Features

Digital Visual Effects, Spring 2008

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Announcements

- Project #1 was due at midnight Friday. You have a total of 10 delay days without penalty, but you are advised to use them wisely.
- We reserve the rights for not including late homework for artifact voting.
- Project #2 handout will be available on the web later this week.

Outline

- Features
- Harris corner detector
- SIFT

Features

Features

- Also known as interesting points, salient points or keypoints. Points that you can easily point out their correspondences in multiple images using only local information.

Desired properties for features

- Distinctive: a single feature can be correctly matched with high probability.
- Invariant: invariant to scale, rotation, affine, illumination and noise for robust matching across a substantial range of affine distortion, viewpoint change and so on. That is, it is repeatable.

Applications

- Object or scene recognition
- Structure from motion
- Stereo
- Motion tracking
- …

Components

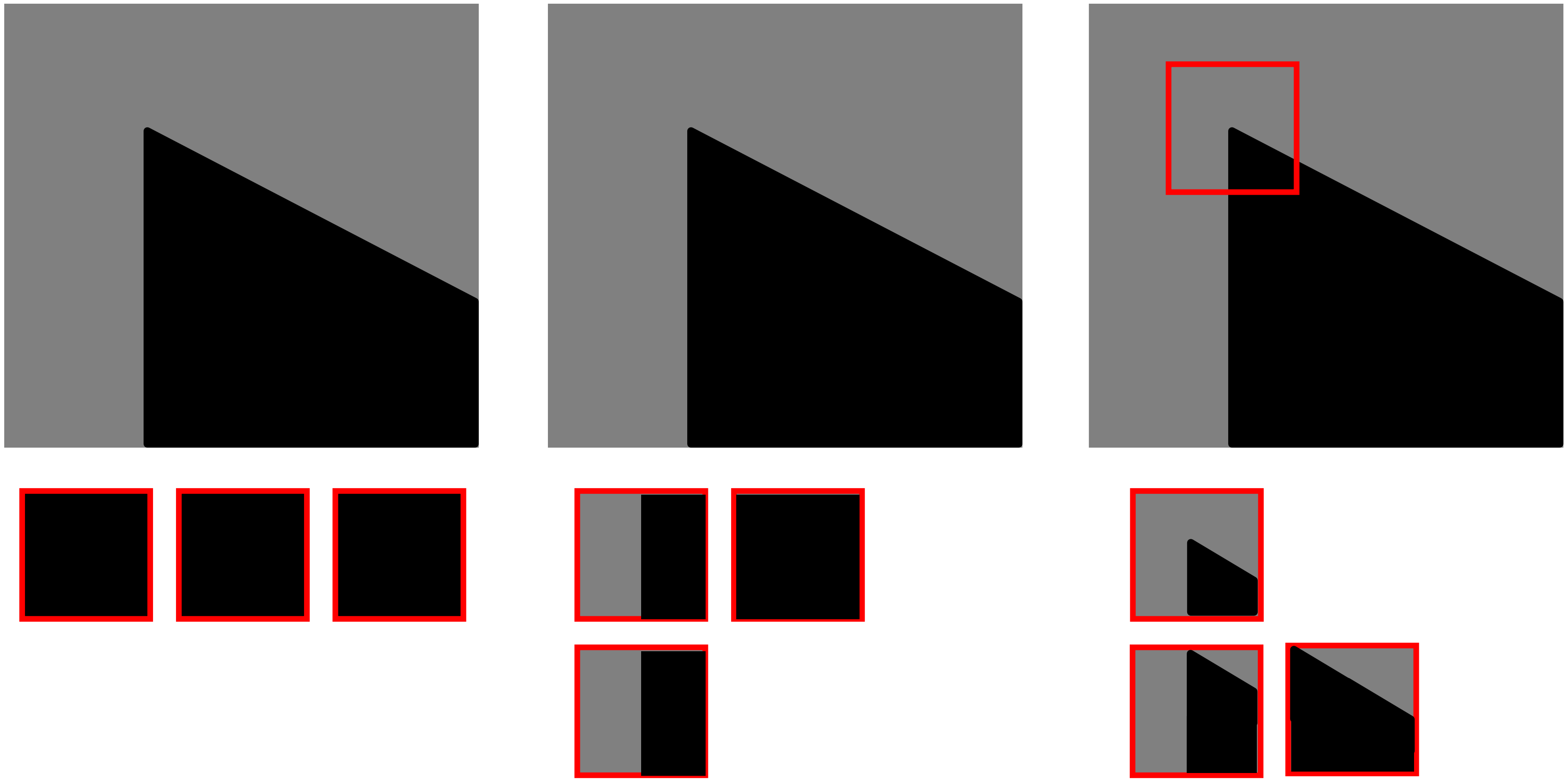
- Feature detection: locate where they are
- Feature description: describe what they are
- Feature matching: decide whether two are the same one

Harris corner detector

Moravec corner detector (1980)

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity

Moravec corner detector



flat

edge

corner
isolated point

Moravec corner detector

Change of intensity for the shift [*u*,*v*]:

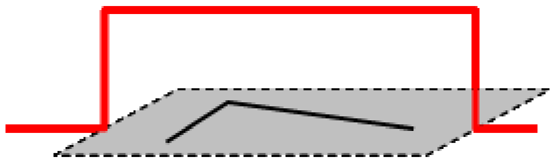
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

window function

shifted intensity

intensity

Window function $w(x,y) =$



1 in window, 0 outside

Four shifts: (u,v) = (1,0), (1,1), (0,1), (-1, 1)

Look for local maxima in $min\{E\}$

Problems of Moravec detector

- Noisy response due to a binary window function
- Only a set of shifts at every 45 degree is considered
- Only minimum of E is taken into account

⇨ Harris corner detector (1988) solves these problems.

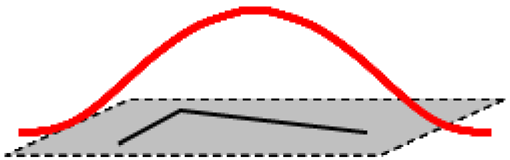
Harris corner detector

Noisy response due to a binary window function

➢ Use a Gaussian function

$$w(x,y) = \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

Window function $w(x,y) =$



Gaussian

Harris corner detector

Only a set of shifts at every 45 degree is considered

- Consider all small shifts by Taylor's expansion

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$
$$= \sum_{x,y} w(x,y)\left[I_x u + I_y v + O(u^2,v^2)\right]^2$$

$$E(u,v) = Au^2 + 2Cuv + Bv^2$$

$$A = \sum_{x,y} w(x,y) I_x^2(x,y)$$

$$B = \sum_{x,y} w(x,y) I_y^2(x,y)$$

$$C = \sum_{x,y} w(x,y) I_x(x,y) I_y(x,y)$$

Harris corner detector

Equivalently, for small shifts [*u*,*v*] we have a *bilinear* approximation:

$$E(u,v) \cong [u,v]\,\mathbf{M}\begin{bmatrix} u \\ v \end{bmatrix}$$

, where $\mathbf{M}$ is a 2×2 matrix computed from image derivatives:

$$\mathbf{M} = \sum_{x,y} w(x,y)\begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Harris corner detector (matrix form)

$$E(\mathbf{u}) = |I(\mathbf{x_0}+\mathbf{u}) - I(\mathbf{x_0})|^2$$

$$= \left| \left(I_0 + \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u} \right) - I_0 \right|^2$$

$$= \left| \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u} \right|^2$$

$$= \mathbf{u}^T \frac{\partial I}{\partial \mathbf{u}} \frac{\partial I}{\partial \mathbf{u}}^T \mathbf{u}$$

$$= \mathbf{u}^T \mathbf{M} \mathbf{u}$$

Harris corner detector

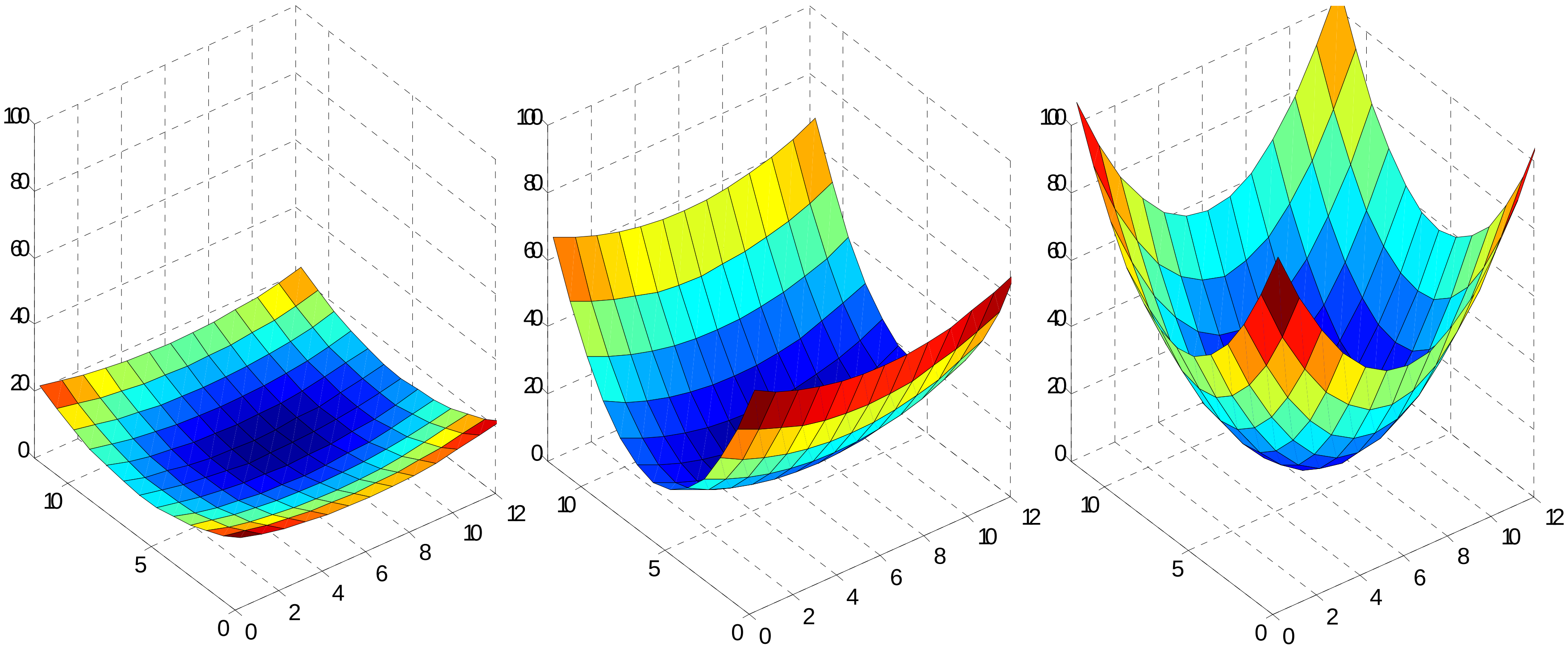
Only minimum of E is taken into account

- A new corner measurement by investigating the shape of the error function

$\mathbf{u}^T\mathbf{M}\mathbf{u}$ represents a quadratic function; Thus, we can analyze E's shape by looking at the property of $\mathbf{M}$

Harris corner detector

High-level idea: what shape of the error function will we prefer for features?



flat　　　　edge　　　　corner

Quadratic forms

- Quadratic form (homogeneous polynomial of degree two) of n variables x_i

$$\sum_{\substack{i=1 \\ i \le j}}^{n} \sum_{j=1}^{n} c_{ij} x_i x_j$$

- Examples

$$4x_1^2 + 5x_2^2 + 3x_3^2 + 2x_1x_2 + 4x_1x_3 + 6x_2x_3$$

$$= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Symmetric matrices

- Quadratic forms can be represented by a real symmetric matrix $\mathbf{A}$ where

$$a_{ij} = \begin{cases} c_{ij} & \text{if } i = j, \\ \frac{1}{2}c_{ij} & \text{if } i < j, \\ \frac{1}{2}c_{ji} & \text{if } i > j. \end{cases}$$

$$\sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \le j}}^{n} c_{ij}x_i x_j = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j$$

$$= \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \mathbf{x}^t A \mathbf{x}$$

Eigenvalues of symmetric matrices

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, $i.e.$, $A = A^T$

fact: the eigenvalues of A are real

suppose $Av = \lambda v$, $v \neq 0$, $v \in \mathbf{C}^n$

$$\overline{v}^T A v = \overline{v}^T (Av) = \lambda \overline{v}^T v = \lambda \sum_{i=1}^{n} |v_i|^2$$

$$\overline{v}^T A v = \overline{(Av)}^T v = \overline{(\lambda v)}^T v = \overline{\lambda} \sum_{i=1}^{n} |v_i|^2$$

we have $\lambda = \overline{\lambda}$, $i.e.$, $\lambda \in \mathbf{R}$

(hence, can assume $v \in \mathbf{R}^n$)

Brad Osgood

Eigenvectors of symmetric matrices

suppose $A \in \mathbf{R}^{n \times n}$ is symmetric, $i.e.$, $A = A^T$

fact: there is a set of orthonormal eigenvectors of A

$$A = Q \Lambda Q^T$$

$$\mathbf{x}^T \mathbf{A} \mathbf{x}$$

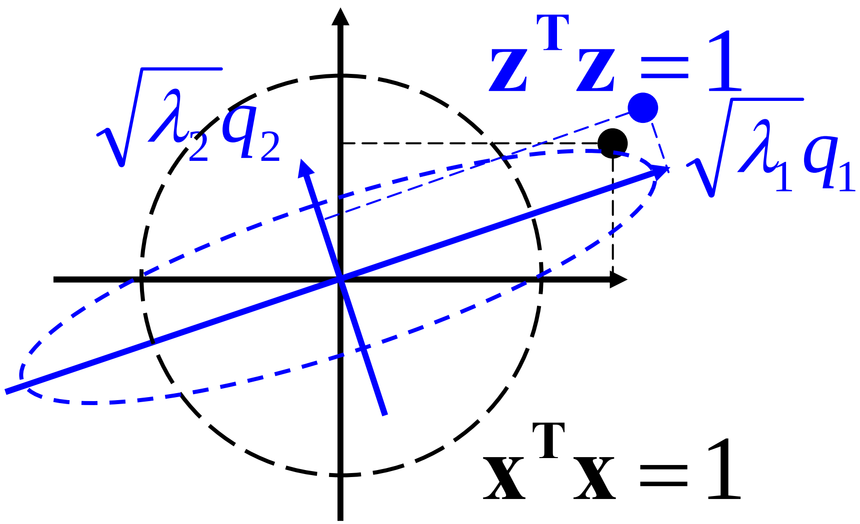
$$= \mathbf{x}^T \mathbf{Q} \boldsymbol{\Lambda} \mathbf{Q}^T \mathbf{x}$$

$$= \left(\mathbf{Q}^T \mathbf{x}\right)^T \boldsymbol{\Lambda} \left(\mathbf{Q}^T \mathbf{x}\right)$$

$$= \mathbf{y}^T \boldsymbol{\Lambda} \mathbf{y}$$

$$= \left(\boldsymbol{\Lambda}^{\frac{1}{2}} \mathbf{y}\right)^T \left(\boldsymbol{\Lambda}^{\frac{1}{2}} \mathbf{y}\right)$$

$$= \mathbf{z}^T \mathbf{z}$$



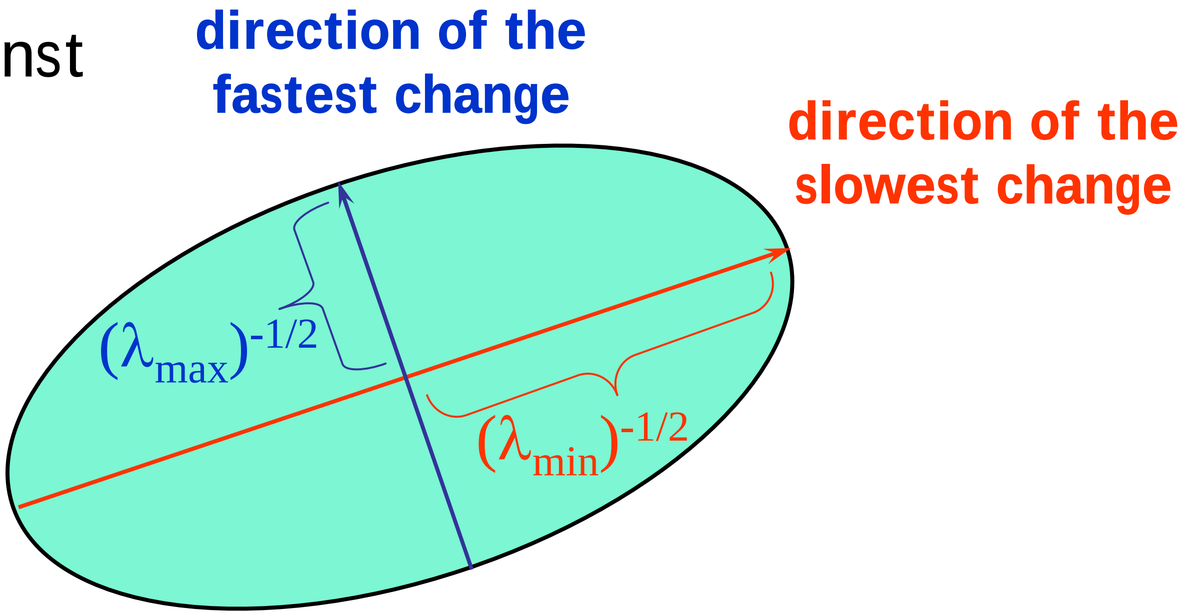
Harris corner detector

Intensity change in shifting window: eigenvalue analysis

$$E(u,v) \cong [u,v]\,\mathbf{M}\begin{bmatrix}u\\v\end{bmatrix}$$

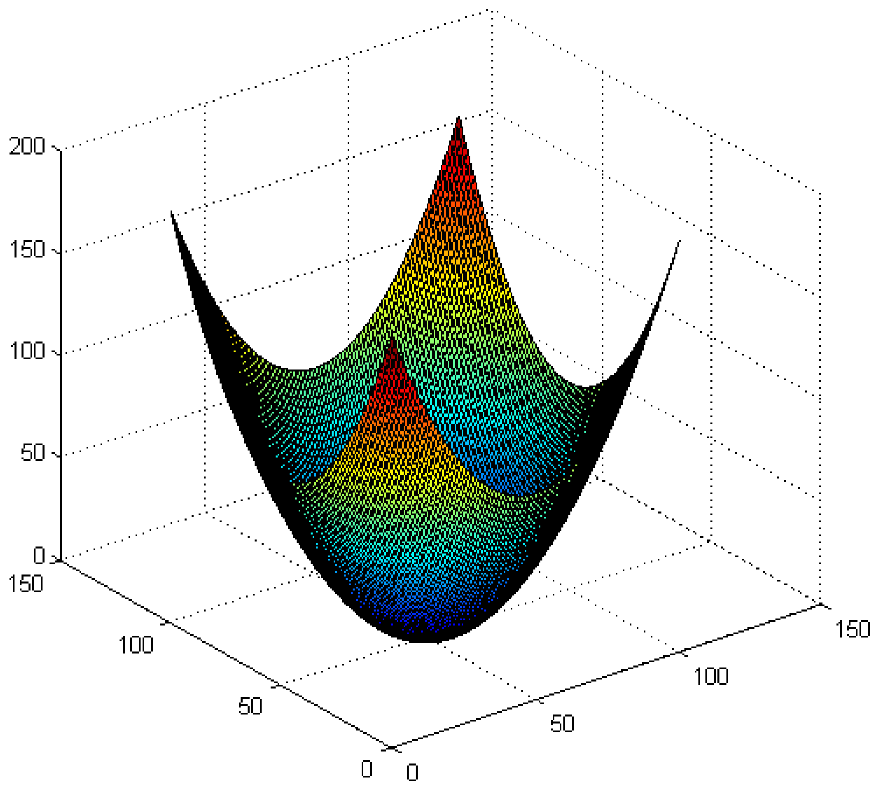
λ_1, λ_2 – eigenvalues of $\mathbf{M}$

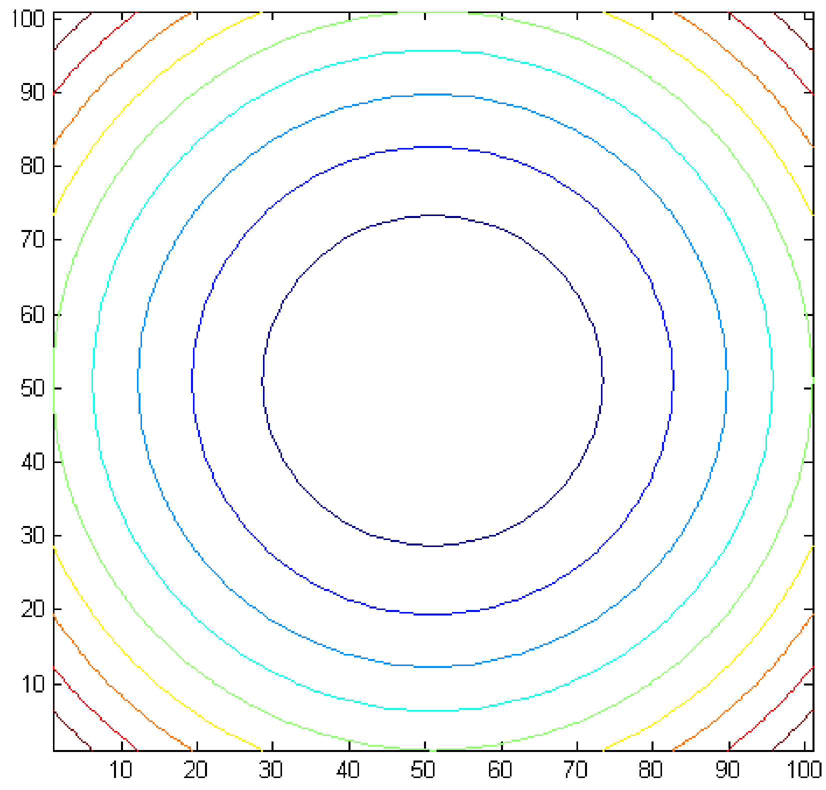
Ellipse $E(u,v)$ = const



Visualize quadratic functions

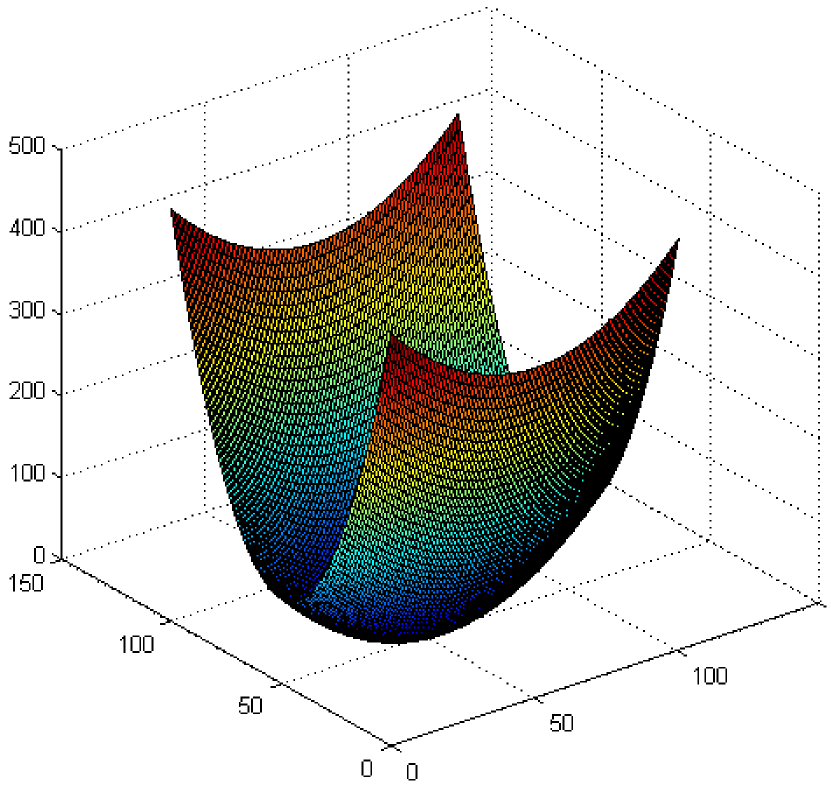
$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

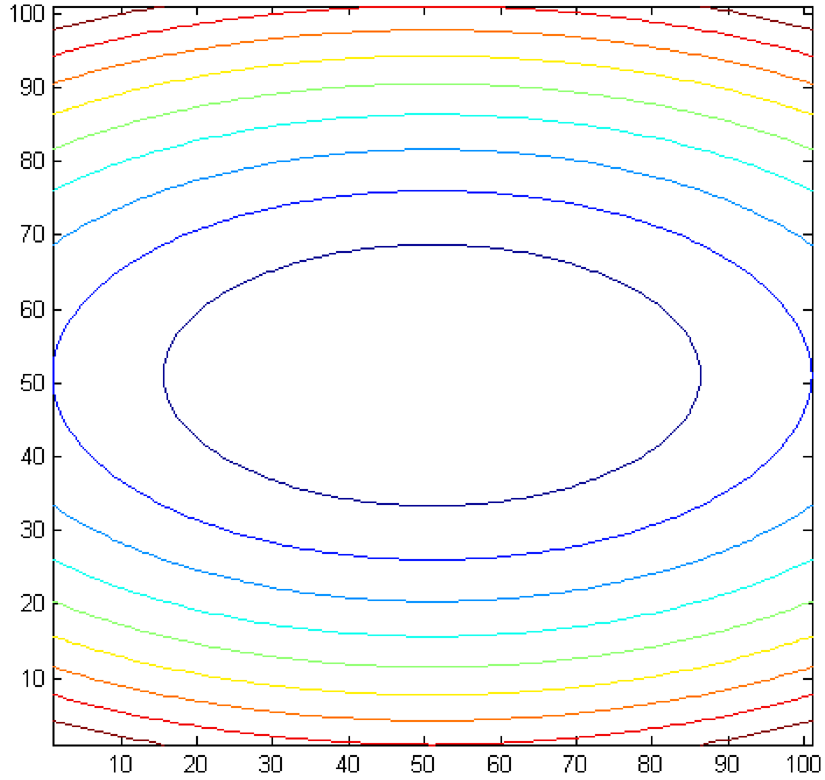




Visualize quadratic functions

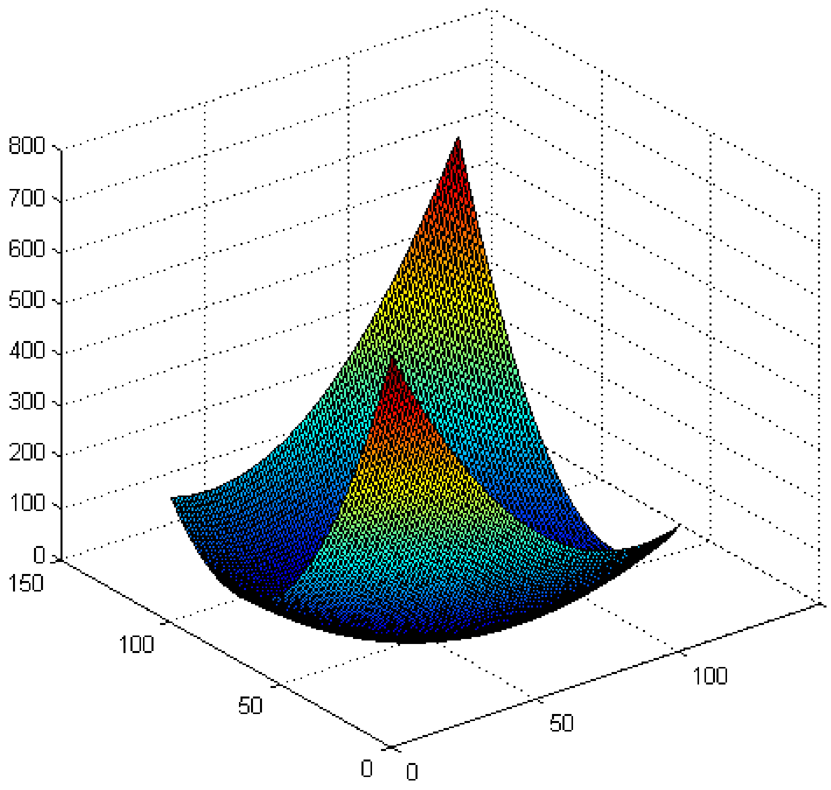
$$\mathbf{A} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

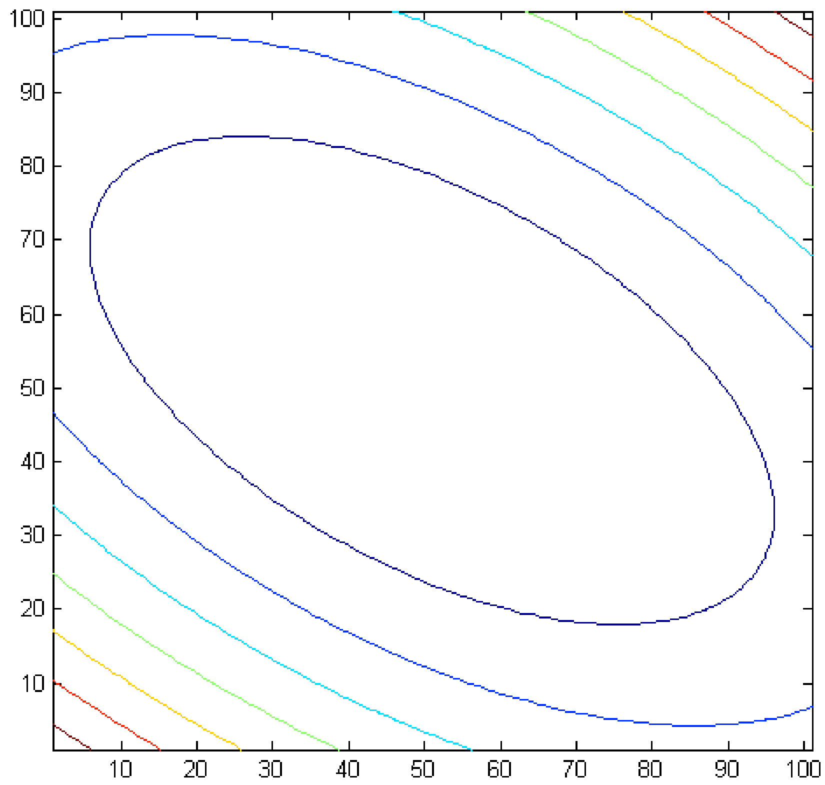




Visualize quadratic functions

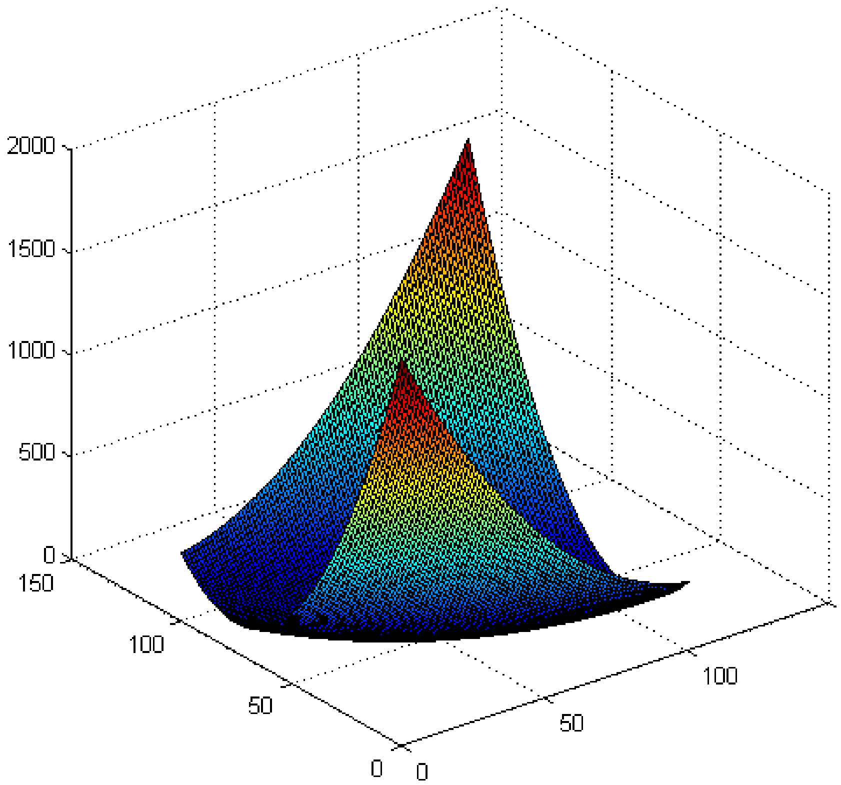
$$\mathbf{A}=\begin{bmatrix}3.25 & 1.30\\ 1.30 & 1.75\end{bmatrix}=\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}^T$$

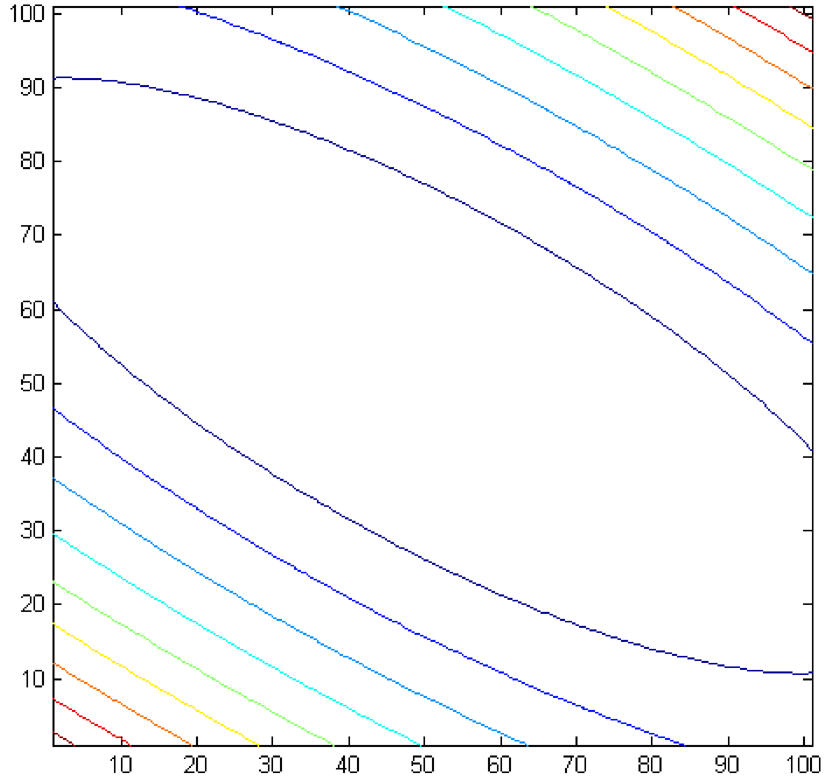




Visualize quadratic functions

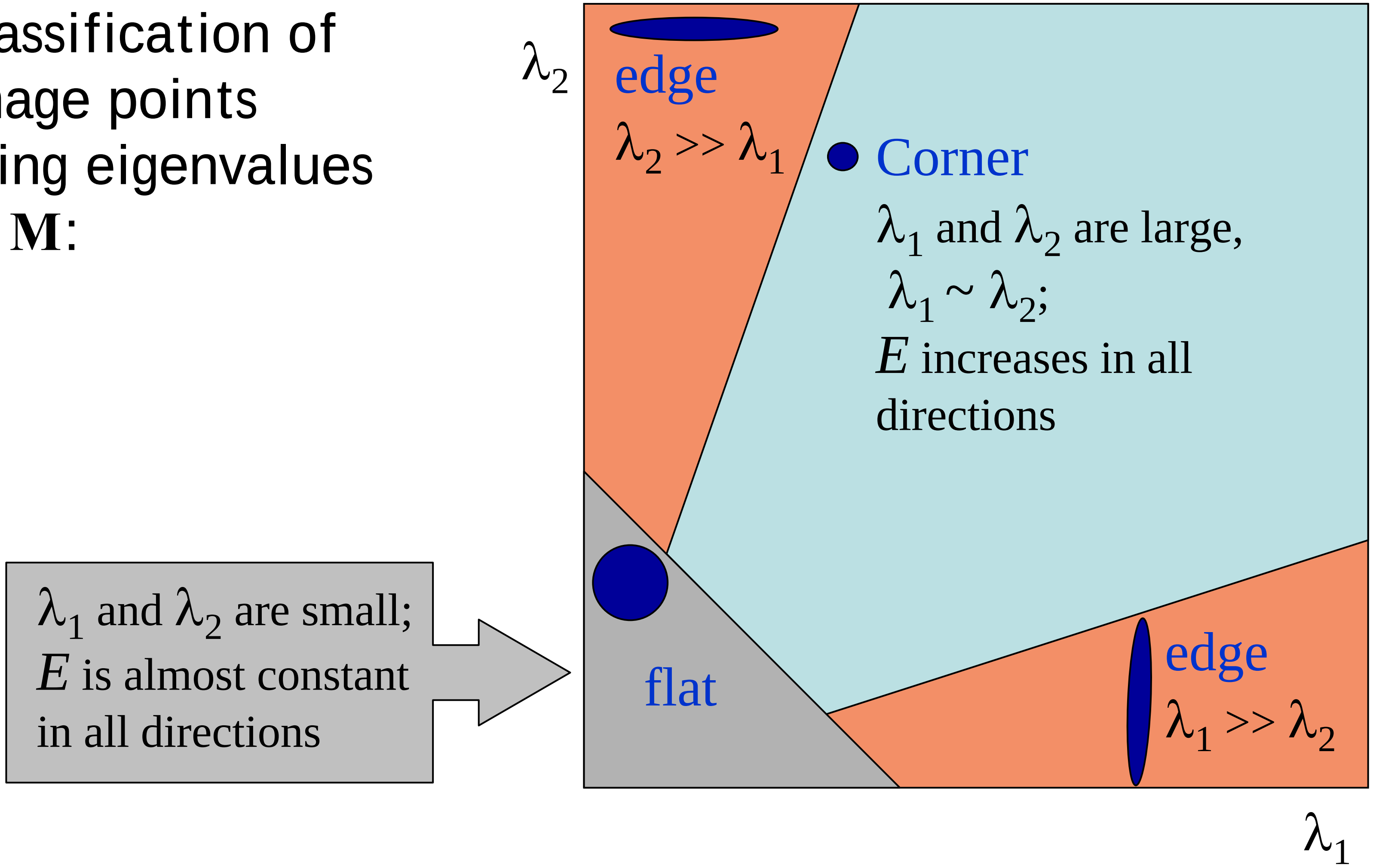
$$\mathbf{A}=\begin{bmatrix}7.75 & 3.90\\ 3.90 & 3.25\end{bmatrix}=\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 10\end{bmatrix}\begin{bmatrix}0.50 & -0.87\\ -0.87 & -0.50\end{bmatrix}^T$$





Harris corner detector

Classification of image points using eigenvalues of $\mathbf{M}$:



Harris corner detector

$$\lambda = \frac{a_{00} + a_{11} \pm \sqrt{(a_{00} - a_{11})^2 + 4a_{10}a_{01}}}{2}$$

Only for reference, you do not need them to compute R

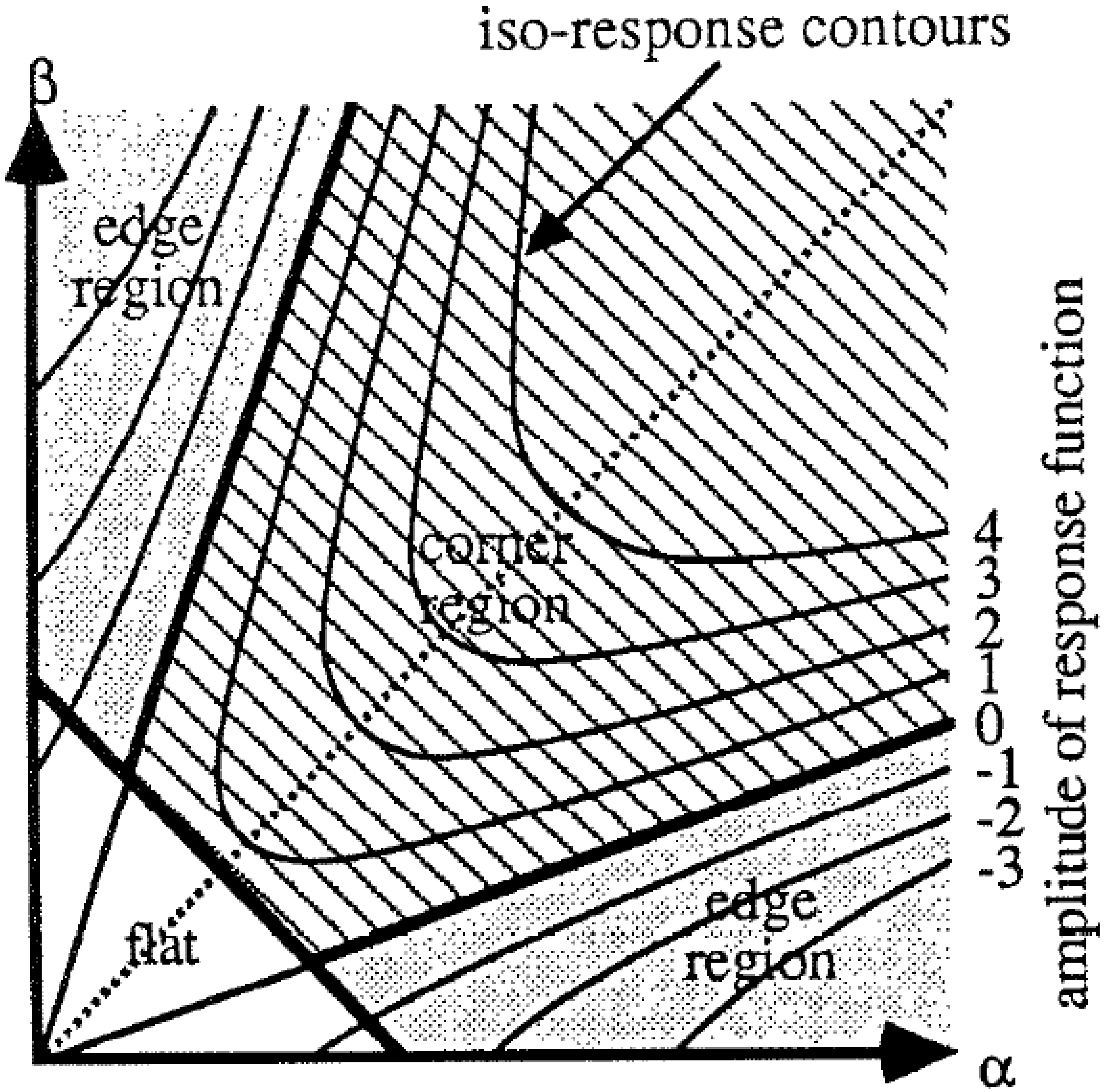
Measure of corner response:

$$R = \det \mathbf{M} - k\left(\operatorname{trace} \mathbf{M}\right)^2$$

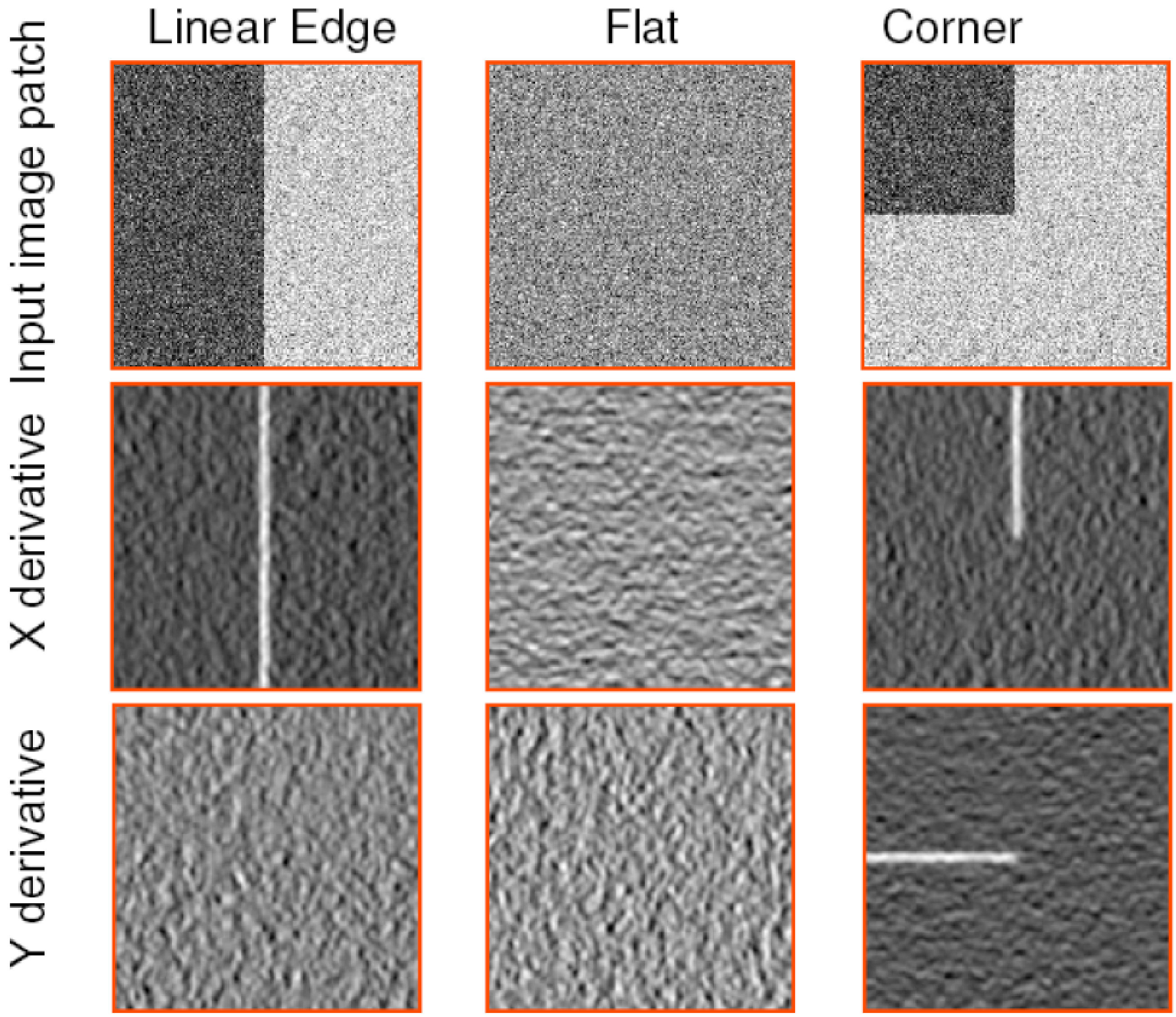
$$\det \mathbf{M} = \lambda_1 \lambda_2$$
$$\operatorname{trace} \mathbf{M} = \lambda_1 + \lambda_2$$

(*k* – empirical constant, *k* = 0.04-0.06)

Harris corner detector

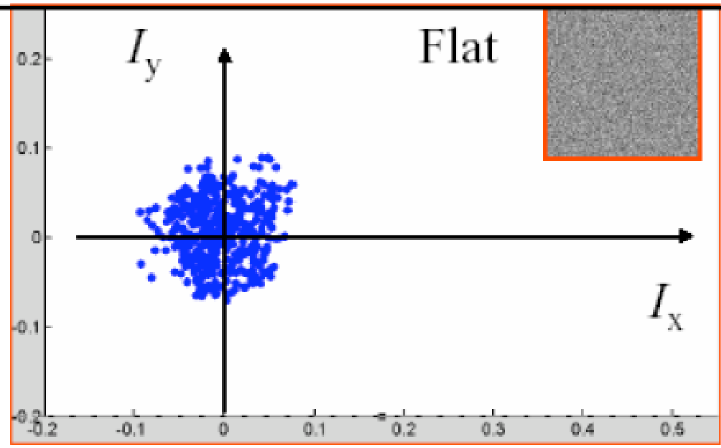


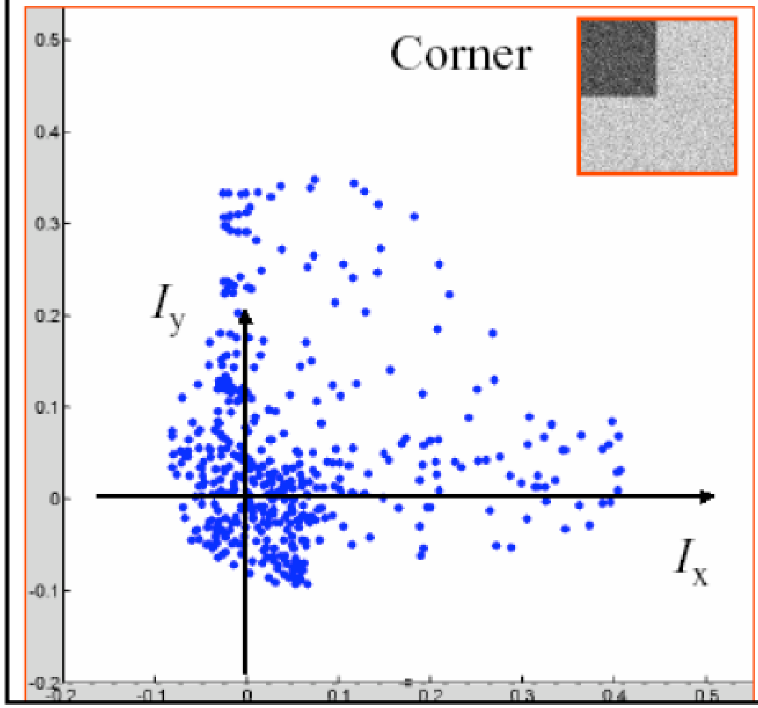
Another view

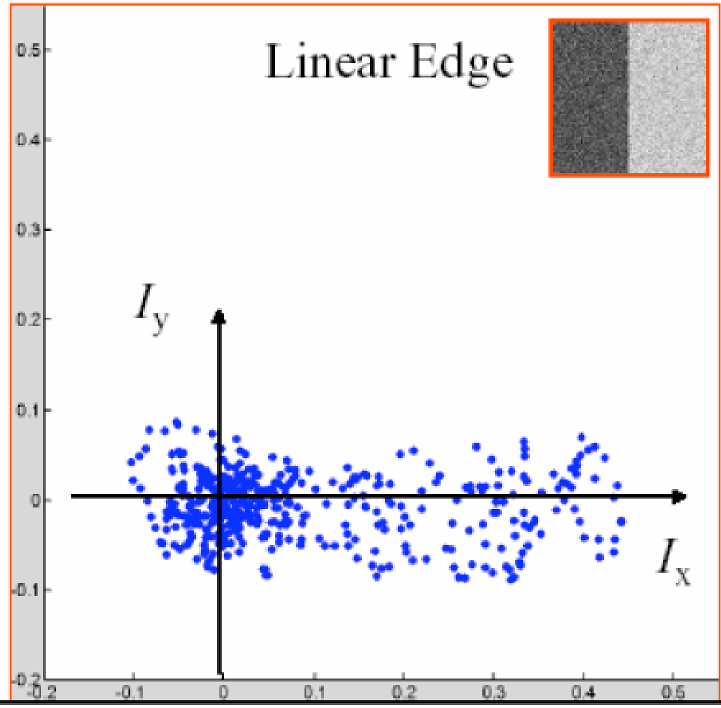


Another view

The distribution of the x and y derivatives is very different for all three types of patches

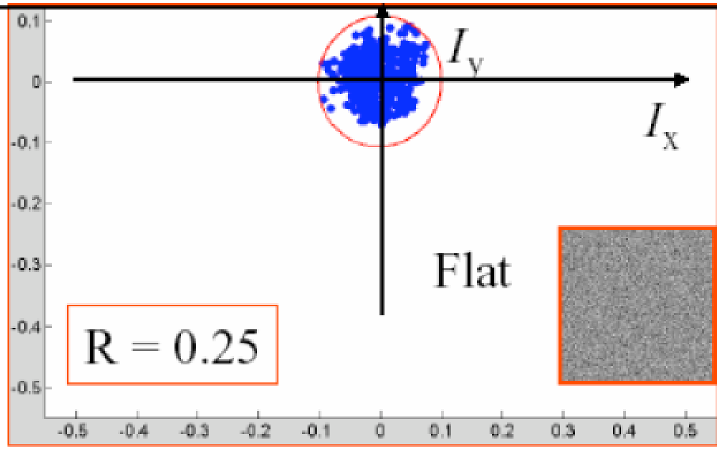


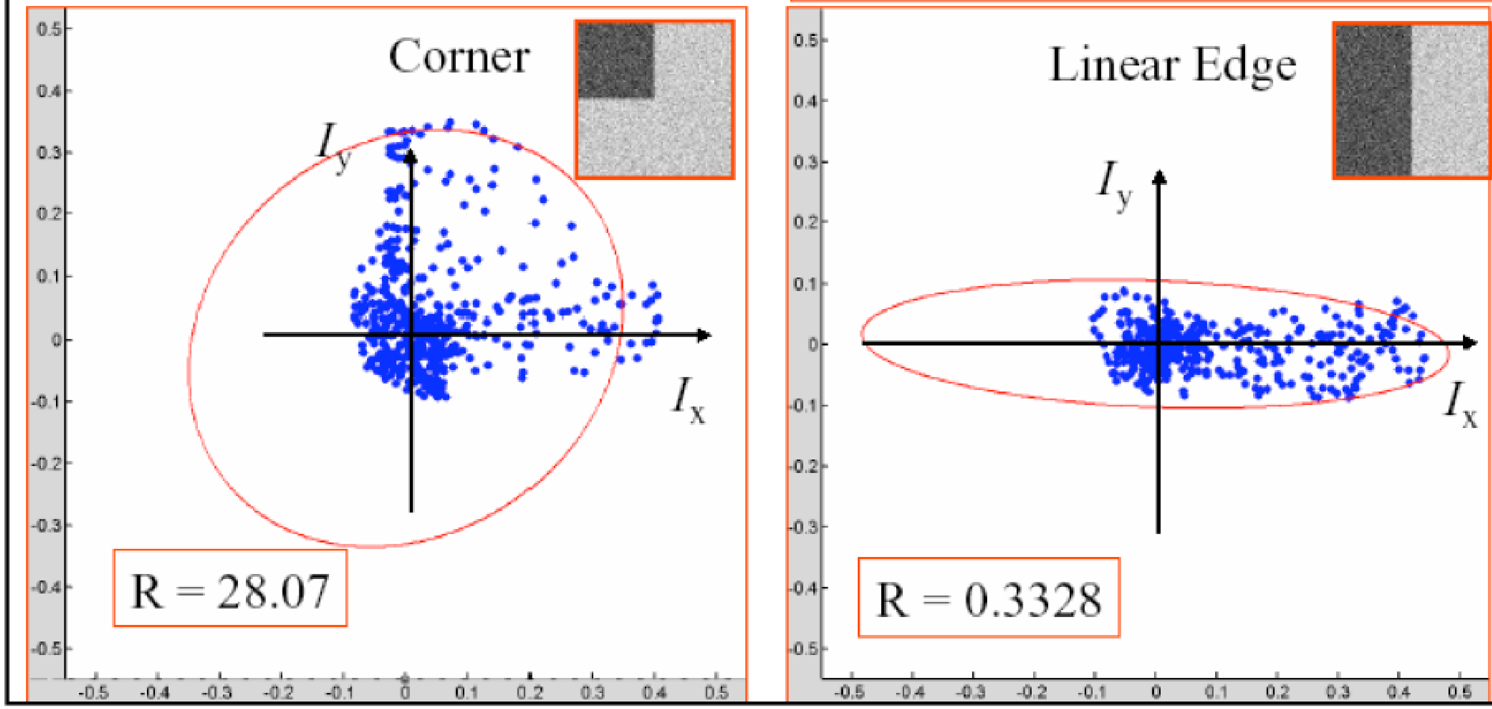




Another view

The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse





Summary of Harris detector

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \qquad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x^2} = I_x \cdot I_x \qquad I_{y^2} = I_y \cdot I_y \qquad I_{xy} = I_x \cdot I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x^2} = G_{\sigma'} * I_{x^2} \qquad S_{y^2} = G_{\sigma'} * I_{y^2} \qquad S_{xy} = G_{\sigma'} * I_{xy}$$

Summary of Harris detector

4. Define the matrix at each pixel

$$M(x,y)=\begin{bmatrix} S_{x^2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y^2}(x,y) \end{bmatrix}$$

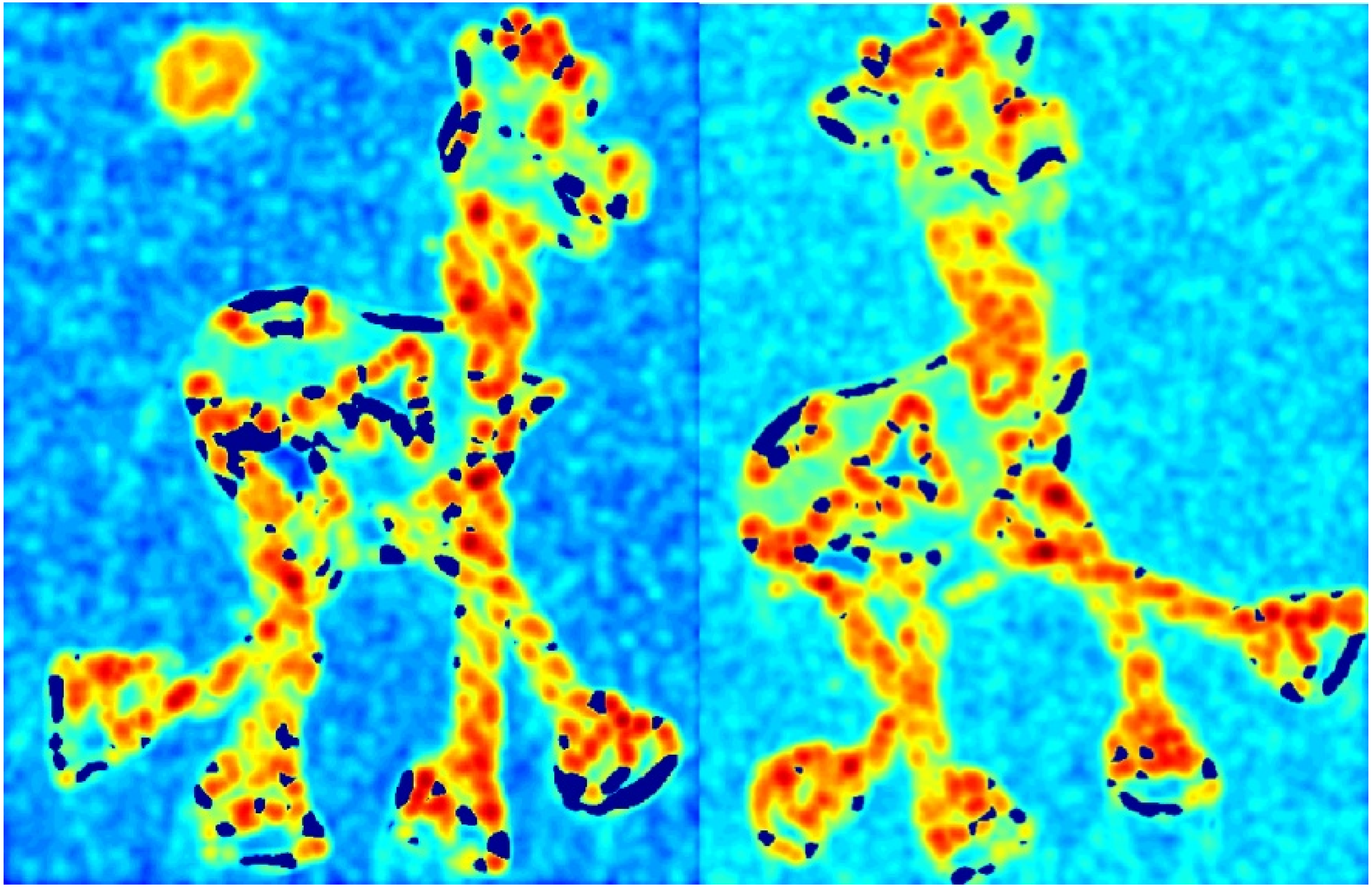
5. Compute the response of the detector at each pixel

$$R=\det M-k(\mathrm{trace}M)^2$$

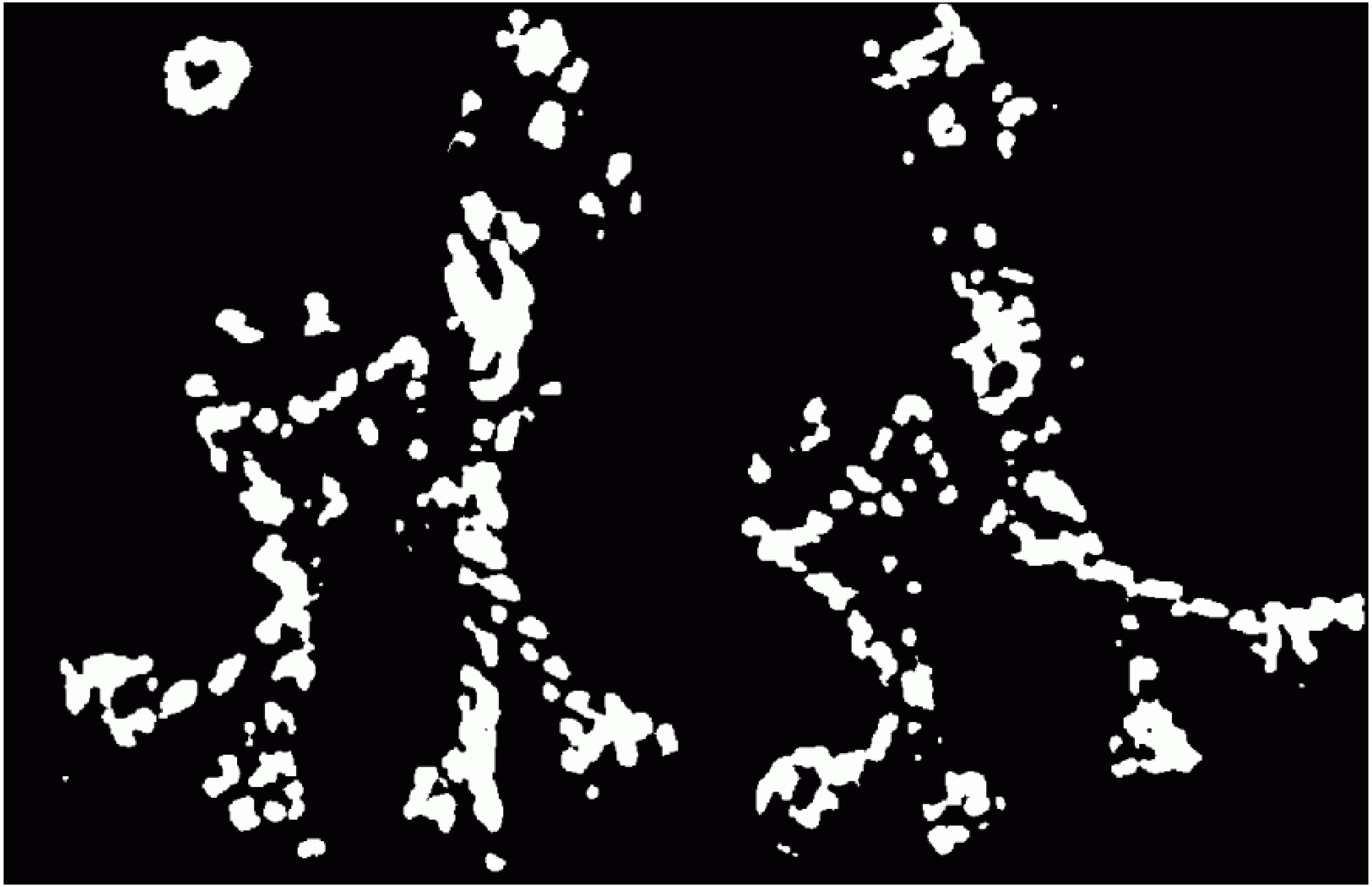
6. Threshold on value of R; compute nonmax suppression.

Harris corner detector (input)

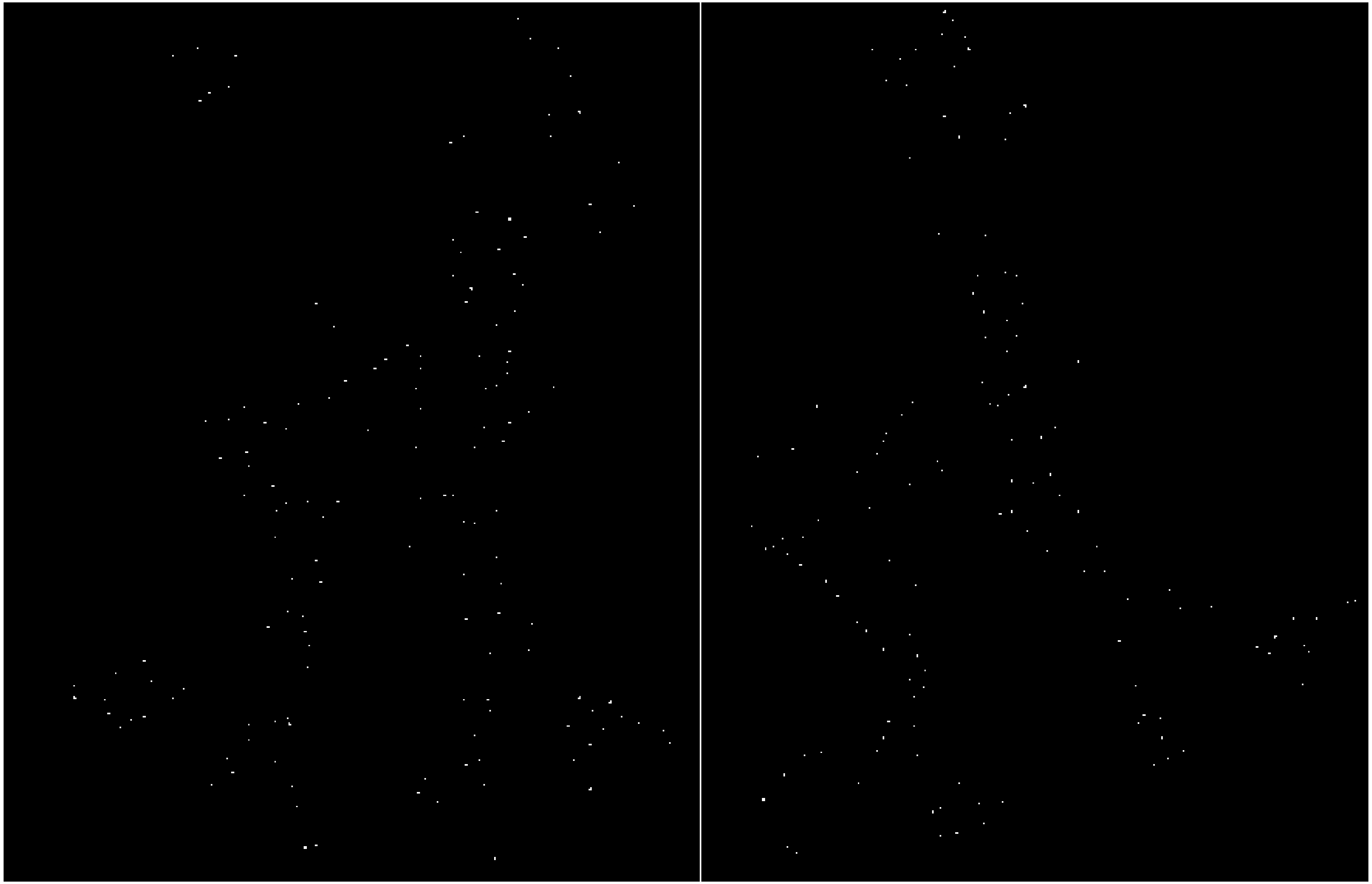
Corner response R



Threshold on R



Local maximum of R



Harris corner detector

Harris detector: summary

- Average intensity change in direction [*u*,*v*] can be expressed as a bilinear form:

$$E(u,v) \cong [u,v]\,\mathbf{M}\begin{bmatrix} u \\ v \end{bmatrix}$$

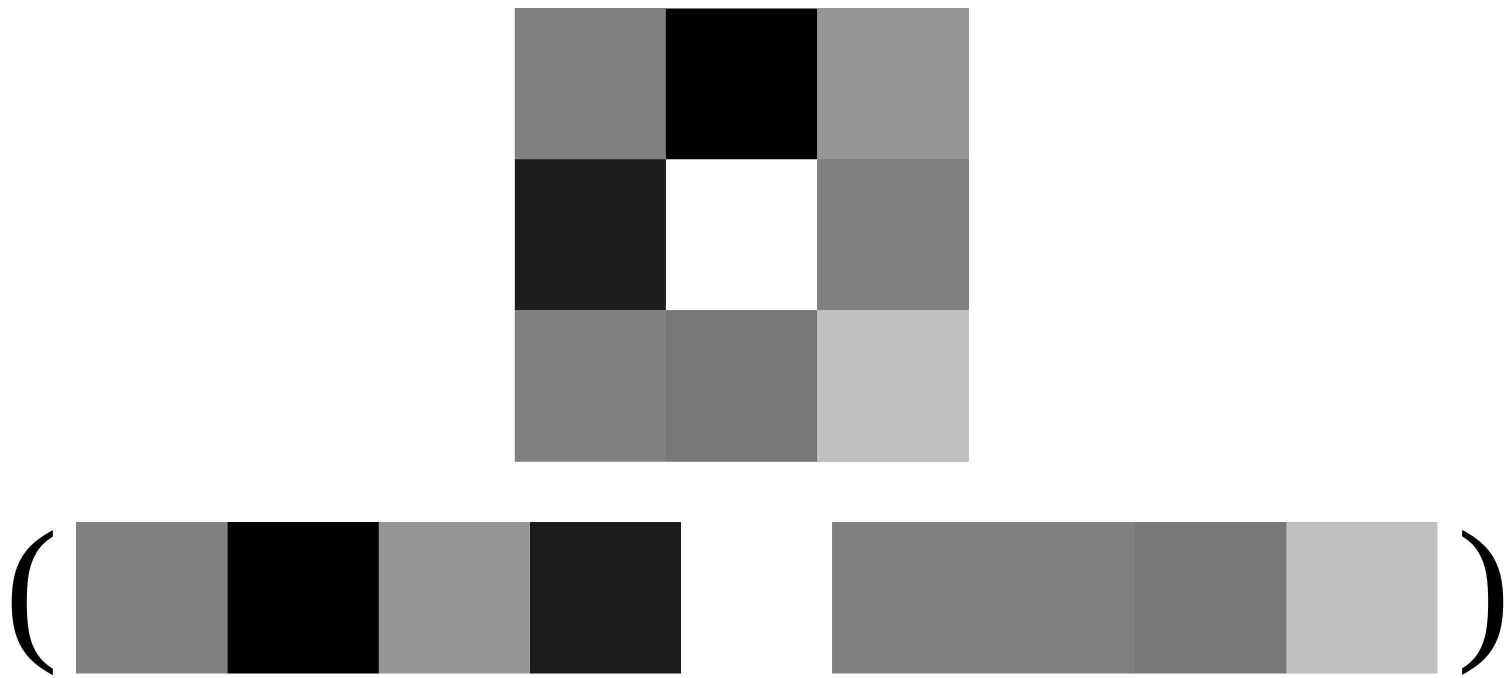
- Describe a point in terms of eigenvalues of *M*: *measure of corner response*

$$R = \lambda_1\lambda_2 - k\left(\lambda_1 + \lambda_2\right)^2$$

- A good (corner) point should have a *large intensity change* in *all directions*, i.e. *R* should be large positive

Now we know where features are

- But, how to match them?
- What is the descriptor for a feature? The simplest solution is the intensities of its spatial neighbors. This might not be robust to brightness change or small shift/rotation.

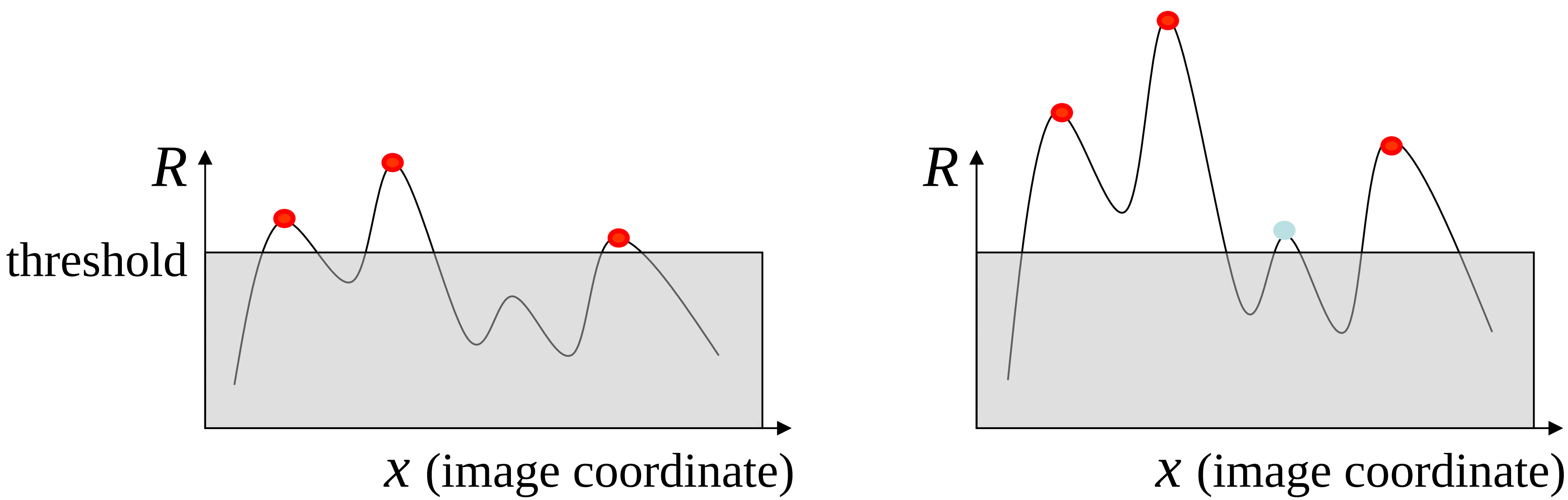


Harris detector: some properties

- Partial invariance to *affine intensity* change

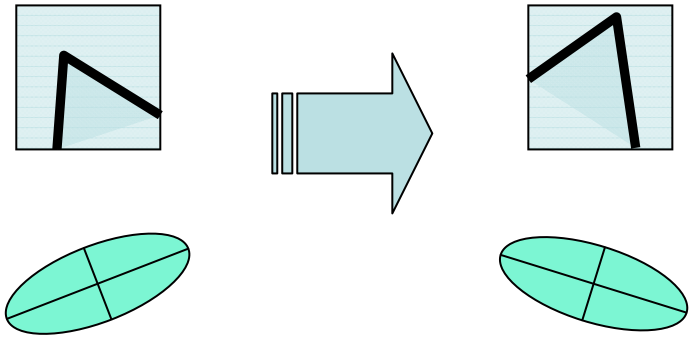
✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

✓ Intensity scale: $I \rightarrow a\,I$



Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

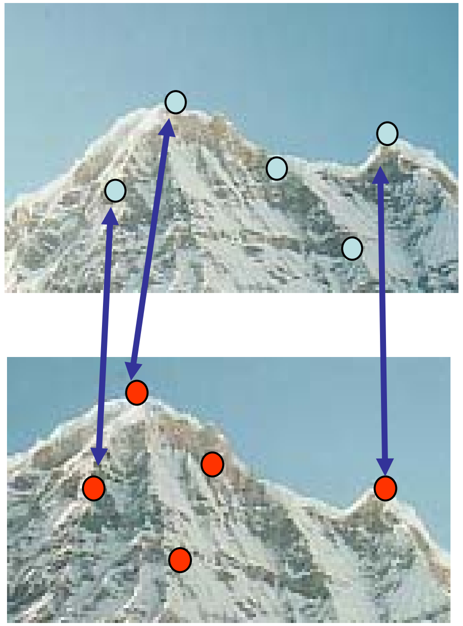
Corner response R is invariant to image rotation

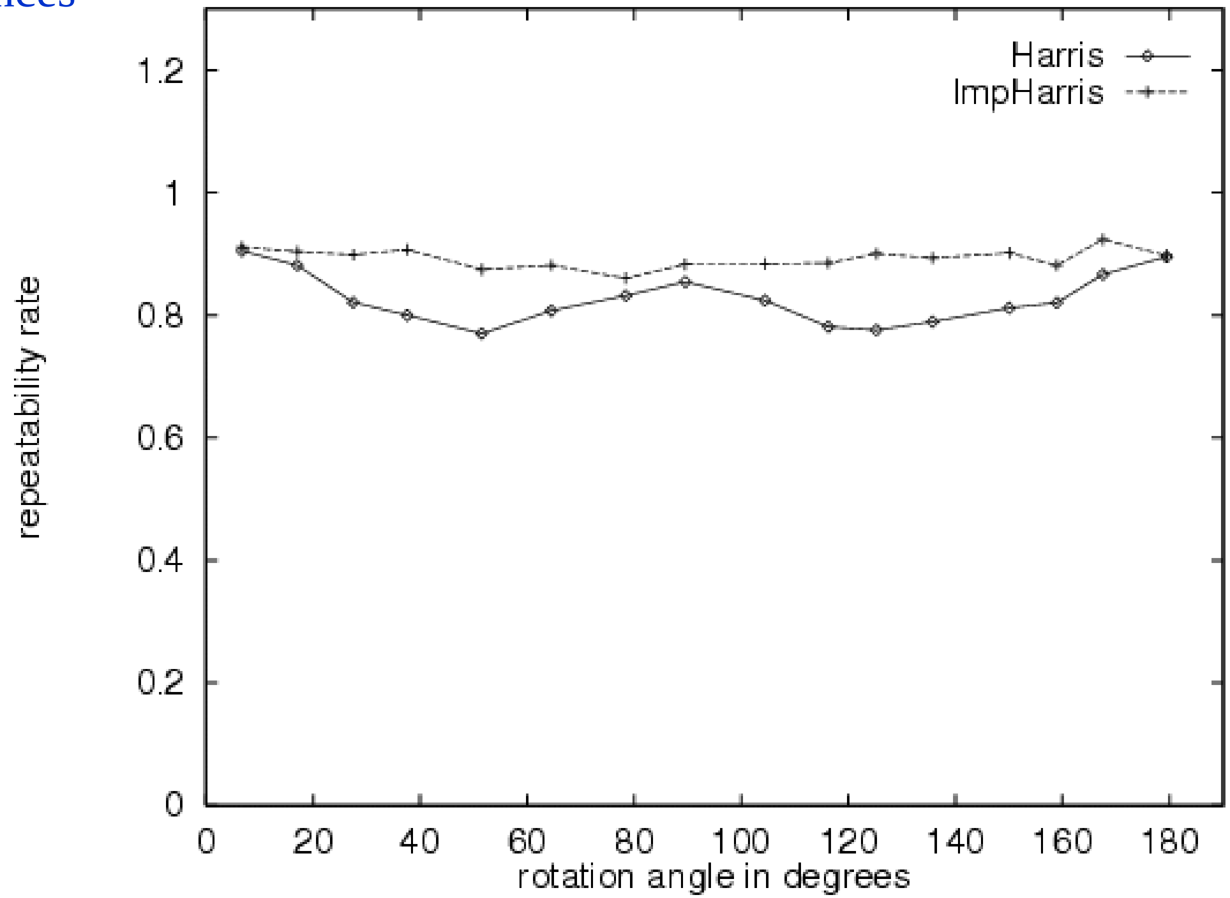
Harris Detector is rotation invariant

DigiVFX

Repeatability rate:

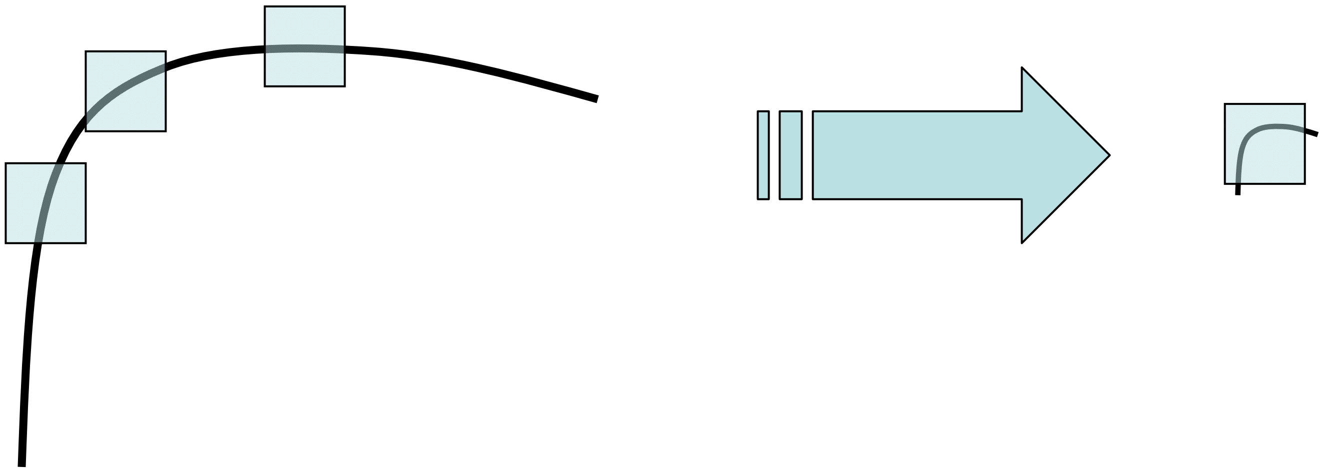
$$\frac{\#\text{ correspondences}}{\#\text{ possible correspondences}}$$





Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as edges

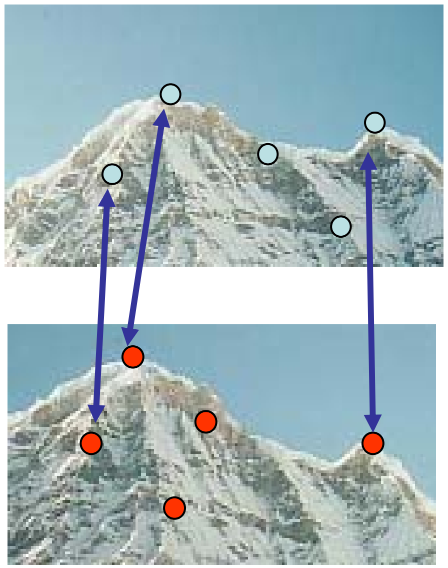
Corner !

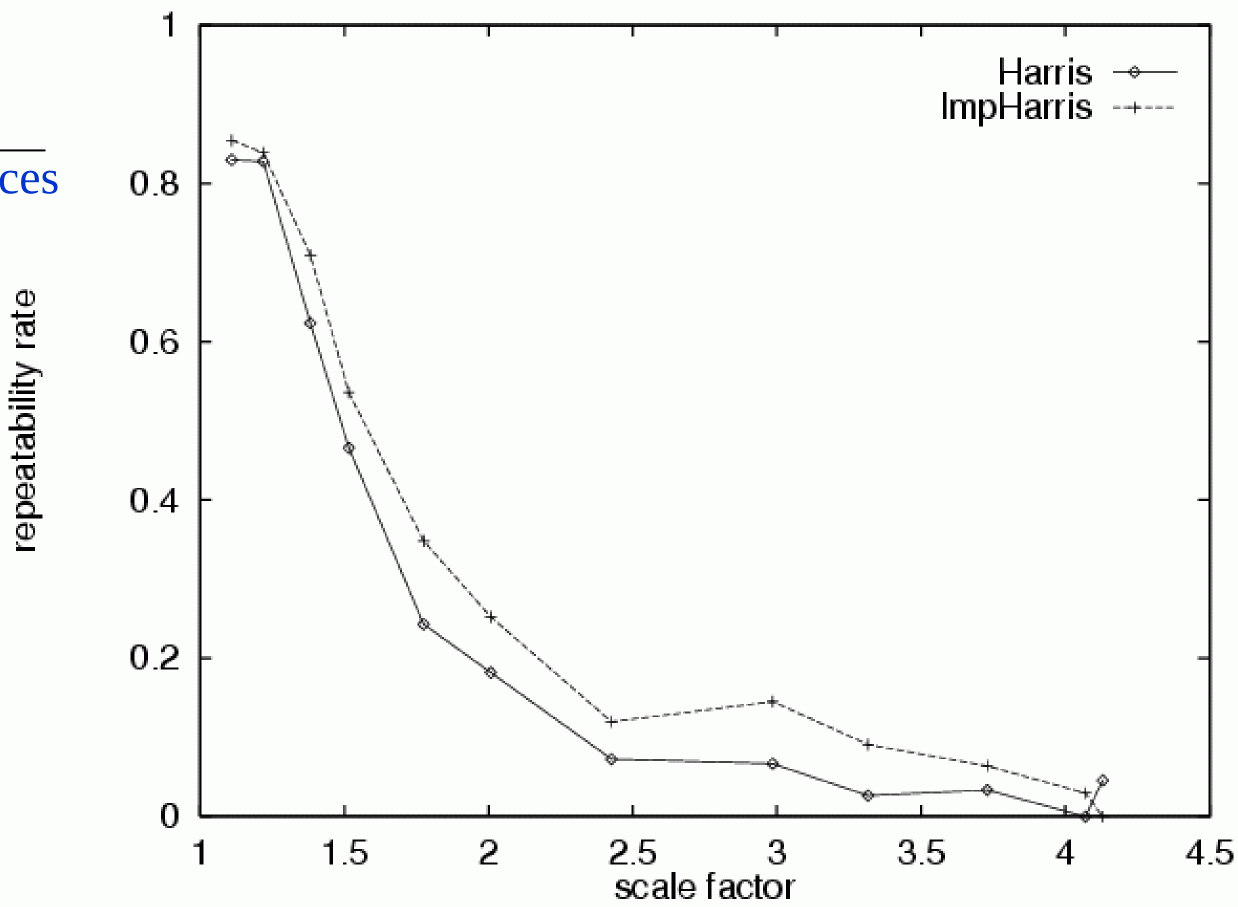
Harris detector: some properties

- Quality of Harris detector for different scale changes

Repeatability rate:

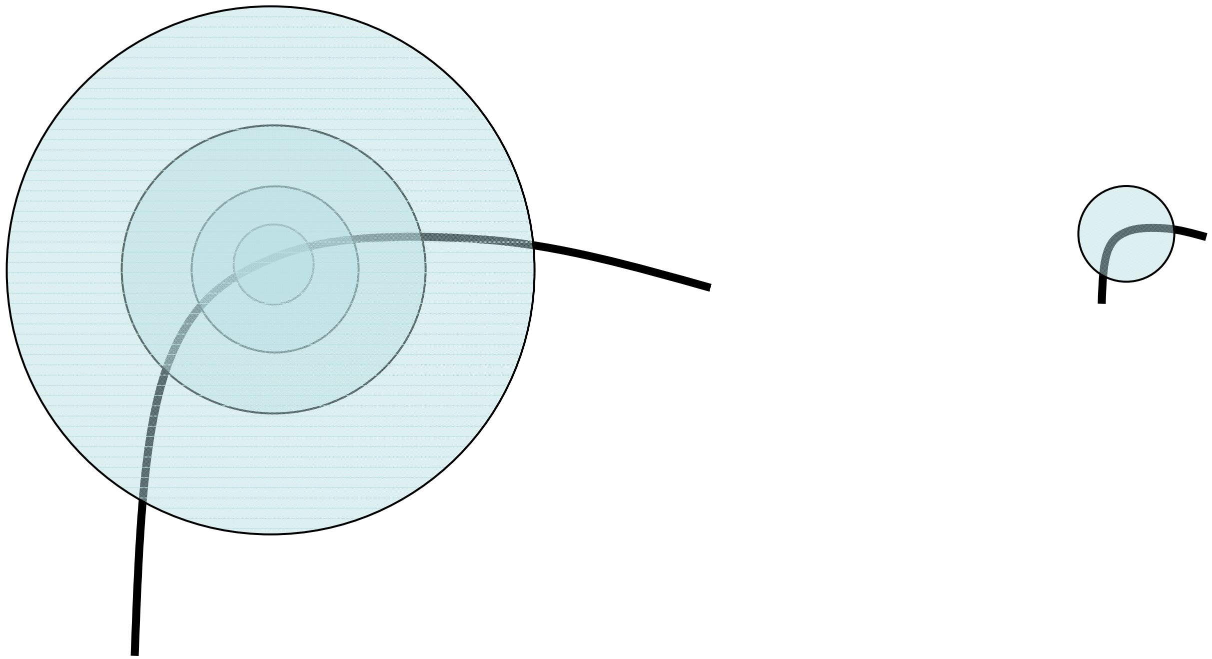
$$\frac{\text{\# correspondences}}{\text{\# possible correspondences}}$$





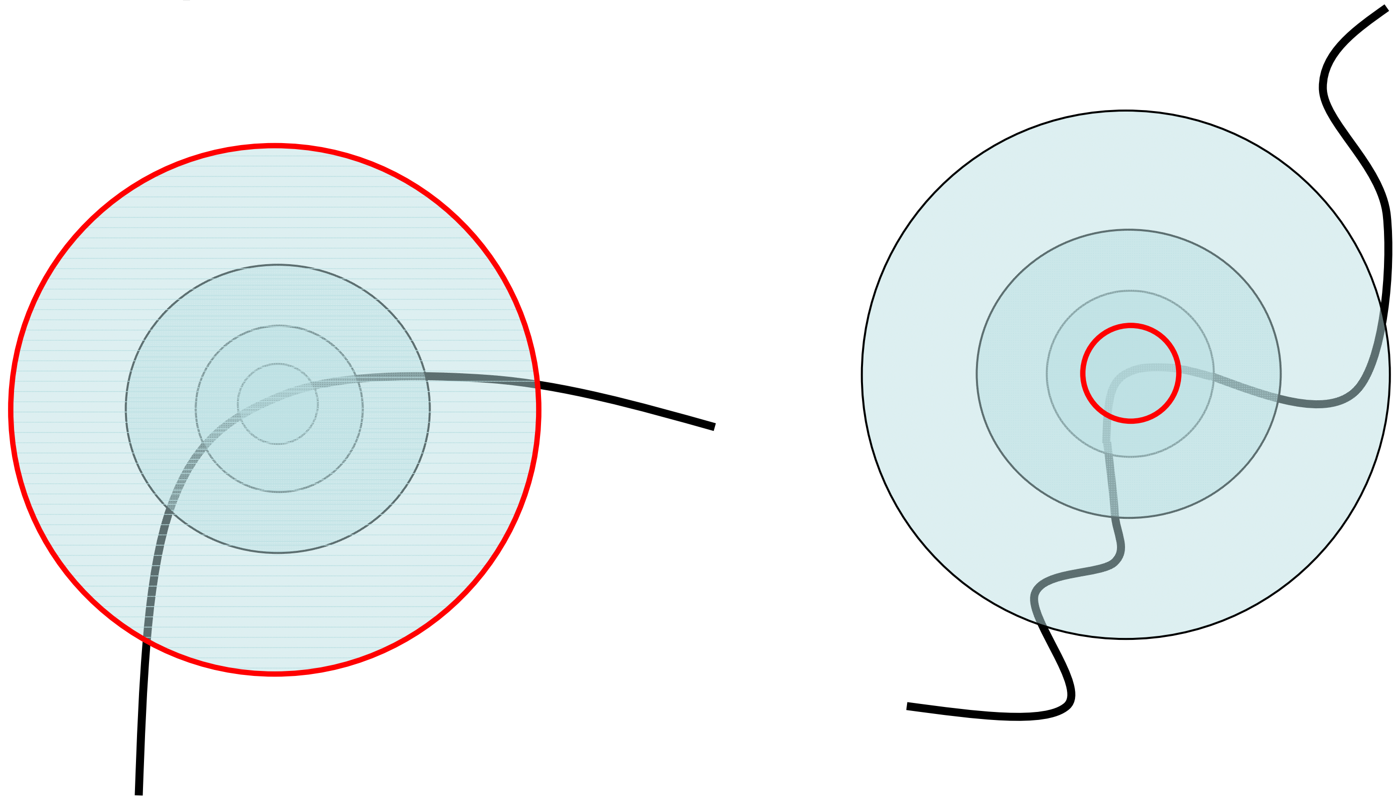
Scale invariant detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



Scale invariant detection

- The problem: how do we choose corresponding circles *independently* in each image?
- Aperture problem



SIFT
(Scale Invariant Feature Transform)

SIFT

DigiVFX

- SIFT is an carefully designed procedure with empirically determined parameters for the invariant and distinctive features.

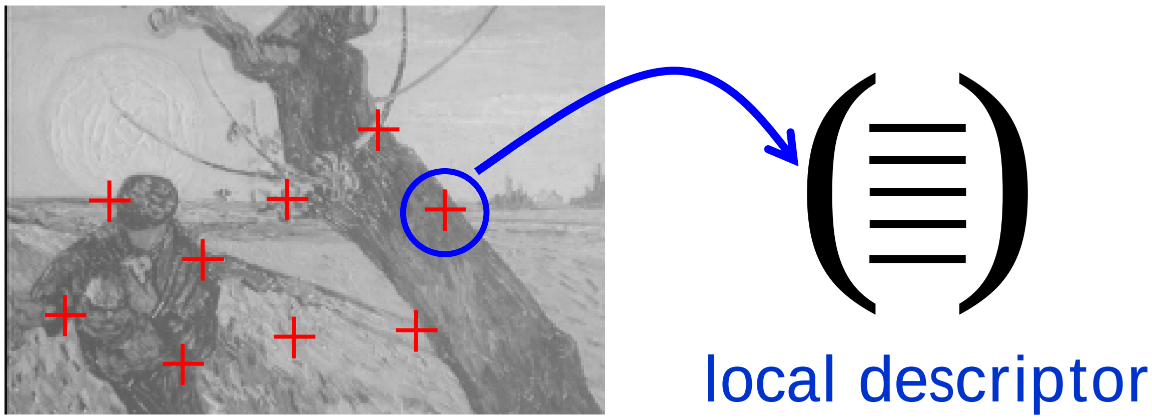
SIFT stages:

- Scale-space extrema detection
- Keypoint localization

detector

- Orientation assignment
- Keypoint descriptor

descriptor



A 500x500 image gives about 2000 features

1. Detection of scale-space extrema

- For scale invariance, search for stable features across all possible scales using a continuous function of scale, scale space.
- SIFT uses DoG filter for scale space because it is efficient and as stable as scale-normalized Laplacian of Gaussian.

DoG filtering

Convolution with a variable-scale Gaussian

$$L(x,y,\sigma) = G(x,y,\sigma) * I(x,y),$$

$$G(x,y,\sigma) = 1/(2\pi\sigma^2)\exp^{-(x^2+y^2)/\sigma^2}$$

Difference-of-Gaussian (DoG) filter

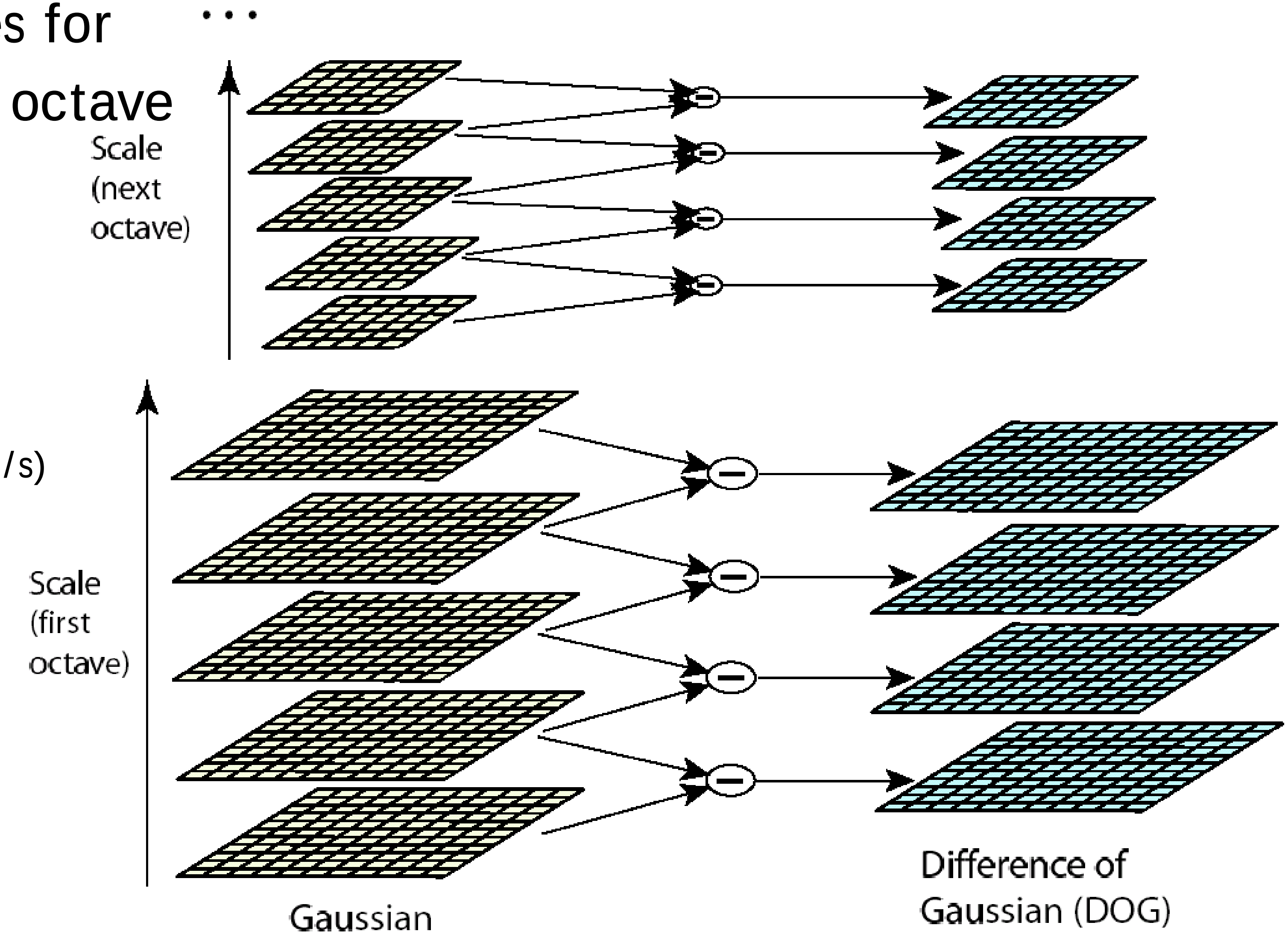
$$G(x,y,k\sigma) - G(x,y,\sigma)$$

Convolution with the DoG filter

$$\begin{aligned} D(x,y,\sigma) &= (G(x,y,k\sigma) - G(x,y,\sigma)) * I(x,y) \\ &= L(x,y,k\sigma) - L(x,y,\sigma). \end{aligned}$$

Scale space

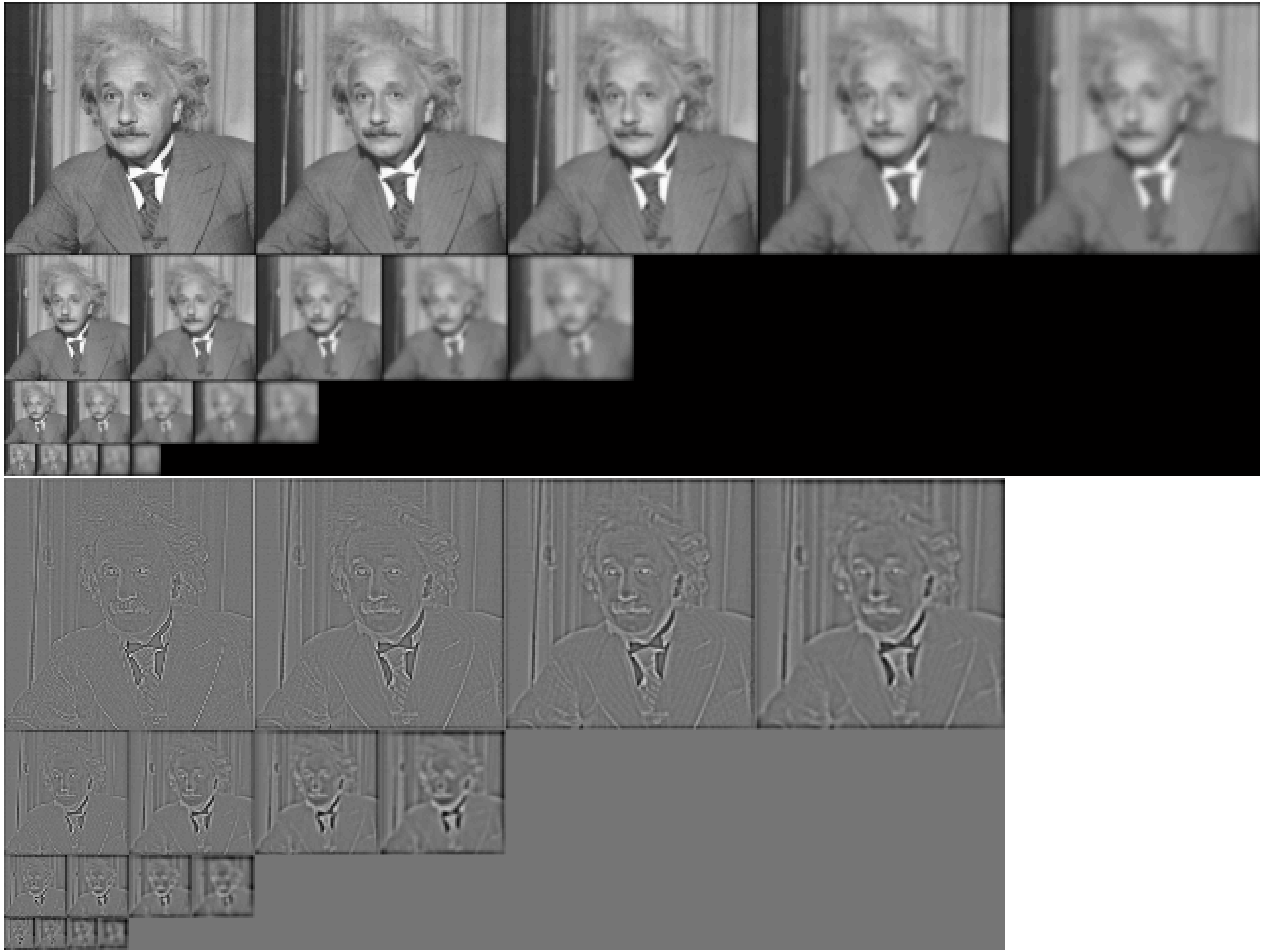
σ doubles for the next octave



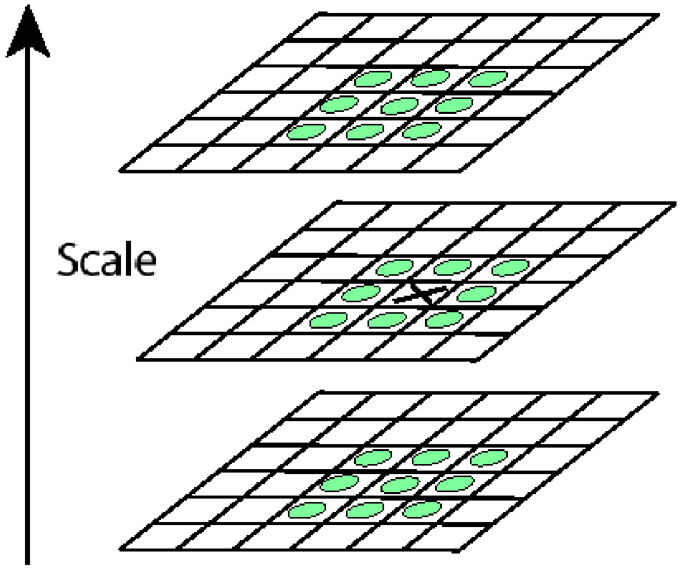
$K=2^{(1/s)}$

Dividing into octave is for efficiency only.

Detection of scale-space extrema



Keypoint localization

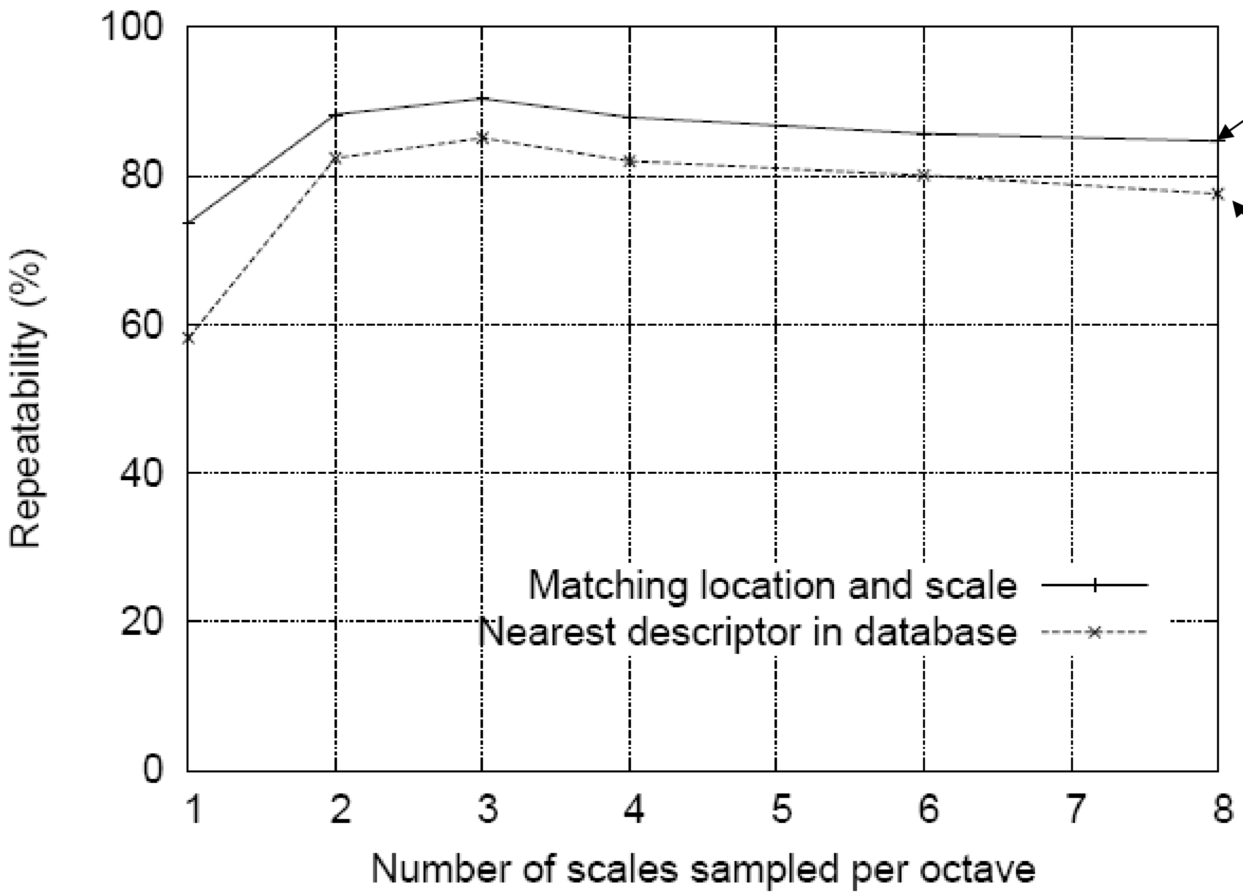


X is selected if it is larger or smaller than all 26 neighbors

Decide scale sampling frequency

- It is impossible to sample the whole space, tradeoff efficiency with completeness.
- Decide the best sampling frequency by experimenting on 32 real image subject to synthetic transformations. (rotation, scaling, affine stretch, brightness and contrast change, adding noise…)

Decide scale sampling frequency

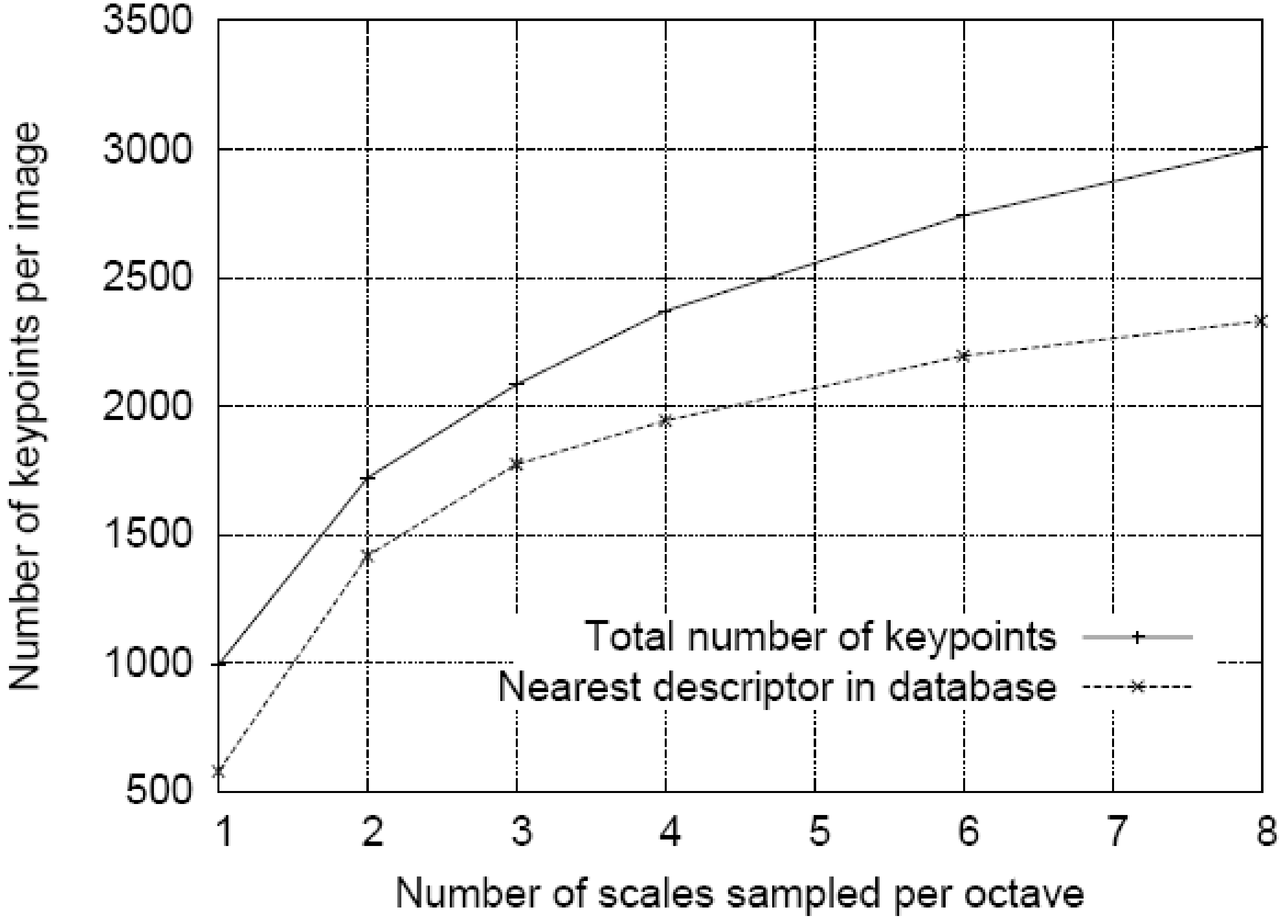


for detector, repeatability

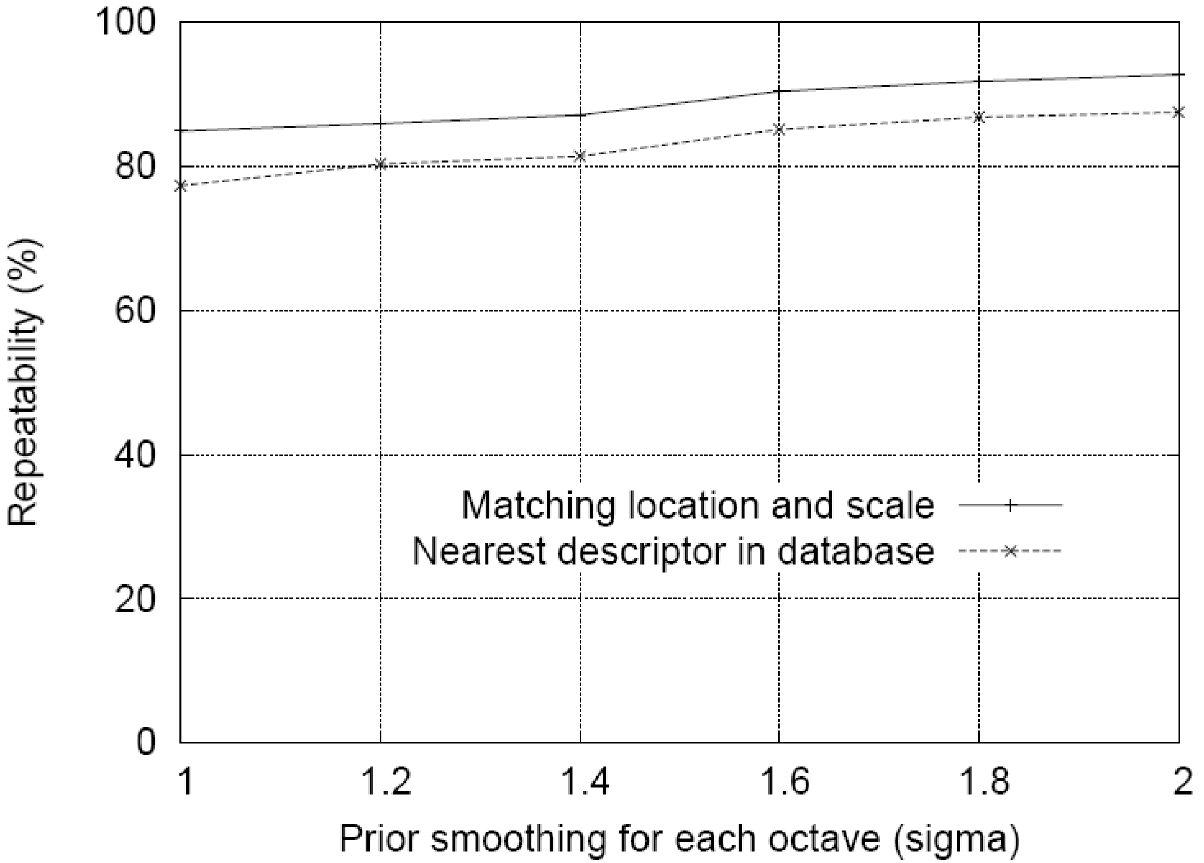
for descriptor, distinctiveness

s=3 is the best, for larger s, too many unstable features

Decide scale sampling frequency

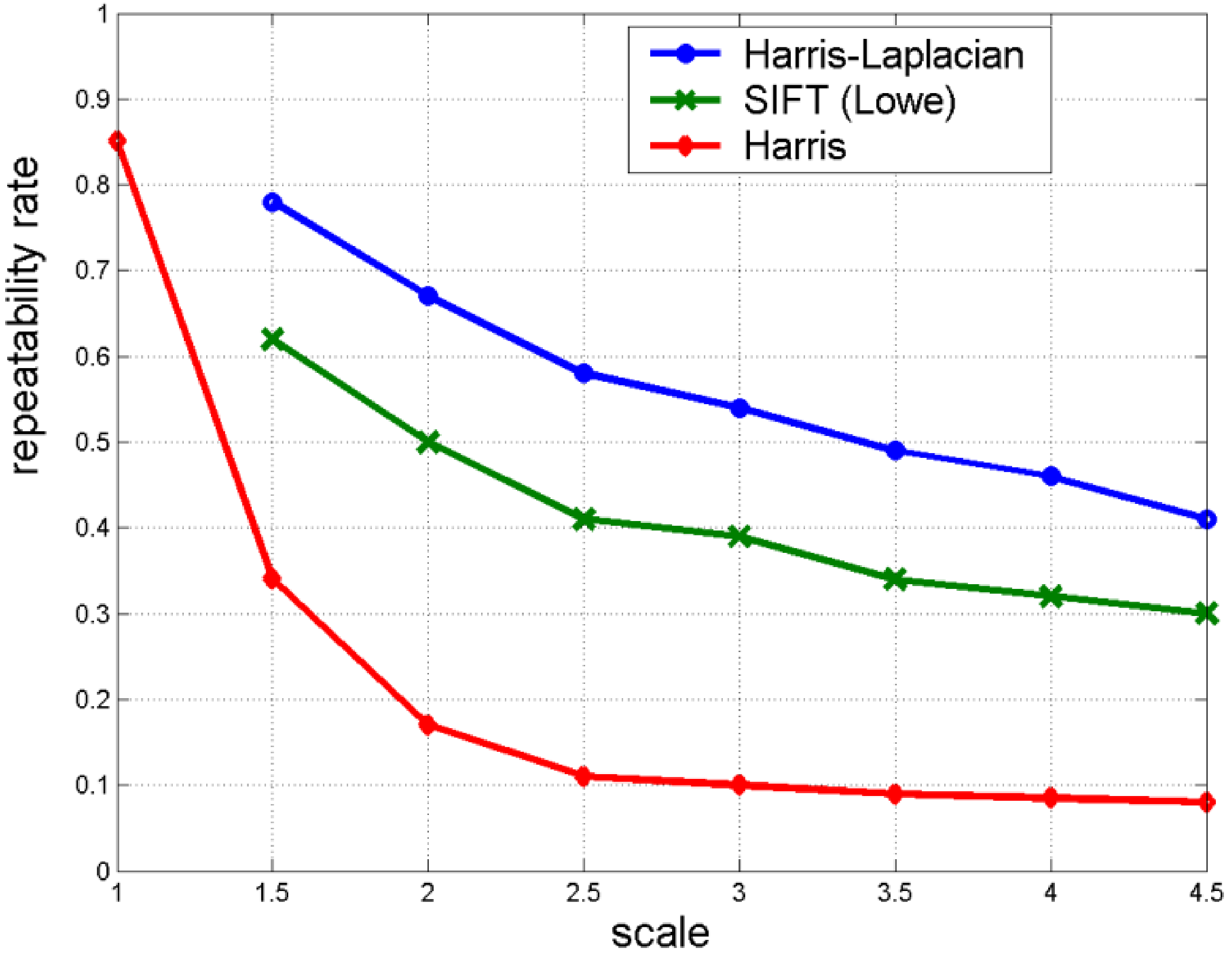


Pre-smoothing



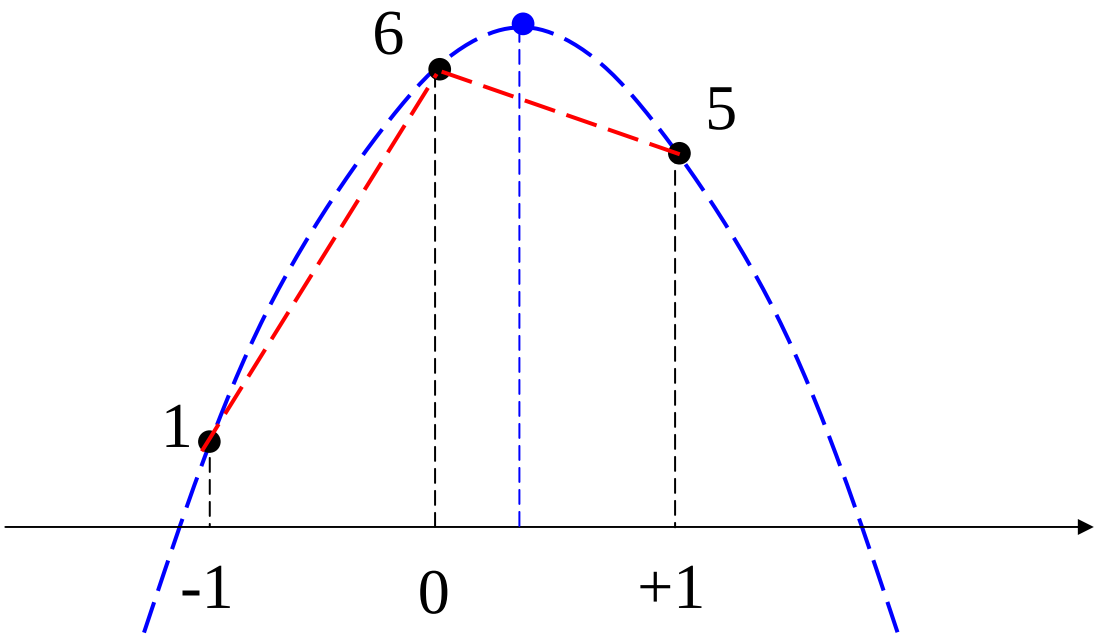
σ =1.6, plus a double expansion

Scale invariance



2. Accurate keypoint localization

- Reject points with low contrast (flat) and poorly localized along an edge (edge)
- Fit a 3D quadratic function for sub-pixel maxima



2. Accurate keypoint localization

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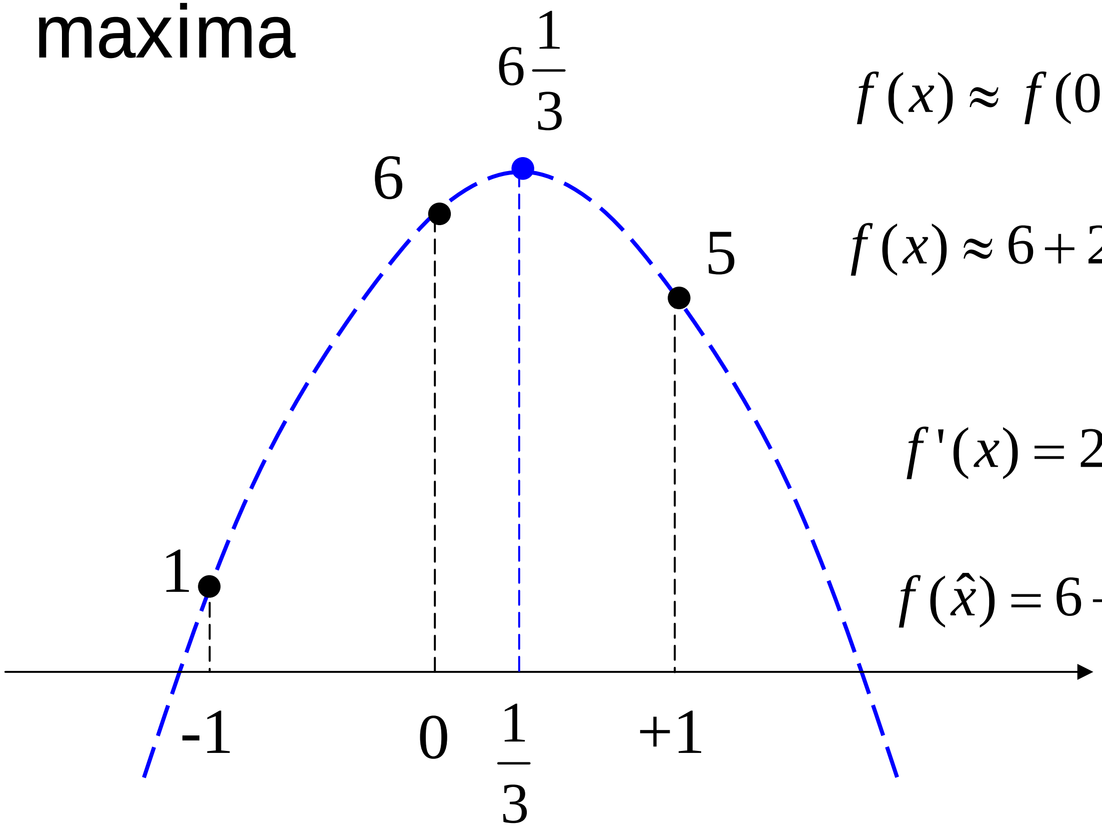


$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

$$f(x) \approx 6 + 2x + \frac{-6}{2}x^2 = 6 + 2x - 3x^2$$

$$f'(x) = 2 - 6x = 0 \longrightarrow \hat{x} = \frac{1}{3}$$

$$f(\hat{x}) = 6 + 2 \cdot \frac{1}{3} - 3 \cdot \left(\frac{1}{3}\right)^2 = 6\frac{1}{3}$$

2. Accurate keypoint localization

DigiVFX

- Taylor series of several variables

$$T(x_1,\cdots,x_d)=\sum_{n_1=0}^{\infty}\cdots\sum_{n_d=0}^{\infty}\frac{\partial^{n_1}}{\partial x_1^{n_1}}\cdots\frac{\partial^{n_d}}{\partial x_d^{n_d}}\frac{f(a_1,\cdots,a_d)}{n_1!\cdots n_d!}(x_1-a_1)^{n_1}\cdots(x_d-a_d)^{n_d}$$

- Two variables

$$f(x,y)\approx f(0,0)+\left(\frac{\partial f}{\partial x}x+\frac{\partial f}{\partial y}y\right)+\frac{1}{2}\left(\frac{\partial^2 f}{\partial x\partial x}x^2+2\frac{\partial^2 f}{\partial x\partial y}xy+\frac{\partial^2 f}{\partial y\partial y}y^2\right)$$

$$f\left(\begin{bmatrix}x\\y\end{bmatrix}\right)\approx f\left(\begin{bmatrix}0\\0\end{bmatrix}\right)+\begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\frac{1}{2}\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}\frac{\partial^2 f}{\partial x\partial x} & \frac{\partial^2 f}{\partial x\partial y}\\ \frac{\partial^2 f}{\partial x\partial y} & \frac{\partial^2 f}{\partial y\partial y}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$f(\mathbf{x})\approx f(\mathbf{0})+\frac{\partial f}{\partial \mathbf{x}}^T\mathbf{x}+\frac{1}{2}\mathbf{x}^T\frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

Accurate keypoint localization

- Taylor expansion in a matrix form, $\mathbf{x}$ is a vector, f maps $\mathbf{x}$ to a scalar

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

Hessian matrix (often symmetric)

gradient
$$\begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

2D illustration

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$f_{-1,1}$	$f_{0,1}$	$f_{1,1}$
$f_{-1,0}$	$f_{0,0}$	$f_{1,0}$
$f_{-1,-1}$	$f_{0,-1}$	$f_{1,-1}$

$$\frac{\partial f}{\partial x} = (f_{1,0} - f_{-1,0})/2$$

$$\frac{\partial f}{\partial y} = (f_{0,1} - f_{0,-1})/2$$

$$\frac{\partial^2 f}{\partial x^2} = f_{1,0} - 2f_{0,0} + f_{-1,0}$$

$$\frac{\partial^2 f}{\partial y^2} = f_{0,1} - 2f_{0,0} + f_{0,-1}$$

$$\frac{\partial^2 f}{\partial x \partial y} = (f_{-1,-1} - f_{-1,1} - f_{1,-1} + f_{1,1})/4$$

2D example

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

-17	-1	-1
-9	7	7
-9	7	7

Derivation of matrix form

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{g}^{\mathbf{T}}\mathbf{x}$$

$$= \begin{pmatrix} g_1 & \cdots & g_n \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^{n} g_i x_i$$

$$\frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial h}{\partial x_1} \\ \vdots \\ \frac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} g_1 \\ \vdots \\ g_n \end{pmatrix} = \mathbf{g}$$

Derivation of matrix form

DigiVFX

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$h(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} = \begin{pmatrix} x_1 & \cdots & x_n \end{pmatrix}^T \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j$$

$$\frac{\partial h}{\partial \mathbf{x}} = \begin{pmatrix} \dfrac{\partial h}{\partial x_1} \\ \vdots \\ \dfrac{\partial h}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} a_{i1} x_i + \sum_{j=1}^{n} a_{1j} x_j \\ \vdots \\ \sum_{i=1}^{n} a_{in} x_i + \sum_{j=1}^{n} a_{nj} x_j \end{pmatrix} = \mathbf{A}^{\mathrm{T}}\mathbf{x} + \mathbf{A}\mathbf{x}$$

$$= (\mathbf{A}^{\mathrm{T}} + \mathbf{A})\mathbf{x}$$

Derivation of matrix form

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

$$\frac{\partial h}{\partial \mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}}^T + \frac{1}{2}\left(\frac{\partial^2 f}{\partial \mathbf{x}^2} + \frac{\partial^2 f}{\partial \mathbf{x}^2}^T\right)x = \frac{\partial f}{\partial \mathbf{x}}^T + \frac{\partial^2 f}{\partial \mathbf{x}^2} x$$

$$\mathbf{x}_m = -\frac{\partial^2 f}{\partial \mathbf{x}^2}^{-1} \frac{\partial f}{\partial \mathbf{x}}$$

Accurate keypoint localization

$$f(\mathbf{x}) = f + \frac{\partial f}{\partial \mathbf{x}}^T \mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 f}{\partial \mathbf{x}^2}\mathbf{x}$$

- $\mathbf{x}$ is a 3-vector
- Change sample point if offset is larger than 0.5
- Throw out low contrast (<0.03)

Accurate keypoint localization

- Throw out low contrast $|D(\hat{\mathbf{x}})| < 0.03$

$$D(\hat{\mathbf{x}}) = D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2} \hat{\mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \hat{\mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2} \left(-\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}} \right)^T \frac{\partial^2 D}{\partial \mathbf{x}^2} \left(-\frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}} \right)$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D}{\partial \mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}^{-T} \frac{\partial^2 D}{\partial \mathbf{x}^2} \frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D}{\partial \mathbf{x}}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}^{-1} \frac{\partial D}{\partial \mathbf{x}}$$

$$= D + \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}} + \frac{1}{2} \frac{\partial D}{\partial \mathbf{x}}^T (-\hat{\mathbf{x}})$$

$$= D + \frac{1}{2} \frac{\partial D}{\partial \mathbf{x}}^T \hat{\mathbf{x}}$$

Eliminating edge responses

DigiVFX

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$ Hessian matrix at keypoint location

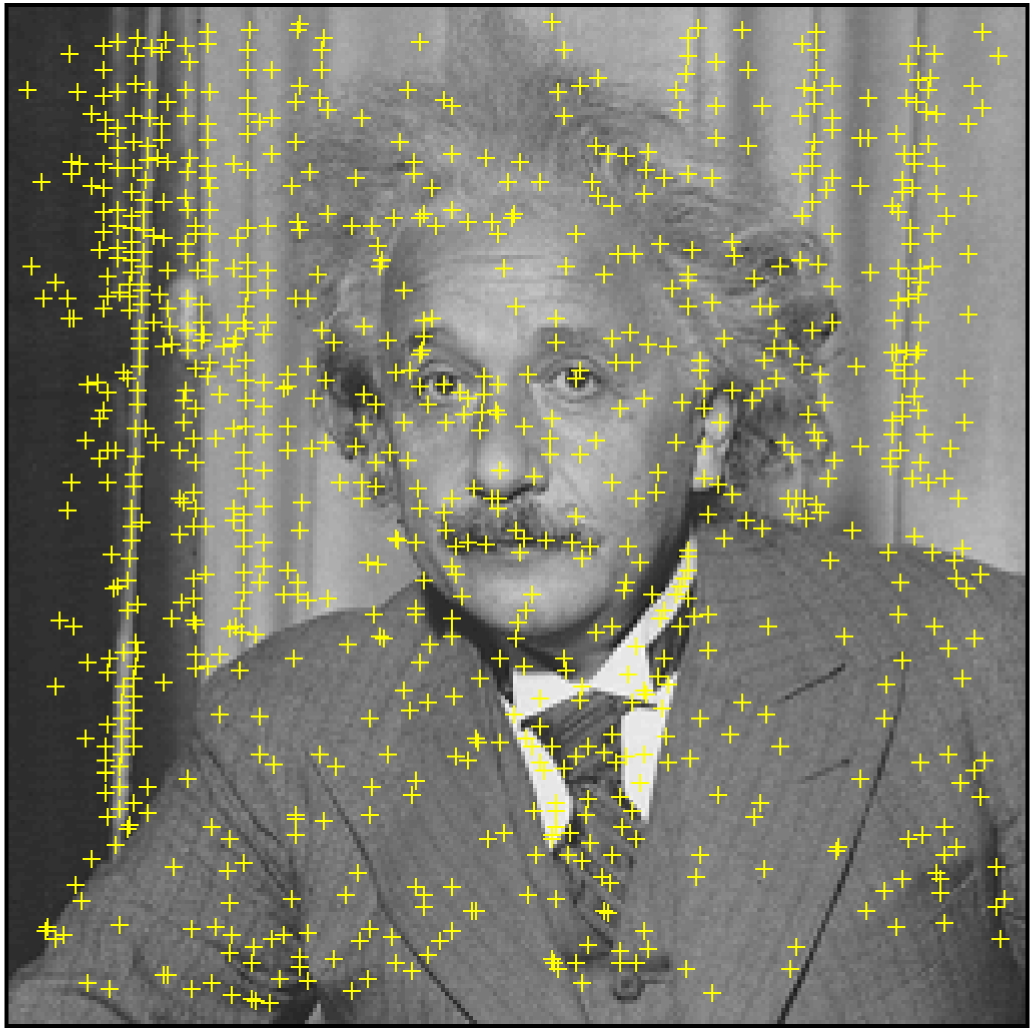
$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

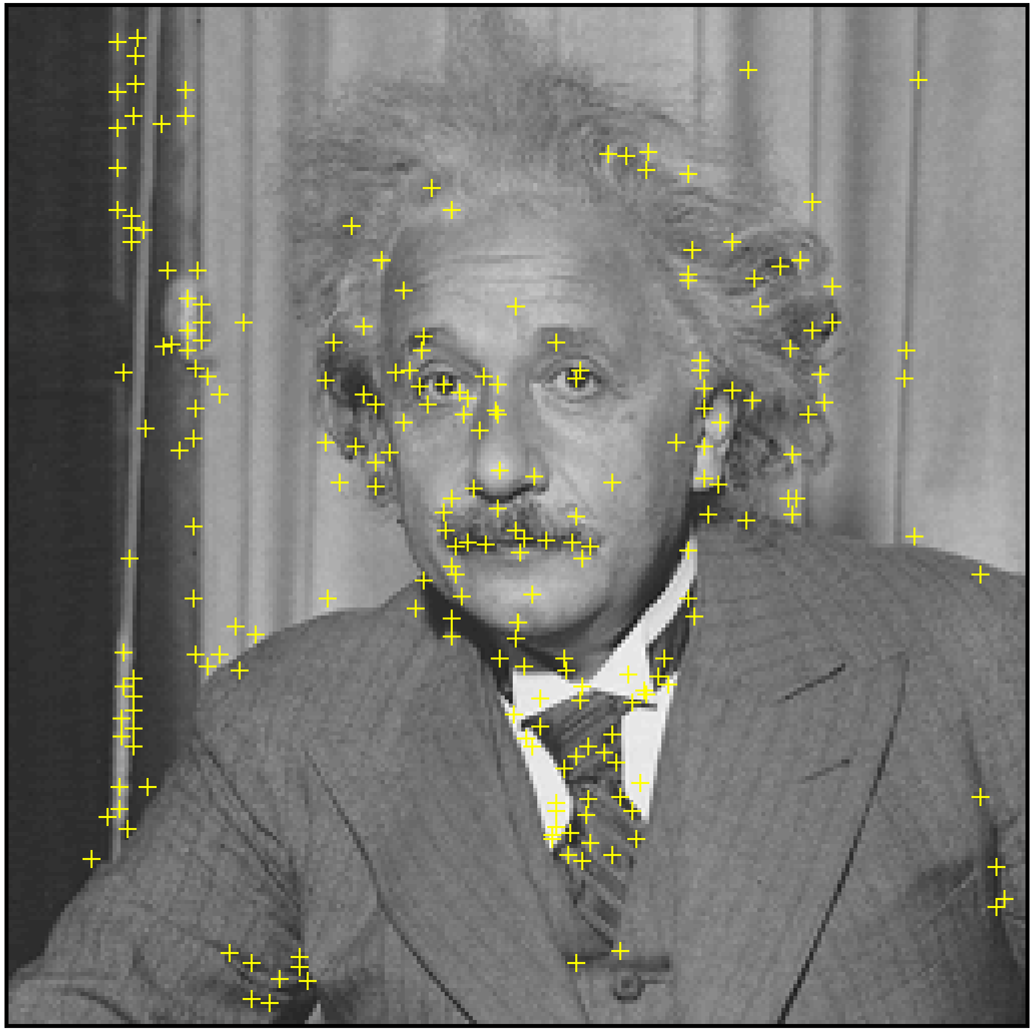
Let $\alpha = r\beta$ $$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}$$

Keep the points with $\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$. r=10

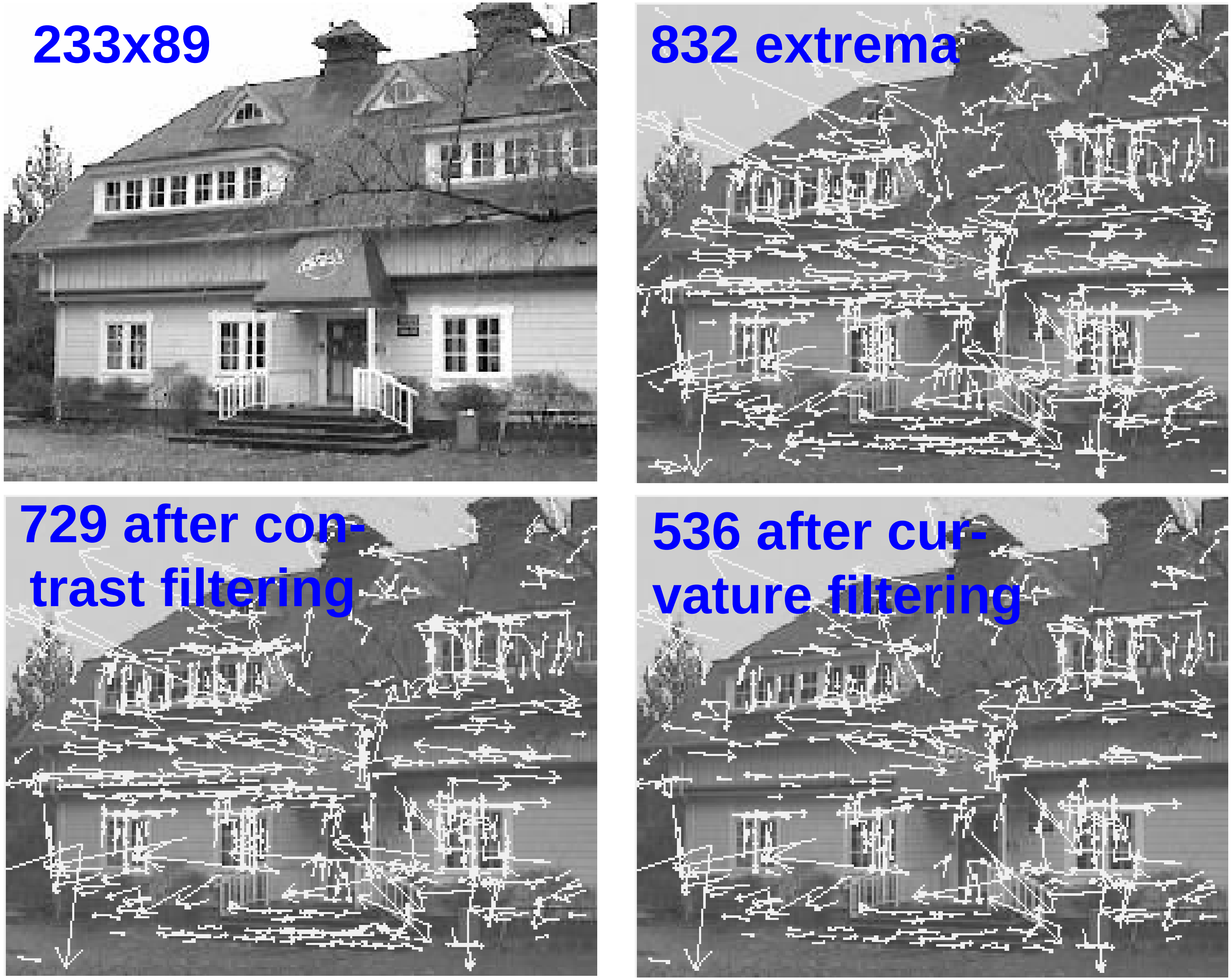
Maxima in D



Remove low contrast and edges



Keypoint detector

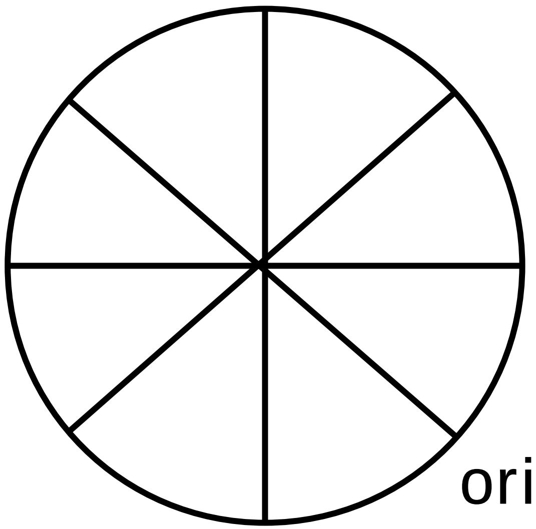


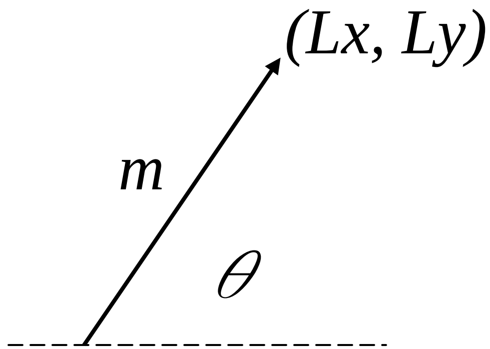
3. Orientation assignment

- By assigning a consistent orientation, the keypoint descriptor can be orientation invariant.
- For a keypoint, L is the Gaussian-smoothed image with the closest scale,

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

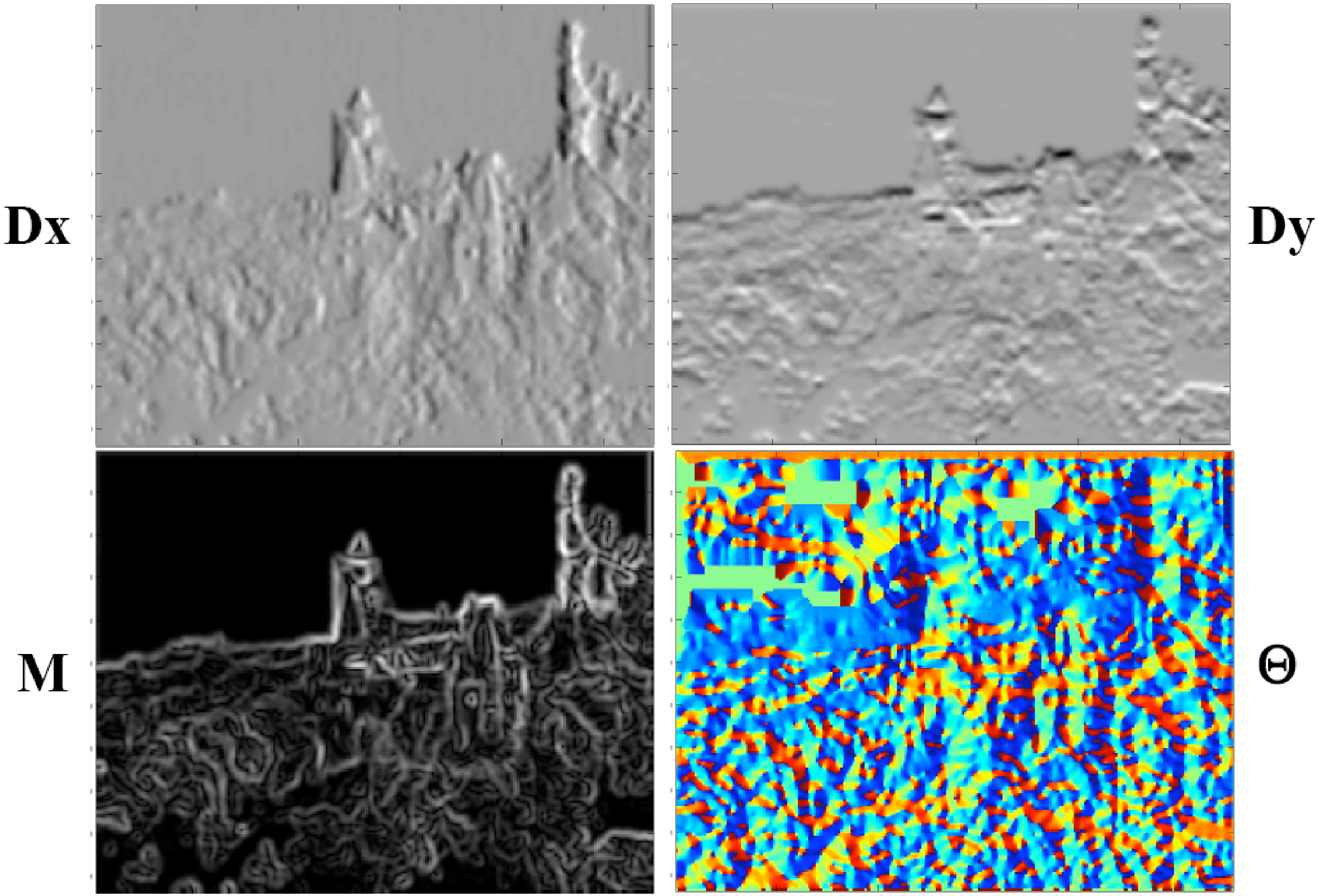
$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$



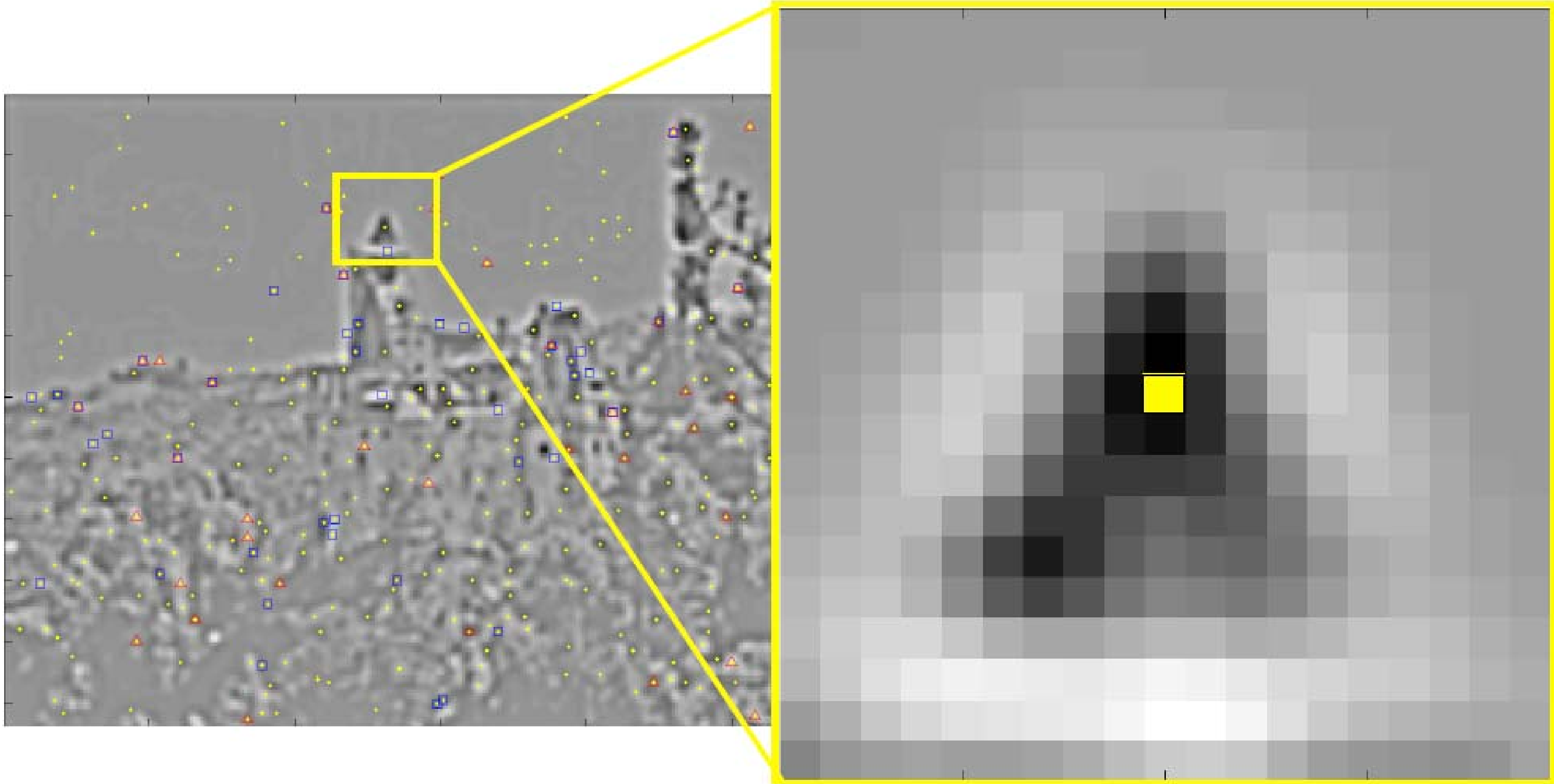


orientation histogram (36 bins)

Orientation assignment

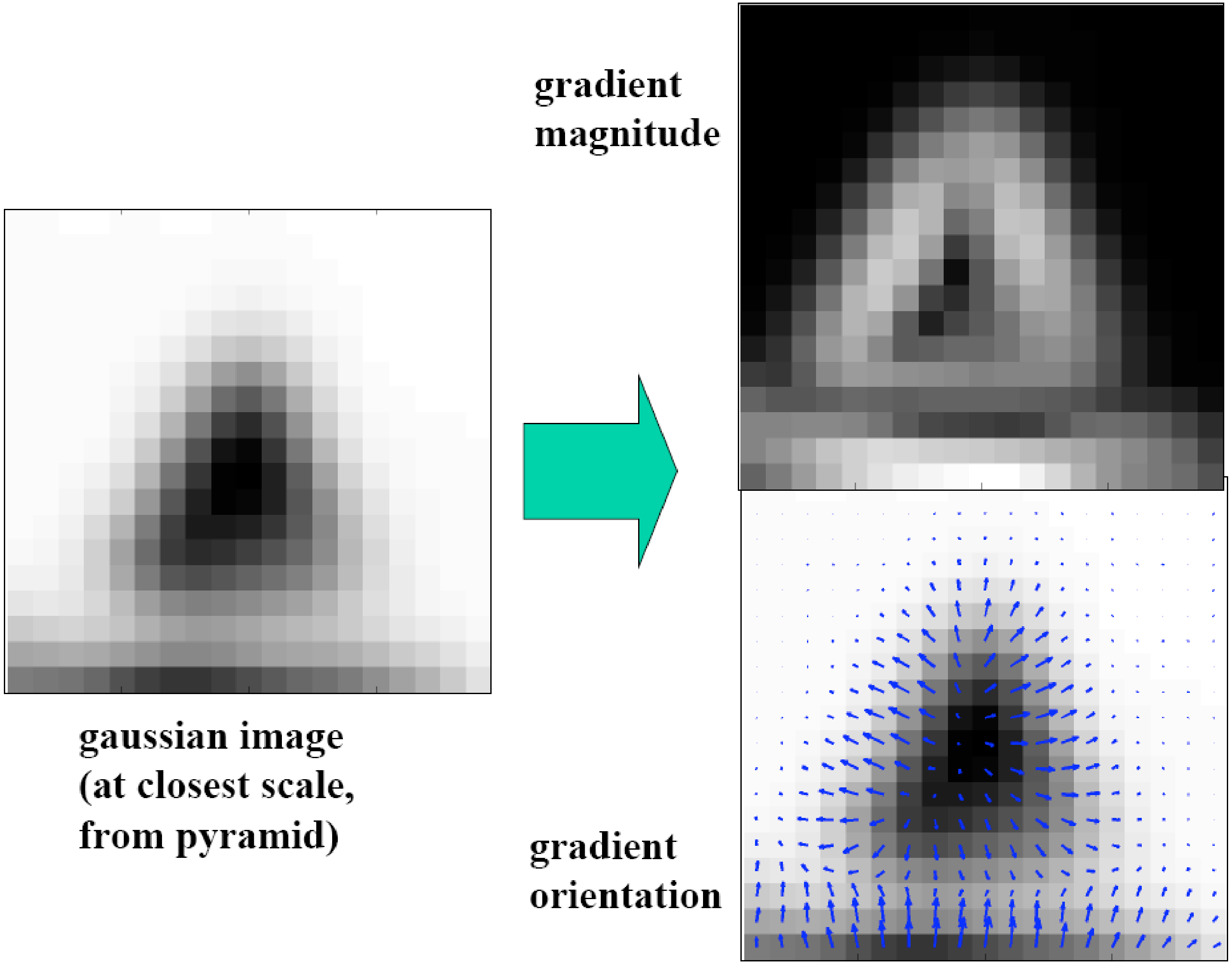


Orientation assignment

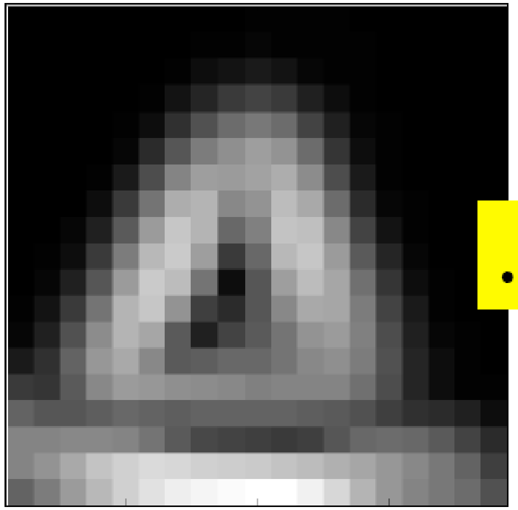


- Keypoint location = extrema location
- Keypoint scale is scale of the DOG image

Orientation assignment

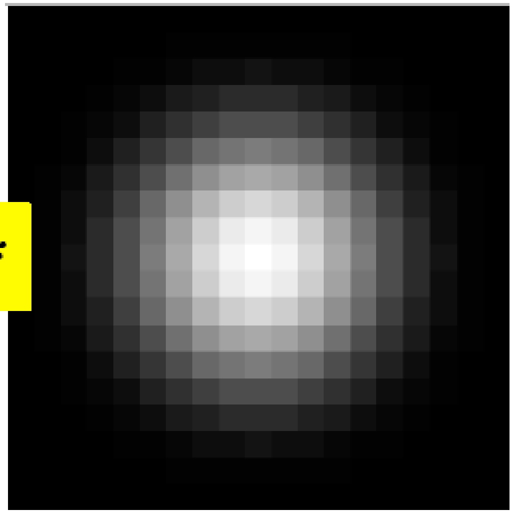


Orientation assignment



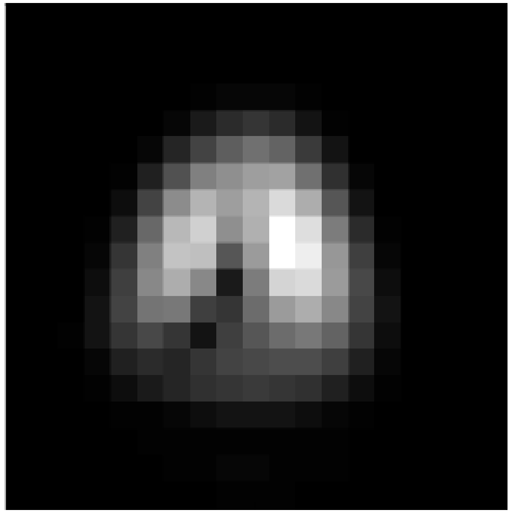
gradient magnitude

.*



weighted by 2D gaussian kernel

=



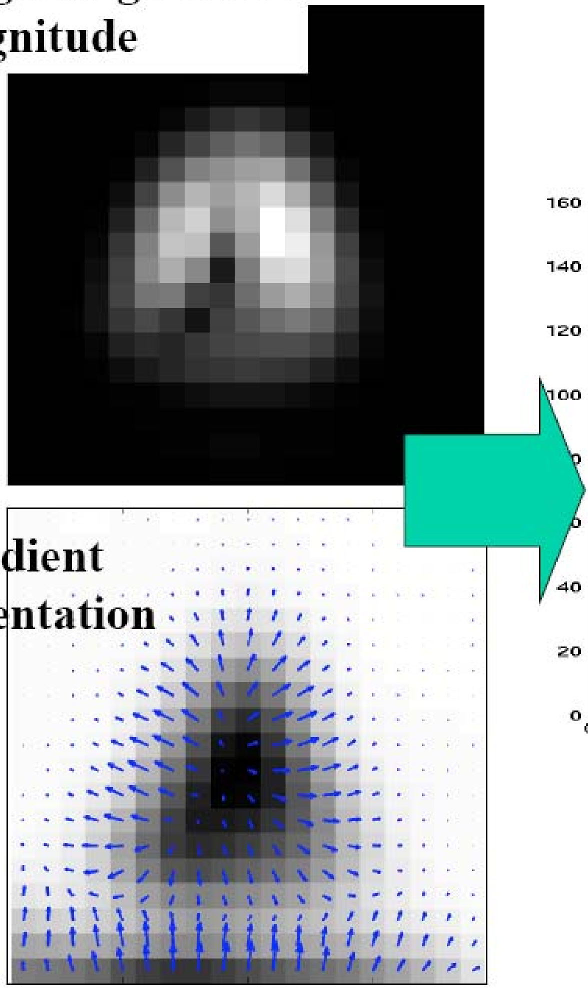
weighted gradient magnitude

σ =1.5*scale of the keypoint

Orientation assignment

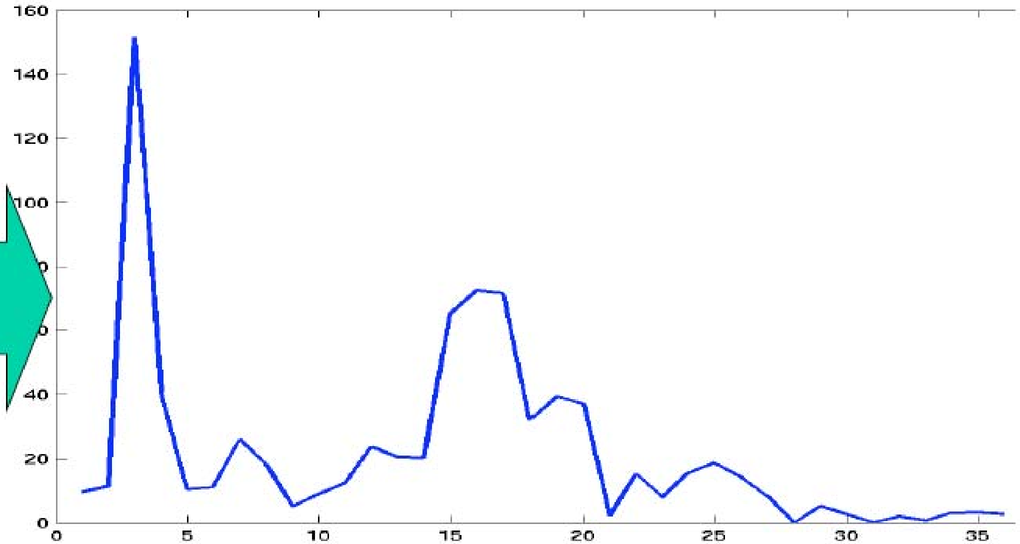
weighted gradient magnitude

gradient orientation



weighted orientation histogram.

Each bucket contains sum of weighted gradient magnitudes corresponding to angles that fall within that bucket.



36 buckets

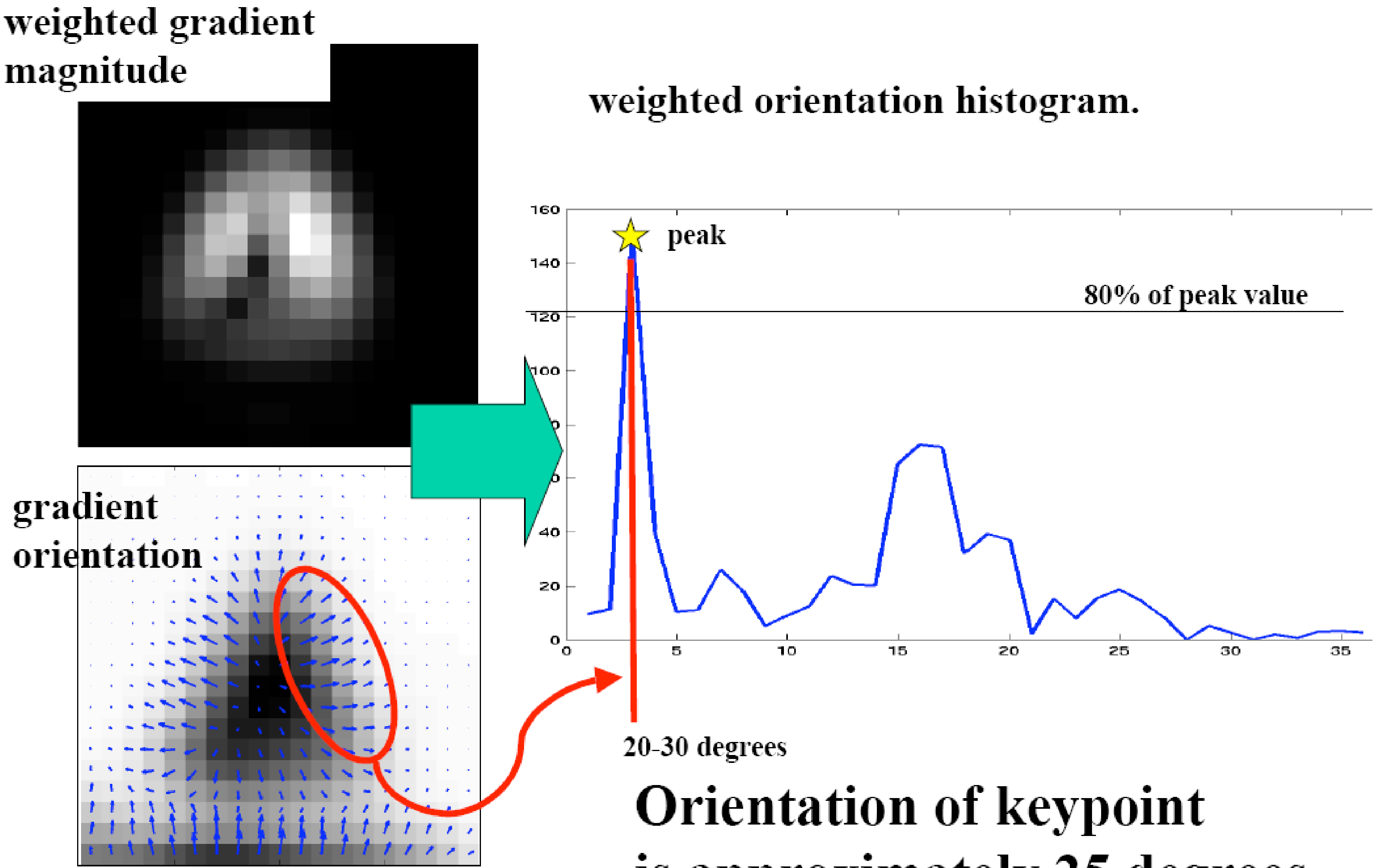
10 degree range of angles in each bucket, i.e.

0 <=ang<10 : bucket 1

10<=ang<20 : bucket 2

20<=ang<30 : bucket 3 …

Orientation assignment

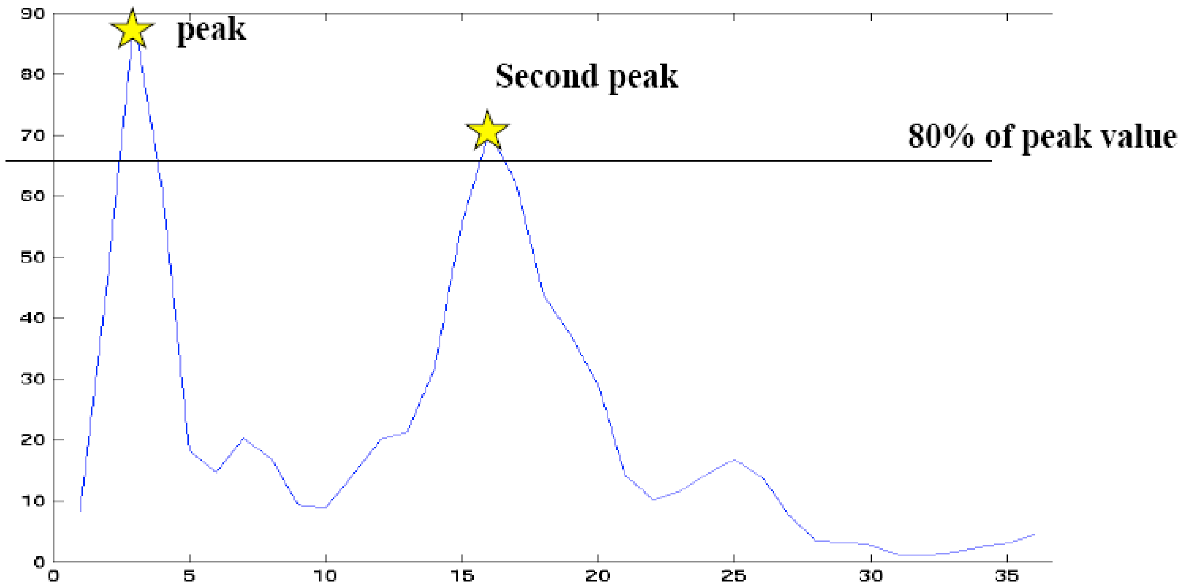


Orientation of keypoint is approximately 25 degrees

Orientation assignment

There may be multiple orientations.

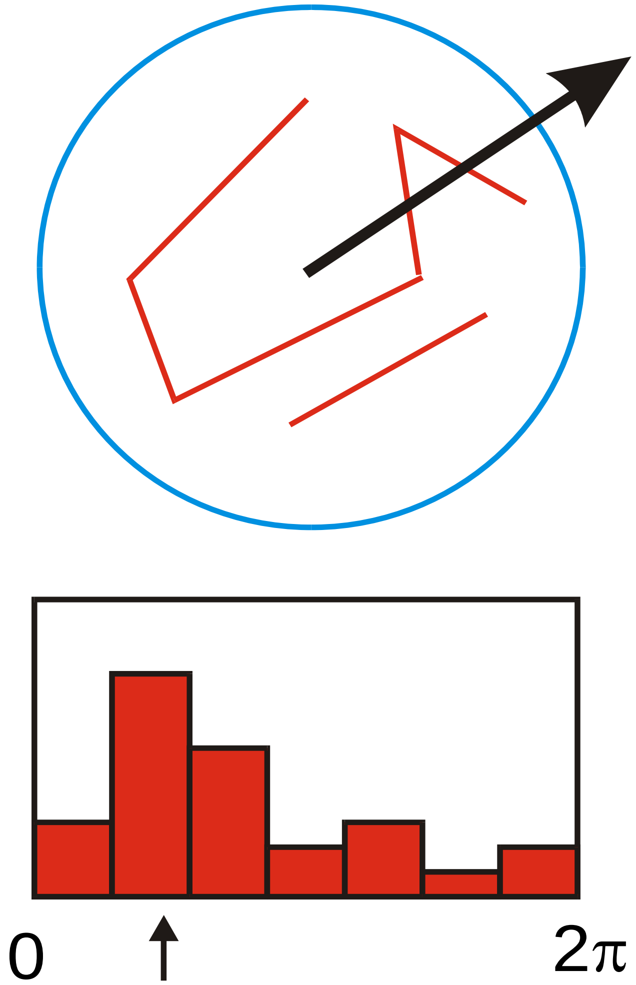
accurate peak position is determined by fitting



In this case, generate duplicate keypoints, one with orientation at 25 degrees, one at 155 degrees.

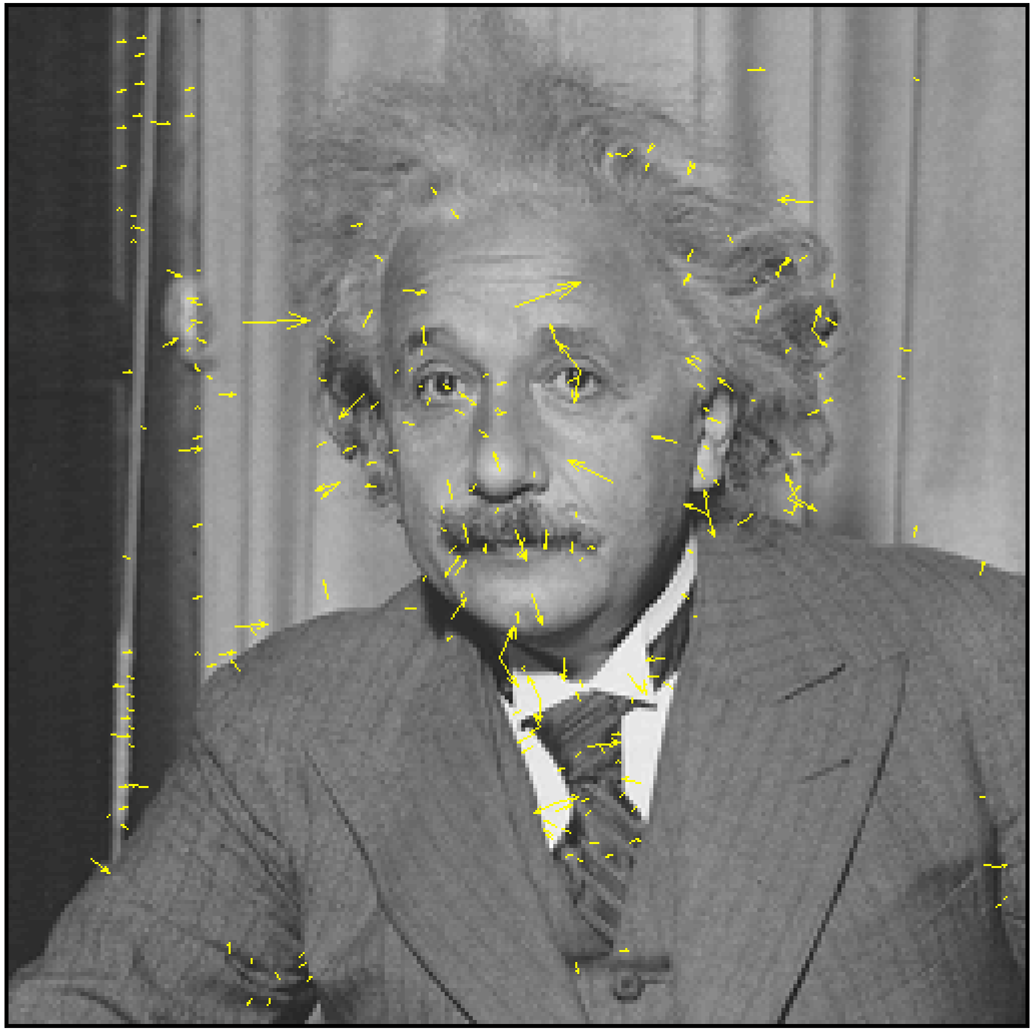
Design decision: you may want to limit number of possible multiple peaks to two.

Orientation assignment



- 36-bin orientation histogram over 360°, weighted by m and 1.5*scale falloff
- Peak is the orientation
- Local peak within 80% creates multiple orientations
- About 15% has multiple orientations and they contribute a lot to stability

SIFT descriptor



4. Local image descriptor

- Thresholded image gradients are sampled over 16x16 array of locations in scale space
- Create array of orientation histograms (w.r.t. key orientation)
- 8 orientations x 4x4 histogram array = 128 dimensions
- Normalized, clip values larger than 0.2, renormalize

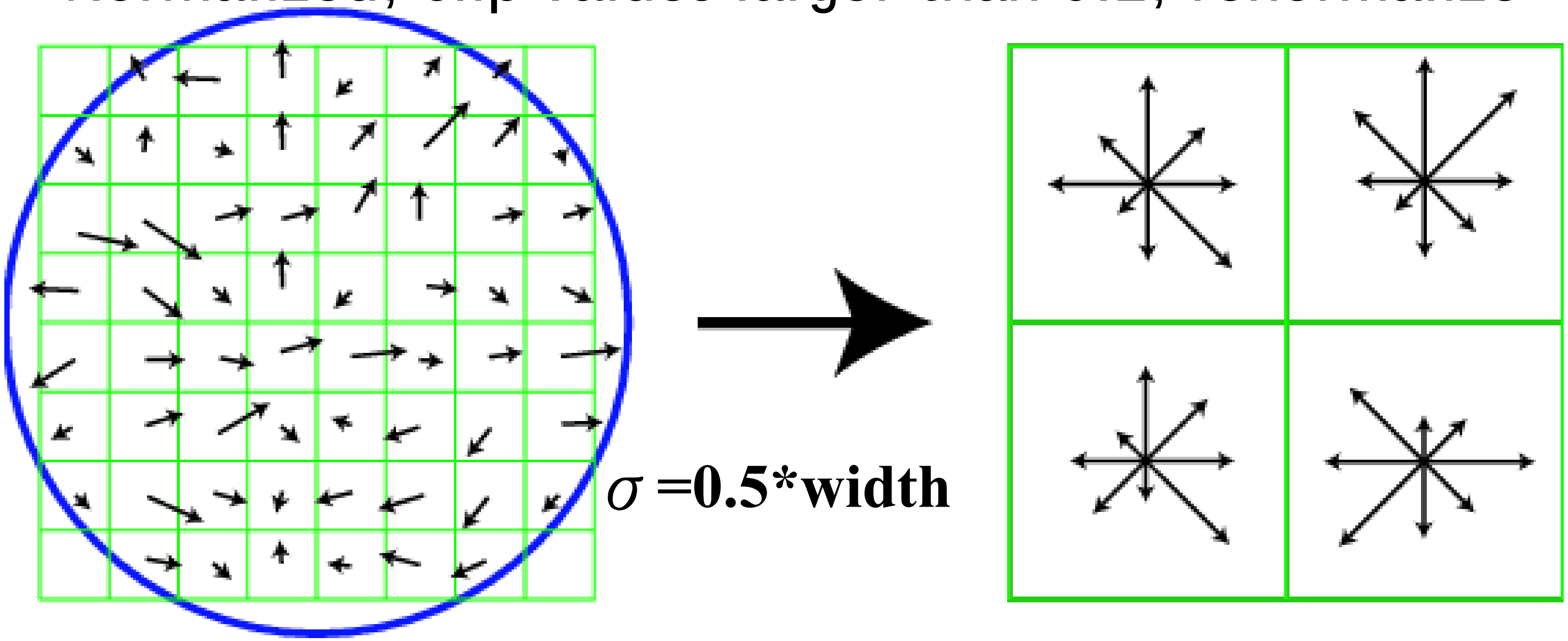
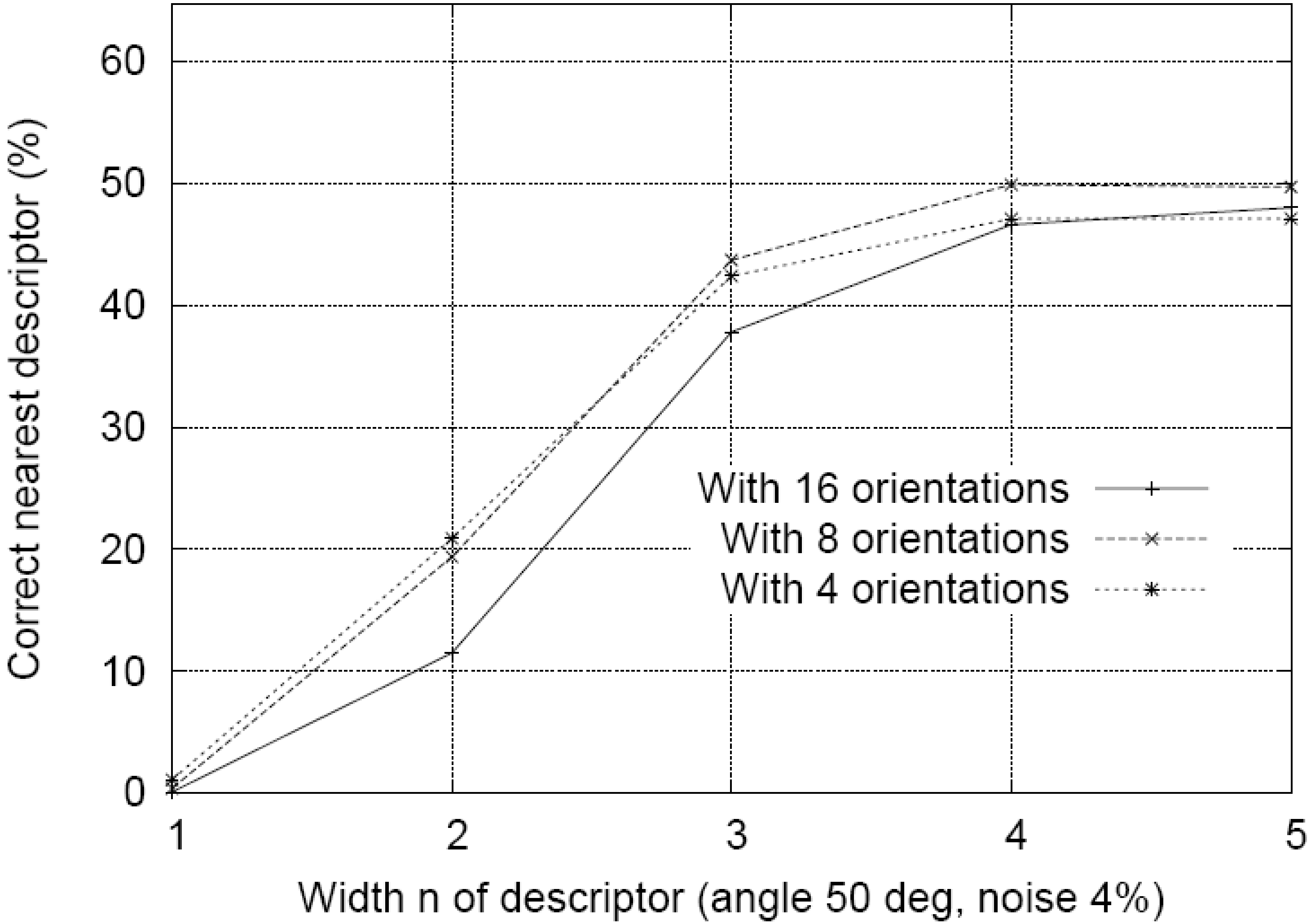


Image gradients

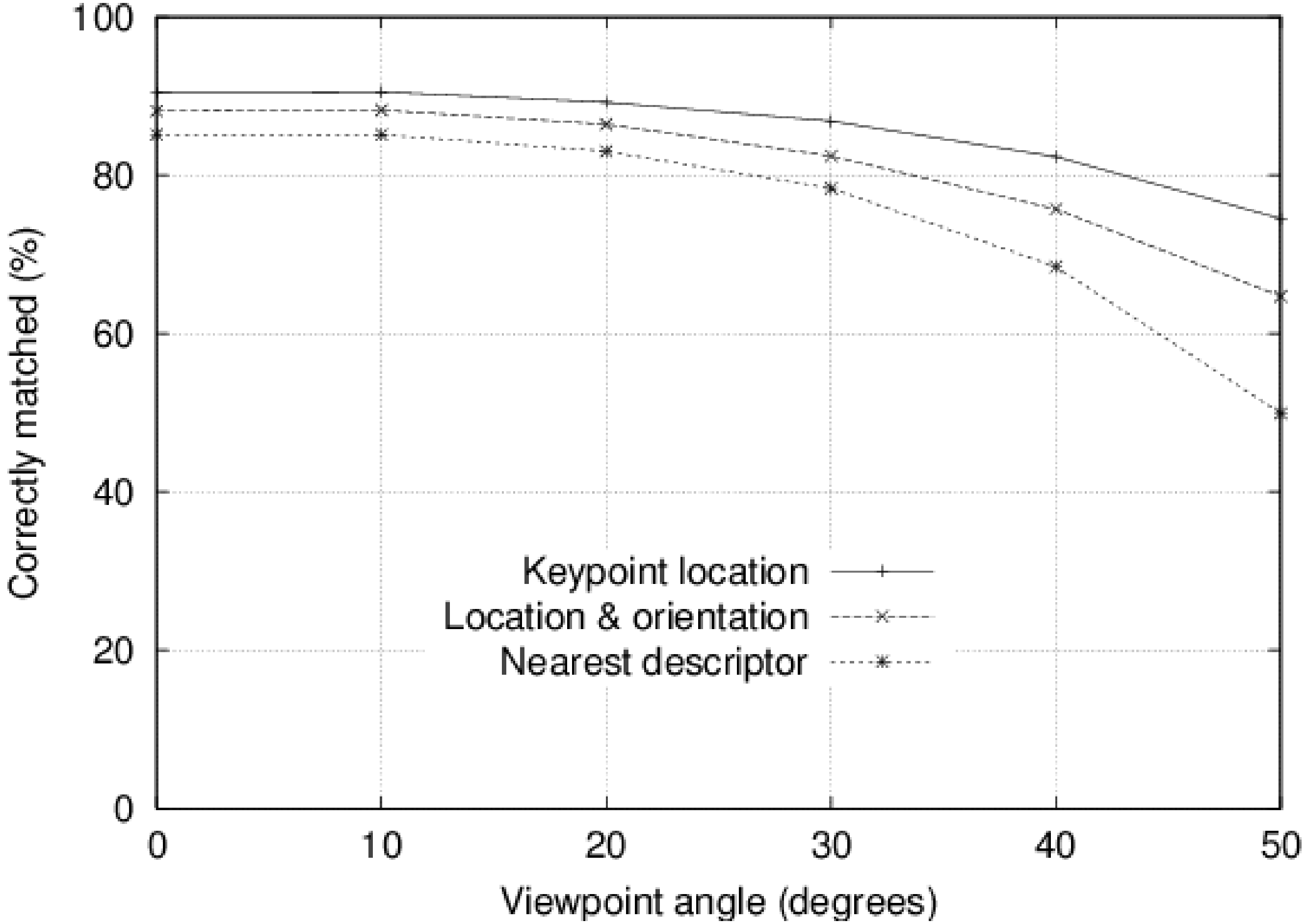
Keypoint descriptor

Why 4x4x8?



Correct nearest descriptor (%)
60
50
40
30
20
10
0
1
2
3
4
5
Width n of descriptor (angle 50 deg, noise 4%)
With 16 orientations
With 8 orientations
With 4 orientations

Sensitivity to affine change

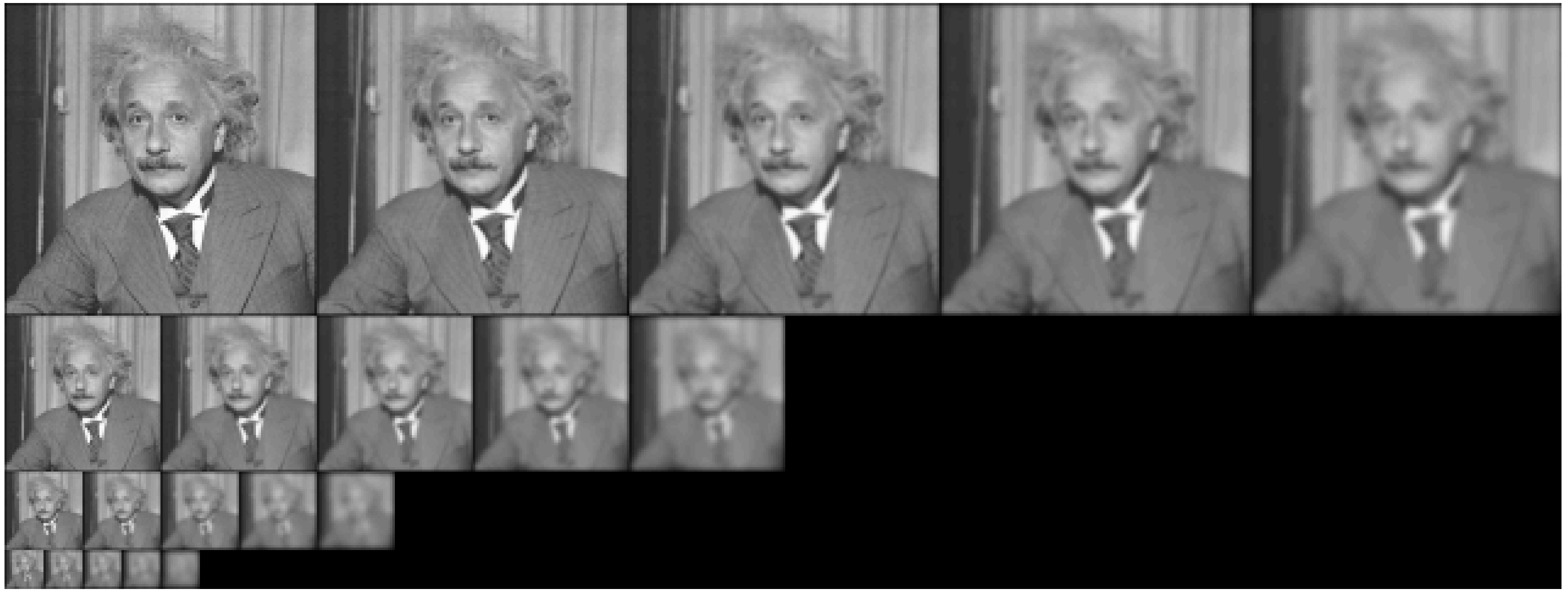


Feature matching

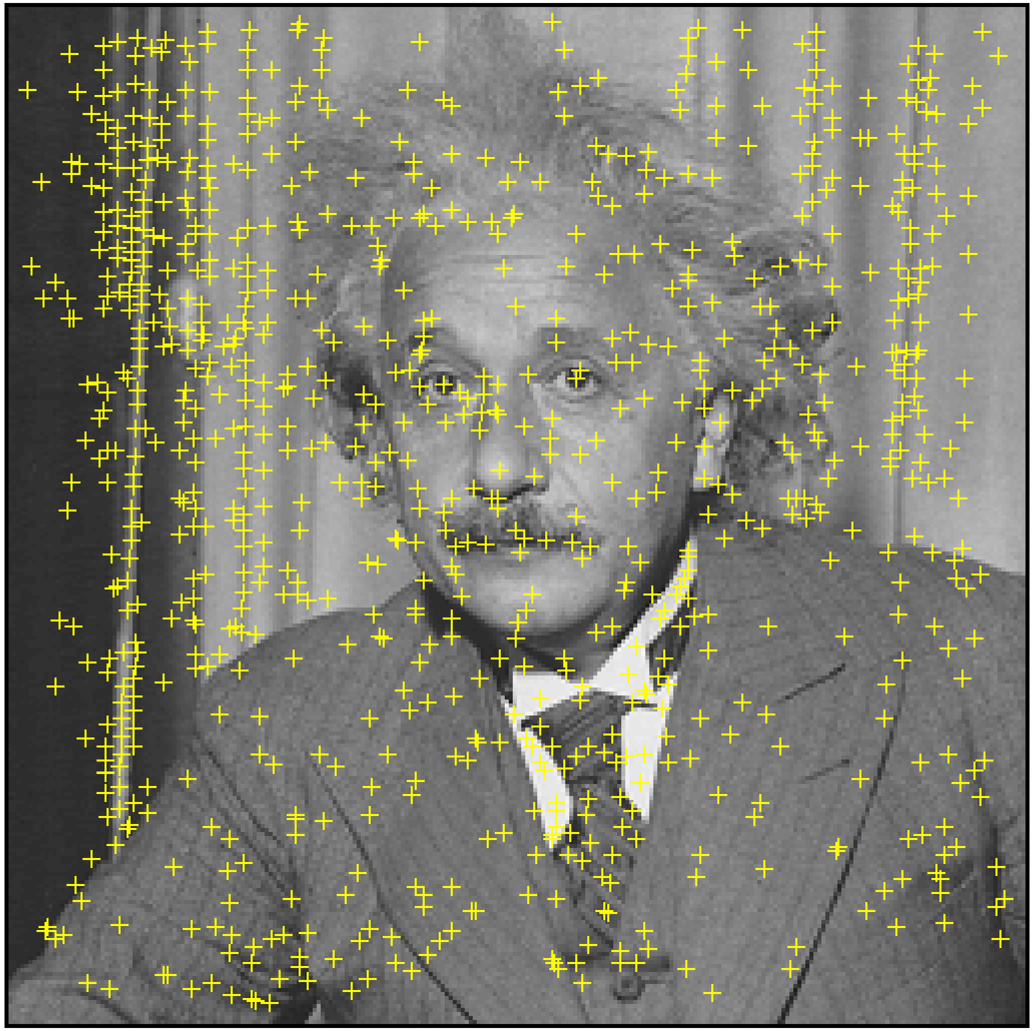
DigiVFX

- for a feature x, he found the closest feature x_1 and the second closest feature x_2. If the distance ratio of $d(x, x_1)$ and $d(x, x_1)$ is smaller than 0.8, then it is accepted as a match.

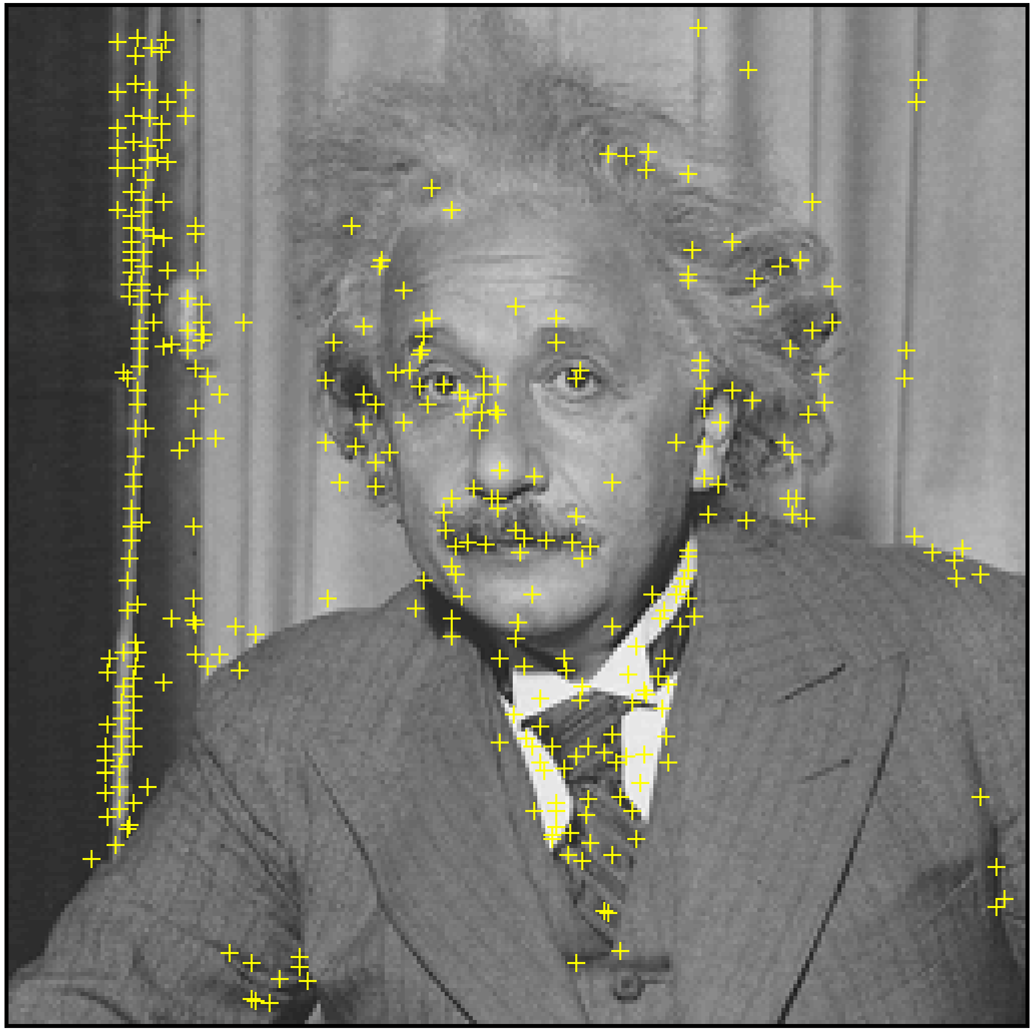
SIFT flow



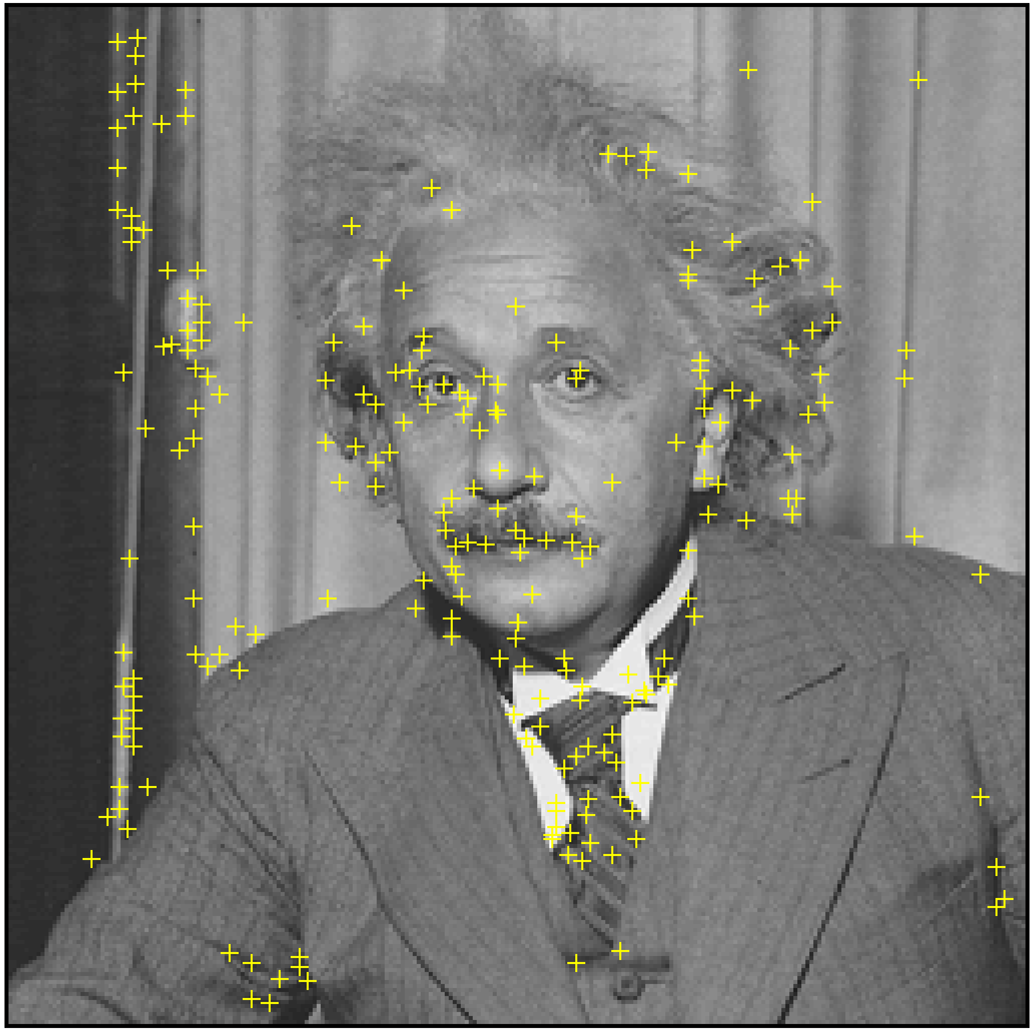
Maxima in D



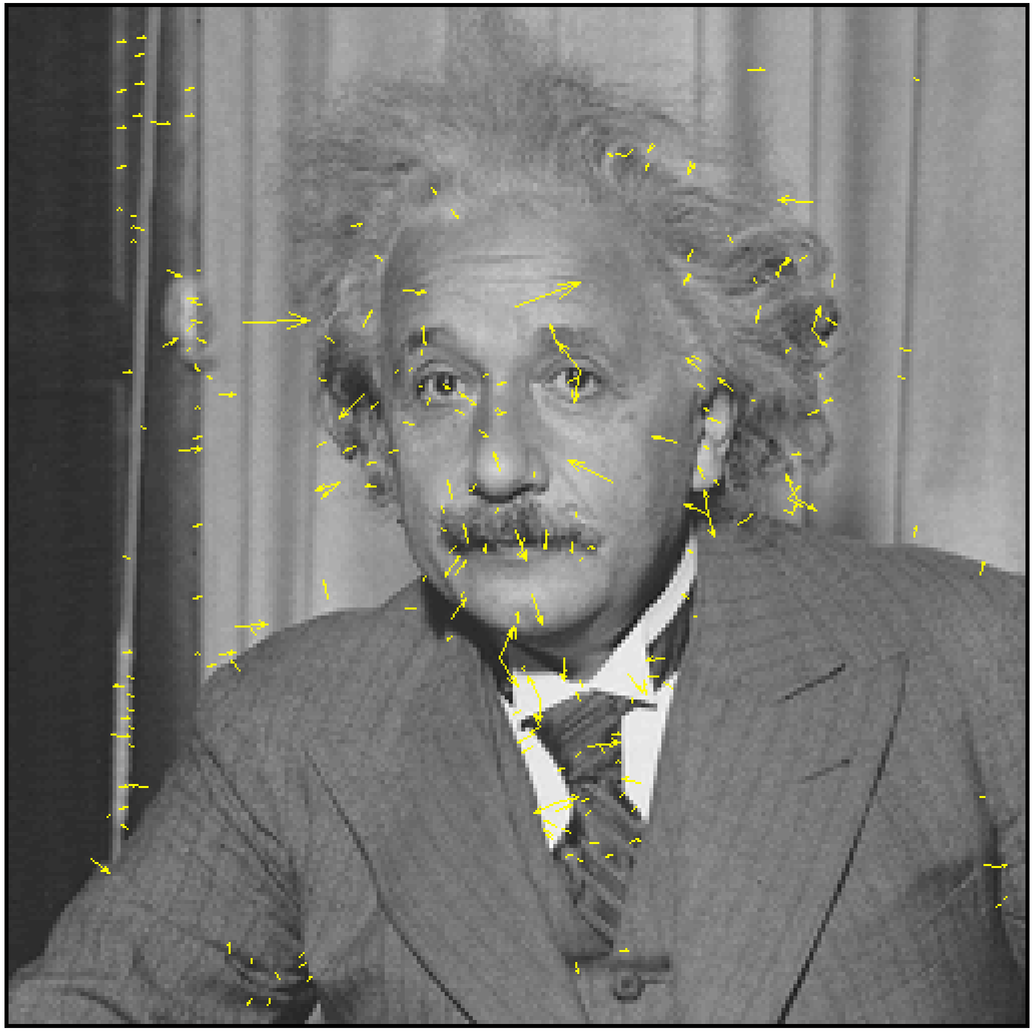
Remove low contrast



Remove edges



SIFT descriptor



Estimated rotation

- Computed affine transformation from rotated image to original image:

0.7060	-0.7052	128.4230
0.7057	0.7100	-128.9491
0	0	1.0000

- Actual transformation from rotated image to original image:

0.7071	-0.7071	128.6934
0.7071	0.7071	-128.6934
0	0	1.0000

Reference

- Chris Harris, Mike Stephens, A Combined Corner and Edge Detector, 4th Alvey Vision Conference, 1988, pp147-151.
- David G. Lowe, Distinctive Image Features from Scale-Invariant Keypoints, International Journal of Computer Vision, 60(2), 2004, pp91-110.
- SIFT Keypoint Detector, David Lowe.
- Matlab SIFT Tutorial, University of Toronto.