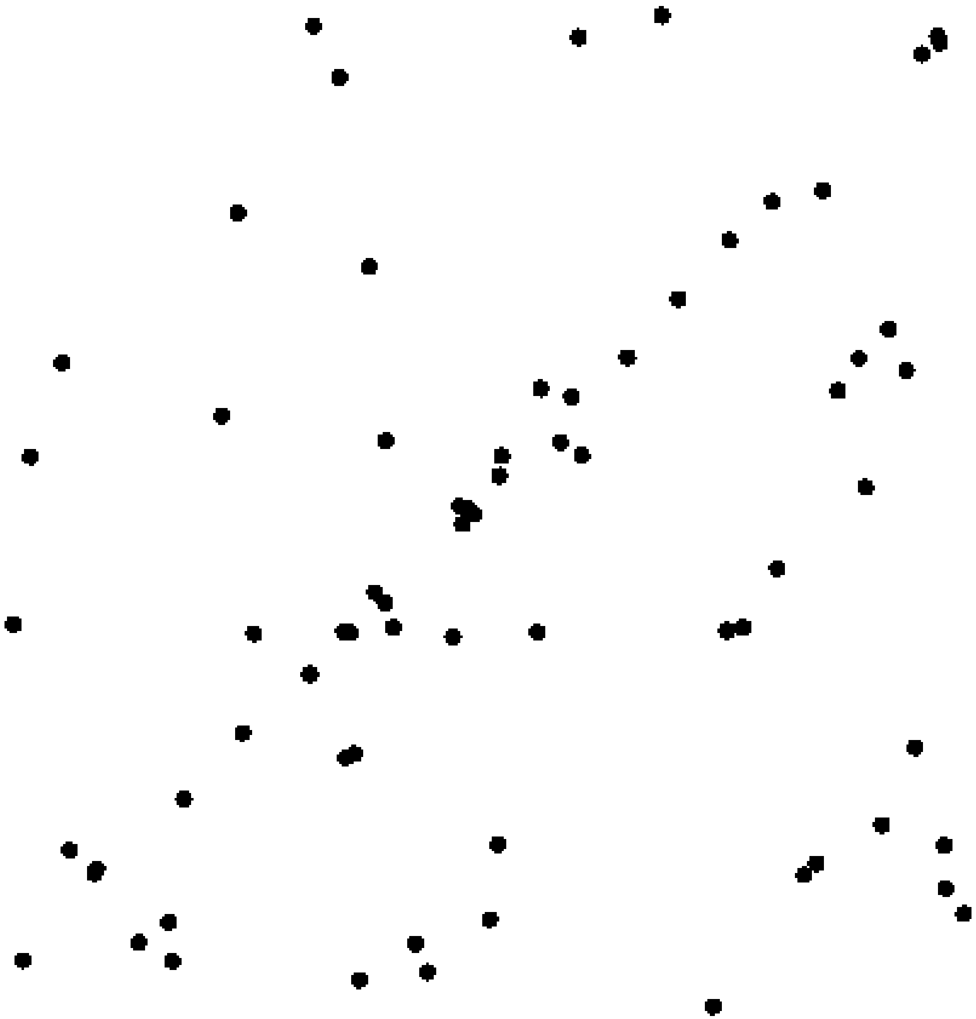
$(1 - p^n)^k$: failure after k trials

$$k = \frac{\log(1-P)}{\log(1-p^n)}$$

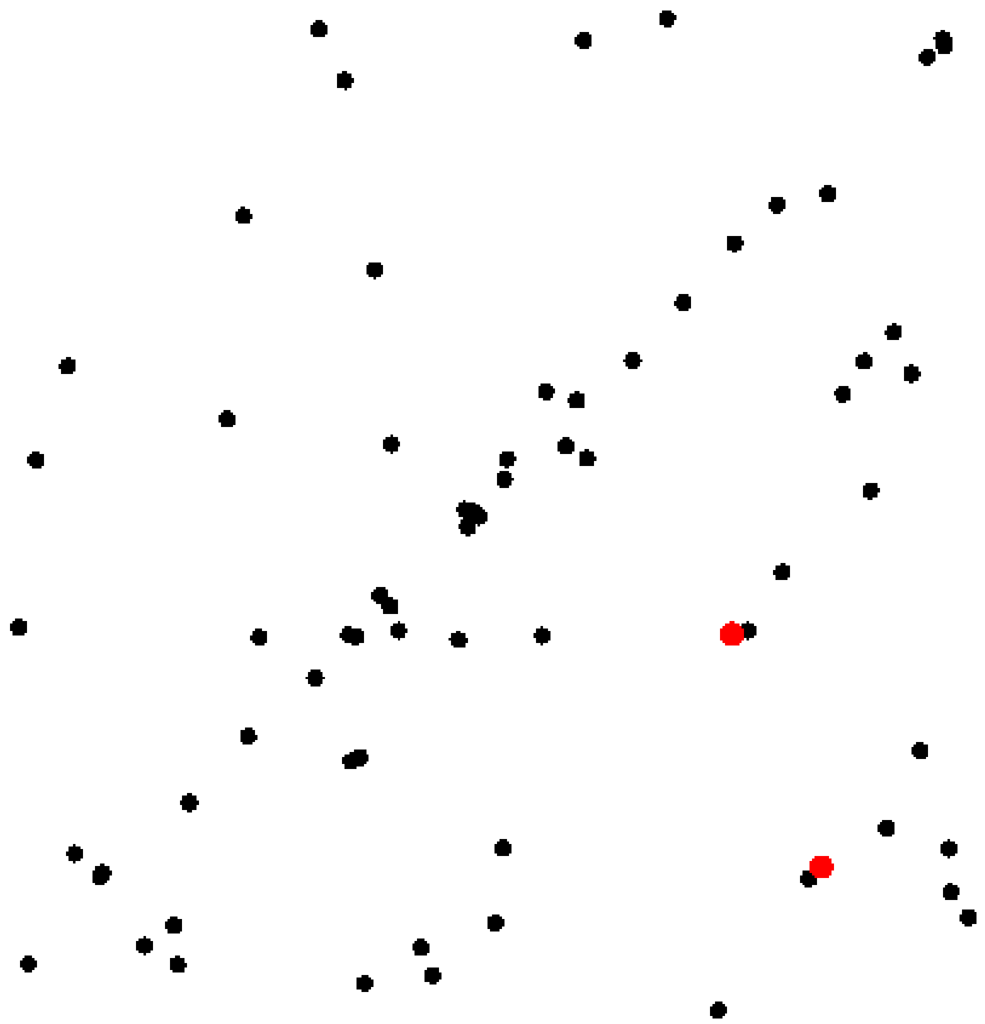
for $P=0.99$

n	p	k
3	0.5	35
6	0.6	97
6	0.5	293

Example: line fitting

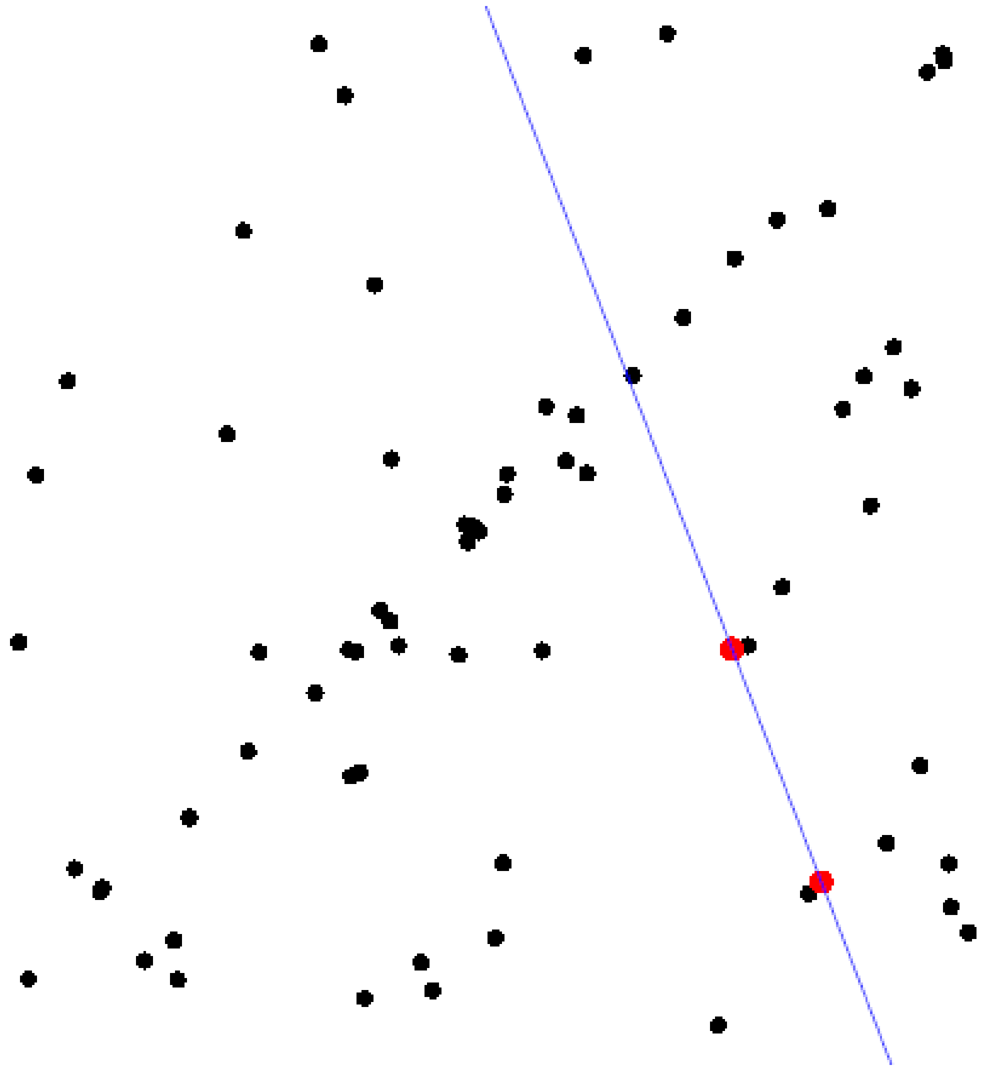


Example: line fitting

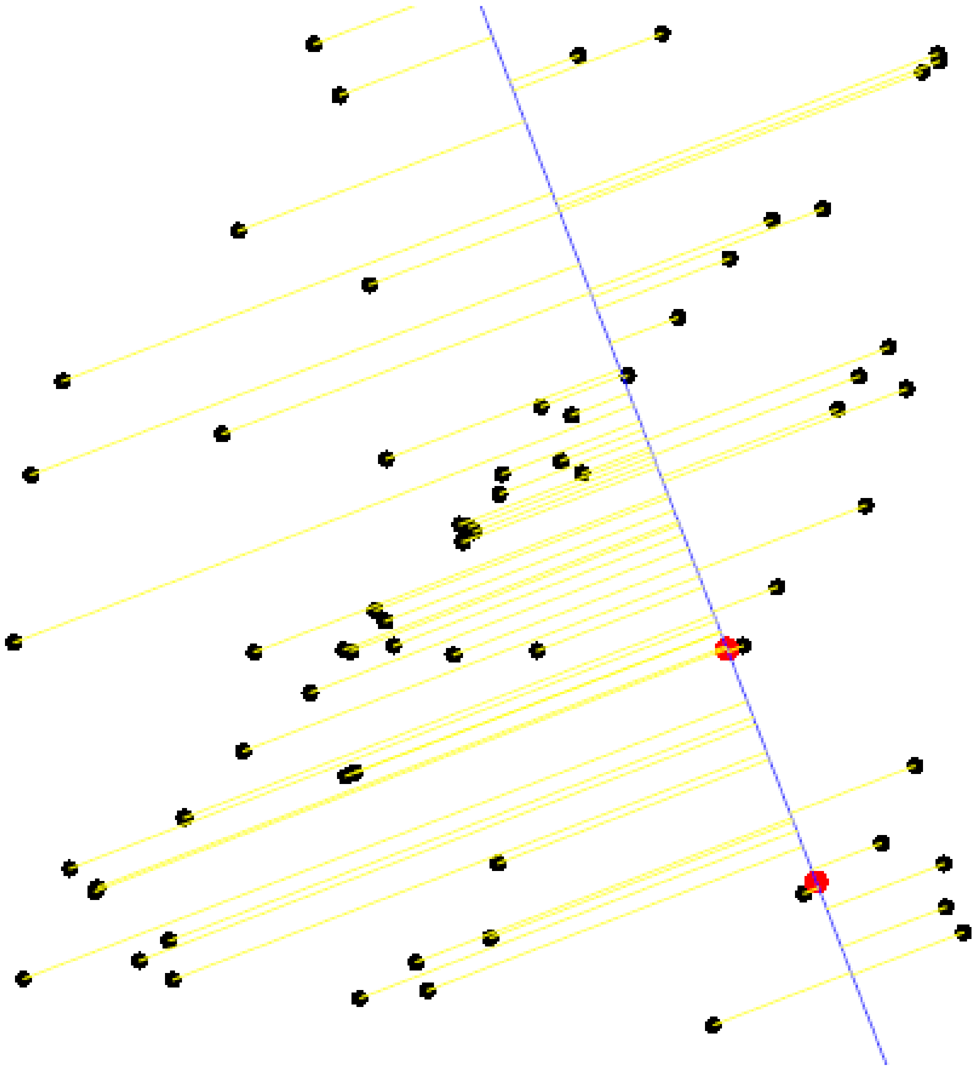


$n=2$

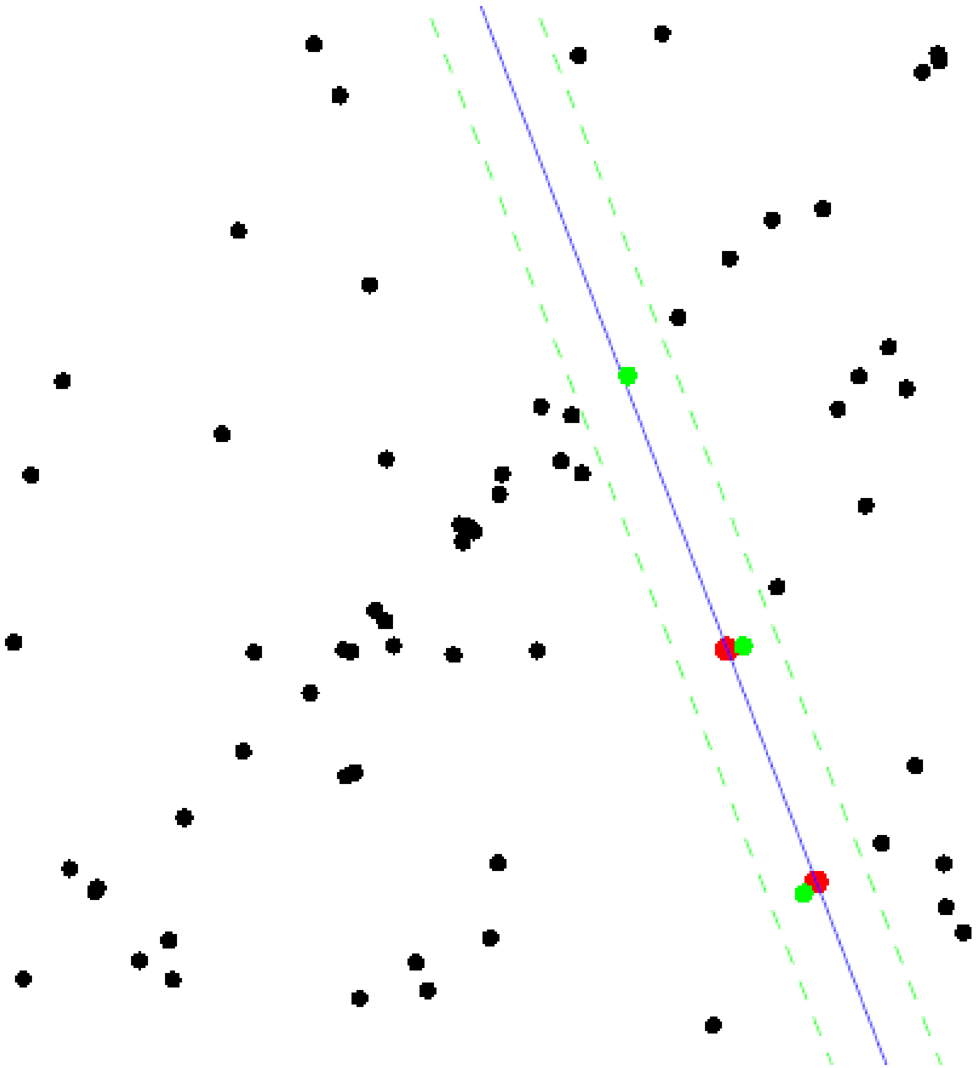
Model fitting



Measure distances

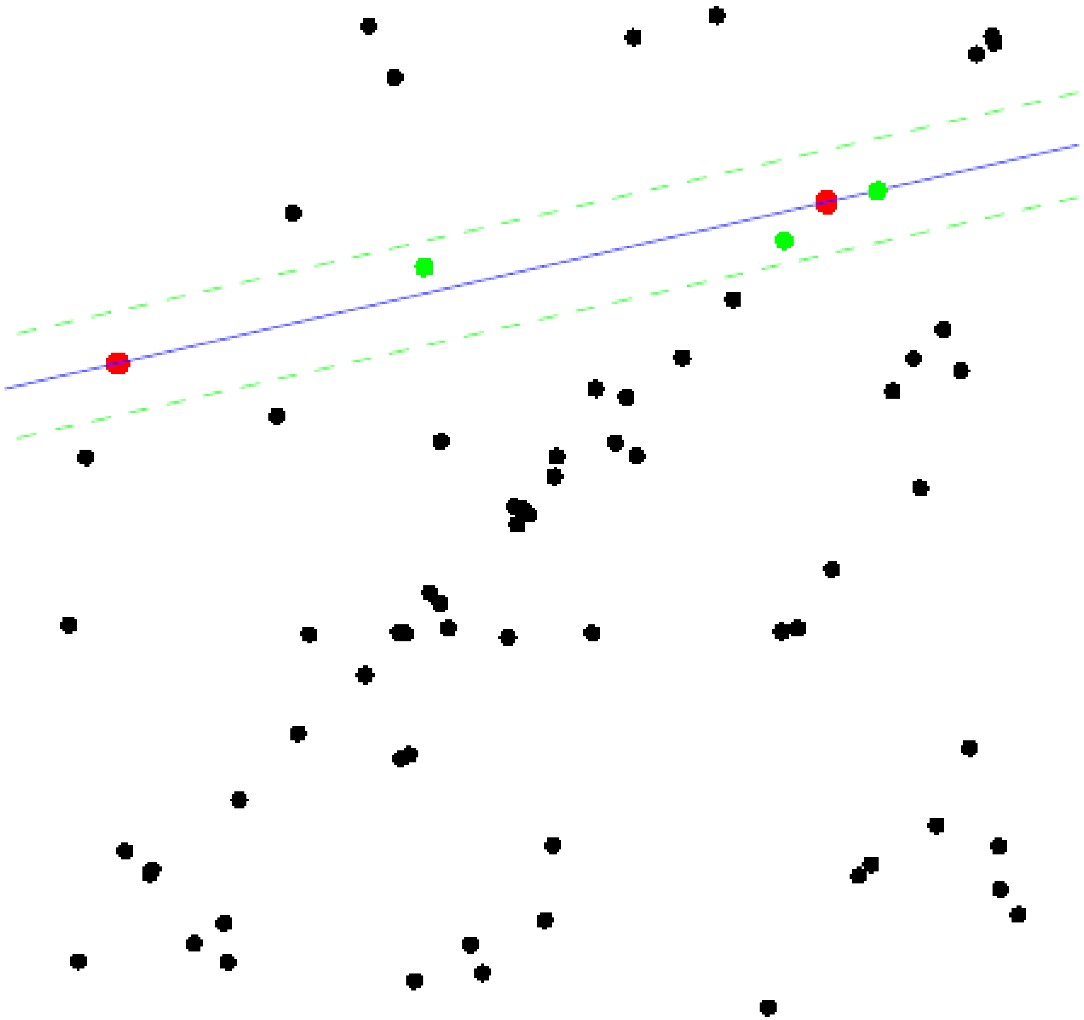


Count inliers



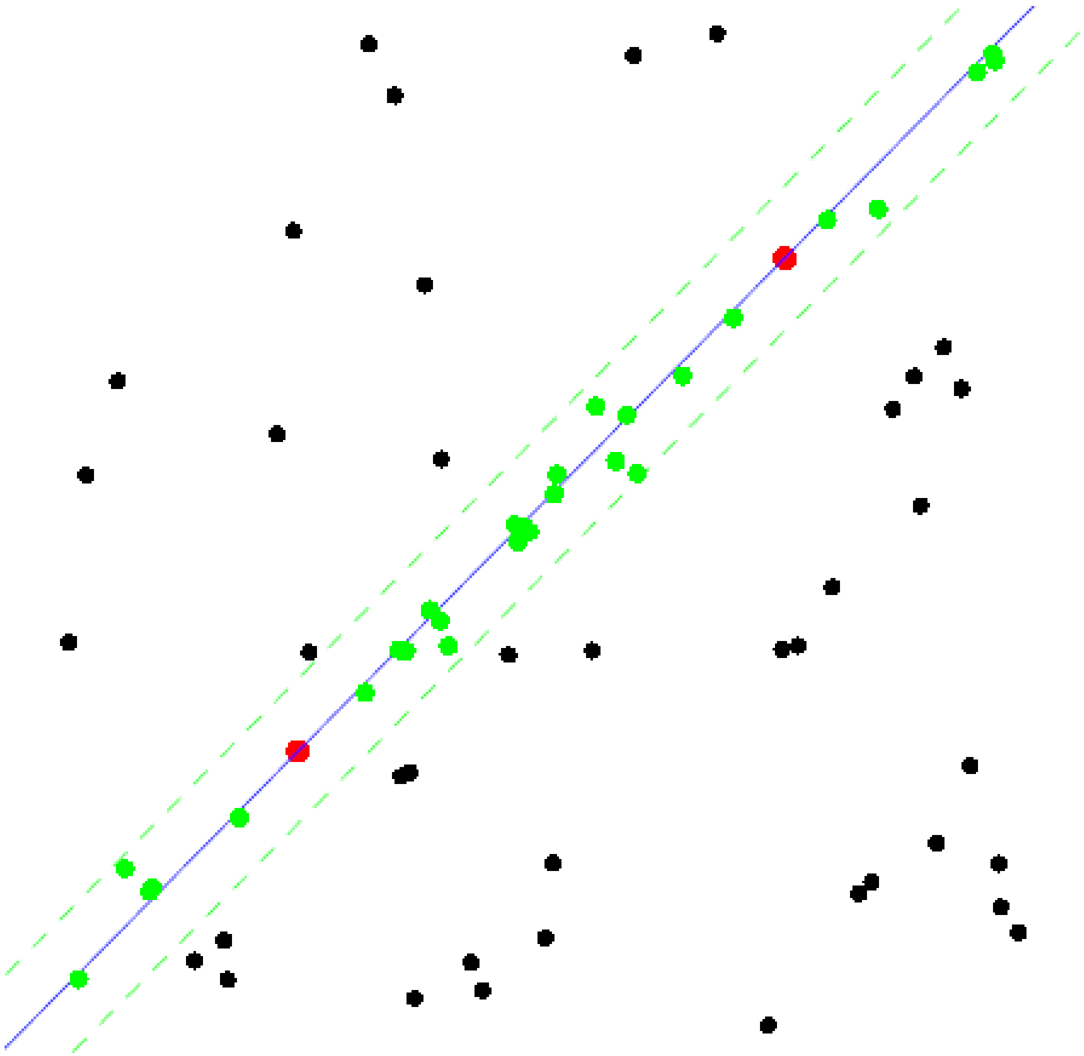
$c=3$

Another trial



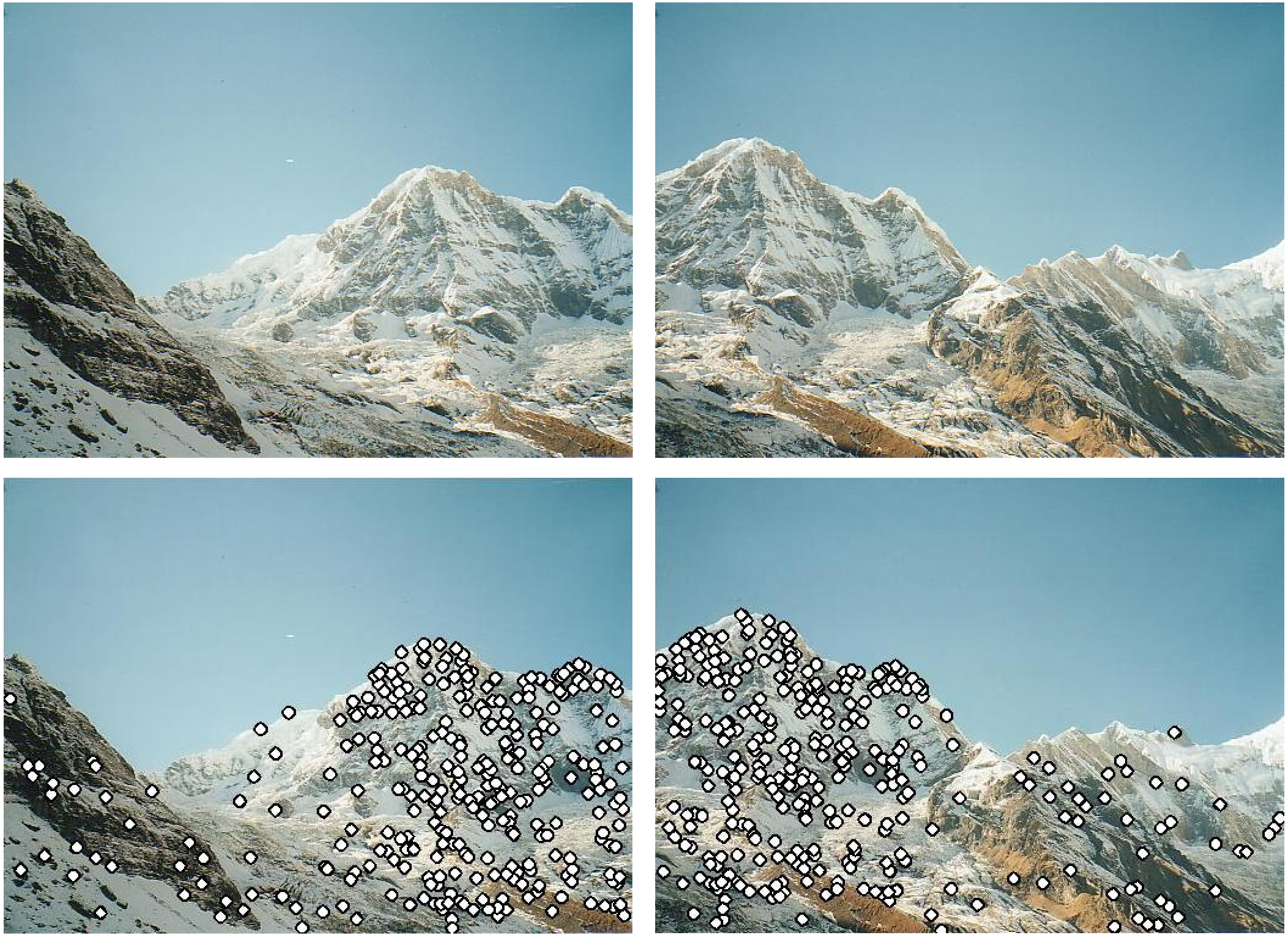
$c=3$

The best model

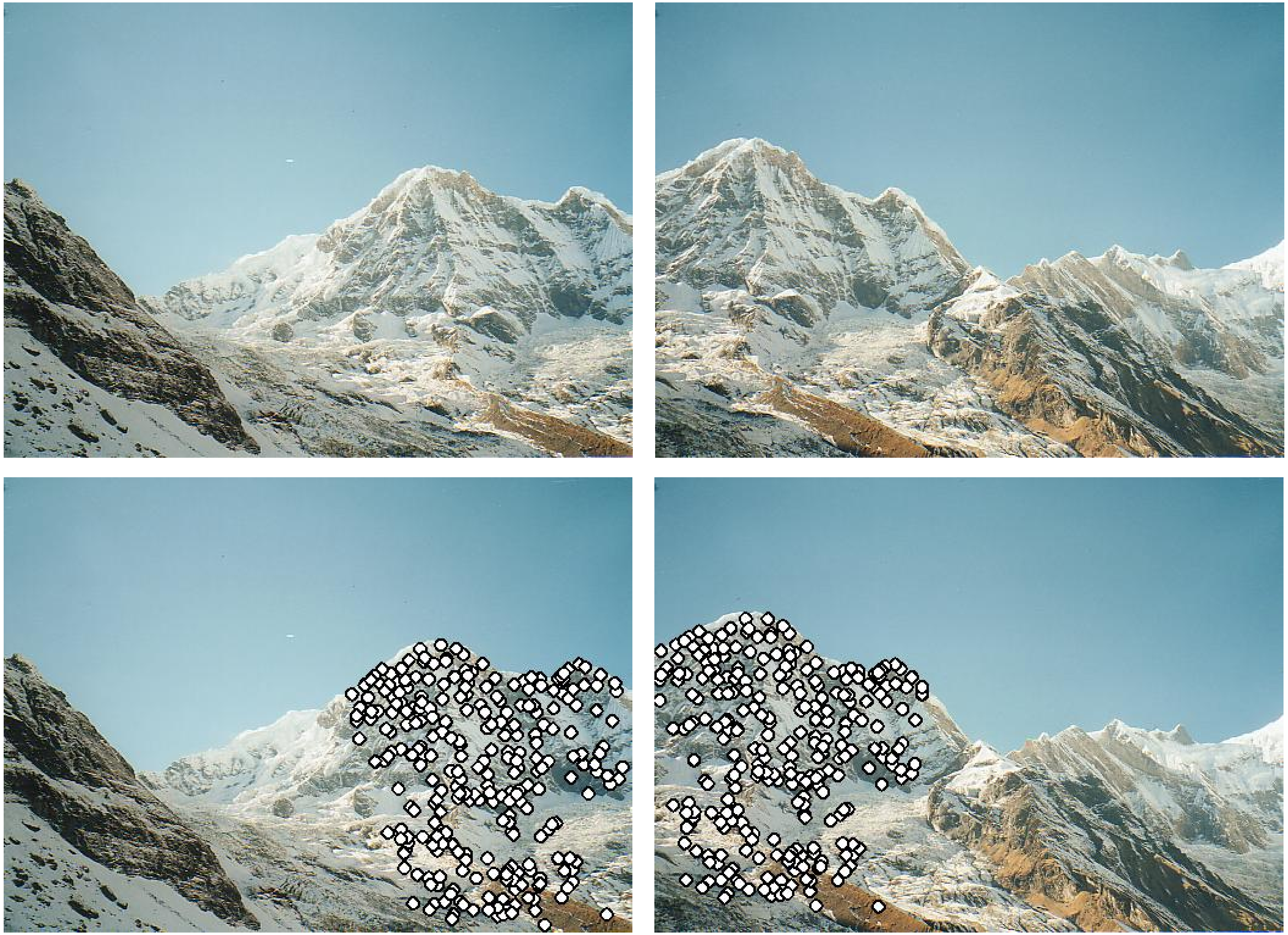


$c=15$

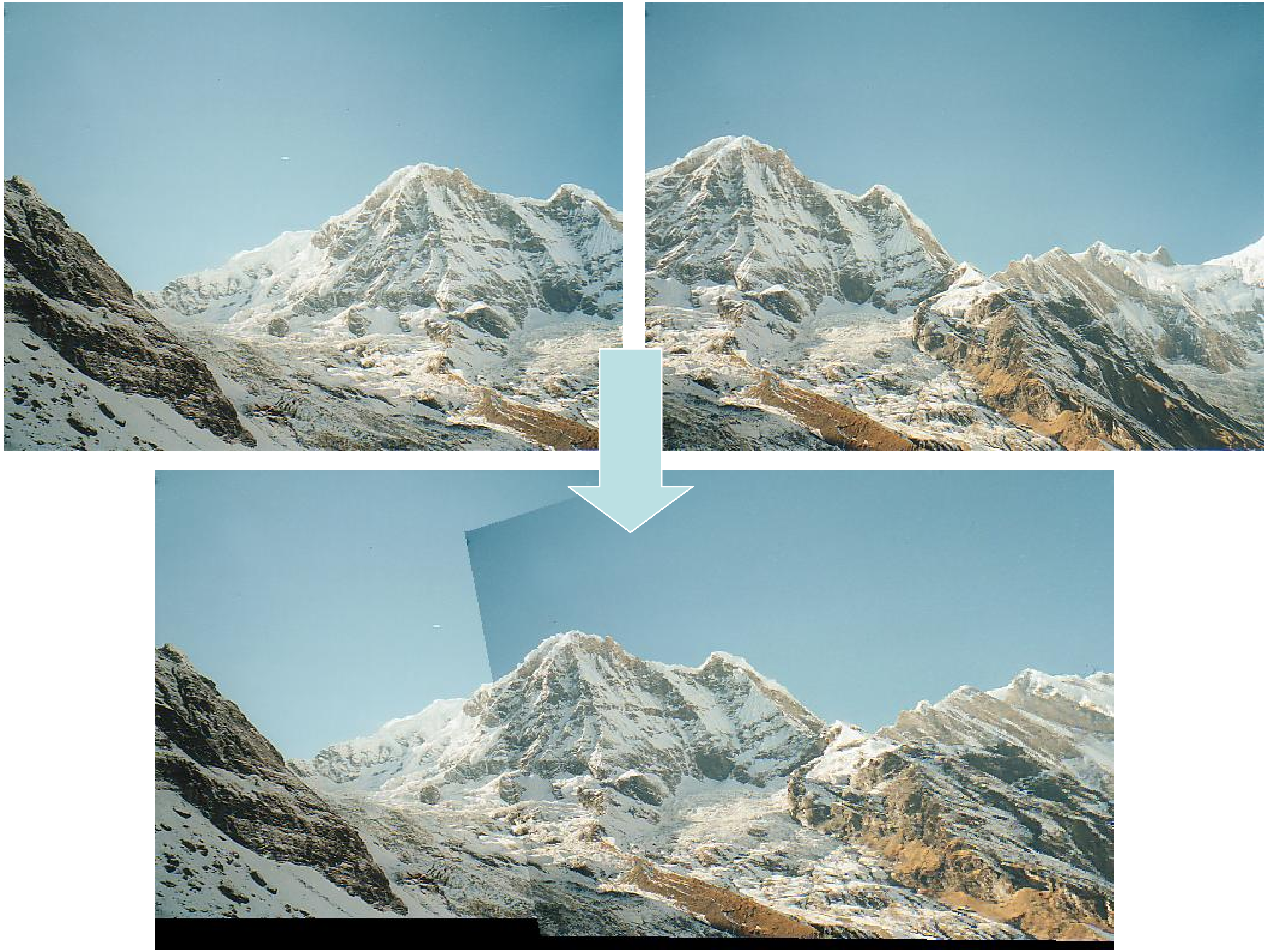
RANSAC for Homography



RANSAC for Homography



RANSAC for Homography



Applications of panorama in VFX

- Background plates
- Image-based lighting

Troy (image-based lighting)

http://www.cgnetworks.com/story_custom.php?story_id=2195&page=4

Spiderman 2 (background plate)