# Tone mapping

Digital Visual Effects, Spring 2008
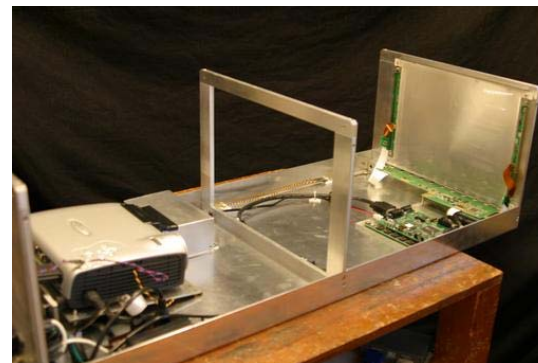
*Yung-Yu Chuang*

2008/3/4

*with slides by Fredo Durand, and Alexei Efros*

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## Display HDR

- Once we have HDR images (either captured or synthesized), how can we display them on normal displays?
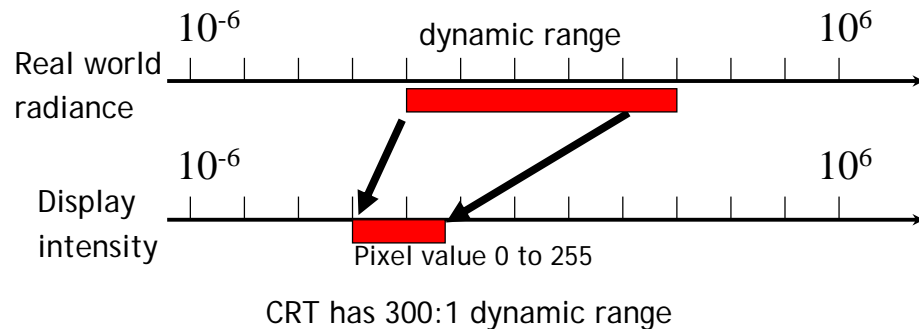
HDR display system, Sunnybrook, 2004

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## Tone mapping

- How should we map scene luminances (up to 1:100,000) to display luminances (only around 1:100) to produce a satisfactory image?

  Linear scaling?, thresholding?

$10^{-6}$       dynamic range       $10^{6}$

Real world radiance

$10^{-6}$       $10^{6}$

Display intensity

Pixel value 0 to 255

CRT has 300:1 dynamic range

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## Eye is not a photometer!

- Dynamic range along the visual pathway is only around 32:1.
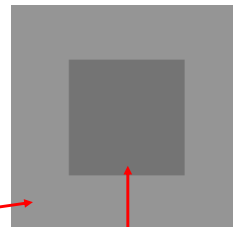- The key is adaptation

## We are more sensitive to contrast

- Weber's law

Just-noticeable
Difference (JND)

$$\frac{\Delta I_b}{I_b} \sim 1\%$$

background
intensity

flash

## How humans deal with dynamic range

- We're more sensitive to contrast (multiplicative)
  - A ratio of 1:2 is perceived as the same contrast as a ratio of 100 to 200
  - Makes sense because illumination has a multiplicative effect
  - Use the log domain as much as possible
- Dynamic adaptation (very local in retina)
  - Pupil (not so important)
  - Neural
  - Chemical
- Different sensitivity to spatial frequencies

## Preliminaries

- For color images

$$\begin{bmatrix} R_d \\ G_d \\ B_d \end{bmatrix} = \begin{bmatrix} L_d \dfrac{R_w}{L_w} \\ L_d \dfrac{G_w}{L_w} \\ L_d \dfrac{B_w}{L_w} \end{bmatrix}$$

- Log domain is usually preferred.

## Tone mapping operators

- Global
- Local
- Frequency domain
- Gradient domain
- 3 papers from SIGGRAPH 2002
  - Photographic Tone Reproduction for Digital Images
  - Fast Bilateral Filtering for the Display of High-Dynamic-Range Images
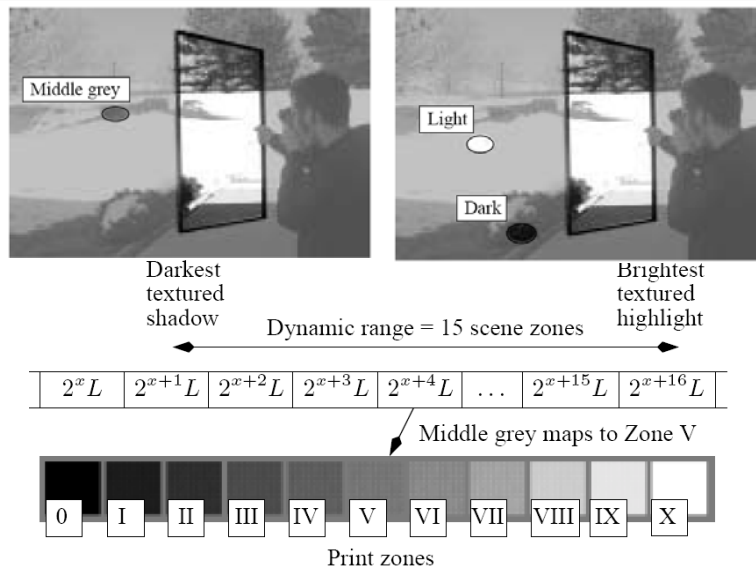  - Gradient Domain High Dynamic Range Compression

# Photographic Tone Reproduction for Digital Images

Erik Reinhard    Mike Stark

Peter Shirley    Jim Ferwerda

SIGGRAPH 2002

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## Photographic tone reproduction

- Proposed by Reinhard et. al. in SIGGRAPH 2002
- Motivated by traditional practice, zone system by Ansel Adams and dodging and burning
- It contains both global and local operators

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## Zone system



Darkest textured shadow — Brightest textured highlight

Dynamic range = 15 scene zones

| $2^x L$ | $2^{x+1}L$ | $2^{x+2}L$ | $2^{x+3}L$ | $2^{x+4}L$ | $\ldots$ | $2^{x+15}L$ | $2^{x+16}L$ |
|---|---|---|---|---|---|---|---|

Middle grey maps to Zone V

| 0 | I | II | III | IV | V | VI | VII | VIII | IX | X |
|---|---|---|---|---|---|---|---|---|---|---|

Print zones

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## Dodging and burning

- During the print
- Hide part of the print during exposure
  - Makes it brighter



From The Master Printing Course, Rudman
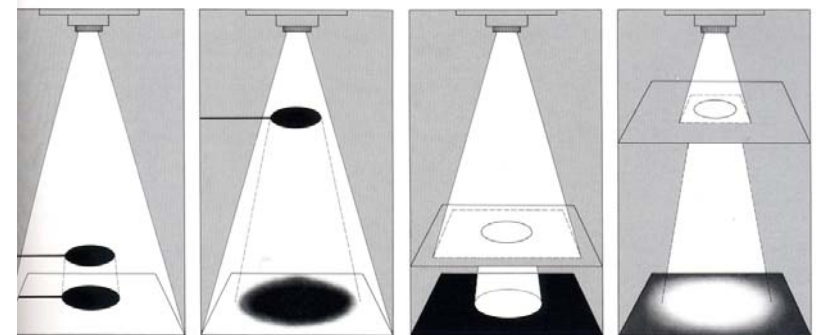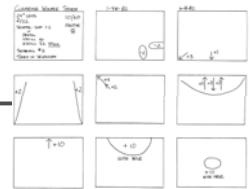
## Dodging and burning
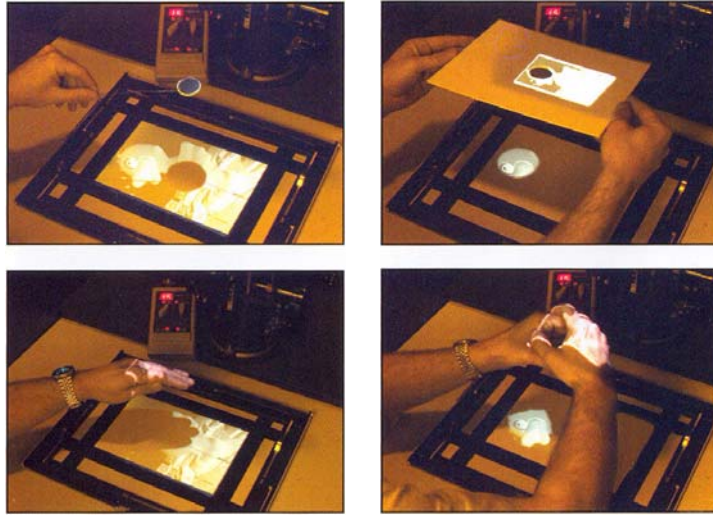


dodging          burning

From Photography by London et al.

## Dodging and burning

• Must be done for every single print!
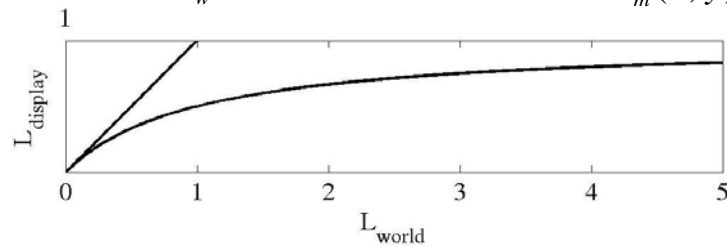


Straight print           After dodging and burning

## Global operator

$$\overline{L}_w = \exp\left(\frac{1}{N}\sum_{x,y}\log(\delta + L_w(x,y))\right)$$

Approximation of scene's key (how light or dark it is). Map to 18% of display range for average-key scene
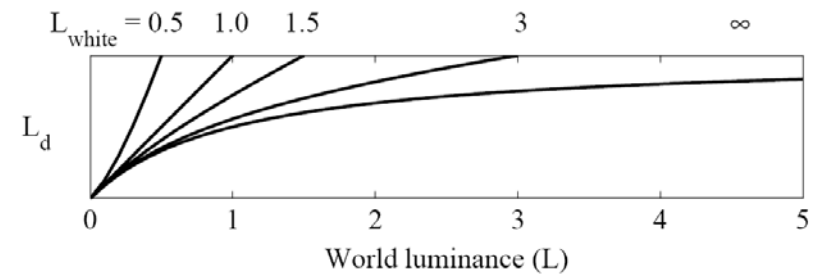
User-specified; high key or low key

$$L_m(x,y) = \frac{a}{\overline{L}_w} L_w(x,y) \qquad L_d(x,y) = \frac{L_m(x,y)}{1 + L_m(x,y)}$$



## Global operator

It seldom reaches 1 since the input image does not have infinitely large luminance values.

$$L_d(x,y) = \frac{L_m(x,y)\left(1 + \dfrac{L_m(x,y)}{L_{white}^2(x,y)}\right)}{1 + L_m(x,y)}$$

low key (0.18)　　　　　high key (0.5)

## Dodging and burning (local operators)

- Area receiving a different exposure is often bounded by sharp contrast
- Find largest surrounding area without any sharp contrast

$$L_s^{blur}(x, y) = L_m(x, y) \otimes G_s(x, y)$$

$$V_s(x, y) = \frac{L_s^{blur}(x, y) - L_{s+1}^{blur}(x, y)}{2^\phi\, a/s^2 + L_s^{blur}}$$

$$s_{max} : \left| V_{s_{max}}(x, y) \right| < \varepsilon$$

## Dodging and burning (local operators)

$$L_d(x, y) = \frac{L_m(x, y)}{1 + L_{s_{max}}^{blur}(x, y)}$$

- A darker pixel (smaller than the blurred average of its surrounding area) is divided by a larger number and become darker (dodging)
- A brighter pixel (larger than the blurred average of its surrounding area) is divided by a smaller number and become brighter (burning)
- Both increase the contrast

## Dodging and burning



Simple operator　　　　　Dodging-and-burning

## Frequency domain

- First proposed by Oppenheim in 1968!
- Under simplified assumptions,

image    = illuminance   *   reflectance

low-frequency     high-frequency
attenuate more    attenuate less



## Oppenheim

- Taking the logarithm to form density image
- Perform FFT on the density image
- Apply frequency-dependent attenuation filter

$$s(f) = (1-c) + c\,\frac{kf}{1+kf}$$

- Perform inverse FFT
- Take exponential to form the final image

## Fast Bilateral Filtering for the Display of High-Dynamic-Range Images

Frédo Durand & Julie Dorsey

SIGGRAPH 2002

## A typical photo

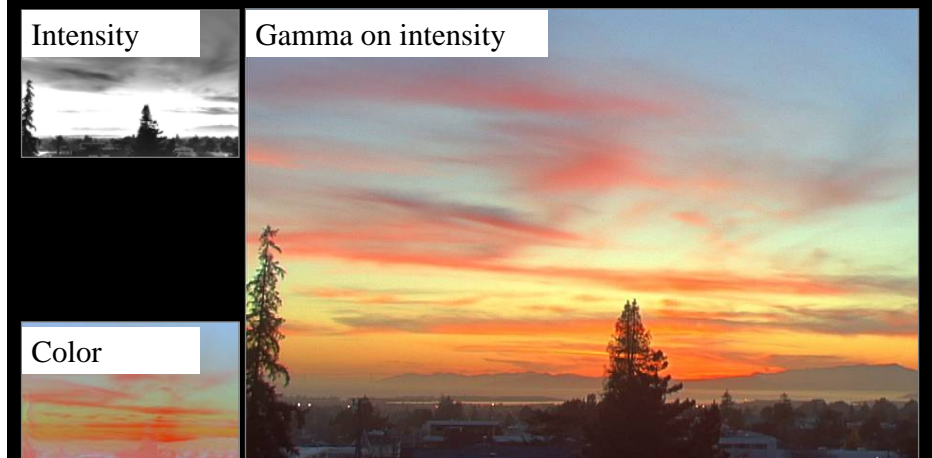- Sun is overexposed
- Foreground is underexposed

# Gamma compression
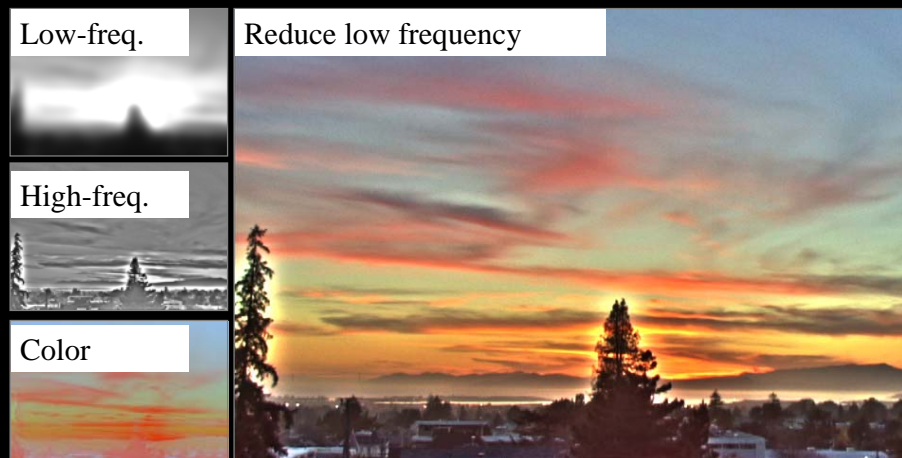
- $X \rightarrow X^{\gamma}$
- Colors are washed-out



Input | Gamma
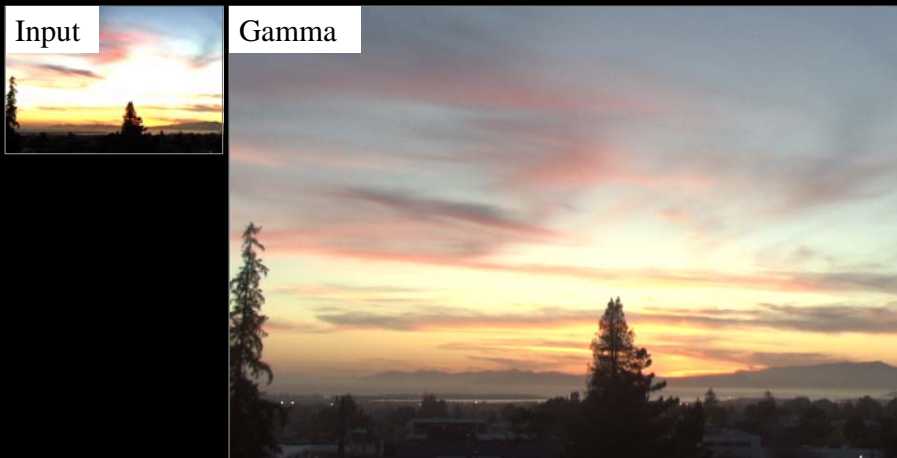
# Gamma compression on intensity

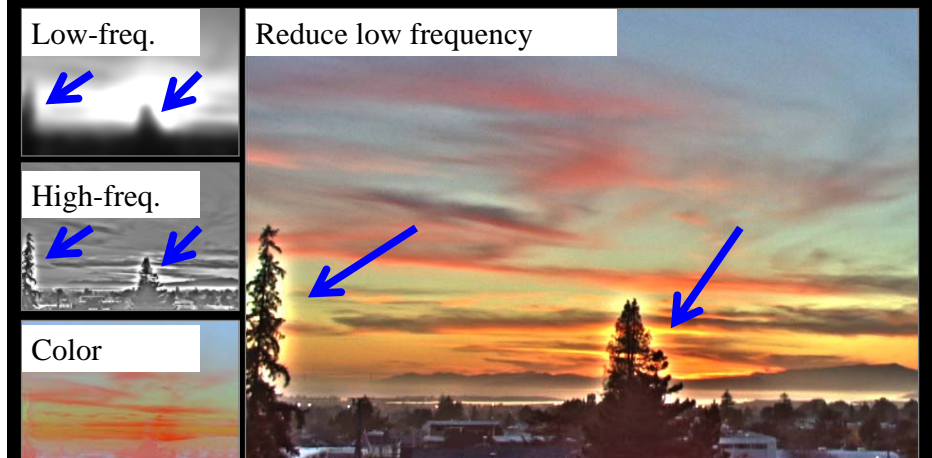- Colors are OK, but details (intensity high-frequency) are blurred



Intensity | Gamma on intensity | Color

# Chiu et al. 1993

- Reduce contrast of low-frequencies
- Keep high frequencies



Low-freq. | Reduce low frequency | High-freq. | Color

# The halo nightmare

- For strong edges
- Because they contain high frequency



Low-freq. | Reduce low frequency | High-freq. | Color

## Durand and Dorsey

- Do not blur across edges
- Non-linear filtering



Large-scale

Detail

Color

Output
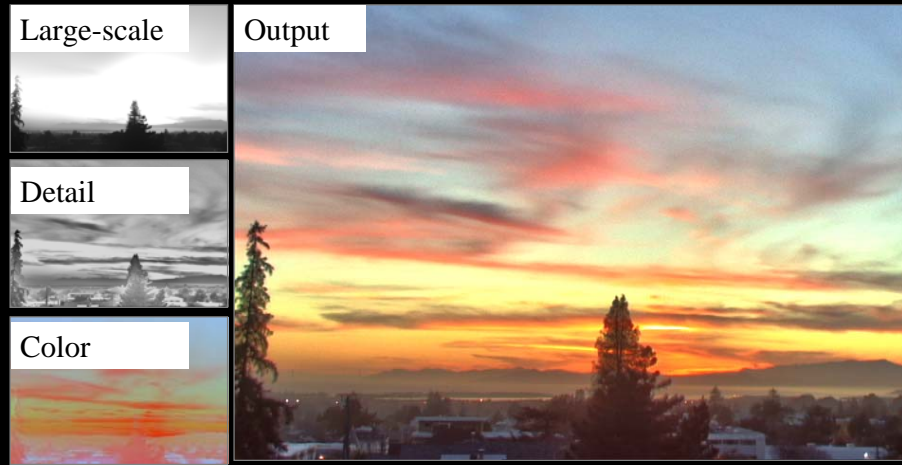
## Edge-preserving filtering

- Blur, but not across edges



Input

Gaussian blur

Edge-preserving
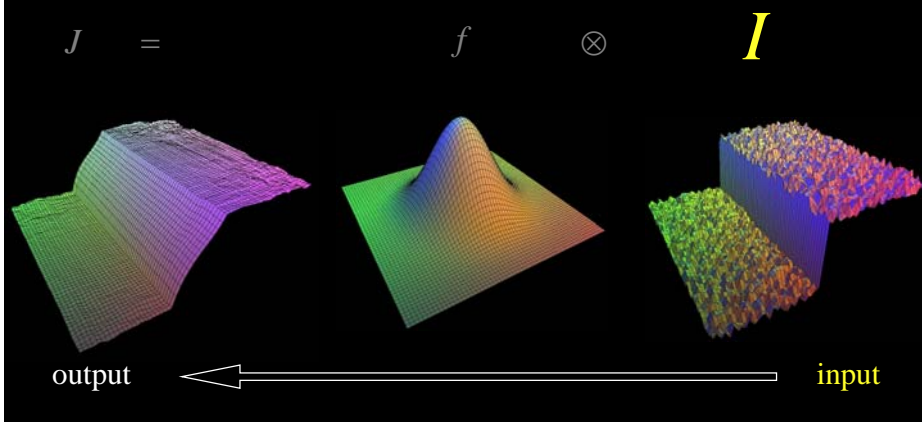
- Anisotropic diffusion [Perona & Malik 90]
  - Blurring as heat flow
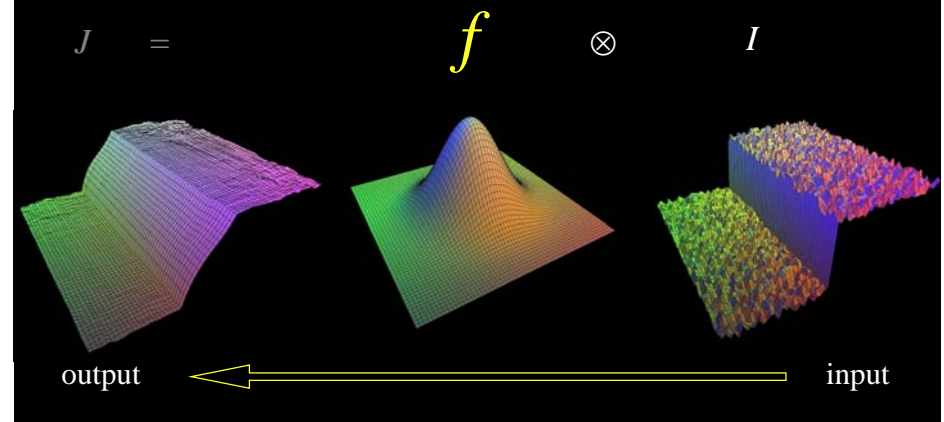  - LCIS [Tumblin & Turk]
- Bilateral filtering [Tomasi & Manduci, 98]

## Start with Gaussian filtering
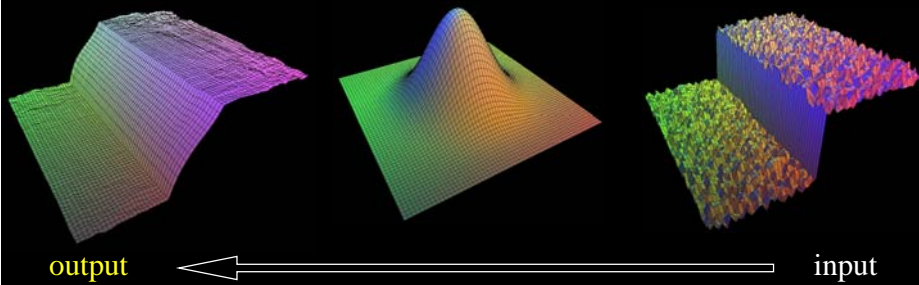
- Here, input is a step function + noise

$$J \quad = \quad f \quad \otimes \quad I$$



output ⟵ input

## Start with Gaussian filtering

- Spatial Gaussian f

$$J \quad = \quad f \quad \otimes \quad I$$



output ⟵ input

## Start with Gaussian filtering

- Output is blurred

$$J \quad = \quad f \quad \otimes \quad I$$



output ⟵ input

## Gaussian filter as weighted average

$$J(x) \quad \sum_{\xi} \quad f(x,\xi) \quad I(\xi)$$



output ⟵ input
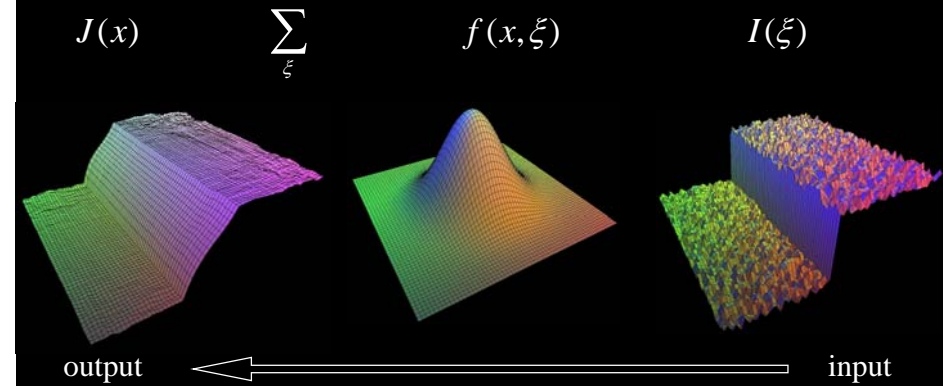
## The problem of edges

- Here, $I(\xi)$ "pollutes" our estimate J(x)
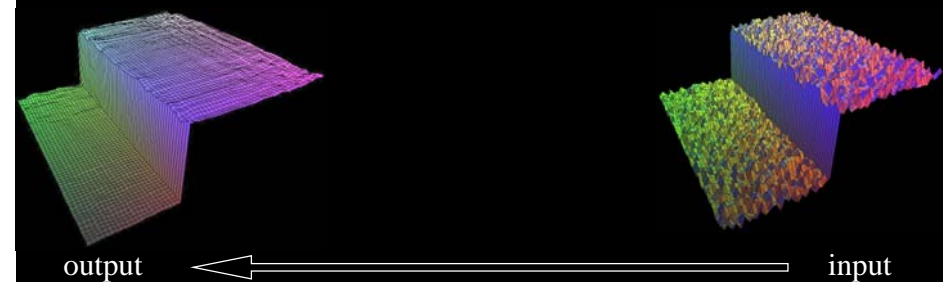- It is too different

$$J(x) \quad \sum_{\xi} \quad f(x,\xi) \quad I(\xi)$$



output ⟵ input
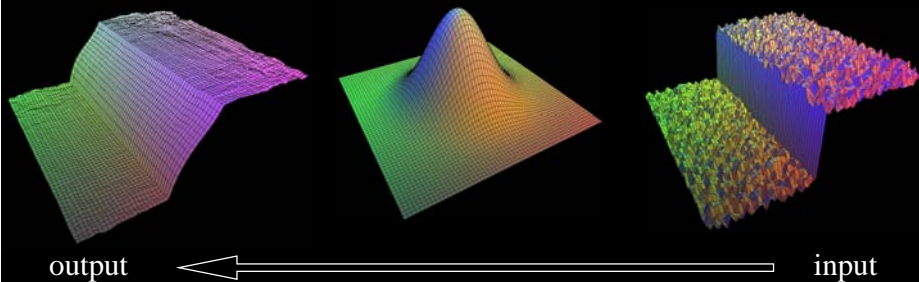
## Principle of Bilateral filtering

- [Tomasi and Manduchi 1998]
- Penalty g on the intensity difference

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi) \quad g(I(\xi) - I(x)) \quad I(\xi)$$



output ⟵ input

## Bilateral filtering

- [Tomasi and Manduchi 1998]
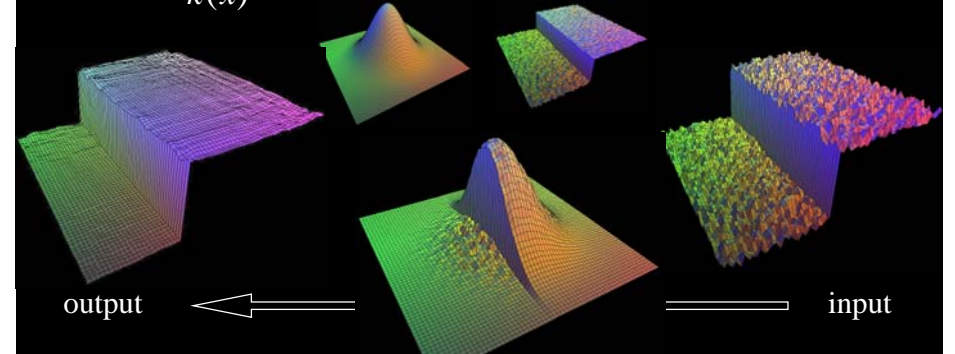- Spatial Gaussian f

$$J(x) \quad \frac{1}{k(x)} \quad f(x,\xi) \quad g(I(\xi)-I(x)) \quad I(\xi)$$



output ⟵ input
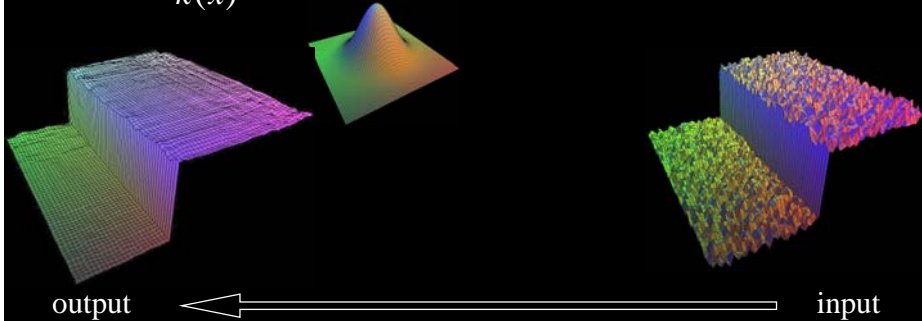
## Bilateral filtering

- [Tomasi and Manduchi 1998]
- Spatial Gaussian f
- Gaussian g on the intensity difference

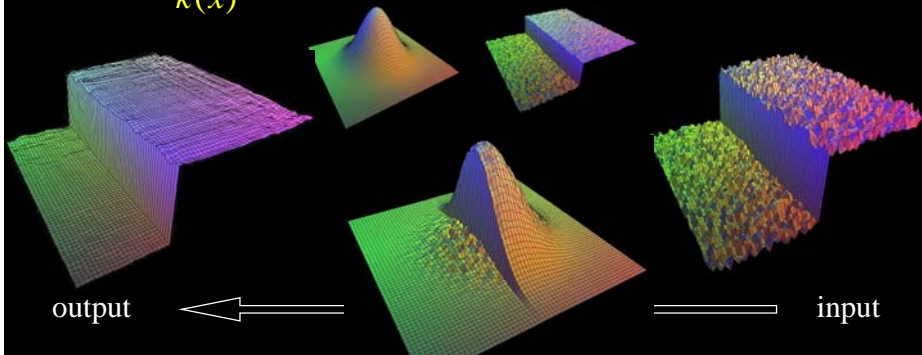$$J(x) \quad \frac{1}{k(x)} \quad f(x,\xi) \quad g(I(\xi)-I(x)) \quad I(\xi)$$



output ⟵ input

## Normalization factor

- [Tomasi and Manduchi 1998]
- $k(x) = \sum_{\xi} \boxed{f(x,\xi) \quad g(I(\xi)-I(x))}$
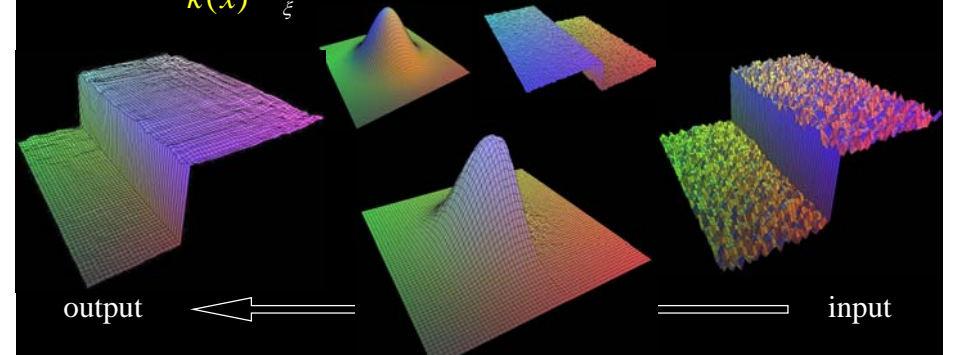
$$J(x) \quad \frac{1}{k(x)} \quad f(x,\xi) \quad g(I(\xi)-I(x)) \quad I(\xi)$$



output ⟵ input

## Bilateral filtering is non-linear

- [Tomasi and Manduchi 1998]
- The weights are different for each output pixel

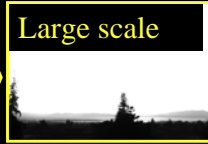$$J(x) \quad \frac{1}{k(x)} \sum_{\xi} f(x,\xi) \quad g(I(\xi)-I(x)) \quad I(\xi)$$



output ⟵ input

Slide 1 — Contrast reduction

Input HDR image
Contrast too high!

Slide 2 — Contrast reduction

Input HDR image
Intensity
Color

Slide 3 — Contrast reduction

Input HDR image
Intensity → Fast Bilateral Filter → Large scale
Color

Slide 4 — Contrast reduction

Input HDR image
Intensity → Fast Bilateral Filter → Large scale
Detail
Color
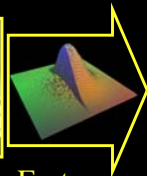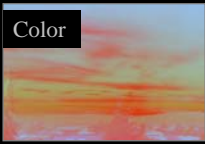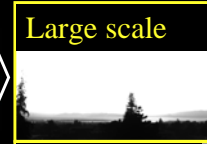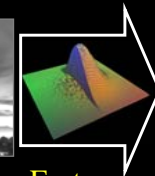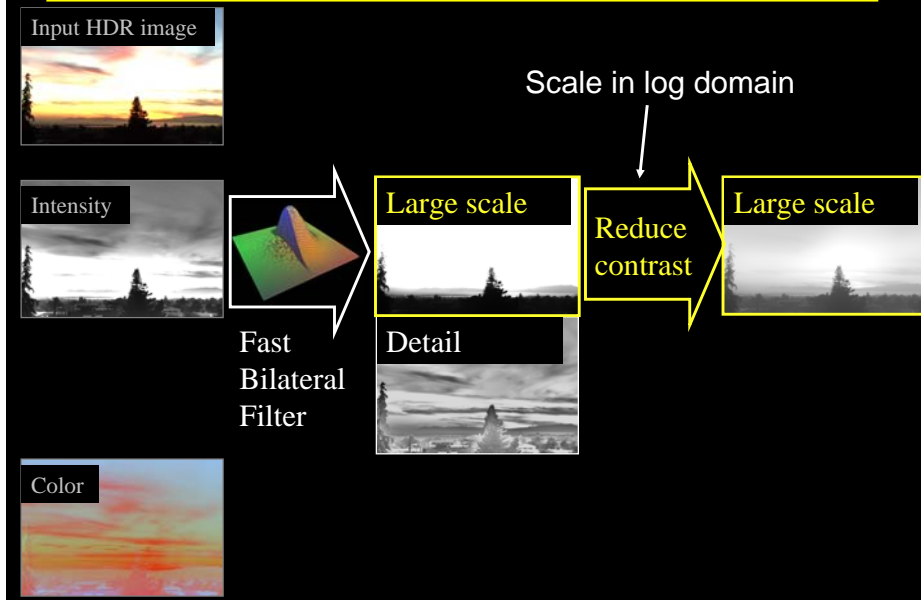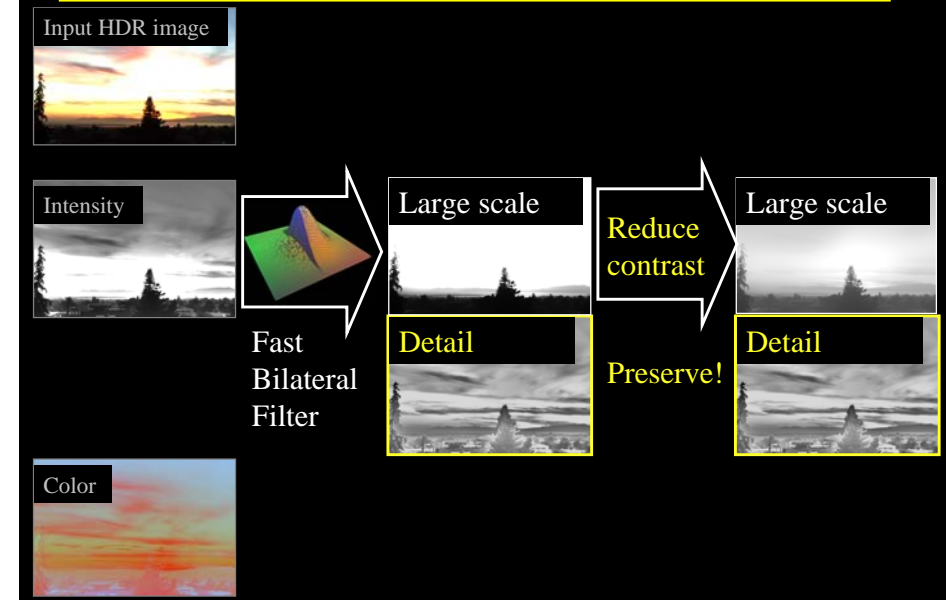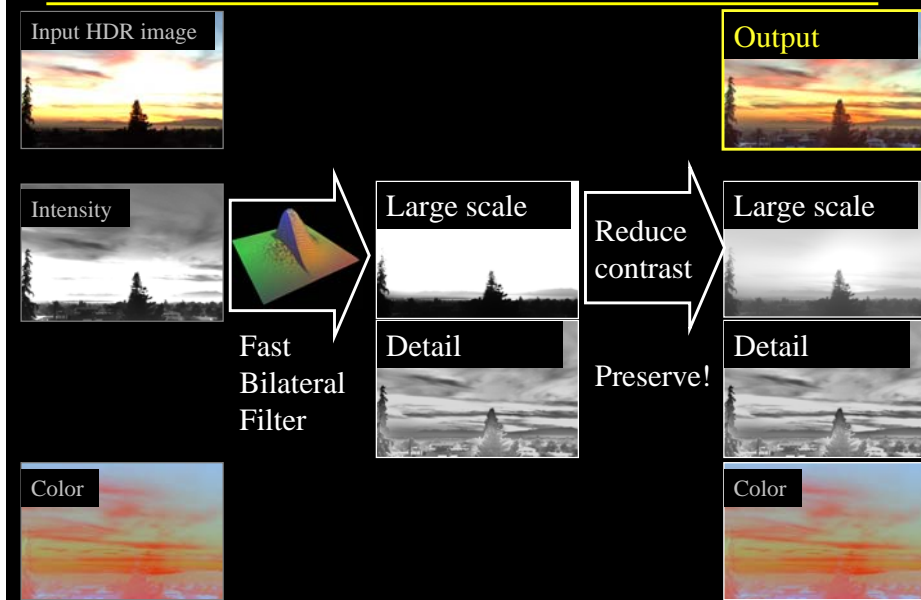
## Bilateral filter is slow!

- Compared to Gaussian filtering, it is much slower because the kernel is not fixed.
- Durand and Dorsey proposed an approximate approach to speed up
- Paris and Durand proposed an even-faster approach in ECCV 2006. We will cover this one when talking about computational photogrphy.

Oppenheim      bilateral

# Gradient Domain High Dynamic Range Compression

Raanan Fattal    Dani Lischinski    Michael Werman

SIGGRAPH 2002

## Log domain

- Logorithm is a crude approximation to the perceived brightness
- Gradients in log domain correspond to ratios (local contrast) in the luminance domain

## The method in 1D

## The method in 2D

- Given: a log-luminance image $H(x,y)$

- Compute an *attenuation map* $\Phi(\|\nabla H\|)$

- Compute an attenuated gradient field $G$:

$$G(x, y) = \nabla H(x, y) \cdot \Phi(\|\nabla H\|)$$

- Problem: $G$ is not integrable!

## Solution

- Look for image $I$ with gradient closest to $G$ in the least squares sense.

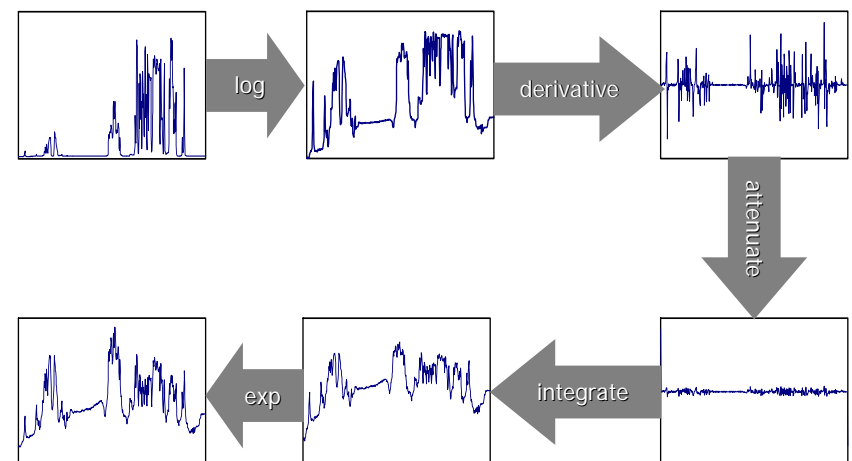- $I$ minimizes the integral: $\iint F(\nabla I, G)\,dxdy$

$$F(\nabla I, G) = \|\nabla I - G\|^2 = \left(\frac{\partial I}{\partial x} - G_x\right)^2 + \left(\frac{\partial I}{\partial y} - G_y\right)^2$$

$$\Longrightarrow \quad \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} = \frac{\partial G_x}{\partial x} + \frac{\partial G_y}{\partial y} \quad \text{Poisson equation}$$

Solve $\dfrac{\partial^2 I}{\partial x^2} + \dfrac{\partial^2 I}{\partial y^2} = \dfrac{\partial G_x}{\partial x} + \dfrac{\partial G_y}{\partial y}$

$$G_x(x, y) - G_x(x-1, y) + G_y(x, y) - G_y(x, y-1)$$

$$I(x+1, y) + I(x-1, y) + I(x, y+1) + I(x, y-1) - 4I(x, y)$$

$$\begin{bmatrix} .. \; 1 \; ... \quad 1\,\text{-}4\,1 \; ... \quad 1 \; .. \end{bmatrix} \begin{bmatrix} \mathbf{I} \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

## Solving Poisson equation

- No analytical solution
- Multigrid method
- Conjugate gradient method

## Attenuation

- Any dramatic change in luminance results in large luminance gradient at some scale
- Edges exist in multiple scales. Thus, we have to detect and attenuate them at multiple scales
- Construct a Gaussian pyramid $H_i$

## Attenuation

$$\varphi_k(x,y) = \left(\frac{\|\nabla H_k(x,y)\|}{\alpha}\right)^{\beta-1} \quad \beta \sim 0.8 \quad \alpha = 0.1\overline{\nabla H}$$



log(Luminance)    gradient magnitude    attenuation map
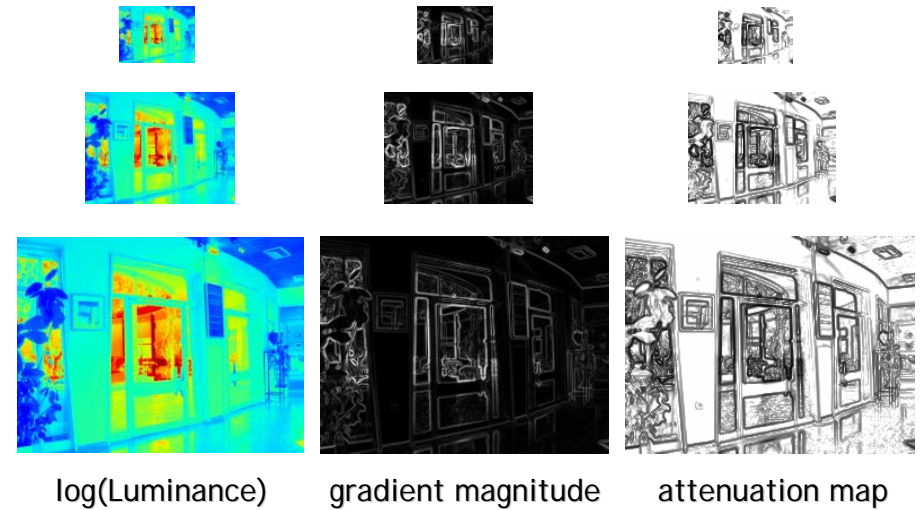
## Multiscale gradient attenuation



interpolate

interpolate
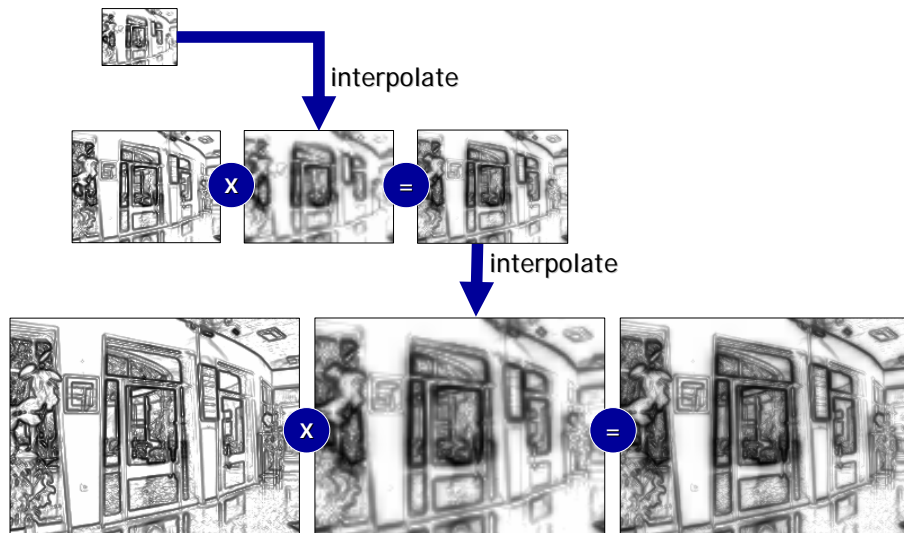
## Final gradient attenuation map

## Performance

- Measured on 1.8 GHz Pentium 4:

    - 512 x 384: 1.1 sec
    - 1024 x 768: 4.5 sec
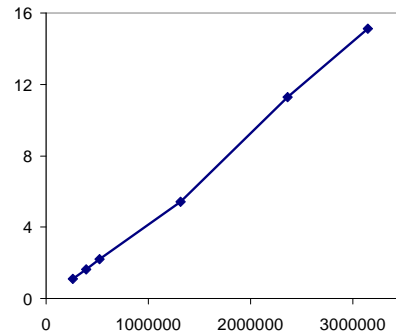


- Can be accelerated using processor-optimized libraries.

## Informal comparison



| Gradient domain [Fattal et al.] | Bilateral [Durand et al.] | Photographic [Reinhard et al.] |

## Informal comparison



| Gradient domain [Fattal et al.] | Bilateral [Durand et al.] | Photographic [Reinhard et al.] |

## Informal comparison



| Gradient domain [Fattal et al.] | Bilateral [Durand et al.] | Photographic [Reinhard et al.] |

# Evaluation of Tone Mapping Operators using a High Dynamic Range Display

Patrick Ledda    Alan Chalmers
Tom Troscinko    Helge Seetzen

SIGGRAPH 2005

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## Six operators

- H: histogram adjustment
- B: bilateral filter
- P: photographic reproduction
- I: iCAM
- L: logarithm mapping
- A: local eye adaption

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## 23 scenes



Scene 1   Scene 2   Scene 3   Scene 4   Scene 5
Scene 6   Scene 7   Scene 8   Scene 9   Scene 10
Scene 11  Scene 12  Scene 13  Scene 14  Scene 15

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## Experiment setting



tonemapping result    HDR display    tonemapping result
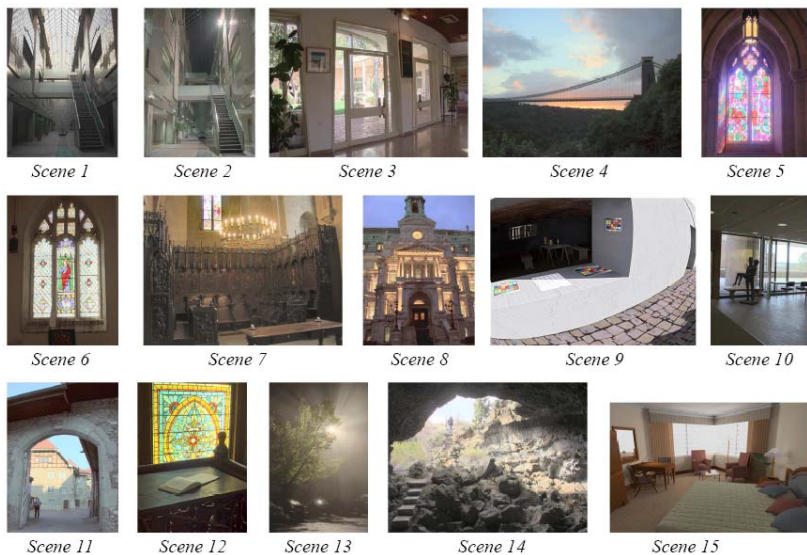
## Preference matrix

- Ranking is easier than rating.
- 15 pairs for each person to compare. A total of 345 pairs per subject.

| | $tmo_1$ | $tmo_2$ | $tmo_3$ | $tmo_4$ | $tmo_5$ | $tmo_6$ | Score |
|---|---|---|---|---|---|---|---|
| $tmo_1$ | - | 1 | 0 | 0 | 1 | 1 | 3 |
| $tmo_2$ | 0 | - | 0 | 1 | 1 | 0 | 2 |
| $tmo_3$ | 1 | 1 | - | 1 | 1 | 1 | 5 |
| $tmo_4$ | 1 | 0 | 0 | - | 0 | 0 | 1 |
| $tmo_5$ | 0 | 0 | 0 | 1 | - | 1 | 2 |
| $tmo_6$ | 0 | 1 | 0 | 1 | 0 | - | 2 |

preference matrix (tmo2->tmo4, tom2 is better than tmo4)

## Statistical measurements

- Statistical measurements are used to evaluate:
  - Agreement: whether most agree on the ranking between two tone mapping operators.
  - Consistency: no cycle in ranking. If all are confused in ranking some pairs, it means they are hard to compare. If someone is inconsistent alone, his ranking could be droped.

## Overall similarity

- Scene 8



| | $P$ | $H$ | $B$ | $L$ | $I$ | $A$ | Total |
|---|---|---|---|---|---|---|---|
| $P$ | - | 24 | 46 | 42 | 10 | 32 | 154 |
| $H$ | 24 | - | 44 | 32 | 8 | 12 | 120 |
| $B$ | 2 | 4 | - | 8 | 2 | 4 | 20 |
| $L$ | 6 | 16 | 40 | - | 4 | 12 | 78 |
| $I$ | 38 | 40 | 46 | 44 | - | 38 | 206 |
| $A$ | 16 | 36 | 44 | 36 | 10 | - | 142 |

## Summary

*Overall Similarity: Color*

| $I$ | $P$ | $H$ | $A$ | $L$ | $B$ |
|---|---|---|---|---|---|
| 3712 | 3402 | 2994 | 2852 | 1902 | 1696 |

*Bright Detail*

| $I$ | $A$ | $P$ | $H$ | $B$ | $L$ |
|---|---|---|---|---|---|
| 823 | 688 | 569 | 549 | 474 | 347 |

*Dark Detail*

| $P$ | $A$ | $I$ | $L$ | $H$ | $B$ |
|---|---|---|---|---|---|
| 815 | 793 | 583 | 491 | 485 | 283 |

## Not settled down yet!

- Some other experiment said bilateral are better than others.
- For your reference, photographic reproduction performs well in both reports.
- There are parameters to tune and the space could be huge.

## References

- Raanan Fattal, Dani Lischinski, Michael Werman, Gradient Domain High Dynamic Range Compression, SIGGRAPH 2002.
- Fredo Durand, Julie Dorsey, Fast Bilateral Filtering for the Display of High Dynamic Range Images, SIGGRAPH 2002.
- Erik Reinhard, Michael Stark, Peter Shirley, Jim Ferwerda, Photographics Tone Reproduction for Digital Images, SIGGRAPH 2002.
- Patrick Ledda, Alan Chalmers, Tom Troscianko, Helge Seetzen, Evaluation of Tone Mapping Operators using a High Dynamic Range Display, SIGGRAPH 2005.
- Jiangtao Kuang, Hiroshi Yamaguchi, Changmeng Liu, Garrett Johnson, Mark Fairchild, Evaluating HDR Rendering Algorithms, ACM Transactions on Applied Perception, 2007.